The Thermal Structure of Tropical Easterly Waves

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ABSTRACT

A procedure is introduced to derive a general balance condition for synoptic-scale disturbances in the tropics. The condition describes a balance between the mean pressure and momentum fields, and the pressure forcing by cumulus clouds. A simple and explicit diagnostic relationship between the mean pressure and momentum fields is obtained when this balance condition is applied to slowly evolving tropical easterly waves. This relationship suggests that the thermal structure of tropical easterly waves reflects not so much the distribution of latent heat release by cumulus clouds, but the thermal structure required to maintain the dynamic structure of the waves. The temperature field of tropical easterly waves calculated from this diagnostic relationship, using the dynamic field of the composite easterly wave presented by Thompson et al., shows reasonably good agreement with observations.

1. Introduction

The thermal structure is one of the many interesting features of the composite tropical easterly wave delineated by Reed et al. (1977) and Thompson et al. (1979) using GATE data. The vertical cross section of the temperature field across one wavelength of the wave is reproduced from Thompson et al. (1979) and shown here as Fig. 1a. For the convention of wave category classifications used in the figure, readers are referred to Reed et al. (1977) and Thompson et al. (1979). The figure shows a very complex thermal structure both in the vertical and in the horizontal direction. The temperature variations across one wavelength of the wave are very small, only of the order of a few tenths of a degree Celsius. The time rate of temperature change in the wave is about 0.5°C day^{-1}, assuming a typical wave period of 3.5 days (Thompson et al., 1979). This is a very slow rate when compared with the rate of heating due to the release of latent heat by cumulus clouds, which is of the order 10°C day^{-1}. It is small even when compared with the heating rate due to radiative processes, which is about 2°C day^{-1}. At present no satisfactory theoretical considerations are available which can explain the complex thermal structure observed in tropical easterly waves.

We present in this paper a theoretical attempt to account for the thermal structure of tropical easterly waves in terms of their thermodynamic and dynamic processes. The base of our theory is the observational fact that the temperature variations in these disturbances are very small, which suggests that the air circulations induced by the heating due to latent heat release possess certain properties which enable the air to retain as thermal energy only a very small fraction of the heat supplied by cumulus clouds. The main purposes of this paper are to explore the implications of these properties and to show that in large-scale tropical disturbances a balance exists between the mean pressure and momentum fields, and the pressure forcing by cumulus clouds.

The possibility of a balance condition in large-scale tropical disturbances has been examined by a number of investigators (Krishnamurti and Baumhefner, 1966; Stevens, 1980). Using simple balanced models without heating and frictional effects, Krishnamurti and Baumhefner (1966) deduced wind fields from the pressure field. They found that the observed and the calculated winds were in quite good agreement, and the balanced models apparently contain adequate dynamics to describe motion fields in low latitudes. Stevens (1980) examined the divergence budget of easterly waves observed in GATE and found considerable similarity in pattern between the Laplacian of geopotential field and the dynamic field. A large residual imbalance was attributed to the effect of cumulus convection, but the author also noted that there could be considerable errors in the calculated geopotential field, especially at the upper levels.

In this paper we attempt to offer a theoretical explanation why there may be a balance condition in synoptic-scale tropical disturbances. The derivation of the balance condition is presented in section 3. In section 4, this balance condition is reduced to an explicit and simple form for tropical easterly waves which are evolving slowly or in steady state. The simplified balance equation suggests that the major portion of the thermal structure in these disturbances reflects what is

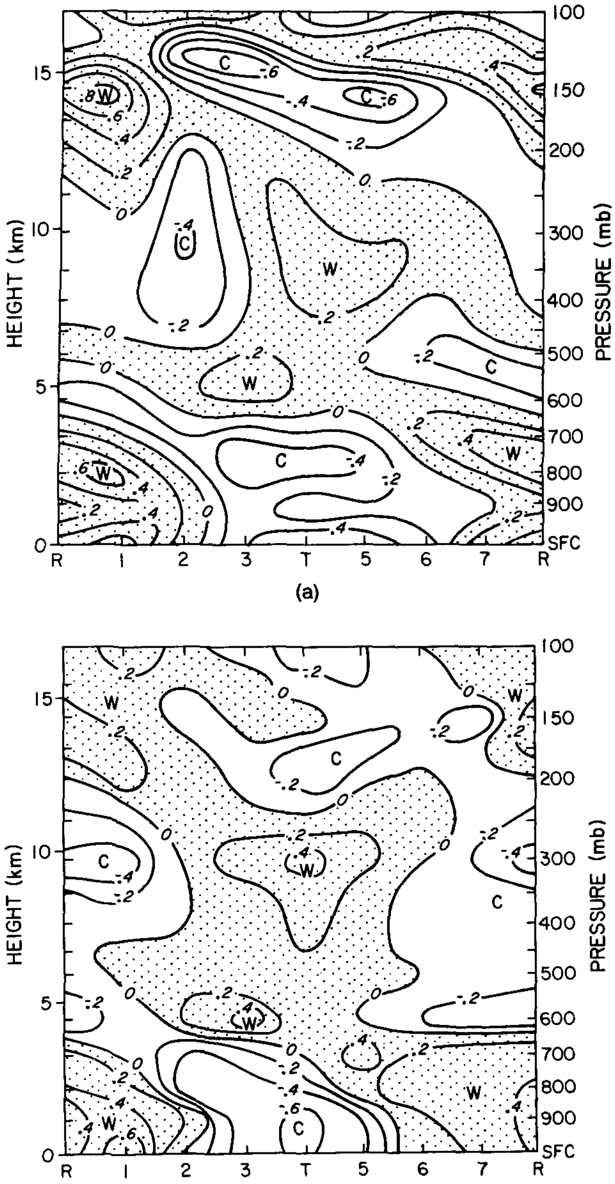

FIG. 1. The vertical cross section of the temperature field across one wavelength of the composite of easterly waves observed during Phase III of GATE: (a) the observed temperature field adopted from Thompson et al. (1979); (b) the calculated temperature field from the derived diagnostic balance condition.

required to maintain the mean dynamic structure rather than the distribution of latent heat release by cumulus clouds.

Attempts were also made to verify this simple balance equation. The temperature field of tropical easterly waves was deduced through the diagnostic relationship from the dynamic field of the composite easterly wave presented in Thompson et al. (1979). The result is presented in Fig. 1b. It is to be compared with the observed thermal structure shown in Fig. 1a. The details of the calculation procedure are discussed in section 5. We shall begin by considering the vertical circulation patterns in easterly waves in the next section.

2. The local vertical circulation pattern in a tropical easterly wave

The fact that the temperature variations in tropical easterly waves should be small can be deduced from a simple scale analysis. Assuming that the pressure gradient force is comparable in magnitude to the Coriolis force, the fractional change of potential temperature θ has a typical magnitude given by the ratio of the Froude number (Fr) to the Rossby number (Ro) (Holton, 1979):

$$\Delta\theta/\theta \sim \mathrm{Fr}/\mathrm{Ro}, \tag{1}$$

where $\mathrm{Ro} = V/fL$ and $\mathrm{Fr} = V^2/gD$. In these expressions, V is the horizontal velocity scale, L and D are the horizontal and vertical length scales of the disturbance, f is the Coriolis parameter and g the gravitational acceleration. In an easterly wave, $\mathrm{Fr}/\mathrm{Ro} \sim 10^{-3}$ which gives a $\Delta\theta \sim 0.3°\mathrm{C}$. This result is in general agreement with observations.

The temperature variations in these disturbances are small despite the large amount of latent heat released by cumulus clouds. This implies a certain approximate balance between thermodynamic processes, and this balance can be deduced from the thermodynamic equation. For the sake of clarity, we shall neglect the effects of radiative heating in the main part of our analysis. Radiative effects will be discussed and treated in the Appendix.

In terms of the potential temperature θ, the first law of thermodynamics can be written as

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\mathbf{V}}\cdot\nabla\bar{\theta} + \bar{\omega}\frac{\partial\bar{\theta}}{\partial p} = -M_c\frac{\partial\bar{\theta}}{\partial p}. \tag{2}$$

The overbar denotes the horizontal areal average which is used to separate the mean flow from small-scale variations induced by cumulus clouds. The cloud effect is represented on the right-hand side of the equation in terms of the total cloud mass flux M_c. This representation was first proposed by Ooyama (1971) and Arakawa and Schubert (1974) and later refined by Cho (1977). If we define

$$\tilde{\omega} = \bar{\omega} + M_c \tag{3}$$

as the mean vertical velocity in the cloud environment, then following the approach adopted by Ooyama (1971), (2) can be rewritten as

$$\frac{\partial\bar{\theta}}{\partial t} + \bar{\mathbf{V}}\cdot\nabla\bar{\theta} + \tilde{\omega}\frac{\partial\bar{\theta}}{\partial p} = 0. \tag{4}$$

In addition to horizontal advection, the mean temperature field is influenced by the vertical motion in the cloud environment. The portion of latent heat released inside cumulus clouds that may be imparted to the mean flow depends on the ability of clouds to influence the vertical velocity field in the cloud environment. The fact that temperature variations in easterly waves are very small also implies that $\tilde{\omega}$ is extremely weak in these disturbances.

If (L/V) is used as the typical time scale of these weather systems, then from (1) and (4), the mean vertical velocity in the cloud environment is estimated as

$$\tilde{\omega} \sim P(V/L)(\mathrm{Fr}/\mathrm{Ro})/S, \tag{5}$$

where P is the vertical pressure scale, and $S = (P/\bar{\theta})(\partial\bar{\theta}/\partial p)$ is a static stability parameter. The magnitude of the mean divergence in the cloud environment $\tilde{\delta}$ can be estimated from (5):

$$\tilde{\delta} \sim \tilde{\omega}/P \sim (\mathrm{Fr}/\mathrm{Ro}S)(V/L). \tag{6}$$

Assuming $\mathrm{Ro} \sim 1$, $\mathrm{Fr} \sim 10^{-3}$ and $S \sim 10^{-1}$, then

$$(\mathrm{Fr}/\mathrm{Ro}S) \sim 10^{-2}. \tag{7}$$

The value of $\tilde{\delta}$ is therefore about two orders smaller than (V/L). The parameter (V/L) is a measure of $\max(\bar{\zeta}, \bar{\delta})$, where $\bar{\zeta}$ is the mean vorticity and $\bar{\delta}$ the mean divergence. Usually $O(\bar{\zeta}) = V/L$ while $O(\bar{\delta}) \leqslant V/L$. So long as $\bar{\zeta} \sim V/L$, (6) and (7) imply that $\tilde{\delta} \sim 10^{-2}\,\bar{\zeta}$. The divergence in the cloud environment is two orders smaller than the mean vorticity. To the order of $(\mathrm{Fr}/\mathrm{Ro}S)$, we may assume that the air flow in the cloud environment is nondivergent. If $\bar{\delta}$ is also of the order V/L, such as near the trough axis of the easterly wave, then $\tilde{\delta} \ll \bar{\delta}$, and (3) can be reduced to

$$\bar{\omega} \approx M_c. \tag{8}$$

The large-scale mean vertical motion is merely a reflection of the vertical mass transport inside cumulus clouds.

3. A general balance condition

If we accept the conclusion reached in the previous section, the existence of a balance condition then becomes obvious. Consider the area A containing a number of cumulus clouds shown in Fig. 2. The properties of air motion in the cloud environment depend on the inertia of air flow in the cloud environment, the pressure force at the boundary L of the area A, and the forcing across the boundaries l_i of cumulus clouds. If airflow in the cloud environment is approximately nondivergent, then the irrotational components of these inertial and pressure forces must be approximately balanced. The precise form of this balance condition may depend on, among other things, the parameterization scheme used to represent the forcing across the cloud boundaries. But this should not obscure the fact that the existence of the balance condition depends only on the discussions given in the previous section.

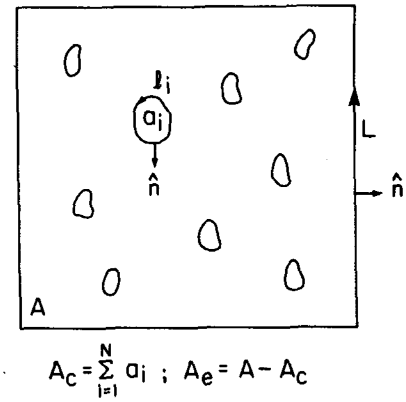

FIG. 2. Schematic geometry of the area of averaging, A. a_i is the cross-sectional area of the ith cloud, $A_c = \sum_{i=1}^{N} a_i$ the total cloud area, and $A_e = A - A_c$ the cloud environmental area. L and l_i are the outer boundaries of A and a_i, respectively, and $\hat{\mathrm{n}}$ is the unit normal vector pointing outward on a closed boundary.

To be specific, let us consider the anelastic form (Ogura and Phillips, 1962) of the horizontal momentum equation:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} + w\frac{\partial \mathbf{V}}{\partial z} + f\mathbf{k}\times\mathbf{V} = -\frac{1}{\bar{\rho}}\nabla p, \tag{9}$$

where $\bar{\rho}$ is the large-scale mean air density. If we denote the sum of the last three terms on the left-hand side of the equation describing inertia and the Coriolis force as $\mathbf{F}$:

$$\mathbf{F} = \mathbf{V}\cdot\nabla\mathbf{V} + w\frac{\partial \mathbf{V}}{\partial z} + f\mathbf{k}\times\mathbf{V}, \tag{10}$$

then the nondivergent nature of airflow in the cloud environment implies that

$$\oint_L \left(\mathbf{F} + \frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}\,dl - \sum_{i=1}^{N}\oint_{l_i}\left(\mathbf{F} + \frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}\,dl \approx 0. \tag{11}$$

Here N is the number of clouds in the area A and $\mathbf{n}$ denotes the unit normal vector pointing outward at a point on a closed boundary. The approximate balance equation given by (11) is valid to the order of $(\mathrm{Fr}/\mathrm{Ro}S)$.

By Gauss' theorem, the first pressure term in (11) can be written as

$$\oint_L \left(\frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}\,dl = A\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right). \tag{12}$$

It represents the forcing for divergent flow over area A by the mean pressure field $\bar{p}$. The other pressure term in (11), i.e.,

$$\sum_{i=1}^{N}\oint_{l_i}\left(\frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}\,dl,$$

represents pressure forcing for divergent flow out of cloud areas. The difference between the two pressure terms gives the forcing due to pressure for divergent flow out of the cloud environmental area A_e.

Similarly, we have by Gauss' theorem,

$$\oint_L \mathbf{F}\cdot\mathbf{n}dl-\sum_{i=1}^{N}\oint_{l_i}\mathbf{F}\cdot\mathbf{n}dl=\int_{A_e}\nabla\cdot\mathbf{F}da. \quad (13)$$

The integral on the right-hand side is carried out over the cloud environmental area A_e. It is a measure of the forcing for divergence due to the inertia of air flow. Inserting (12) and (13) into (11), we get

$$\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right)=-\frac{1}{A}\int_{A_e}\nabla\cdot\mathbf{F}da+\frac{1}{A}\sum_{i=1}^{N}\oint_{l_i}\left(\frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}dl. \quad (14)$$

The equation describes a balance between the mean pressure forcing, the inertia forcing in the cloud environment and the pressure forcing at the boundaries of clouds.

The inertia forcing term in (14) can be made more explicit by recognizing that

$$\nabla\cdot(\mathbf{V}\cdot\nabla\mathbf{V})=\mathbf{V}\cdot\nabla\delta-\frac{1}{2}(\zeta^2-\delta^2-A^2-B^2), \quad (15)$$

where ζ is the vertical component of relative vorticity and $A=(\partial v/\partial x+\partial u/\partial y)$ and $B=(\partial v/\partial y-\partial u/\partial x)$, the two components of deformation. Here we have adopted the convention that x and y are the horizontal coordinates in the east and north directions, respectively, and u and v are the east and north components of the horizontal velocity. Using (15), we obtain

$$\int_{A_e}\nabla\cdot\mathbf{F}da=\int_{A_e}\left[\mathbf{V}\cdot\nabla\delta-\frac{1}{2}(\zeta^2-\delta^2-A^2-B^2)-f\zeta+\beta u+\nabla\cdot\left(w\frac{\partial\mathbf{V}}{\partial z}\right)\right]da.$$

In this expression $\beta=\partial f/\partial y$. Since in the cloud environment the divergence field δ is very weak, to the order of (Fr/RoS), the terms in (16) involving δ and vertical velocity w may be neglected. Equation (16) can be simplified to give

$$\int_{A_e}\nabla\cdot\mathbf{F}da=\int_{A_e}\left[-\frac{1}{2}(\zeta^2-A^2-B^2)-f\zeta+\beta u\right]da. \quad (17)$$

Using $(\tilde{\cdot})$ to denote the cloud environmental mean value of a variable, direct substitution of (17) into (14) gives

$$\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right)=\frac{A_e}{A}\left\{\frac{1}{2}[(\widetilde{\zeta^2})-(\widetilde{A^2})-(\widetilde{B^2})]+f\tilde{\zeta}-\beta\tilde{u}\right\}+\frac{1}{A}\sum_{i=1}^{N}\oint_{l_i}\left(\frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}dl. \quad (18)$$

Equation (18) is based entirely on the assumption or approximation discussed in section 2, i.e., the airflow in the cloud environment is approximately non-divergent; its validity depends only on (2). The radiative heating effect is neglected in deriving (18). In the Appendix we will show that (18) is still valid when radiative heating is considered, but to a lesser degree of approximation.

A few other approximations may be introduced into (18), but they are not essential. One such approximation is that the fractional area covered by cumulus clouds $\Sigma=A_c/A$ is very small, of the order of 10% or less; therefore,

$$A_e/A\approx 1. \quad (19)$$

Another approximation is concerned with the values of $\tilde{u}$ and $\tilde{\zeta}$. By definition of areal averaging,

$$\bar{u}=\sum u_c+(1-\sum)\tilde{u}, \quad (20)$$

$$\bar{\zeta}=\sum\zeta_c+(1-\sum)\tilde{\zeta}, \quad (21)$$

where $(\cdot)_c$ represents the average of a variable over the cloud areas. Since Σ is small,

$$\tilde{u}\approx\bar{u} \quad (22)$$

$$\tilde{\zeta}\approx\bar{\zeta}, \quad (23)$$

if u_c and ζ_c are of the same order of magnitude as $\tilde{u}$ and $\tilde{\zeta}$, respectively. Results from observational and numerical modeling studies seem to suggest that it is not unreasonable to assume that u_c and $\tilde{u}$ are of the same order in magnitude (Clark, 1979; Schlesinger, 1984). By considering vorticity dynamics of cumulus clouds, Cho and Cheng (1980), and Cho and Clark (1981) showed that mean cloud vorticity ζ_c is comparable in magnitude to the vorticity in the cloud environment, even though point values of cloud vorticity can be much larger than that of the large-scale mean flow. Furthermore, results from these studies suggest that large fluctuations in vorticity are associated with strong divergence/convergence and vertical motion which take place mainly inside clouds. In the cloud environment, the fluctuations in ζ are small. We can therefore assume also that point values of ζ in the cloud environment are essentially the same as the cloud environmental mean, i.e.,

$$\zeta=\tilde{\zeta} \quad (24)$$

in cloud environment. With these approximations, (18) can be simplified to give

$$\nabla\cdot\left[\frac{1}{\bar{\rho}}(\nabla\bar{p})\right]=\frac{1}{2}[\zeta^2-(\widetilde{A^2})-(\widetilde{B^2})]+f\bar{\zeta}-\beta\bar{u}+\frac{1}{A}\sum_{i=1}^{N}\oint_{l_i}\left(\frac{1}{\bar{\rho}}\nabla p\right)\cdot\mathbf{n}dl. \quad (25)$$

The equation relates the mean pressure field to the large-scale mean and the cloud environmental mean dynamic properties, and the pressure forcing at the cloud boundaries. If any of these secondary approximations just discussed is not valid, then the original form of the balance condition given by (18) should be used. We note also that (23) does not imply that the momentum field in the cloud environment is uniform, only that the cloud environmental mean values of air momentum are about the same as the large-scale mean values.

4. The thermal structure of slowly evolving easterly waves

Although the general balance condition derived in the previous section is formally interesting, it is difficult to apply in practice unless the nature of the pressure forcing at the cloud boundaries is understood and the physical processes it represents can be explicitly parameterized. We will show in this section, however, that a more explicit balance condition can be derived if we limit our study to only "slowly evolving" easterly waves.

To define a "slowly evolving" easterly wave, let us consider a wave perturbation in the zonal mean flow of the form $\Psi = A \exp[\alpha t + i(\Omega t - kx)]$. Here Ω is the angular frequency and k the wavenumber; α is the growth rate of the wave. For a simple sinusoidal wave of this form,

$$\frac{\partial \Psi}{\partial t} = (\alpha + i\Omega)\Psi \qquad (26)$$

$$\frac{\partial \Psi}{\partial x} = -ik\Psi. \qquad (27)$$

$c = \Omega/k$ gives the phase speed. The local time change of the wave contains two components: one component is due to the growth of the amplitude of the wave, the other to the wave propagation, i.e.,

$$\frac{\partial \Psi}{\partial t} = \alpha\Psi - c\frac{\partial \Psi}{\partial x}. \qquad (28)$$

The wave will be considered "slowly evolving" in this analysis if

$$\alpha \ll \Omega. \qquad (29)$$

In slowly evolving waves, the local changes of wave properties are dominated by the propagational translation of wave fields, i.e., $\partial\Psi/\partial t \approx -c\partial\Psi/\partial x$.

For easterly waves observed during Phase III of GATE, the average east–west wavelength λ_x is about 2500 km with a typical period $\tau \sim 3.5$ days (Reed et al., 1977; Thompson et al., 1979). These correspond to a wave angular frequency $\Omega \sim 1.8$ day^{-1} and a phase speed $c \sim 8$ m s^{-1}. The growth rate of these waves in their source region over west Africa was examined in a number of theoretical studies (Rennick, 1976; Mass, 1979). The maximum growth rate α was found to be about 0.36 day^{-1} by Rennick (1976) and about 0.52 day^{-1} by Mass (1979). Using these growth rates, the ratio α/Ω can be estimated to be in the range

$$\alpha/\Omega \sim 0.2\text{–}0.3. \qquad (30)$$

It appears that even during the most rapidly growing phase of these waves, they may be considered slowly evolving in the sense defined in this paper.

We shall attempt to derive here an approximate balance condition for this slowly evolving type of disturbance. Two assumptions will be introduced in the derivation: 1) the airflow in the cloud environment is approximately nondivergent, as discussed in section 2, and 2) the horizontal eddy flux of momentum associated with cumulus clouds can be neglected, i.e., $\overline{\mathbf{V}'\mathbf{V}'} = 0$. Because of the condition of slowly evolving disturbances, no explicit parameterization of cumulus scale processes is necessary in the derivation. To do this, let us apply the horizontal equation of motion (9) to the area of averaging A. Line integrations of the terms in the equation along the boundary L gives

$$\oint_L \left(\mathbf{F} + \frac{1}{\rho}\nabla p\right)\cdot \mathbf{n}dl = -A\frac{\partial \bar{\delta}}{\partial t} \approx A\mathbf{c}\cdot\nabla\bar{\delta}. \qquad (31)$$

The last equality is obtained from the assumption that we are dealing only with slowly evolving waves. Here $\mathbf{c}$ is the phase velocity vector of the waves.

The inertial forcing term in (31) is given by

$$\oint_L \left[\mathbf{V}\cdot\nabla\mathbf{V} + w\frac{\partial \mathbf{V}}{\partial z} + f\mathbf{k}\times\mathbf{V}\right]\cdot\mathbf{n}dl = \oint_L \left[\nabla\cdot\mathbf{V}\mathbf{V} - \mathbf{V}\delta + w\frac{\partial \mathbf{V}}{\partial z} + f\mathbf{k}\times\mathbf{V}\right]\cdot\mathbf{n}dl. \qquad (32)$$

The outer boundary L of the area of averaging lies mainly in the cloud environment where the divergence and vertical motion fields are very weak. The vertical advection of momentum, for example, should be a very weak process along L. Therefore, terms involving δ or w in (32) can be neglected. This leads to the result

$$\oint_L \mathbf{F}\cdot\mathbf{n}dl = \oint_L [\nabla\cdot\mathbf{V}\mathbf{V} + f\mathbf{k}\times\mathbf{V}]\cdot\mathbf{n}dl = \int_A \nabla\cdot[\nabla\cdot\mathbf{V}\mathbf{V} + f\mathbf{k}\times\mathbf{V}]da = A[\overline{\nabla\cdot(\nabla\cdot\mathbf{V}\mathbf{V})} + \overline{\nabla\cdot(f\mathbf{k}\times\mathbf{V})}]. \qquad (33)$$

Since

$$\overline{\nabla\cdot(\nabla\cdot\mathbf{V}\mathbf{V})} = \nabla\cdot[\nabla\cdot(\bar{\mathbf{V}}\bar{\mathbf{V}} + \overline{\mathbf{V}'\mathbf{V}'})] \qquad (34)$$

$$\overline{\nabla\cdot(f\mathbf{k}\times\mathbf{V})} = \nabla\cdot(f\mathbf{k}\times\bar{\mathbf{V}}) = -f\bar{\zeta} + \beta\bar{u}, \qquad (35)$$

we obtain by substituting (12) and (33)–(35) into (31)

$$\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right) + \nabla\cdot[\nabla\cdot(\bar{\mathbf{V}}\bar{\mathbf{V}} + \overline{\mathbf{V}'\mathbf{V}'})] - f\bar{\zeta} + \beta\bar{u} = \mathbf{c}\cdot\nabla\bar{\delta}. \qquad (36)$$

If we introduce the mathematical identity

$$\nabla\cdot(\nabla\cdot\bar{\mathbf{V}}\bar{\mathbf{V}})=\nabla\cdot(\bar{\mathbf{V}}\cdot\nabla\bar{\mathbf{V}}+\bar{\mathbf{V}}\bar{\delta})$$

$$=2\bar{\mathbf{V}}\cdot\nabla\bar{\delta}-\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2-\bar{A}^2-\bar{B}^2), \quad (37)$$

then (38) becomes

$$\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right)=-(2\bar{\mathbf{V}}-\mathbf{c})\cdot\nabla\bar{\delta}+\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2$$

$$-\bar{A}^2-\bar{B}^2)+f\bar{\zeta}-\beta\bar{u}-\nabla\cdot\overline{\mathbf{V}'\mathbf{V}'}. \quad (38)$$

The last term in (38) represents the effect of the horizontal eddy flux of momentum caused by cumulus clouds. The assumption is often made in cumulus parameterization studies that

$$\nabla\cdot\overline{\mathbf{V}'\mathbf{V}'}=0. \quad (39)$$

It is usually justified on the intuitive basis that cumulus clouds transport horizontal momentum effectively only in the vertical direction because it is in the vertical direction the mass transport by clouds takes place. A more rigorous justification of (39) was recently given by Cho (1985). If this approximation is introduced, then (38) can be further reduced to

$$\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla\bar{p}\right)=-(2\bar{\mathbf{V}}-\mathbf{C})\cdot\nabla\bar{\delta}$$

$$+\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2-\bar{A}^2-\bar{B}^2)+f\bar{\zeta}-\beta\bar{u}. \quad (41)$$

It gives a diagnostic relationship between the pressure field and the mean momentum field in a slowly evolving easterly wave.

Equation (40) can also be written in terms of the geopotential ϕ in a pressure p coordinate system:

$$\nabla^2\bar{\phi}=-(2\bar{\mathbf{V}}-\mathbf{c})\cdot\nabla\bar{\delta}$$

$$+\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2-\bar{A}^2-\bar{B}^2)+f\bar{\zeta}-\beta\bar{u}. \quad (41)$$

Through the hydrostatic relation

$$\frac{\partial\phi}{\partial p}=-\frac{RT}{p}, \quad (42)$$

where T is the temperature and R the gas constant, (41) also implies that

$$\nabla^2\bar{T}=\frac{p}{R}\frac{\partial}{\partial p}[(2\bar{\mathbf{V}}-\mathbf{c})\cdot\nabla\bar{\delta}$$

$$-\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2-\bar{A}^2-\bar{B}^2)-f\bar{\zeta}+\beta\bar{u}], \quad (43)$$

which is a diagnostic relationship between the thermal field and the momentum field in a slowly evolving easterly wave.

We should note that while the balance condition given by (25) derived in section 3 is a general balance equation, (41) and (43) derived in this section for slowly evolving easterly waves might be somewhat limited in their universal applicability. Equations (41) and (43) merely indicate that in a slowly evolving tropical system, the major portion of the thermal structure reflects not so much the heating processes but the thermal structure required to maintain a balance in the airflow in a manner described by these equations. It is not clear from our analysis whether these equations also provide an adequate approximation for the study of evolution or the growth of these disturbances.

5. Application to a composite easterly wave

The composite easterly wave presented by Reed et al. (1977) and Thompson et al. (1979) represents the average of all easterly waves observed during Phase III of GATE. It probably reflects more the structure of an easterly wave in a steady state than that of a growing or decaying wave. Therefore, the balance condition derived in the previous section can be applied to examine the thermal structure presented by these authors, even though the condition in the form given by (43) might be limited in its general applicability.

If we let I represent the terms on the right-hand side of (43) which describe the effects of inertia of airflow:

$$I=\frac{p}{R}\frac{\partial}{\partial p}\Big[(2\bar{\mathbf{V}}-\mathbf{c})\cdot\nabla\bar{\delta}$$

$$-\frac{1}{2}(\bar{\zeta}^2-3\bar{\delta}^2-\bar{A}^2-\bar{B}^2)-f\bar{\zeta}+\beta\bar{u}\Big], \quad (44)$$

the balance equation (43) can be written in the compact form:

$$\nabla^2\bar{T}=I. \quad (45)$$

To examine whether the equation indeed gives the correct relationship between the thermal field and the momentum field of an easterly wave in steady state, we shall attempt to determine the temperature field from the inertia field through (45), using the inertia field of the composite easterly wave presented in Thompson et al. (1979). The procedure we used in these calculations is as follows.

To separate the wave perturbation from the zonal mean state, a quantity $(\cdot)$ will be first averaged zonally on a constant pressure surface across one wavelength of the wave. Such an average will be denoted by $(\bar{\cdot})$. The wave perturbations will be described as $(\cdot)''$. Thus

$$\bar{T}=\bar{\bar{T}}+T'', \quad (46)$$

$$I=\bar{I}+I''. \quad (47)$$

A zonal average of (45) gives

$$\nabla^2\bar{\bar{T}}=\bar{I}. \quad (48)$$

Subtracting (48) from (45) we get

$$\nabla^2 T''=I''. \quad (49)$$

To determine $\nabla^2 T''$, a three-dimensional structure of the temperature field is needed. Results presented in Reed et al. (1977) shows a clear meridional structure of the wave, even though only the east–west cross section of the wave structure was presented by Thompson et al. (1979). The horizontal spatial scale of the wave in the meridional direction estimated from Reed et al. (1977) corresponds to a meridional half wavelength $\lambda_y/2 \approx 1800$ km. This gives a meridional wavenumber

$$k_y = \frac{2\pi}{\lambda_y} \approx 1.7 \times 10^{-6}\,\mathrm{m}^{-1}.$$

For zonal wavenumber k_x, we assume $\lambda_x = 2500$ km. This gives

$$k_x = \frac{2\pi}{\lambda_x} \approx 2.5 \times 10^{-6}\,\mathrm{m}^{-1}.$$

To calculate the Laplacian of the wave perturbation temperature field, we simply assumed that

$$\nabla^2 T'' = -(k_x^2 + k_y^2)T''. \quad (50)$$

Combining (49) and (50), we have

$$T'' = -\frac{1}{(k_x^2 + k_y^2)} I''. \quad (51)$$

In Thompson et al. (1979), the values of the total vorticity and divergence, i.e., $\bar{\zeta}$ and $\bar{\delta}$, were given in each region of the wave, together with the wave perturbation wind field u'' and v''. In addition, the vertical profile of the Phase III mean zonal wind speed $\bar{u}$ was also given. Using these data, the terms in the expression for I involving $\frac{1}{2}\bar{\zeta}^2$, $\frac{3}{2}\bar{\delta}^2$, $f\bar{\zeta}$ and $\beta\bar{u}$ can be calculated directly. Noting that the zonal wind speed $\bar{u}$ is usually much larger than the meridional wind speed $\bar{v}$ in easterly waves, the following approximation is made in evaluating the term $(2\bar{\mathbf{V}} - \mathbf{c}) \cdot \nabla\bar{\delta}$:

$$(2\bar{\mathbf{V}} - \mathbf{c}) \cdot \nabla\bar{\delta} \approx (2\bar{v} - c)\frac{\partial\bar{\delta}}{\partial x}. \quad (52)$$

The approximation is necessary because no data were presented in Thompson et al. (1979) to enable us to determine the meridional gradient of $\bar{\delta}$. The wave speed $c = -8$ m s^{-1} is used throughout our calculations. Furthermore, since no information was provided about the deformation field, we simply assumed that

$$\bar{A} = 0, \quad \bar{B} = 0. \quad (53)$$

Using the procedure just outlined, the value of I in each region of the wave can be determined. A zonal average of I over one wavelength of the wave gives $\bar{\bar{I}}$ from which I'' can also be determined. The wave perturbation temperature field T'' is then determined from (51). The result is presented in Fig. 1b. It is to be compared with the observed temperature field shown in Fig. 1a, adopted from Thompson et al. (1979).

Overall, there is a fair degree of agreement between the calculated temperature field and the temperature field determined from observations. Equation (43) is able to provide the correct outline of major regions of warm and cold masses of air even though there are some differences in the magnitude of the temperature anomalies and in the shape of the contours. Over most regions of the wave, the agreement is quantitatively acceptable as well, with perhaps two major exceptions. First, the column of cold air observed over wave region 2 in the layer between 500 and 200 mb levels is shifted to the ridge region of the wave in the calculated temperature field. Secondly, the amplitude of the calculated temperature variations in the upper layer between 200 and 100 mb is considerably smaller than that of the observed field. The reasons for these discrepancies are not entirely clear. The temperature variations in the wave are very small and the measured wind and temperature data might have considerable errors, especially at the upper levels. The upper level flow fields of the wave presented by Reed et al. (1977) suggest that there may be a large deformation component in airflow at these levels. Furthermore, the composite procedure used by Reed et al. (1977) and Thompson et al. (1979) was based on wave structure at 700 mb. All of these could be contributing factors to the discrepancies. But we have no way of making quantitative assessment of these factors using the composite easterly wave data presented in these papers.

As a measure of discrepancy between the calculated and the observed temperature field, we show in Fig. 3a the difference field obtained by subtracting the observed temperature field from the calculated temperature field. The difference is quite substantial at levels above 300 mb, with peak values ~ 0.6°C. The observed temperature field at these levels has peak values about 0.6°–0.8°C. Below 300 mb, the observed temperature field has peak values in the range of 0.4°–0.6°C, while the magnitude of the difference field is generally about 0.2°C. There are three regions below 700 mb where the difference exceeds 0.4°C, but they occupy only very small areas. Furthermore, the difference field does not seem to have any systematic patterns at levels below 300 mb. Generally speaking, this difference field confirms the earlier observation that below 300 mb, the calculated temperature field more or less reproduced the observed temperature field.

In a study by Chen and Ogura (1982), a composite analysis of easterly waves observed during Phase III of GATE was also made. The composite temperature field obtained by these authors is very similar to that obtained by Thompson et al. (1979). Since the same GATE dataset was used in both analyses, any difference between them can only be attributed to differences in analysis procedures and any possible corrections made to the data. As a measure of uncertainties in the observed temperature field, we show in Fig. 3b the difference between Chen and Ogura's analysis and the analysis by Thompson et al. The difference is generally

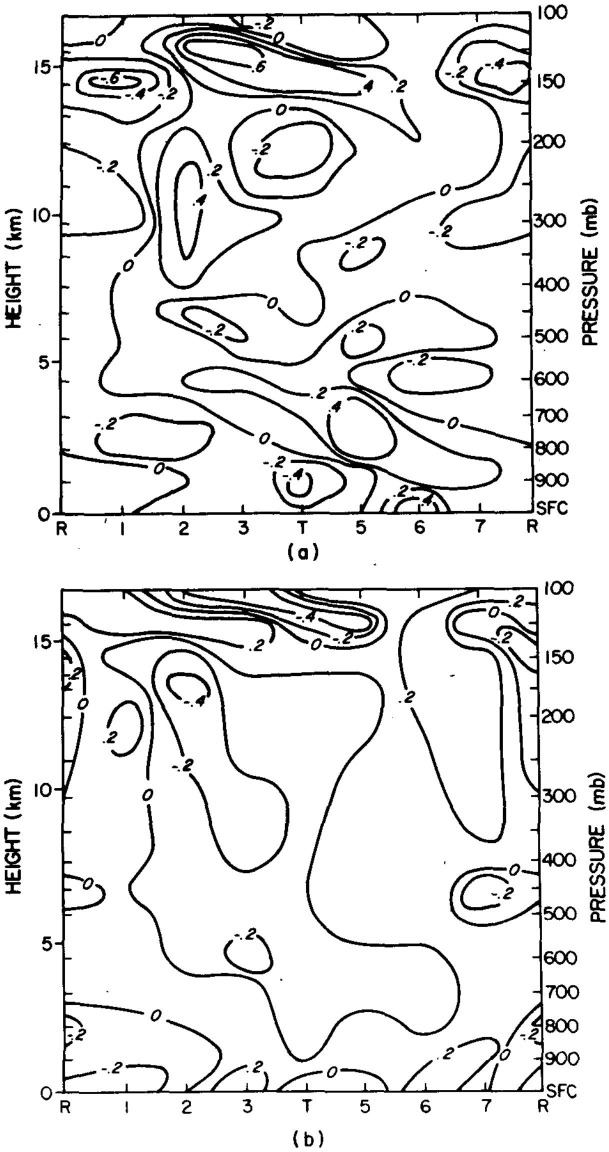

FIG. 3. The difference (a) between the calculated temperature field from the derived balance condition, and the observed temperature field analyzed by Thompson et al. (1979); (b) between the observed temperature field analyzed by Chen and Ogura (1982) and that analyzed by Thompson et al. (1979).

very large above 300 mb, with peak values exceeding 0.4°C. Below 300 mb, the magnitude of the difference is about 0.2°C. Comparing Fig. 3a with 3b, the difference between the observed and the theoretically calculated temperature fields is somewhat larger than the difference between the two temperature fields obtained by two separate analyses of the same set of observational data, but only by about 0.2°C, which is not excessively large considering the fact that the temperature variation from wave trough to wave ridge is about 1°C.

In view of the uncertainty in the composite dataset itself, the observational evidence we show to support the derived balance equation is only suggestive and does not provide a full proof of the theoretical result. The dataset may not be good enough, for example, to verify any particular assumptions made in the derivation, or to discriminate between various forms of the balance equation derived from a reasonable set of cumulus parameterization schemes. A more careful analysis of the balance condition should be made using original observational data. Even then, the difficulty remains that the measurements of the temperature and geopotential fields are subject to relatively large errors.

6. Conclusion

In this paper a procedure is introduced to derive a general balance condition for large-scale tropical disturbances. The condition suggests that in these disturbances, a balance exists between the mean pressure field and the momentum field, and the pressure forcing across the boundaries of cumulus clouds.

We showed that a simple and explicit form of this balance condition can be obtained when it is applied to a slowly evolving or steady state tropical easterly wave. The equation suggests that the thermal structure of such disturbances reflects not so much the distribution of latent heat release which is believed to be one of the main energy sources for tropical disturbances, but the thermal structure required to maintain their dynamic structures. The balance equation in this simple form, however, may be limited in its general applicability. It is not clear at present whether the approximations introduced in obtaining this simplified balance equation are adequate for the study of the evaluation or intensification of large-scale tropical disturbances.

An attempt was made to deduce the thermal structure of easterly waves from this simplified balance equation using the momentum field of the composite easterly wave presented by Thompson et al. (1979). The results show reasonably good agreement with the observed thermal structure presented by these authors. The main weakness of the calculated temperature field is in the upper troposphere in the layer between 200 and 100 mb levels where the temperature variations in the calculated field are much smaller than those observed. This discrepancy could be caused by observational data error, the approximations made in the diagnostic calculations, or physical processes which are not well represented in the derived balance equation. More careful analyses should be made in the future using original observational data to calculate all of terms in the balance condition directly, and if possible demonstrate how much error is in the calculated result.

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APPENDIX

The Effect of Radiative Heating

In the analysis presented in sections 2–4, the effect of radiative heating was neglected. But the atmosphere is not in radiative balance, and the radiative heating usually cannot be ignored in the thermodynamic energy equation in the tropics. In this Appendix, we will analyze the effect of radiative heating on the balance equation derived in the main part of this paper.

By including the effect of radiative heating Q_R, the thermodynamic equation (2) becomes

$$\frac{\partial\bar{\theta}}{\partial t}+\bar{\mathbf{V}}\cdot\nabla\bar{\theta}+\bar{\omega}\frac{\partial\bar{\theta}}{\partial p}=-M_c\frac{\partial\bar{\theta}}{\partial p}+Q_R. \quad \text{(A1)}$$

The effect of Q_R can be described in terms of a vertical motion field ω_R defined through the equation

$$\omega_R\frac{\partial\bar{\theta}}{\partial p}=Q_R. \quad \text{(A2)}$$

ω_R may be interpreted as the vertical motion field required to balance the radiative effect. The corresponding divergence field will be denoted as $\delta_R = -\partial\omega_R/\partial p$.

Using a similar analysis as given in section 2, one can show readily that

$$\tilde{\delta}-\delta_R\sim(\mathrm{Fr/Ro}S)\bar{\zeta}. \quad \text{(A3)}$$

Comparing (A3) with the large-scale mean vorticity, the divergence field in the cloud environment is essentially the same as δ_R, to within the order of (Fr/RoS). Or equivalently, we have

$$\tilde{\omega}\approx\omega_R. \quad \text{(A4)}$$

To the order of (Fr/RoS), the vertical velocity in the cloud environment is essentially equal to that induced by radiation.

In order to determine if the balance conditions derived in sections 3 and 4 are still valid in the presence of radiative heating, we need to compare δ_R with the large-scale mean vorticity $\bar{\zeta}$. If $\delta_R \ll \bar{\zeta}$ and the cloud environment is still approximately nondivergent, then the balance conditions remain valid.

The observed values of $\bar{\zeta}$ in tropical easterly waves are of the order 10^{-5} s^{-1}. To estimate the magnitude of δ_R, we use the values of radiation flux convergence calculated for Phase III of GATE by Cox and Griffith (1979). The values of the radiative flux convergence varied, depending on the time of the day and the weather conditions, but generally they ranged between 10 and 30 W m^{-2} (100 mb)$^{-1}$. These values give

$$Q_R\sim 1\text{–}3\times10^{-5}\ ^\circ\mathrm{C\,s^{-1}}.$$

Assuming $\partial\theta/\partial p \sim 3\times10^{-2}$ °C mb^{-1}, one finds

$$\delta_R\sim0.3\text{–}1\times10^{-6}\,\mathrm{s}^{-1}.$$

δ_R is one order of magnitude smaller than $\bar{\zeta}$. Therefore the balance condition is valid even in the presence of radiative heating, but the approximation is to the order of $\delta_R/\bar{\zeta}\sim10^{-1}$, instead of the order (Fr/RoS) $\sim 10^{-2}$.

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