The Relationship between Tropopause Potential Temperature and the Buoyant Energy of Storm Air

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ABSTRACT

It has been suggested that the potential temperature of the tropical tropopause may be linked to the energy of air flowing into storms, and previous studies have sought to correlate the subannual fluctuations of the two quantities. The formulations used in these studies also provide estimates of the mean tropopause potential temperatures, but such temperatures are much lower than the observed values. In this work we reexamine the problem. The earlier thermodynamic formulation is replaced by a more accurate form, and the appropriate humidity content of storm air is introduced and justified. The data now show that, averaged over the tropical ocean regions, the mean potential temperature of the tropopause is determined accurately and directly by the mean sea surface temperature. Continental conditions are significantly different and require further study. It appears desirable to undertake detailed investigations of the local relationship between surface storm conditions and tropopause characteristics.

1. Introduction

The tropopause marks a fundamental division of the lower atmosphere. Below it is the troposphere, characterized by vigorous mixing and large values of eddy diffusivity. Above it is the stratosphere, the lower levels of which are characterized by much weaker mixing and small diffusivities. At the interface of the two regions there is a change in the lapse rate accompanying the transition from energy transports dominated by turbulence to transports dominated by radiation (Goody, 1949). There is also an abrupt change in the potential vorticity (Reed, 1955), which is possibly of greater dynamical significance than the change in the lapse rate.

Storms and severe convection provide much of the character of the troposphere, and Riehl and Malkus (1958) deduced that the vertical extent of the tropical Hadley cell is controlled by the cumulus towers within storm cells. A generalization of this concept suggests that the buoyancy of storm air may be a major factor in determining the tropopause characteristics. This is assumed in most simple radiative-convective models of the lower atmosphere, although its application is somewhat tentative. Schneider (1977) clearly identifies the moist static energy of the troposphere as a major element in his computation of the tropopause height, but its contribution is merged with that of the radiation; there is nothing in the final result that is specific to the moisture alone. In a more direct approach, Held (1982) identifies the moist static energy in storm systems with the potential temperature of the tropopause, but he defers to a GCM study that uses a more complex procedure and concedes that the direct identification "may be a significant oversimplification."

Whatever the merits of GCMs, the definitive test must be made against atmospheric observations. It is necessary to compare the observed tropopause potential temperature with the observed energy content of near-surface air. We are aware of two such studies: Reid and Gage (1981) and Krishna Murthy et al. (1986). These appear to severely undermine the direct identification of the two quantities, at least in the mean. Reid and Gage, primarily interested in the subannual fluctuations, must adjust the mean surface temperature of their model to very unrealistic values to reproduce the observed mean tropopause value. However, they find a correlation of the variations about these means. Krishna Murthy et al. apply the formulation used by Reid and Gage to a diverse set of tropical soundings obtained in and around the Indian continent. They seek correlations between tropopause and surface values in a local sense rather than through the spatial averages that Reid and Gage employ. The observed values of the tropopause potential temperature are always much higher than those computed from the formulation. Subannual variations seem to display much less correlation than revealed in the Reid–Gage analysis.

The formulation employed by Reid and Gage is well suited to their purpose (seeking correlations) but is not appropriate to direct quantitative studies. Its definition of moist static energy ignores a significant portion of the buoyant energy available to a rising parcel and understates the moisture content of storm air. In this study we reexamine the situation. We conclude that for marine conditions, the appropriate formulation provides an excellent correspondence between the means of tropopause potential temperature and buoyant energy entering a storm. For continental conditions the correspondence seems reasonable, but critical observations are lacking.

Recomputed results are presented for the datasets of Reid and Gage and of Krishna Murthy et al. Computations are also presented for the classical dataset of Newell et al. (1972). Results from aircraft observations confirm the appropriate moisture content for marine storm air.

If confirmed by further testing, this relationship between the means of surface conditions and tropopause potential temperature will provide a fundamental constraint on models and computations of the lower atmospheric structure.

2. Concepts and assumptions

The essential concept is that the tropopause is defined by the level of buoyant equilibrium for favored parcels of air entering the storm systems.

The air in the surface layer contains the latent heat that drives the tropospheric convection on all its scales. Entrainment with air at mid- and upper levels is an essential feature of storms, but not all surface parcels have the same evolution. In any thermodynamic process every physically possible interaction path will be followed by a certain number of elements. One such path allows a parcel to rise through the storm column without mixing. The present hypothesis is that the tropopause is defined by such parcels.

Formulating a closed relationship between storm air and tropopause air requires the selection of an appropriate thermodynamic expression and information about the humidity of storm air. We find that estimates of mean surface conditions are not adequate for the present application. What is required is the expected extremes of humid air entering storms. Very few such observations are presently available, but those that are suggest a simple relationship for tropical marine conditions. It appears that the storm-air state is, essentially, one of saturation at the temperature of the underlying sea surface. If this is so, the thermodynamic relationship can be made very precise.

Support for this relationship is found in the measurements reported by Meyer and Rao (1985) as part of the 1979 MONEX (Summer Monsoon Experiment) work. The investigations were performed over the Arabian Sea and consisted of detailed measurements of conditions in the air below cloud base. Figure 1, reproduced from their paper, shows that the probing aircraft was directed to regions of intense cloud cover where it sampled the air below the storm systems. Table 1, also derived from their results, includes our computation of the temperature and dewpoint of the air. Climatological data (such as Newell et al.) show a sea surface temperature of 27.5°C for the Arabian Sea at this season, so the final column of Table 1 indicates relative humidities from 94% to supersaturation at this temperature. We suspect that within the errors of the measurements it is not possible to discriminate between such results and 100% relative humidity.

The published literature does not contain enough further information about tropical storms to allow us to judge the general applicability of the above results, but some of the published GATE data indicate similar conditions to those found by Meyer and Rao. Fitzjarrald and Garstang (1981), Houze (1977), and Johnson and Nicholls (1983) present complementary studies that tell much about one particular tropical squall line and its accompanying boundary layer. Ahead of this line of storms the shipborne instruments recorded relative humidities ranging from 85% to 95%. No aircraft measurements were made below the active storm clouds, but Fitzjarrald and Garstang report, as a general feature, that the humidity of the mixed layer rises in regions of light-to-moderate rainfall. This strongly suggests that at least some of the air in the storm updrafts must be saturated at the temperature of the underlying sea surface, but direct confirmation is not available. Similarly, studies of hurricanes suggest that the air in their more active regions may be saturated over a lower layer of considerable depth (Jordan, 1961). But the data do not allow us to compute actual humidity contents, and reference them to saturation at normal sea surface conditions.

It can be argued that the expected boundary-layer state below a heavy cloud layer with light rain must be saturation at the local surface temperature. The rain supplies a continuous source of moisture to the air between cloud base and sea surface, while the infrared radiation emitted by the sea surface is trapped and returned by the enhanced cloud cover.

The boundary-layer air over the warmest ocean regions may be below saturation but approach that state if it advects into adjacent regions that are somewhat cooler. Moreover, the warmer regions will generally be expected to produce proportionally more storms. Thus, when we take averages over the entire tropical ocean and relate mean storm-air water content to mean surface temperature we are probably introducing a somewhat spurious relationship that requires a higher "relative humidity" than would be discovered by a direct average of actual relative humidity. This simply provides an instance in which the average of a product differs from the product of the averages. This is an interesting problem, but for the somewhat restricted purposes of the present study we will use the assumption of 100% relative humidity to connect the means of sea surface temperature and storm-air energy.

3. Formulation

We take the surface air to define the character of the air that eventually ends up at the very top of the tro-

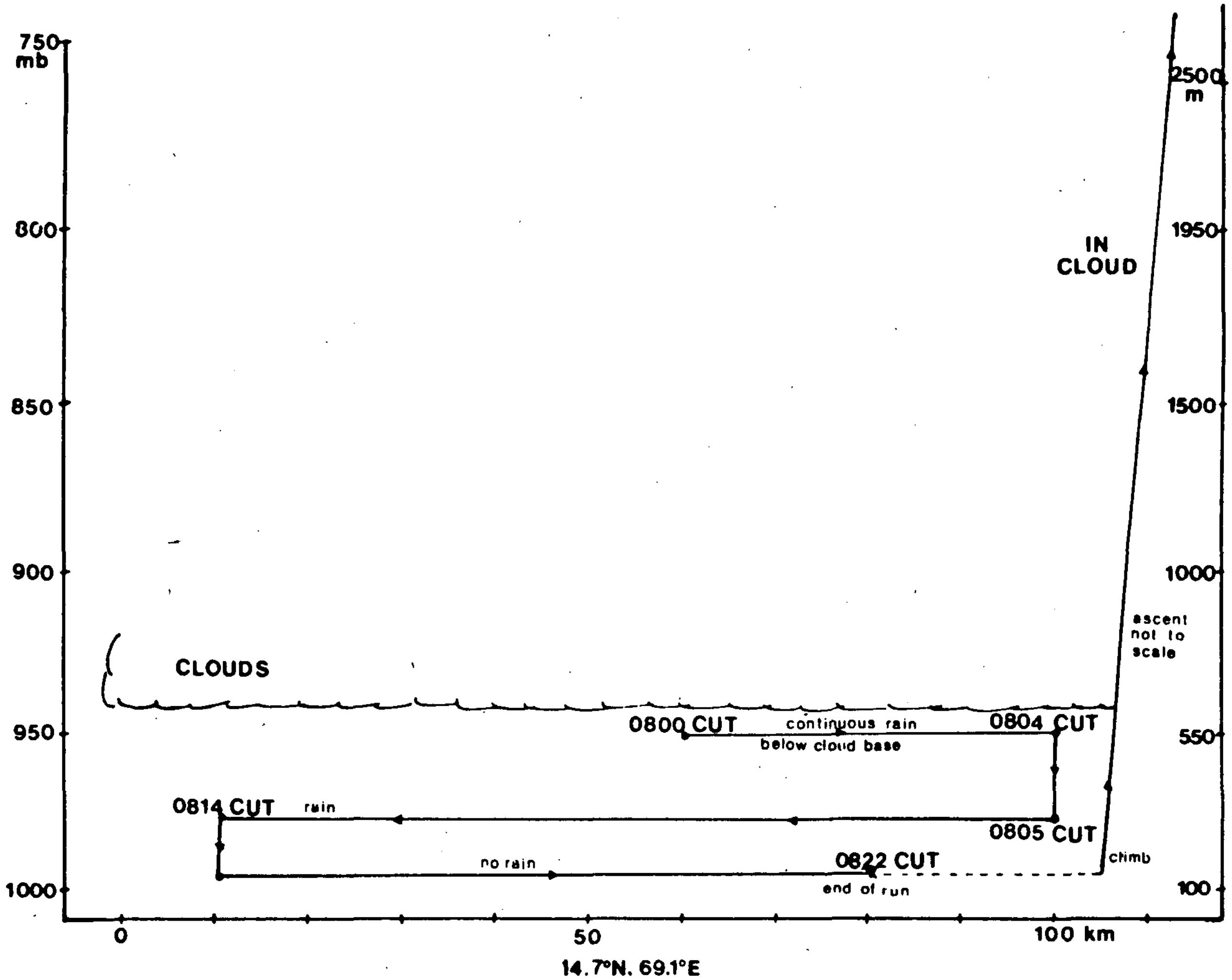

FIG. 1. The NCAR Electra's ascent and race track maneuvers below a monsoon cloud at 14.7°N, 69.1°E over the Arabian Sea on 20 June 1969 (Meyer and Rao, 1985). Analysis of such data shows that the air below the cloud has a humidity content well above mean boundary-layer values (Table 1).

posphere. The initial and final states of the air are linked by the thermodynamics of the lifting process. As a parcel of surface air rises within the storm column, latent heat is released in the changes of state, adding to the buoyancy of the air parcel; the liquid and ice materials also give up heat to the expanding gas, aiding its ascent. We assume that the ice falls out when the parcel reaches -10°C, which is a conservative value for the onset of

TABLE 1. Conditions beneath monsoon clouds over the Arabian Sea. The first four columns reproduce data from Meyer and Rao (1985). We deduced temperature from data of columns 1 and 4, and dewpoint from the data of column 3. Columns 4 and 5 have estimated accuracies of ±0.5°C, but no error estimate is available for the dewpoint. Sea surface temperature from climatological sources is 27.5°C. The lowest two rows of data were obtained on the lowest two tracks shown in Fig. 1.

Flight level (m)	Duration of run (min)	Specific humidity (gm kg^{-1})	Potential temperature (°C)	Estimated temperature (°C)	Dewpoint temperature (°C)
90	6	22.2	27.9	27.0	26.6
180	6	22.3	28.4	26.6	26.6
355	6	22.2	29.0	25.5	26.6
405	6	22.3	29.0	25.0	26.6
85	3	25.4	28.4	27.5	28.8
190	3	23.8	26.3	26.4	27.8

freezing. These parcels leave the storm with the highest potential temperature available to unentrained air originating in the troposphere.

The calculation of the final potential temperature of the lifted air, a relatively simple exercise in thermodynamics, is presented in the Appendix. It is slightly different from most of the standard definitions of equivalent potential temperature—the water is removed at a "fall-out" temperature imposed by the storm, rather than at absolute zero or at some other arbitrary stage. There are also subtleties concerning the reference values of the latent heats, although these probably have no significance in the present state of meteorology. The final result is a relationship between the potential temperature of the lifted air and its initial state:

$$\theta_{\mathrm{FINAL}} = \theta_{\mathrm{INITIAL}} \exp\left\{\frac{r}{c_A}\left[\frac{L_C}{T_D} + \frac{(L_S - L_C)}{T_F} + c_L \ln\left(\frac{T_D}{T_F}\right) + c_S \ln\left(\frac{T_F}{T_0}\right)\right]\right\}. \quad (1)$$

All symbols have their usual thermodynamic meaning and are explicitly defined in the Appendix.

Figure 2 plots the relationship

$$\theta = T_D \exp\left\{\frac{r}{c_A}\left[\frac{L_C}{T_D} + \frac{(L_S - L_C)}{T_F} + c_L \ln\left(\frac{T_D}{T_F}\right) + c_S \ln\left(\frac{T_F}{T_0}\right)\right]\right\} \quad (2)$$

assuming a surface pressure of 1000 mb. The function θ_{FINAL} can be recomputed using this plot when the actual initial conditions are given. In the present application, (1) and (2) are identical since we assume

$$\theta_{\mathrm{INITIAL}} = T_D = T = \text{surface temperature} \quad (3)$$

and Fig. 2 can be read as the relationship between tropopause potential temperature and the underlying sea surface temperature. As suggested by a referee, it could also be regarded as a mapping between surface wet-bulb potential temperature θ_w and the equivalent potential temperature θ_e used in this paper.

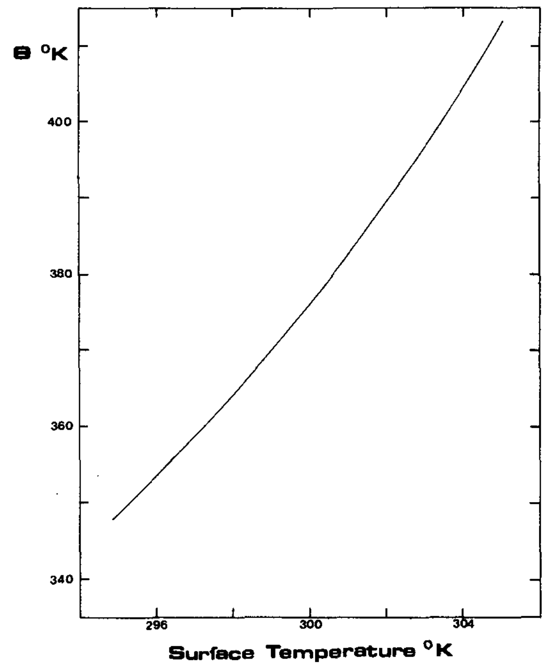

FIG. 2. The plot of final potential temperature available to a lifted air parcel that starts saturated at temperature T and pressure 1000 mb [cf. formulas (1)–(3)].

4. Test cases

a. The Reid–Gage dataset

Reid and Gage (1981) examine a relationship between an average sea surface temperature and an average tropopause potential temperature. Their work is formulated about a model for the sea surface temperature averaged over the latitude belt 20°N to 20°S. To relate sea surface temperature and tropopause potential temperature, they employ the formula

$$\theta_e = T_S \exp(wL_C/c_A T_D) \quad (4)$$

where w is the humidity mixing ratio and T_S is the sea surface temperature. The θ_e is identified as "the maximum potential temperature attainable by an ascending air parcel." The mixing ratio w and, hence, the dewpoint temperature T_D are obtained by "the assumption that the relative humidity of the boundary-layer air that is ultimately pumped into the hot towers has a constant value of 85%" (referenced to T_S), as "the boundary-layer air over the tropical oceans is not saturated."

Comparison of (1) and (4) shows that the Reid–Gage formulation has omitted part of the heat that is available for transfer between the water and the dry air component. The frequent use in the literature of (4), or its equivalent form as the moist static energy, is correct within the context of general circulation problems. Interest centers on the transfer of energy between different air masses; this transfer usually involves only the latent heat of condensation as the effect of higher-level freezing and lower-level thawing cancel out within the rainout air mass. And even in this context, forms such as (4) underestimate the heat transfer to some degree (Bolton, 1980). In the present context we need to know the heat that is available to the air parcel that carries the water vapor aloft; the additional terms contributing to (1), particularly the latent heat of fusion, now become important. Moreover, while marine boundary-layer air is not always saturated, we have presented evidence to support the position that a significant part of the air below a storm system will be

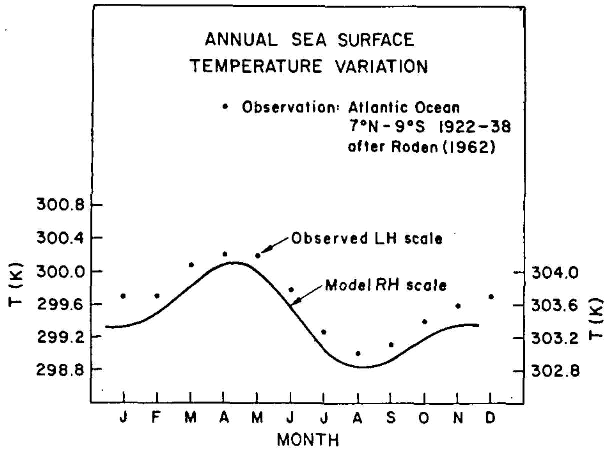

FIG. 3. Latitudinal average of sea surface temperature data and a fitted model variation. The model is for an average over 20° about the equator, while data are from a more equatorial band. Note offset of the two scales (from Reid and Gage, 1981).

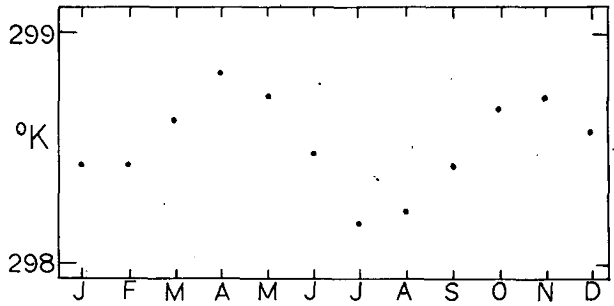

FIG. 4. Average sea surface temperature of the six stations within 20° of the equator presented by Roden (1962). Note the reduced mean and variation of this data compared with the narrower band used in Fig. 3.

saturated. Thus (4), as used by Reid and Gage, provides a considerable underestimate of "the maximum potential temperature attainable" and also of the observed tropopause potential temperatures.

The Reid–Gage dataset is now reexamined to see if the formulation (1)–(3) provides a better fit. It is first necessary to define the basis of a comparison since Reid and Gage vary their base. Thus, Fig. 3, reproduced from their work, shows the correlation between their model of sea surface temperature and observed sea surface temperature. However, the model was constructed as an average for the latitude belt 20°N to 20°S, while the observations, selected from Roden (1962), are for latitudes from 7°N to 9°S.

But Roden's dataset includes other latitudes (Table 2). Within the context defined by Reid and Gage it seems that the data for 19°N and 15.2°S should be included in the averaging procedure. When this is done, the observations provide Fig. 4. The observed variation is now much reduced, and the discrepancy between the means of observation and model sea surface temperature is increased.

The Reid–Gage formulation of the ocean response to insolation contains a free parameter. This was used to adjust the model sea surface temperature so that θ_e, obtained via (4), agrees with the observed tropopause value in the mean. We see from Fig. 3 that this requires a model sea surface temperature of 30.5°C, while the observed value (Fig. 4) is much lower (25.7°C). This in no way affects the validity of the correlation analysis for the subannual variations—the point pursued by Reid and Gage. Yet it very much affects our concern—a quantitative computation of tropopause potential temperature from actual surface conditions. The dataset of Krishna Murthy et al. (1986) (examined later) also shows that a literal application of the Reid–Gage approach severely underestimates the potential temperature of the tropopause.

TABLE 2. Locations of stations for which Roden (1962) reports marine conditions.

Latitude	Longitude
25.0°N	17.5°W
19.0°N	22.5°W
7.0°N	28.0°W
1.0°N	30.0°W
4.0°S	32.3°W
9.0°S	34.5°W
15.2°S	36.8°W
23.5°S	42.2°W

Reid and Gage defined their study in terms of averages over the ±20° latitude band. Accepting this, Roden's dataset used consistently provides Fig. 4 for the average sea surface temperature with an annual mean value of 298.7 K. The connection with the tropopause dataset will now be made directly, without further reference to the sea state model interposed by Reid and Gage.

The tropopause observations presented by Reid and Gage are reproduced in Fig. 5. From their curve we estimate an annual mean value of 368 K for the tropopause potential temperature. The relationships (1)–(3), as plotted in Fig. 2, predict that the sea surface temperature of 298.7 K corresponds to a tropopause potential temperature of 368.5 K. This agreement between theory and observation would be quite spectacular—if we could believe it.

However, the comparison involves sea surface temperatures obtained along the major shipping route between 19°N and 15°S in the Atlantic, and tropopause temperatures from five Pacific island sites in the band 7°N to 13.5°N. Such a limited dataset cannot provide definitive conclusions. We regard the previous agreement as encouraging but preliminary.

b. *The dataset of Newell et al.*

Newell et al. (1972) compiled a dataset of tropical conditions that is still one of the most complete sources

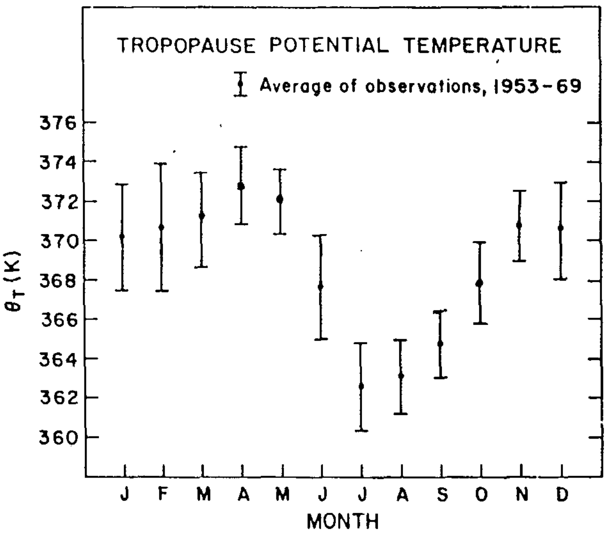

FIG. 5. The averaged tropopause potential temperature from five tropical island sites (Reid and Gage, 1981).

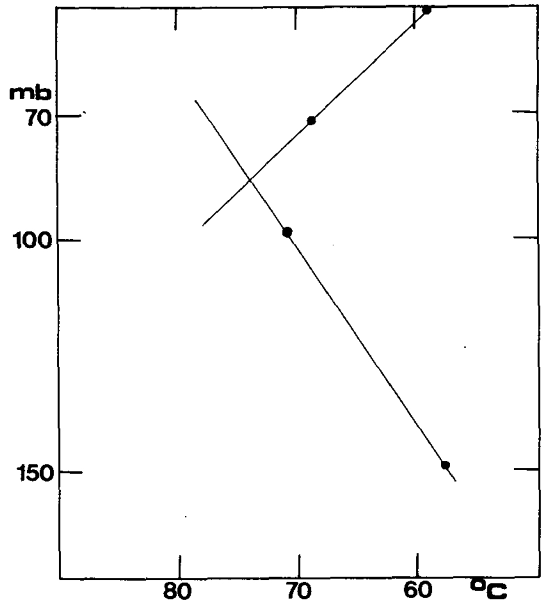

FIG. 7. Interpolation intended to reconstitute tropopause level from the gradients between the surrounding recorded data levels.

available. Initially, it appeared that we would be able to test the procedure (1)–(3) using this source. But instead, it demonstrates the need to keep marine and continental data separate in these studies.

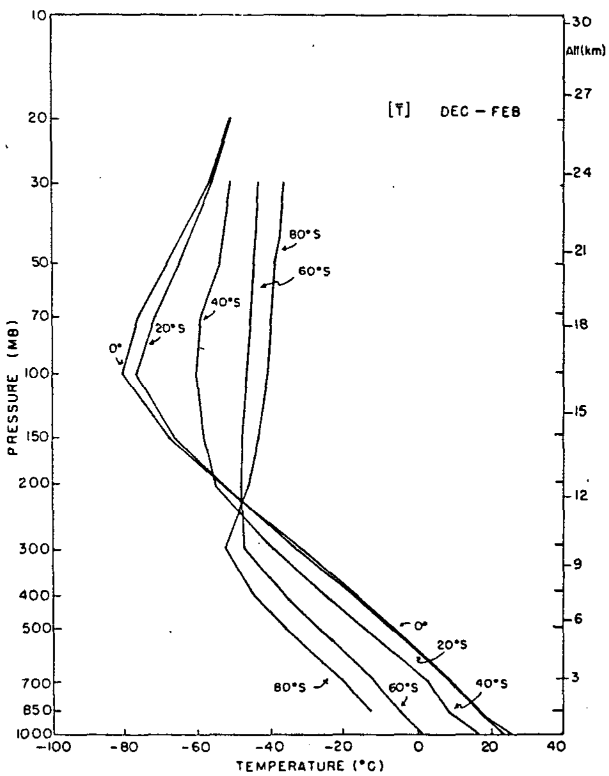

FIG. 6. Zonal-average temperature profiles presented by Newell et al. (1972) for December through February. Significant data levels (150, 100, 75 mb) have been joined by straight lines. Within 40° of the equator, the 100 mb level appears as the coldest point on the profiles. The tabulation did not include actual tropopause levels.

The data are presented as zonal-mean temperatures arranged by season, latitude, and pressure level (Table 3.1 of Newell et al.). Some are also plotted as profiles, such as Fig. 6. However, data have been tabulated only at select intervals—150, 100, and 70 mb in the region about the tropopause—and the profiles are just line sections joining these levels.

We attempted to compute the tropopause potential temperature by two methods. The simple method uses temperature and pressure from the coldest data level, which always turns out to be the 100 mb level. The other method attempted an interpolation using slopes from the four data points about the tropopause. An example is shown in Fig. 7. The interpolation method does not work, however; it yields potential temperatures that are significantly higher than those from all other consulted sources.

Using the 100 mb data as the best available estimate of tropopause conditions may not be very accurate, but it does appear to reveal credible trends in the data. We have compiled these estimates of potential temperature in Table 3, along with the "surface temperatures" implied by the (marine) thermodynamic relationships (1)–(3).

The tropopause potential temperature and consequently the "surface temperature" increases with latitude, contrary to the tendencies of observed sea surface temperatures. The latitudinal increase in the tabulated temperatures can be understood when the locations of the meteorological stations used by Newell et al. are examined. Around 0°, about half of the stations are island or coastal. At higher latitudes, an increasing number are continental, until around 20° the data are

TABLE 3. Tropopause potential temperature θ deduced from the 100 mb data in Newell et al. (1972) as a function of latitude and season. The dewpoint T_D is the value that reproduces θ when used in the formulation (1)–(3).

Latitude	θ (K)	T_D (°C)
(a) Averages for June–August		
0°N	377.6	27.1
10°N	380.5	27.6
20°N	382.4	27.9
(b) Averages for December–February		
0°N	370.8	26.0
10°N	374.7	26.7
20°N	384.3	28.2

dominated by inland stations. The tropopause over a continent has a higher potential temperature than the tropopause over ocean at the same latitude. This is known from the plots presented by Crutcher and Davis (1969), and it appears in the local data presented by Krishna Murthy et al. which we examine next. The values in Table 3 are consistent with a sequence that is increasingly weighted towards continental stations.

The conclusion to be drawn is that the contrast between continental and marine stations will overwhelm any trends in the marine data alone. Tests of the formulation (1)–(3) (or an equivalent for continental regions) must keep separate accounts of marine and continental sites.

c. *The dataset of Krishna Murthy et al.*

This dataset is different from the previous two in that it introduces no spatial averages. Results from 11 meteorological sites in and around the continent of India are presented separately. For each site, monthly means of a 9-yr record have been compiled. The different characteristics of the continental and the marine locations can be identified. The marine data prove remarkably similar to average marine data of the first analysis.

Figure 8 reproduces the monthly mean data for Minicoy, a site in the Arabian Sea. This particular site is in the general area probed by the aircraft experiments of Meyer and Rao (1985) and is also in the same latitude band as the Pacific sites used by Reid and Gage (1981).

Krishna Murthy et al. use a literal interpretation of the Reid–Gage formulation. They take the surface measurements of mean humidity and temperature and use (4) to produce a model tropopause potential temperature. This has been plotted in Fig. 8 along with the observed mean values. The annual averages of model and observation differ by about 20 K. This great a difference is well outside the errors that can reasonably be associated with the mean of the observations. The observed potential temperature, 369 K, is within 0.5° of the value that we have already deduced from (1)–(3) with the observed tropical marine storm conditions (see the latter part of case study A above).

Comparing Fig. 8 and Table 1, it is seen that the standard meteorological observations give no indication of the saturation that was observed in the detailed aircraft studies. Perhaps a different type of analysis of the standard data could isolate stormtime conditions. Our formulation of the connection between tropopause conditions and surface conditions explicitly recognizes that the surface conditions must be those encountered during storms: the connection cannot be made otherwise. We estimate that the error in the Reid–Gage formulation comes in equal parts from using (4) rather than (3) and using average, rather than stormtime, conditions.

Table 4 summarizes the data from the 11 sites analyzed by Krishna Murthy et al. We have ordered the meteorological sites in a sequence that starts from the most marinelike (island sites) and ends with the most continental-like (most-interior sites). There is some arbitrariness in this ordering. Most obviously, we have set Cochin, a coastal site, below three other sites that are up to 150 miles inland to take account of the prevailing wind direction; the trade winds reach Cochin after crossing the entire continent, while the other three sites are not far downwind of the Bay of Bengal.

Arranged in this manner, the sequence shows a steady increase in the tropopause potential temperature. Within the tropical region as a whole, downwind distance from the ocean has a greater influence on tropopause conditions than latitude. This continental influence was referred to earlier in our explanation of the dataset of Newell et al.

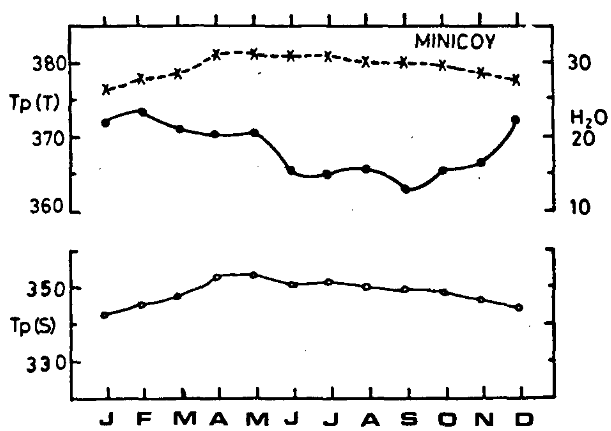

FIG. 8. Data for the island site of Minicoy. Two upper curves: solid line is observed potential temperature, and dashed line is surface water vapor partial pressure. Lower curve: model tropopause potential temperature deduced from the formulation of Reid and Gage [Eq. (4) of text] with the humidity mixing ratio and surface temperatures taken from the means of the observations (Krishna Murthy et al., 1986).

TABLE 4. Mean potential temperature (degrees K) of the tropopause at 11 Indian meteorological stations. Sequence ordered from marine to continental, using normal trade wind direction to define marine influence for an inland station. Final column is the temperature that must be fed into (3) of text to produce recorded tropopause value.

Location	Description	Tropopause temperature (K)	Surface temperature (K)
Minicoy 8.4°N, 73.0°E	Island in Arabian Sea	369	298.8
Port Blair 11.6°N, 92.8°E	Island in Bay of Bengal	368	298.6
Trivandrum 8.5°N, 77.0°E	Coast near south tip of India	369	298.8
Madras 13.1°N, 80.4°E	West coast of Bay of Bengal	369	298.8
Visakhapatnam 17.7°N, 83.2°E	North coast of Bay of Bengal	372	299.3
Hyderabad 17.3°N, 78.5°E	150 miles inland of Bay of Bengal	371	299.1
Bangalore 13.0°N, 77.5°E	150 miles inland of Bay of Bengal	373	299.5
Calcutta 22.5°N, 88.5°E	80 miles north of Bay of Bengal	375	299.8
Cochin 10.0°N, 76.3°E	West coast of Indian Continent	377	300.1
Lucknow 26.8°N, 81.0°E	North India remote from any ocean	380	300.5
Delhi 28.6°N, 77.1°E	North India remote from any ocean	385	301.3

The final column in Table 4 gives the saturated 1000 mb temperature that must be used in (3) to produce the observed tropospheric potential temperature. (This can be read from Fig. 2.) Recall that Roden's Atlantic data, averaged over 20° latitude about the equator, gave a mean sea surface temperature of 298.7 K. The coincidence with the values at the marinelike end of Table 4 is remarkable and supports the same agreement noted earlier in our reanalysis of the Reid–Gage data for the Pacific sites. The result must be fortuitous to some degree; there is no reason to believe that Roden's data sites—located on the main Atlantic shipping route—present a proper statistical sample of the latitude belt. More data are needed.

We know of no studies of storm air over tropical continents equivalent to the aircraft studies noted for the marine environment. Also, the continental meteorology is not dominated by one surface parameter equivalent to mean sea surface temperature—averages and statistical distributions of local surface wet-bulb temperatures can be obtained, but the simplicity and generality of the marine formulation will probably not be recaptured. Thus the "sea surface temperature" given in the final column of Table 4 for the continental stations has no physical basis. It is simply the value that must be fed into (3) to produce the observed tropopause value. These values increase in very much the same manner as the "surface temperatures" derived from the data of Newell et al. (Table 3). We have no dataset for continental stormtime conditions that would allow us to investigate a formulation equivalent to the marine formulation (1)–(3).

5. Conclusions and discussion

We believe that our results strongly support the direct computation of tropopause potential temperature from the buoyant energy of air entering storms, at least in tropical marine regions. The tropopause over the tropical continents has a significantly higher potential temperature and at this time we do not know if it can be simply related to surface conditions.

It may well be that the tropopause characteristics are defined by the buoyancy of air entering local storms in any region—marine or continental, tropical or extratropical. A simple, direct, empirical relationship of this kind deserves thorough investigation. If it can be firmly established, even if only for particular surfaces or geographical locations, it introduces a check that can be incorporated into models of the atmosphere.

Much further investigation is justified but requires careful treatment of the data. The surface measurements must specifically identify the most-moist air entering the storms; average boundary-layer conditions are not relevant. Tropopause data must be divided into marine and continental types as well as by latitude. In the present study, no latitudinal effects could be detected in the marine data, supporting the suggestion of

Reid and Gage that the storms introduce an averaging over the tropical region. However, this could also result in part from our method of analysis. If the surface-air energy in local storms could be compared directly with the local tropopause potential temperature, new correlations might be discovered. Some of these may represent short-term or geographical variations, and some may represent dynamical associations. In particular, we anticipate that the tropopause in the vicinity of a hurricane will be much disturbed from its normal state. The hurricane is associated with particularly warm ocean surfaces and induces a surface pressure well below the nominal 1000 mb level. Both factors should provide abnormally high effective potential temperatures for the surface air and, thus, abnormally high tropopauses. This suggests that the hurricane may contribute to the troposphere/stratosphere exchange process.

The surface parameter governing continental conditions has not been identified, so it is not clear that any form of spatial averaging can be used in this case. So far, only the annual means of surface and tropopause conditions have been linked by a quantitative thermodynamic formula. The variations about these means may be similarly linked, or they may be dominated by stratospheric effects, but this too presents an interesting and worthwhile problem.

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APPENDIX

The Potential Temperature of Lifted Air

Assume that within the parcel the dry air component, the water vapor, the liquid water and the ice are in equilibrium at a common temperature, and there is no heat flow to or from the surroundings. Then the heat balance determining the temperature T in the parcel is

$$c_A d\ln\theta + (r_V c_V + r_L c_L + r_S c_S)\frac{dT}{T} - r_V R_V \frac{dp_V}{p_V} - \frac{L_C dr_L}{T} - \frac{L_S dr_S}{T} = 0. \quad \text{(A1)}$$

Subscripts A, V, L and S refer to the dry air, water vapor, liquid water, and solid (ice) components respectively. R denotes a gas constant, L_c is the latent heat of evaporation and L_S is the latent heat of sublimation. The mass of water per unit mass of dry air is r, with a suscripted r denoting the contribution from a particular phase, c is specific heat at constant pressure, and pressure itself is p. The potential temperature θ of the dry air component is defined as

$$\theta = T(p_A)^{-R_A/c_A} \quad \text{(A2)}$$

with pressure measured in bars.

If the air is unsaturated, (A1) gives a trivial change to the potential temperature of the dry air component during a lifting process as it exchanges heat with the water vapor due to their different specific heats. We ignore this step and calculate effects that begin with saturation of the air. Then p_V will be the saturation vapor pressure given by the Clausius–Clapeyron equation:

$$\frac{dp_V}{dT} = L_C p_V/(R_V T^2). \quad \text{(A3)}$$

We also need the second equation of latent heat:

$$\frac{dL_C}{dT} = (c_V - c_L). \quad \text{(A4)}$$

In the early part of the lifting process where condensation takes place but the temperature exceeds the freezing point, these relations allow (A1) to be cast in the form

$$c_A d\ln\theta + d(L_C r_V/T) + c_L r\frac{dT}{T} = 0, \quad T_F < T. \quad \text{(A5)}$$

Almost all the water condenses out by the time the freezing point temperature T_F is reached, so we set r_V to zero at this limit of (A5). The freezing process is assumed to occur over a minute temperature interval, so (A1) can be computed for this stage as

$$\Delta(c_A \ln\theta) + (L_S - L_C)r/T_F = 0 \quad \text{(A6)}$$

where Δ signifies the change for this interval.

Finally, heat from the ice is transferred to the now-dry gas during further expansion of the parcel until the ice falls out. In this stage (A1) reduces to

$$c_A d\ln\theta + (rc_S/T)dT = 0, \quad T < T_F. \quad \text{(A7)}$$

After this the potential temperature is unchanged by further lifting.

Equations (A5) and (A7) can be integrated through the temperature ranges for which they apply and combined with (A6) to yield

$$\theta_{\text{FINAL}} = \theta_{\text{INITIAL}} \exp\left\{\frac{r}{c_A}\left[\frac{L_C}{T_D} + \frac{(L_S - L_C)}{T_F} + c_L \ln\left(\frac{T_D}{T_F}\right) + c_S \ln\left(\frac{T_F}{T_0}\right)\right]\right\} \quad \text{(A8)}$$

relating the final and initial potential temperatures of the dry air. Here T_D is the temperature at which condensation begins, T_F the temperature at which the water freezes, and T_0 the temperature at which the ice falls from the parcel.

For the computations, we assume saturation at the surface with a dry-air partial pressure of 1 bar, so

$$\theta_{\text{INITIAL}} = T_D = T_S \quad \text{(A9)}$$

where T_S is the surface kinetic temperature. The initial mixing ratio is taken from the Clausius–Clapeyron equation as

$$r = 3.8 \times 10^{-3} \exp\left[\frac{L_C}{R_V}\left(\frac{1}{273} - \frac{1}{T_D}\right)\right]. \quad \text{(A10)}$$

Equations (A8)–(A10) allow computation of the model tropopause potential temperature as a function of sea surface temperature.

The sequence (A8)–(A10) is computed assuming that the ice falls out at $-10°C$, while freezing occurs at $0°C$, and the laboratory values of latent heats and specific heats for pure water apply. In the atmosphere, freezing may start at lower temperatures than this, but changes of latent heat and specific heat should also change, producing the same net thermodynamics. Values used include $c_A = 1004$, $c_V = 1952$, $c_L = 4218$, $c_S = 2106$ J/deg C/kg, $L_C = 2.500\ 10^6$, $L_S = 2.834\ 10^6$ J/kg, $R_A = 287$ and $R_V = 461$ J/deg C/kg.

It must be remarked that water is a complex substance with properties that depart from the idealizations of simple thermodynamic models. Bolton (1980) offers an excellent treatment of the liquid/vapor phase transition that is more accurate than the one just offered. His formulation should certainly be used when the grosser uncertainties in our computations (notably the lack of information on storm humidities and sea surface temperatures) have been eliminated. Bolton's work and that of other authors he cites comment on the underestimate of θ_e given by formulas such as (4). We are indebted to an unknown referee for bringing these references to our attention.

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