A Resonant Wave in a Numerical Model of the 1979 Sudden Stratospheric Warming

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ABSTRACT

A simple numerical model of the stratosphere has been used to examine the possibility that a resonant growth of wave 2 was responsible for the 1979 major sudden warming. The model solves for linear steady state solutions to the quasi-geostrophic wave equation in the presence of realistic damping. The basic state is taken from observations (NMC and LIMS), and the frequency of the wave forcing is varied over a wide range. The model results show that in the days during the initial observed amplification of wave 2 (14–15 February), a clear resonant mode existed. The maximum response is for a wave moving eastward with a period of 12–16 days. Another peak at very low frequency (period greater than 100 days) occurs on 22 February. Other days during the period 12–24 February show weaker, but nevertheless significant, peaks for particular frequencies. The frequency of the maximum is lower for later days and is nearly stationary at the height of the warming around 21 February. This frequency shift found in the model corresponds closely to the observed wave behavior.

Although the details of the results vary with changes in the model resolution or lower boundary position, the resonant wave does not disappear. However, when the wave forcing is applied at the earth's surface rather than in the tropopause region, no resonance occurs. To test the effect of the lower boundary, the troposphere–stratosphere model was run with an internal vorticity forcing similar to the structure of the observed wave 2 in the troposphere. In this case the frequency dependence of the amplitude within the stratosphere was similar to that of the model with a tropopause boundary, although the magnitude was considerably smaller. This suggests that for resonance to have occurred, a planetary scale disturbance that did not propagate from the surface must have been maintained in the upper troposphere. The two well-developed blocking ridges present in the troposphere during this period may have contributed enough to planetary wave 2 to provide the necessary boundary conditions.

1. Introduction

The stratospheric sudden warming is generally believed to result from the interaction of planetary waves with the mean zonal wind. This basic interpretation is supported by numerous modeling studies. Matsuno (1971) showed that a simple mechanistic model of the stratosphere can simulate sudden stratospheric warmings that in many ways resemble observations of actual warmings. In mechanistic models such as Matsuno's, an atmosphere that is initially zonally symmetric is forced at the lower boundary (in the upper troposphere or lower stratosphere) with a prescribed wave forcing that grows with time. Eddy heat and momentum flux convergences and the secondary circulation associated with the growing wave cause polar warming and deceleration or reversal of the mean westerly wind in high latitudes.

Sensitivity studies (Bridger and Stevens, 1982; Butchart et al., 1982) show that the structure of the initial mean wind field is important in getting a model warming that develops in a realistic manner. The zonal wind is said to be preconditioned when its structure is such that planetary wave activity tends to be focused into the high latitude stratosphere. A common feature of the preconditioned flows of Bridger and Stevens and Butchart et al. appears to be a more poleward position of the jet in the middle stratosphere. Wave–mean flow and wave–wave interactions occurring previous to the warming are instrumental in preconditioning the mean state (Palmer and Hsu, 1983; Matsuno, 1984).

One question about warmings that has not been answered concerns the initiation and persistence of wave forcing from the troposphere (McIntyre, 1982). A possible mechanism which has been proposed to maintain the strong forcing is a resonant wave that amplifies in the troposphere and stratosphere. For a particular basic state configuration, resonance exists when there is some wave that, in the absence of damping, has an infinite response (amplitude) for a finite forcing. The terms used by others to refer to this phenomenon include "normal mode" and "instability" as well as resonance. In this paper the term resonance will be used to avoid confusion. The kind of event being considered here is a local, i.e., latitudinally trapped, rather than a global mode and depends very strongly on the details of the basic state structure and the scale and frequency of the perturbation. In these ways it differs from certain other types of phenomena that can also be described by the general terms normal mode and instability.

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Resonance has been found in a number of simple analytical and numerical models of the stratosphere (Clark, 1974; Geisler, 1974; Tung and Lindzen, 1979a,b; Plumb, 1981; Haynes, 1985). Tung and Lindzen used a linear model to compute the vertical structure of the zonal wind for which a stationary resonant wave would exist. They found certain situations that were conducive to larger wave growth, which they linked to both blocking in the troposphere and sudden warmings in the stratosphere. Their results indicate that it is possible for stationary waves 1 and 2 to become resonant when the stratospheric jet maximum was lowered to about 40 km. In fact, the vertical structure of this wind profile in the stratosphere is similar to that of the preconditioned flow found by Butchart et al. (1982) from observations.

Plumb (1981) used a simple interactive model in which a traveling wave near resonance and a stationary wave forced by topography interacted nonlinearly with the mean wind. Plumb's results indicate that strong deceleration of the mean flow occurs when the wave is initially not quite resonant. As the wave interacts with and changes the basic state, it adjusts to remain close to resonance and continues to propagate. In a small amplitude limit, regular oscillations in the wave and mean flow develop. In a finite amplitude situation, the wave–mean flow interactions are not reversible, and a full-scale breakdown of the vortex can occur. In either case, the self-tuning action, in which the resonant wave adjusts as the mean flow changes, is an important factor in maintaining the resonant traveling wave.

Haynes (1985) found resonant instability in a simple semianalytic model in which Rossby waves were confined to high latitudes by a moveable boundary representing the "surf zone" (McIntyre and Palmer, 1983). The surf zone is the transition between the region of strong potential vorticity gradients and a region where the gradients are weak and wave propagation is therefore inhibited. Haynes' conditions for resonance depended on wave–mean flow interaction in a manner similar to the self-tuning described by Plumb. A point emphasized in his study is that the boundary represented by the surf zone could cause sufficient reflection of Rossby waves to act as the low latitude edge of a resonant cavity.

The three previous model studies just described demonstrate the possibility that resonant modes could be involved in sudden warmings. However, all three studies allowed waves to propagate in only one dimension. Variations in the basic state structure were also limited. Since waves in the stratosphere propagate both upward and equatorward and depend on the two-dimensional structure of the basic state, these approximations limit the validity of the models. In one dimension, waves can be trapped by a reflecting boundary extending in only one direction. For a two-dimensional situation, the reflecting boundary needs to form an actual cavity rather than a wall; this situation is less likely to occur.

The work presented in this paper investigates the link between resonant waves and sudden warmings in the context of an actual observed warming. For this study, a steady state model is used to search for enhanced wave responses in two-dimensional basic state wind fields taken from observations. The wind data cover the period before and during the observed wave 2 major warming of 1979. Both traveling and stationary waves are considered in the model. Substantial damping is applied to eliminate modes that could not exist under normal atmospheric conditions.

Because of the model simplicity, the results of this study are only suggestive of what might be occurring during sudden warmings. Wave perturbations traveling with a particular frequency are not necessarily present in the atmosphere at all times, so this kind of test can only indicate that a resonant response would occur if finite forcing were present. While a linear study cannot address the question of how a resonant wave grows with time or interacts with the mean flow, it is able to show the dependence of the steady-state wave amplitude on small variations in the basic state wind and wave frequency. Further justification of the linear treatment is provided by Plumb's (1981) finding that a sudden warming was able to develop in his model only when the traveling wave was initially near the linear resonant solution.

Section 2 gives a description of the governing equations and numerics of the model. Section 3 describes features of the observed February 1979 warming. Section 4 presents results from the basic model described in section 2, and section 5 discusses the effect of variations in the numerics of the model. For each of the experiments discussed in sections 4–5, the model is run with a prescribed basic state wind and forcing frequencies that range between zero (stationary wave) and 0.1 day^{-1} (10 day eastward traveling wave). The experiments differ in the zonal wind used and in the formulation of the model. Section 6 contains a summary and further discussion.

2. Model and data

The model used in this study is similar to those used by Matsuno (1970) and Schoeberl and Clark (1980) to simulate stationary and traveling waves respectively. It differs from Matsuno's model in that Rayleigh friction and Newtonian cooling are explicitly included. The model is derived from the quasi-geostrophic, global vorticity and thermodynamic equations:

$$\left(\frac{\partial}{\partial t}+\bar{\omega}\frac{\partial}{\partial\lambda}\right)\zeta'+\frac{v'}{a}\frac{\partial\bar{\zeta}}{\partial\theta}-fe^{z}\frac{\partial}{\partial z}e^{-z}w'=-\beta\zeta'+F' \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\bar{\omega}\frac{\partial}{\partial\lambda}\right)T'+\frac{v'}{a}\frac{\partial\bar{T}}{\partial\theta}+w'\frac{s}{R}=J'-\alpha T' \quad (2)$$

where

$$\zeta = f + \frac{1}{a\cos\theta}\frac{\partial v}{\partial\lambda} - \frac{1}{a\cos\theta}\frac{\partial}{\partial\theta}u\cos\theta.$$

The primes refer to wave quantities and the overbars to zonal means. Symbols are defined as

λ	longitude
θ	latitude
z	$\ln(p/p_0)$, where $p_0 = 1000$ mb
T	temperature
u, v, w	zonal, meridional and vertical winds, respectively
ω	$u/(a\cos\theta)$
s	static stability
a	radius of the earth
α	Newtonian cooling coefficient
β	Rayleigh friction coefficient
R	gas constant for dry air
J	diabatic forcing
F	internal vorticity forcing
Ω	angular rotation rate of the earth
f	Coriolis parameter

Using the geostrophic approximation for f' and the addition of the isallobaric component for v' (as in Matsuno, 1970) (1) and (2) can be combined into a single equation in geopotential ϕ'. We define a new variable

$$\Psi' = \frac{e^{-z/2}}{\sqrt{s}}\phi'$$

and assume a wave form in longitude and time:

$$\Psi(\lambda,\theta,z,t) = \Psi_{m,\sigma}(\theta,z)e^{i(m\lambda-\sigma t)}.$$

The internal forcing (J and F) can also be expressed in wave form in an equivalent manner. We then have an equation with wavelike solution in the meridional and vertical directions and complex index of refraction squared Q:

$$\frac{\sin^2\theta}{\cos\theta}\frac{\partial}{\partial\theta}\left(\frac{\cos\theta}{\sin^2\theta}\frac{\partial\Psi_{m,\sigma}}{\partial\theta}\right) + \frac{f^2a^2}{s(m\omega-\sigma-i\beta)}\frac{\partial}{\partial z}\left[(m\omega-\sigma-i\alpha)\frac{\partial\Psi_{m,\sigma}}{\partial z}\right] + Q\Psi_{m,\sigma} = -\frac{if^2a^22\Omega a e^{z/2}}{(m\omega-\sigma-i\beta)\sqrt{s}}\frac{\partial}{\partial z}\frac{e^{-z}RJ_{m,\sigma}}{s} - \frac{ifa^22\Omega a e^{-z/2}}{(m\omega-\sigma-i\beta)\sqrt{s}}F_{m,\sigma} \quad (3)$$

where

$$Q = \frac{m}{\cos\theta(m\omega-\sigma-i\beta)}\frac{\partial\bar{q}}{\partial\theta} - \frac{m^2}{\cos^2\theta} + \frac{f^2a^2}{s(m\omega-\sigma-i\beta)}\left(\frac{1}{4}+\frac{1}{2}\frac{\partial}{\partial z}\right)(m\omega-\sigma-i\alpha)$$

and

$$\frac{\partial\bar{q}}{\partial\theta} = \cos\theta\left[2(\Omega+\bar{\omega}) + 3\tan\theta\frac{\partial\bar{\omega}}{\partial\theta} - \frac{\partial^2\bar{\omega}}{\partial\theta^2} - f^2a^2e^z\frac{\partial}{\partial z}\left(\frac{e^{-z}}{s}\frac{\partial\bar{\omega}}{\partial z}\right)\right].$$

The domain of the basic model (referred to as model A) is global and extends from 15 to 100 km. (For discussion and for plotting, the vertical coordinate will be referred to in kilometers based on a constant scale height of 7 km.) The lower boundary is specified one level below the lowest model level; for the basic model it is at 12.5 km. There are 26 grid points in the meridional direction and 35 in the vertical. Results from other versions with different meridional and vertical resolution and boundaries will also be discussed. The wave amplitude is set to zero at the top and at the poles. The zonal wind is prescribed from daily observations and a single planetary wave is forced. The diabatic heating is in most cases set to zero and the wave is forced by a geopotential perturbation at the lower boundary. The meridional structure of the forcing is as simple as possible; values are constant (no amplitude variation or phase tilt).

Damping consists of Rayleigh friction, taken from Holton and Wehrbein (1980) and Newtonian cooling adapted from Dickinson (1969), both of which have been extended smoothly into the troposphere and mesosphere. These are shown in Fig. 1. Static stability in the model has been kept constant; tests with the initial version of the model showed that the effects of its variation are small.

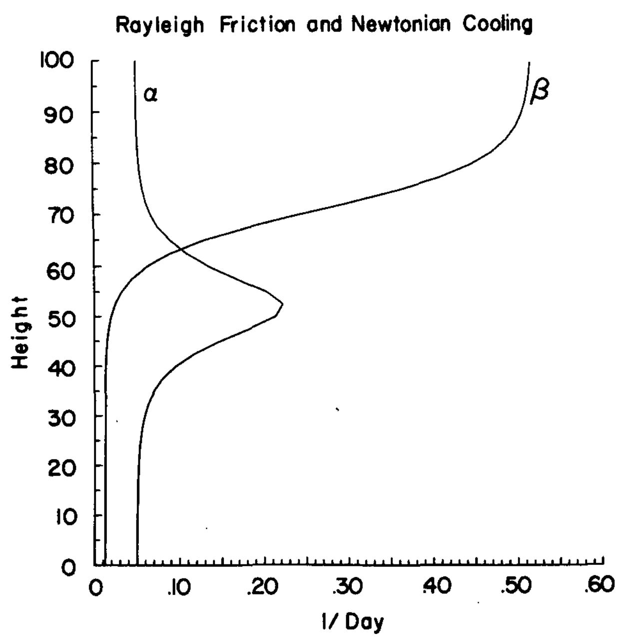


FIG. 1. Newtonian cooling (α) and Rayleigh friction (β) coefficients used in the model.

Tests show that structure of the simulated wave does not change when the model domain is limited to the Northern Hemisphere. Also, as long as the upper boundary is at or above 100 km, its position has no effect on the structure of the simulated wave in the stratosphere. There are, however, several features of the model that can have a strong influence on the results. The wave structure is sensitive to the location of the lower boundary and to the meridional and vertical resolution of the model. Results from several different cases are discussed in Section 5 for comparison with the basic model results.

The basic state winds were derived geostrophically from geopotential heights measured by NMC (1000–100 mb) and LIMS (100–0.1 mb; 64°S–84°N). The LIMS data are available for the 1978/79 winter, and the study is limited to the major sudden warming during this winter. The winds in the stratosphere were interpolated to zero at the north and south poles. The CIRA (1972) monthly climatology was used above 80 km, and the winds were interpolated between 0.1 mb (~65 km) and 80 km. Winds were also interpolated across the equator from 8°S to 8°N. The overall character of the results in the troposphere and stratosphere was found to be not sensitive to the winds above 0.1 mb or the method of extrapolation in high latitudes.

3. Unusual features of the 1979 sudden warming

Observations of the February 1979 major sudden warming are presented in Palmer (1981), Quiroz (1979), and Gille and Lyjak (1984). The warming was notable because of the unusually simple structure of the perturbation fields; the wave structure in the initial phase of the warming was almost completely dominated by wave 2. Also, the observational network for the stratosphere was denser than for any previous warmings, so more accurate estimates of the stratospheric fields were obtained.

Butchart et al. (1982) were able to simulate the warming quite well using a primitive equation stratospheric circulation model initialized with observations and forced from below. They isolated two factors essential for the successful simulation. It was necessary to use a basic state initialized from observations just before the warming rather than from climatology, and it was necessary to include the observed eastward phase progression of wave 2 at the lower boundary. Both of these factors have a major impact on the quasi-geostrophic refractive index, and thus affect the ability of waves to propagate and the direction of propagation. Butchart et al. found that when the basic state was initialized from climatology, the mean flow deceleration was strongest in low latitudes, unlike observed warmings. In another test, when the wind was initialized from observations but stationary wave forcing was applied, the wave penetrated into high latitudes for a short period only, and no high latitude warming developed. When observations were used for both the initial and boundary conditions, the wave activity, as determined by the direction of the EP flux, was focused into high latitudes and the model results were similar to the observations reported by Palmer (1981).

The observations of wave 2 phase presented by Butchart et al. (their Fig. 1) indicate that wave 2 at 100 mb, 57.5°N and 67.5°N, moved eastward 55° during the period 14–20 February. At the same time, the amplitude at 62.5°N increased steadily from about 240 to 600 m. By 20 February the wave had become stationary, and a few days later it began to move slightly westward. Figure 2 shows the geopotential height perturbation due to waves 1–6 at 100 mb during the period 10–26 February, based on NMC data. The dominance of wave 2 and the eastward progression of the wave can be clearly seen. The frequency of this regular phase progression of wave 2 does not match that expected from any of the global normal modes (e.g., Madden, 1979).

The circulation in the troposphere for February 1979 was characterized by a strong omega-shaped blocking ridge over the north central Pacific Ocean (Dickson, 1979). In addition the observations indicate a blocking ridge over the western Atlantic Ocean during the period 6–18 February. Although these two blocking ridges contribute to a wave 2 pattern, it was not until about 14 February that wave 2 propagated into the stratosphere. Figure 3 shows a longitude by height cross section of the geopotential height perturbation from the zonal mean at 60°N for 3 days. The observations are

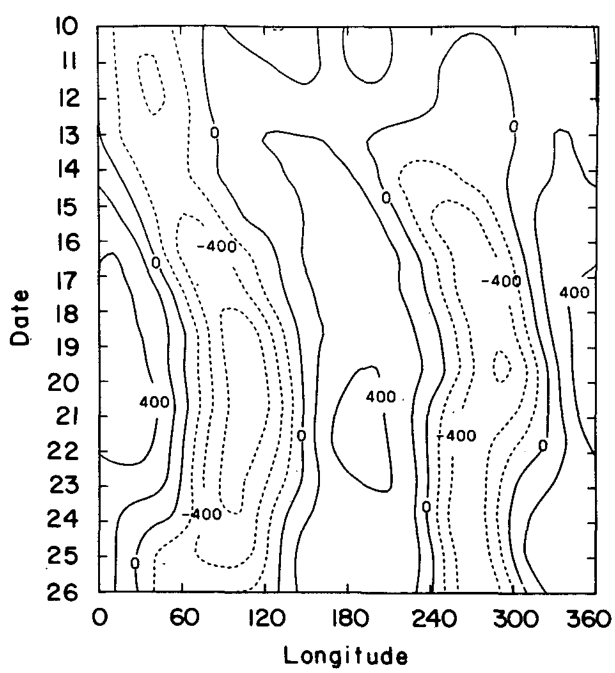
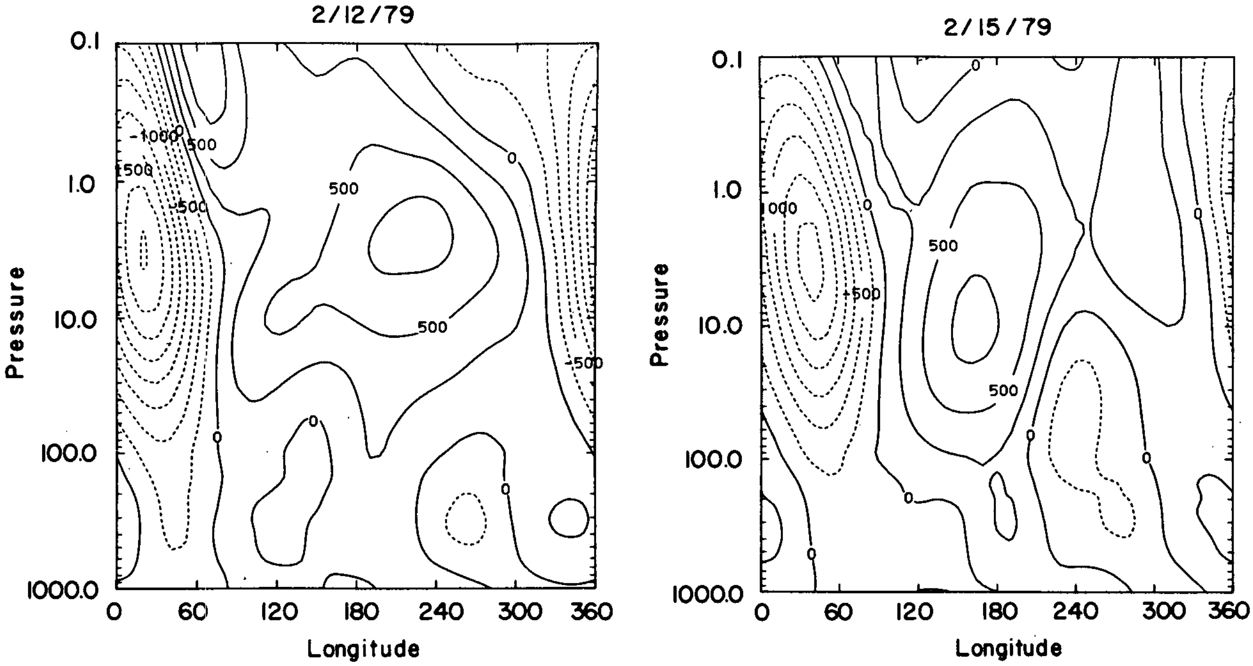


FIG. 2. Perturbation height for waves 1–6 at 60°N and 100 mb plotted against day (10–26 February) and longitude.



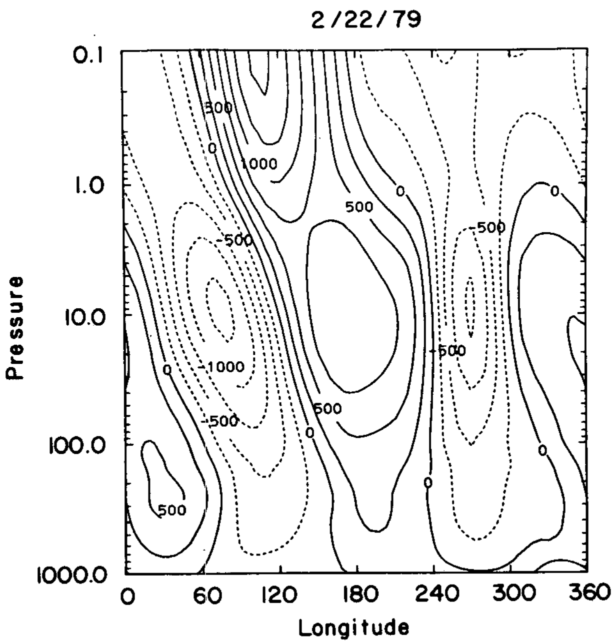


FIG. 3. Perturbation height for waves 1–6 at 60°N plotted against pressure and longitude for (a) 12 February, (b) 15 February, and (c) 22 February.

taken from NMC and LIMS. On 12 February, even though there are multiple positive and negative values in the troposphere, the stratosphere is dominated by wave 1 (a single positive and negative value). On 15 February, wave 2 is evident in the upper troposphere and lower stratosphere, and on 22 February wave 2 dominates from the surface to 1 mb. The temporal development of the wave structure suggests that the presence of the two blocking ridges was not sufficient to lead to wave 2 propagation in the stratosphere before 15 February. Also, it appears from Fig. 3 that the rapid growth of wave 2 occurred first in the lower stratosphere and then spread downward as well as upward. As an additional note, the strong growth of wave 2 in the troposphere coincided with the severe Presidents' Day storm in the eastern United States (18–19 February), which has been the focus of several studies (e.g., Bosart, 1981).

Without further analysis of observations, it is not possible to tell whether an eastward traveling wave 2

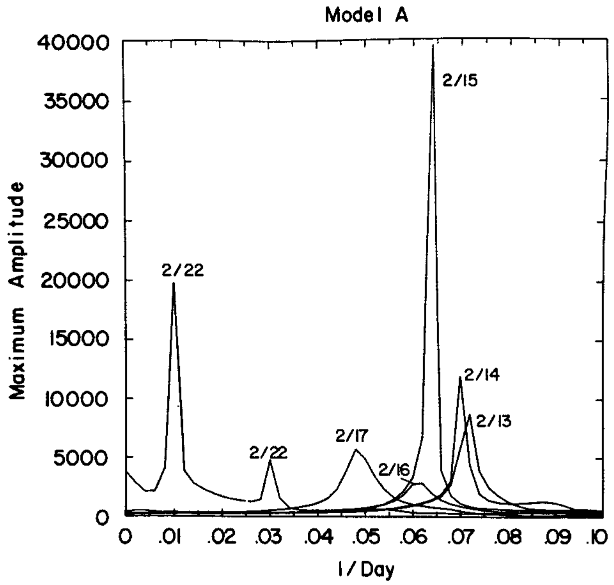


FIG. 4. Maximum amplitude of wave 2 in the Northern Hemisphere stratosphere (0–90°N, 15–60 km) for frequencies from 0 to 0.1 day^{-1} computed with model A. The six curves correspond to different basic states, observed on the days 13–17 and 22 February. See text for the method of determining maximum amplitude.

was important only in the 1979 warming, or played a major role in others of the limited number that have been observed. Eastward movement of disturbances has been noted before several other warmings. Quiroz (1975) presented evidence that there was a regular eastward movement of a center of high radiance (local temperature maximum) in the stratosphere leading up to the major warmings of 1969–70, 1970–71, and 1972–73. Hirota (1968) found an eastward moving wave 2 at 100 mb prior to the February 1966 major warming. The development of those warmings involved both waves 1 and 2, but all would be classified as wave 1 warmings.

4. Results from the basic model

The linear growth of wave amplitude observed at 100 mb during the period 14–20 February (Butchart et al., 1982) suggests the presence of a resonant wave. To examine this possibility, the linear steady state model described in section 2 was run with a variety of frequencies using as basic state the observed winds from each of the days during this period. Figure 4 shows the amplitude of the maximum response in the Northern Hemisphere stratosphere as a function of frequency for 13–17 and 22 February. The maximum amplitude is determined by using a fixed value (100 m) for the amplitude at the lower boundary and selecting the highest amplitude for all grid points in the region between 0° and 90°N and 16 and 60 km. As can be seen from Fig. 4, the maximum amplitude is extremely large for some frequencies and days, and the peak shifts towards slower frequencies for successive days. This trend continues during the interval 18–21 February, but the amplitude peaks are small and very broad relative to the values shown and have been omitted from the figure. On 22 February the peak corresponds to a period of more than 100 days. There are also small peaks on 23–24 February, also at low frequencies.

The amplitude for the peak on 15 February at 0.064 days^{-1} (or 15.6 day period) is 40 000 m, which is ridiculously large. (Although, as will be demonstrated in section 5, the magnitude of the peak varies when different model formulations are used; no tuning of the model characteristics has been done to isolate the case with the strongest resonance.) These very large amplitudes occur in the presence of damping (described in section 2). For comparison, Schoeberl and Clark (1980) tested the frequency response in a similar model that used climatological winds for a basic state. They found that the resonant amplitudes for all except a few modes were sharply reduced when a damping time of 30 days (roughly comparable to the values used here) was used. Even for the strongest peaks found in their model, the enhancement of resonant over background response was substantially smaller than that shown in Fig. 4 for 14–15 February. These qualitatively different results are due to the different basic state used by Schoeberl and Clark. The present model also gave relatively small variations in wave amplitude with frequency when a climatological mean wind was used for the basic state.

The amplitude structure of wave 2 in the stratosphere for 15 February and 0.064 day^{-1} is shown in Fig. 5a. The structure of the wave amplitude plot does not look unusual for the winter stratosphere; it is only the magnitude that is unrealistic. Note that the wave grows with height only up to about 35 km and decays above there. The phase at 60°N is shown in Fig. 5b. There is very little vertical phase tilt in the region (15–35 km) where the amplitude is growing rapidly.

The wave structure in this model is determined entirely from the complex index of refraction and the boundary forcing. Since, for any given model run, the latter varies only in frequency, the resonant peak must be a result of the refractive index structure. Charney and Drazin (1961) and Dickinson (1968) showed that the refractive index depends on the mean wind speed and structure and the wave frequency. Figure 6 shows several fields relevant to the basic state refractive index. (The plot extends down to the earth's surface although this particular version of the model has its lowest level at 15 km.) Figure 6a gives the mean wind for 15 February and Fig. 6b gives the Doppler shifted wind ($u - c$, where c is the wave velocity) for the frequency corresponding to the resonant wave. Figure 6c shows the meridional gradient of quasi-geostrophic potential vorticity (q_θ).

The refractive index squared Q is dominated by a term proportional to $q_\theta/(u - c)$. A wavelike solution

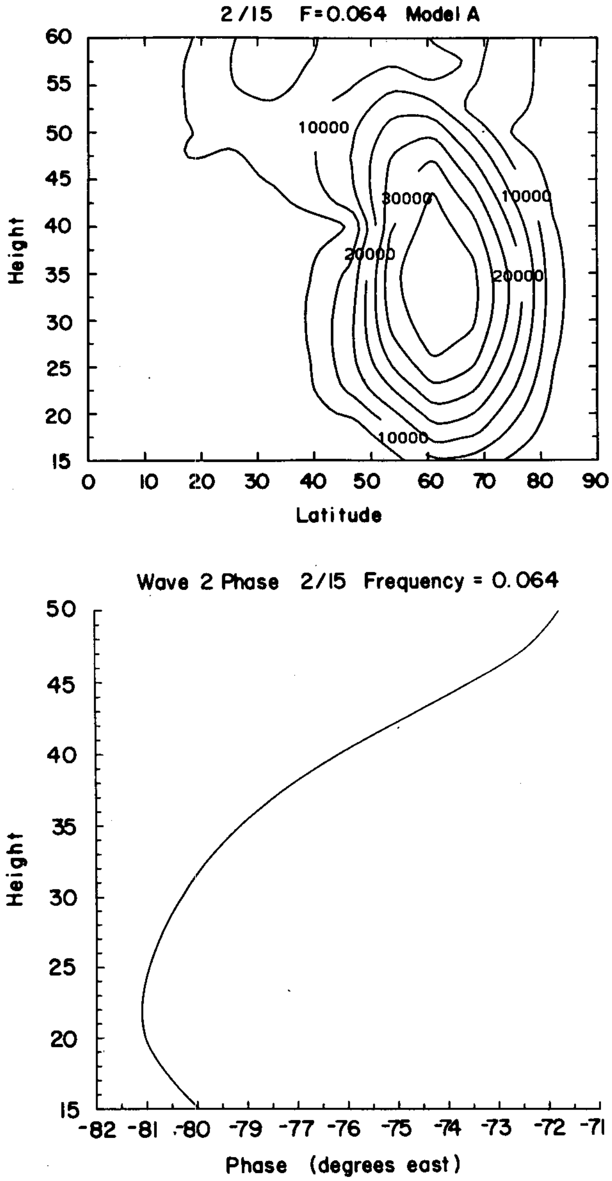


FIG. 5. (a) Amplitude cross section in meters, and (b) phase at 60°N in degrees of wave 2 for 15 February and a frequency of 0.064 days^{-1}, computed with model A.

exists when Q is positive. Because the group velocity is, roughly, inversely proportional to Q (Smith, 1983) waves propagate more slowly and are therefore more likely to be damped when Q is large. From the plot of the real part of Q for the traveling wave, shown in Fig. 6d, it is evident that there is a region of small positive Q in the lower to middle stratosphere between about 50° and 70°N. This positive region is almost completely surrounded by negative values, making a cavity. The southern and bottom edges of the cavity result from regions where the Doppler shifted wind speed is less than zero, i.e., where the wind is easterly with respect to the wave (see Fig. 6b). The upper boundary results from the potential vorticity structure, due to the meridional and vertical curvature of the zonal jet (see Fig. 6c). The polar boundary is a consequence of the wavenumber dependence of refractive index described by Charney and Drazin (1961).

The presence of this cavity is undoubtedly associated with the existence of a resonant wave. However, the shape of this cavity varies only slightly when the wave frequency changes by a small amount, and the cavity structure does not disappear when the winds from an adjacent day are used. The very strong resonance must depend on details of the wave structure within the region of propagation rather than just the general cavitylike structure of the refractive index.

In order for the steady state assumption to be valid, wave group velocities must be fast enough that the wave is reflected within the cavity before the mean wind changes substantially. A WKB calculation of group velocity applies to a wave propagating through a medium that changes slowly in space and time, and is therefore not applicable to the resonant wave case. An evaluation of the time scale for the resonant wave to be established must await a time dependent model calculation.

Another problem with interpreting steady state resonance is that the results are meaningful only if the resonant growth time is substantially faster than the nonresonant growth time over a period during which the mean wind does not change significantly. Simmons (1974) found, in a linear time-dependent model with one meridional mode, that resonant and nonresonant waves had the same amplitude growth rate for about the first 10 days after turn-on of the wave. His results suggest that it may not be possible to identify actual resonance in a complicated time-dependent system.

5. Model variations

It is possible for simple models to find modes, such as resonant waves, that cannot exist in the real atmosphere. The limited resolution of models and the assumptions that go into the model formulation restrict the kinds of motion and can lead to spurious trapping. A number of tests have been performed with the model used here to determine the dependence of the resonance on characteristics of the model. All of these tests have been done on the same basic model, which is limited to linear, steady state, quasi-geostrophic dynamics. The applicability of these basic assumptions has not been tested.

Table 1 summarizes the results of the tests. All of the factors listed were independently varied from their value in the basic model. The table indicates whether the effect was to modify or eliminate resonance, or whether there was no change. Notice that the upper and southern boundaries, which are located far outside the cavity in refractive index squared Q, have no effect on the resonant wave. It was, however, important to

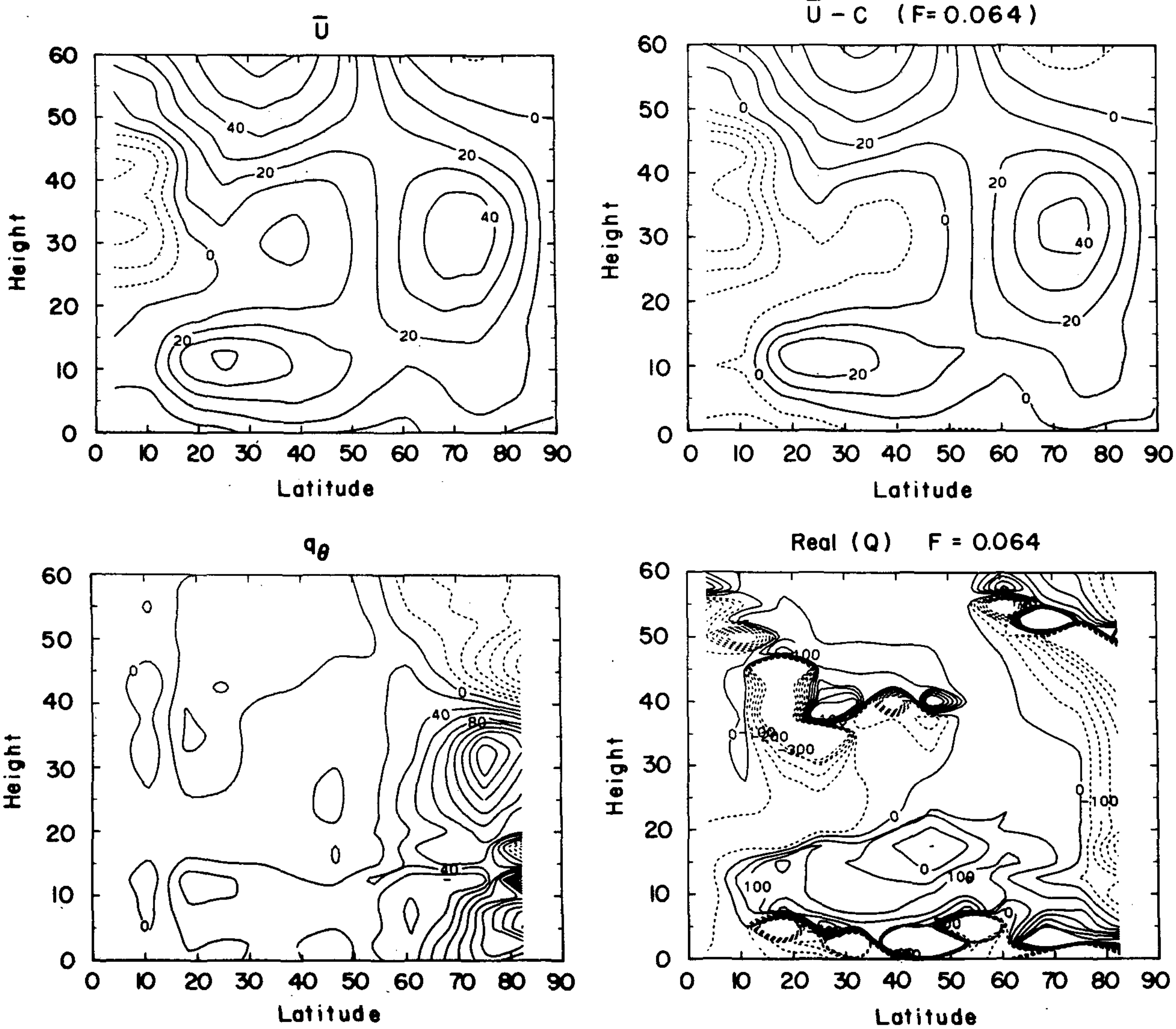


FIG. 6. Basic state fields for 15 February: (a) zonal wind speed; (b) Doppler shifted wind speed for a wave with frequency of 0.064 day^{-1}; (c) meridional gradient of quasi-geostrophic potential vorticity; and (d) the real part of the refractive index squared Q for a frequency of 0.064 day^{-1}.

have this broad region beyond the cavity with substantial damping. When the upper boundary was moved down to 80 km, very strong peaks in wave amplitude occurred, which appear to be caused by reflection of the wave from the top. This indicates that, to some extent, the resonance wave has a dependence on the atmospheric beyond, as well as within, the cavity walls.

As indicated in Table 1, the horizontal and vertical resolution and the location of the lower boundary all affect the results. To summarize the kinds of changes involved, Figs. 7–8 show some results of tests with a version (referred to as model B) in which all three of these factors were changed. The horizontal resolution was reduced to 4° latitude, the vertical resolution to 2 km and the lower boundary was set at 14 km (with the lowest model level at 16 km). Figure 7 shows the amplitude of wave 2 forced with 0.064 day^{-1} frequency using the basic state for 15 February. The structure is similar to that shown in Fig. 5a for the identical case using model A, but the amplitude is smaller by a factor of almost 100. The reason is partly because the resonance frequency has shifted with the different model formulation, so this is no longer the frequency of the peak. Figure 8 shows the maximum amplitudes, as in Fig. 4, but with model B instead of model A. The peak amplitude for 15 February is substantially smaller (by a factor of about 20), and for this model a larger amplitude appears for 14 February than for 15 February. In addition, the resonant frequencies for days 13–16 February are higher than they were for model A. The

TABLE 1. Summary of the effect of various changes in the model formulation or parameters on the occurrence of wave-2 resonance.

Formulation/parameter	Eliminate resonance	Modify resonance	No change
Horizontal resolution		×	
Vertical resolution		×	
Top boundary			×
Bottom boundary	×	×	
Southern boundary			×
Phase tilt at lower boundary			×
Structure of damping		×	
Structure of internal forcing (for model C)	×	×	

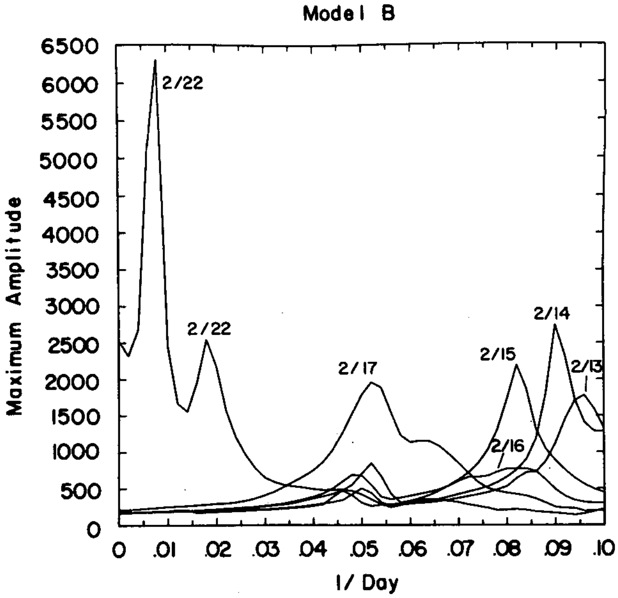


FIG. 8. As in Fig. 4 except for model B.

peaks on 17 and 22 February are smaller by only about a factor of 3 and are at approximately the same frequency.

Despite the differences, the experiments with model B provide support for the basic results found using model A. In both versions, there is a small peak on 12 February (not shown), a larger one at the same frequency on 13 February, and sharp maxima on 14 and 15 February. The frequency of the peak tends to move to slower frequencies on each subsequent day beginning on 13 February. Both model versions show a sharp peak at low frequency (almost stationary) on 22 February. Other tests show that the frequency shift in the resonant wave for a comparable change in the model resolution decreases, but does not completely disappear, for finer resolution versions of the model.

Table 1 indicates that resonance was in some cases eliminated by changing the position of the bottom boundary or the structure of the internal forcing. Modification of resonance occurred when the boundary was left somewhere in the region of the upper troposphere or lower stratosphere and the wave was forced by a geopotential specified at the lower boundary, as in the model A and B experiments. When the lower boundary was moved to the earth's surface, some model experiments gave no resonance.

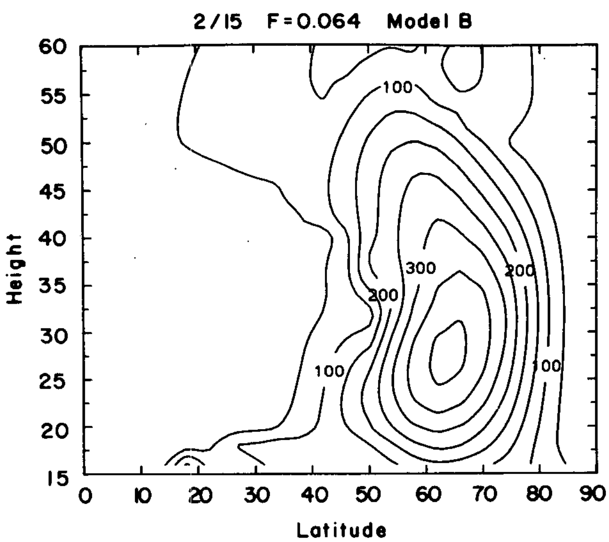


FIG. 7. As in Fig. 5a except for model B.

The troposphere–stratosphere model, referred to as model C, had the same resolution as model A, but the lower boundary was at the surface and several different methods for forcing the wave were used. A version of model C forced by vertical velocity at the surface was tested and gave no traveling wave penetration into the stratosphere for most of the frequencies and days studied here. The reason for this is clear from Fig. 6; the refractive index squared is negative throughout most of the troposphere. There is, however, an interesting result of the test with the vertical velocity forcing. Model C was run with a stationary vertical velocity forcing at the surface using basic state winds for all days in February. The peak stratospheric amplitudes (calculated as in Figs. 3 and 7) are shown in Fig. 9. This figure indicates that the penetration of stationary wave 2 into the stratosphere was quite weak through most of the month. However, for the period 21–25 February, wave 2 propagation from the surface was much greater. This period corresponds to the peak of the sudden warming, when the observed wave 2 in the troposphere and stratosphere was large and close to stationary.

In another experiment with model C, the wave was forced by applying a heating function in the troposphere and stratosphere, given by

$$J_{m,\sigma} = A\frac{(15-z)}{15}\sin\frac{(15-z)\pi}{15}$$

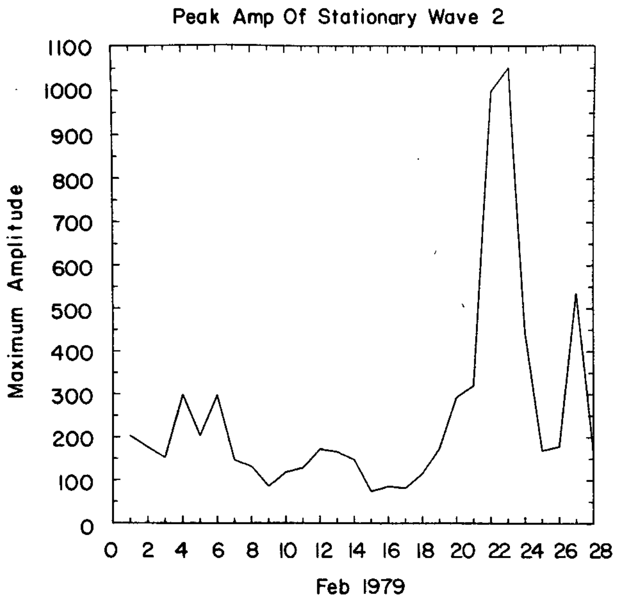


FIG. 9. Maximum amplitude of stationary wave 2 in the Northern Hemisphere for all days during February 1979, calculated in model C with a vertical velocity forcing at the earth's surface.

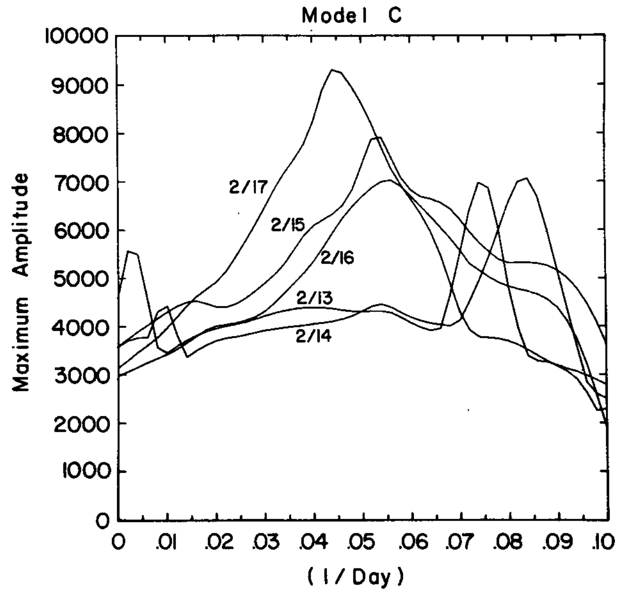


FIG. 11. As in Fig. 4 except for model C. The curve for 22 February has been omitted; see text for a description.

where z is height in kilometers and A is an arbitrary amplitude. While the wave in the stratosphere had a structure quite similar to that shown in Figs. 5a and 7 for models A and B, respectively, no resonance appears for this version extending to the earth's surface. Two possible reasons for the lack of resonance in model C are that the structure of the wave is substantially different from the case in which the geopotential is fixed at the tropopause, or that the presence of resonance depends on the artificial lower boundary at 15 km. To

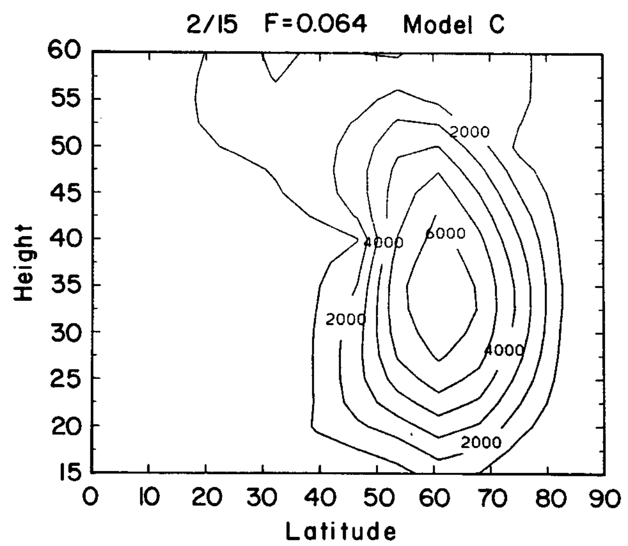


FIG. 10. As in Fig. 5a except for model C with internal vorticity wave source.

investigate this, a case was run with a wave 2 internal vorticity forcing, given by

$$F = -k(\zeta - \zeta_0).$$

This function gives relaxation back to a specified vorticity ζ_0, determined from a geopotential field with no latitudinal variation and with a vertical structure similar to the observed midlatitude tropospheric structure of wave 2 during this period. The relaxation time ($1/k$) is a few days in the lower troposphere and decays to zero just above the tropopause.

Figure 10 shows the wave amplitude in the stratosphere for a wave 2 that corresponds to the resonant frequency shown in Figs. 5a and 7. Again, wave 2 has similar structure in all models. The maximum amplitude for 13–17 February (shown in Fig. 11) indicates a dependence of amplitude on frequency that resembles those from models A and B, but with substantially weaker enhancement of peaks over background values. The maximum amplitude for the period 22–23 February is at zero frequency (stationary) and has been omitted from the graph because it is larger by a factor of 5 to 10 than the peak values shown. Further tests with this model indicate that there is a fairly strong dependence of the maximum amplitude results on the structure of ζ_0. Also, the maximum amplitude peaks shift in frequency when the model resolution is varied, in a manner analogous to that in models A and B. As in the stratosphere model results, no special tuning of the troposphere–stratosphere model has been done to attain the frequency dependent behavior in wave amplitude.

Apparently, in order to have a resonant wave during this period, it is necessary to have a slowly traveling geopotential perturbation somewhere within the upper troposphere or lower stratosphere that did not propagate from below. The disturbance must be maintained in such a way that it can serve as an effective lower boundary to the stratosphere. The wave-2 component associated with the dual blocking ridges in the troposphere (Dickson, 1979) may provide the necessary forcing if the ridges are maintained by interaction with other scales of motion rather than by propagation from below.

The tests described herein indicate the effect of the model domain, resolution, lower boundary position and wave forcing mechanism on the resonant wave. In addition, two other features of the model were tested. Several runs in which the geopotential lower boundary condition was given a meridional phase tilt were performed. These had no significant effect on the results. Another test was to vary the structure of the damping by adjusting the parameterization constants slightly. This did result in significant changes in the resonance, although smaller than those due to changes in the model resolution. The effect in this case was to alter the amplitude of the resonant peak but not to shift its frequency. The reason for the change can be seen from the wave equation (3). The refractive index has terms that depend on the magnitude and vertical structure of the damping parameters α and β. As suggested by a comparison of the results in section 4 and this section, the resonant wave depends sensitively on the structure of Q. Variations in static stability are expected to lead to changes similar to those due to damping coefficient variations.

6. Summary and discussion

The model experiments presented here provide evidence that a resonant mode was capable of existing in the stratosphere during mid-February 1979. In the models with a lower boundary in the tropopause region, the results show that the maximum wave-2 amplitude depends quite strongly on frequency for about 12 days, beginning on 12 February. The wave period that has the largest amplitude is initially in the range of 10–15 days eastward (variations depend on model formulation) and shift to slower frequencies with succeeding days. On 14–15 February the peaks are very strong and have a frequency dependence characteristic of resonance. A week later (22 February) there is again a strong peak, this one at a much lower frequency. On other days (before 14 February, between 16 and 21 February, and after 22 February), there appears to be a broad preferred frequency range rather than a sharp resonance. The refractive index for eastward moving wave 2 has a cavity, with negative values surrounding the region 45°–70°N, 10–35 km. The model with a lower boundary at the earth's surface shows a weaker frequency dependence of amplitude when the wave is forced with an internal vorticity field in the troposphere similar to that observed. Other methods of wave forcing (vertical velocity at the surface and diabatic heating) give no resonant type behavior.

The observations for the sudden warming show that wave 2 in the lower stratosphere was growing linearly with time over a period of about a week. The phase of wave 2 was initially moving eastward and became stationary on about 21 February. The wave was evident first in the lower stratosphere and then appeared to spread downward as well as upward.

The model used in this study is based on steady state linear equations and gives only a simplified view of the processes in the atmosphere. The most important problem is probably the use of steady state equations for a period during which both the wave and mean state are changing rapidly. Nevertheless, the similarity between the model predictions and the observations suggests that the resonance theory is relevant to the 1979 warming.

There are two ways that the observations can be interpreted in light of the model results. These are that the growth of eastward traveling wave 2 in mid-February is a result of selective growth of a resonant wave with only minimal forcing, or that there is a certain more favorable frequency range, which can change with time and within which wave propagation into the stratosphere is enhanced. The latter requires a stronger forcing from the troposphere than a true resonance would, since it is not really an instability. At the same time, the range of frequencies that can propagate is expected to be broad because of the fairly slow variation of refractive index with wave frequency. Both the experiments with the present model and the work of Butchart et al. (1982) give strong support for the importance of the preferred frequency range in wave-2 propagation during the 1979 major sudden warming. Further tests with more complete numerical models are needed to understand in detail the role of resonance in the generation and propagation of wave 2 during this period.

The resonance in this model occurs because of the particular structure of the zonal wind prior to the warming. That basic state is said to be preconditioned for sudden warmings (Butchart et al., 1982), a situation that also depends on the structure of the zonal wind. This suggests that there could be a direct link between preconditioning and resonance. The conditions for a major warming to occur require that EP flux vectors be directed upward into high latitudes rather than toward the equator. The EP flux vectors tend to point up the gradient of refractive index Q and to avoid regions where Q is negative. One way for a preconditioned state to exist is for there to be a region of negative Q in midlatitudes that restricts equatorward wave propagation. Butchart et al. discuss this in comparing the refractive indices in different runs of their warming simulation model. Such a region of negtive Q can also act as a partially reflecting wall for planetary waves, leading to resonance. The conditions for a resonant wave in Haynes' (1985) simple barotropic model were based on McIntyre and Palmer's (1983) qualitative description of changes in the basic state due to wave breaking. The same condition that was assumed by McIntyre and Palmer to be responsible for preconditioning (i.e., a poleward movement of the surf zone) was necessary for the existence of a resonant instability in Haynes' study. A corresponding link between a preconditioned flow and a vertical barrier to Rossby wave propagation is less obvious.

The shift of the resonance towards slower frequencies as the warming progresses is easily understood in terms of quasi-geostrophic theory. The refractive index squared is dominated by a term proportional to $q_\theta/(u - c)$. If only the speed and not the structure of the background wind changes (i.e., q is constant with time), a simultaneous decrease in the wind speed and the wave frequency will leave the refractive index the same. Once the resonant cavity is established *for an eastward moving wave,* a decrease in wind speed will not destroy the cavity but merely shift the resonance to a lower frequency. If the presence of the cavity can be maintained long enough for the resonant mode to become stationary, it can then interact with the forcing always present due to the earth's topography and distribution of heating. The shift in resonance characteristics is similar to the self-tuning mechanism proposed by Plumb (1981) and can maintain resonant wave growth even in the presence of wind variations due to wave–mean flow interaction. In contrast to Plumb's theory, topographically forced waves did not appear to play a role in the early stages of the 1979 warming. Another difference is that the resonant wave in Plumb's model moved westward, rather than eastward as in the present study.

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