On a Theory of the Evolution of Surface Cold Fronts

GAD LEVY
Department of Atmospheric Sciences, The University of Washington, Seattle, WA 98195

CHRISTOPHER S. BRETHERTON
Applied Mathematics Program, The University of Washington, Seattle, WA 98195

(Manuscript received 1 December 1986, in final form 5 June 1987)

ABSTRACT

The governing vorticity and divergence equations in the surface layer are derived and the roles of the different terms and feedback mechanisms are investigated in semigeostrophic and nongeostrophic cold-frontal systems. A planetary boundary layer model is used to perform sensitivity tests to determine that in a cold front the ageostrophic feedback mechanism as defined by Orlanski and Ross tends to act as a positive feedback mechanism, enhancing vorticity and convergence growth. Therefore, it cannot explain the phase shift between convergence and vorticity as simulated by Orlanski and Ross. An alternative plausible, though tentative, explanation in terms of a gravity wave is offered. It is shown that when the geostrophic deformation increases, nonlinear terms in the divergence equation may become important and further destabilize the system.

1. Introduction

Since the early 1970s, much attention has been given to the study of fronts and frontogenesis, and major advances in our understanding of frontogenetic processes have been made primarily by employing the semigeostrophic approximation (Eliassen, 1959; Hoskins, 1971; Hoskins and Bretherton, 1972). Most of those treatments have neglected part of the ageostrophic circulation and the role of friction in the mature steady state front. Williams (1974) has shown that the addition of an Ekman layer to one of the models treated by Hoskins and Bretherton (1972) resulted in realistic quasi-steady fronts. Ross and Orlanski (1982) and Orlanski and Ross (1984) (hereafter referred to as RO and OR, respectively) studied the evolution of an observed front with a three-dimensional primitive equation model in order to determine whether other ageostrophic mechanisms besides dissipation act to balance frontogenetic effects. Their results showed a significant phase shift at the surface (up to 400 km) between the front as defined by the line of maximum vorticity and the line of maximum convergence. They explained this phase shift based on an equation for the horizontal divergence tendency at the surface, which was derived from the primitive equations. They assumed: 1) that where the surface vertical vorticity q was becoming very large, it increased less rapidly than the geostrophic vertical vorticity q_g, leading to a negative ageostrophic vertical vorticity $q_a = q - q_g$ at the surface, and 2) that the nonlinear and frictional terms in the surface divergence equation were small in comparison to the stretching term fq_a. Where q_a is negative there is a positive divergence tendency which halts convergence and, they suggest, ultimately leads to a steady state front in which the vorticity maximum lags and does not significantly overlap with the convergence maximum, as they have observed in their simulation.

The primary point of this paper is to critically examine these two assumptions. As a preamble, in section 2 we rederive OR's surface divergence and vorticity equations and show that these equations hold approximately in the lower part of the boundary layer if there is surface friction. In section 3 we examine the assumptions in the context of a semigeostrophic model of frontogenesis (Hoskins and Bretherton, 1972) in which surface friction is not included. In this model $q_a = 0$, and assumptions 1 and 2 do not hold.

Orlanski and Ross' simulated front and observed fronts are not everywhere in cross-front geostrophic balance, so semigeostrophic theory may not be applicable, especially near the surface where frictional effects are important. The effect of a surface Ekman pumping on fronts was studied by Williams (1974), who obtained steady state fronts but found that the maxima of vorticity and convergence were almost colocated, casting some doubt on the generality of OR's phase shift. In section 4 we examine assumption 1 in a similarity model of the planetary boundary layer (Brown, 1982), where we allow q_a to be influenced by friction. Under almost all circumstances, we find q_a is positive when the stratification and thermal wind across the boundary layer are typical of a surface cold front. Other effects of surface friction may be frontolytic, but the stretching term appears to enhance convergence near the front.

In section 5 we comment on an alternative expla-

© 1987 American Meteorological Society

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC

nation of OR's simulated phase shift: gravity waves produced during the breakdown of cross-front geostrophic balance in the front (Ley and Peltier, 1978). We conclude with some thoughts as to how the mechanism of the phase shift could be resolved by future work.

2. The governing equations at the surface

We follow OR in deriving the vorticity and divergence equations near the surface from the anelastic momentum equation

$$\frac{dV}{dt}+f\mathbf{k}\times\mathbf{V}+C_p\theta\nabla_h\pi-\nabla\cdot\tau=0, \qquad (1)$$

where **V** is the horizontal wind velocity, θ the potential temperature, τ the Reynolds stress tensor, and π is the Exner pressure function.

We assume that in the lower part of the boundary layer (consisting of the surface and matching layers) the wind has a logarithmic profile. We now show, based on a scaling argument, that the twisting terms $W_xV_z - W_yU_z$, and $W_xU_z + W_yV_z$ in the vorticity and divergence equations, respectively, may be neglected in this layer compared to the much larger vortex-stretching (fD), horizontal friction, and pressure $[C_p\theta\nabla_h^2\pi = O(fq)]$ terms; for u_z in the surface layer one can apply the relation $u_z = u_*/kz$ where u_* is the friction velocity [$O(0.35$ m s^{-1} for 10 m s^{-1} winds)], and k is the von Kármán constant (0.35). Using the continuity equation and neglecting variations in density, one can substitute Hd/L for w_x, where d is a characteristic scale of the horizontal divergence $D = u_x + v_y$, H is the height scale and L the horizontal length scale. The twisting term is then $\leqslant O(U*d/kL)$, while the vortex-stretching term is $(q+f)D$ and is $O(fd)$ (q the relative vorticity and f the Coriolis parameter). The ratio of the twisting term to the vortex stretching term is then $U*/kfL$. With $L = 100$ km and $f = 10^{-4}$ s^{-1}, this ratio is $\leqslant 1/10$. The friction terms are the divergence and the curl of the vertical shear of the Reynolds stresses. In a steady-state homogeneous atmosphere near the surface, an approximate balance between $f(v - v_g)$ and the vertical shear of the Reynolds stresses is maintained, and they scale as fV. The friction terms will then scale as $fV/L = fd$ and are of the same order of magnitude as the stretching and pressure terms in the respective equations. The resulting equations are identical to the ones used by OR and are valid throughout the surface and matching layers:

$$dq/dt=-(f+q)D+F_q \qquad (2)$$

$$dD/dt=fq-fq_g-[U_x^2+2V_xU_y+V_y^2]+F_D \qquad (3)$$

then $q_g = (C_p\theta\nabla_h^2\pi)/f = \nabla_h^2\Phi/f$ is the geostrophic relative vorticity, Φ is the geopotential and F_q, F_D represent dissipation in terms of frictional forces.

Based on the set of equations (2)–(3), OR arrived at the following explanation for the simulated phase shift: The primary balance is between the first two terms on the right in (3). As frontogenesis proceeds, q increases relative to q_g, resulting in a decrease in convergence in the frontal zone. Other nondissipative terms on the right-hand side of (3) are quadratic and thus are of higher order than this effect. Hence the ageostrophic difference $f(q - q_g)$ represents a negative feedback mechanism to limit growth due to vortex stretching. If all other terms may be neglected and the convergence is initially in phase with the vorticity, the negative feedback, then, will be maximum at the peak of the convergence, thereby limiting vortex growth which will lead eventually to reduction of the vorticity tendency to zero. If dissipation is absent, an approximate equilibrium condition is possible in the mature front only if vorticity and divergence are out of phase.

In order to investigate the validity of OR's assumptions and explanation, it is helpful to manipulate Eq. (3) and bring the nonlinear terms to a form that is easier for interpretation; the term in the square brackets in (3) can be written as $D^2 + 2J(V, U)$ where J is the Jacobian operand $[J(u, v) = u_xv_y - u_yv_x]$. Using the definition of the total horizontal deformation δ:

$$\delta=[(V_x+U_y)^2+(U_x+V_y)^2]^{1/2}; \qquad (4)$$

after substitution and rearrangement, Eq. (3) may be written in the form:

$$dD/dt=f(q-q_g)-\frac{1}{2}D^2-\frac{1}{2}\delta^2+\frac{1}{2}q^2+F_D. \qquad (5)$$

In this form the roles of the nonlinear terms involving divergence deformation and vorticity are elucidated. As shown in the next section, there is large cancellation between the quadratic deformation and vorticity terms.

3. Feedback mechanisms in semigeostrophic and primitive equation systems

The semigeostrophic momentum equations from Hoskins and Bretherton, 1972 (hereafter denoted as HB) are

$$fV=\Phi_x \qquad (6)$$

$$dV/dt+fU+\Phi_y=0, \qquad (7)$$

where U and V are the velocity components in geostrophic coordinates, and Φ is the geopotential. Equations (6) and (7) state that a cross-front geostrophic balance is maintained.

As in HB we assume a solution

$$U=-\alpha x+u(x,z,t) \qquad (8)$$

$$V=\alpha y+v(x,z,t) \qquad (9)$$

$$\Phi=f\alpha xy-(\alpha^2+d\alpha/dt)y^2/2-(\alpha^2+d\alpha/dt)x^2/2 + \Phi'(x,z,t) \qquad (10)$$

where α is the background geostrophic deformation

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC

and may be a function of time. The quadratic terms in x and y are required to produce a purely deformational basic flow in a primitive equation model. In a semigeostrophic model only the second term on the right of Eq. (10) is required because a cross-front geostrophic balance is assumed.

OR state that the semigeostrophic approximation holds only when

$$(q+f)/f \ll (f/\delta_g)^2. \qquad (11)$$

As the simulated front intensifies, both δ and δ_g increase within the frontal zone to the order of f. Since (11) no longer holds, they suggest that the semigeostrophic approximation is no longer valid.

In fact, the semigeostrophic approximation can hold even after (11) has broken down. If the curvature of the front is neglected, the semigeostrophic approximation is valid as long as a cross-front geostrophic balance is maintained (i.e., $dU/dt \ll fV$). In the zero potential vorticity solution of the semigeostrophic model, HB suggest that this implies

$$(q+f)/f \ll (f/2\alpha)^2. \qquad (12)$$

Here α is the background geostrophic deformation, not the total geostrophic deformation in the frontal zone. Using Φ in the semigeostrophic version of (10) the geostrophic wind components are

$$U_g = -f^{-1}\Phi_y = -\alpha x + f^{-1}(\alpha^2 + d\alpha/dt)y \qquad (13)$$

$$V_g = f^{-1}\Phi_x = \alpha y + f^{-1}\Phi'_x; \qquad (14)$$

substitution into (4) yields

$$\delta_g = \{[f^{-1}\Phi'_{xx} + (\alpha^2 + d\alpha/dt)]^2 + [2\alpha]^2\}^{1/2} \qquad (15)$$

when

$$\alpha \ll f \quad \text{and} \quad d\alpha/dt = O(\alpha^2)$$

$$\delta_g \approx \{[f^{-1}\Phi'_{xx}]^2 + 4\alpha^2\}^{1/2} = \{q_g'^2 + 4\alpha^2\}^{1/2}. \qquad (16)$$

Since the geostrophic vorticity can become much larger than α without violating the semigeostrophic approximation, (12) is much less stringent than the criterion (11) used by OR to check the validity of semigeostrophy; it is not clear whether (12) would be satisfied for their simulated front. In a three-dimensional numerical simulation it is not even obvious what an appropriate estimate for α should be. Thus, we will first determine the dominant terms in a semigeostrophic model of frontogenesis and then we will comment on what changes could be expected if α is too large for (12) to hold.

The semigeostrophic divergence and vorticity equations for a two-dimensional front are found by differentiating Eqs. (6) and (7) w.r.t. x [the components resulting from differentiating (6) and (7) w.r.t. y are identically zero as may be verified by substitution of the assumed solution (8–10)]. The divergence equation reduces to a statement of cross-frontal geostrophic balance,

$$fV_x = fq = \Phi_{xx} = fq_g. \qquad (17)$$

This balance equation, in common with the quasi-geostrophic equations, does not describe gravity wave motion. The vorticity equation in this system is

$$dq/dt = -D(q+f) + \alpha q. \qquad (18)$$

This form provides for an unlimited vorticity growth through vortex stretching and through geostrophic deformation. In such a system only friction can limit growth and slow frontogenesis. This, as shown by Williams (1974) in a simulation using a drag coefficient not much larger than that of OR, may lead to a quasi-steady state front (though not with OR's phase shift).

Assuming a solution of the form (8) and (9) for U and V, the full divergence equation (5) derived from the horizontal momentum Eq. (1) will take the form

$$dD/dt = fq_a - D^2 - 2\alpha^2 + 2\alpha D + F_D \qquad (19)$$

where subscript a denotes the ageostrophic component. We have again used the definition (4) of the total deformation, letting $U = U_g + U_a$, $V = V_g + V_a$, and substituted it into (5) recalling that $q = q_g + q_a$. In the pure deformational basic state $D = q = 0$ but $q_a = -2\alpha^2$ allowing $dD/dt = 0$.

The first term on the right-hand side of (19) is the ageostrophic feedback mechanism postulated by OR. The explanation given by OR for the phase shift depends on fq_a dominating the other terms on the right-hand side of (19). However, until Kelvin–Helmholtz instability, surface friction, or cross-frontal acceleration become important, we expect the semigeostrophic solution will remain accurate. In this solution, as frontal collapse proceeds, q_a remains zero. It seems unlikely that fq_a will dominate the right-hand side of (19). When α becomes large, the nonlinear terms on the right of (19) all act to make D more negative. In order for q_a to act as a negative feedback mechanism it must be positive where D is negative. This condition implies that the surface vorticity is supergeostrophic, which is rarely observed in a cold frontal situation. In addition, near the surface the ageostrophic flow is highly dependent on the surface friction. Hence neglecting the friction term, as OR do in order to explain the phase shift between the vorticity and divergence maxima, is not justified.

4. Evaluating the ageostrophic vorticity with a similarity boundary layer model

Equations (2) and (3) do not constitute a closed set of ordinary differential equations. They could be closed if F_D, F_q, and q_g were specified in terms of q and D. Our scale analysis showed that the frictional terms cannot be neglected in determining the evolution of q and D. However, following OR we concentrate on the role of q_a by neglecting the frictional terms. The ageostrophic vorticity is computed using a similarity solution for the boundary layer flow.

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC

We shall now examine how q_a can be expressed in terms of q_g using a boundary layer similarity model. We define a parameter:

$$C \equiv q_g/q$$

and examine its dependence on the geostrophic and cross-isobaric flow.

Assume now that the geostrophic wind with magnitude G and direction β, has components:

$$U_g = G\cos\beta$$

$$V_g = G\sin\beta.$$

The surface wind with speed S is related to it and has the components

$$U = C_1 G\cos(\beta - \Gamma)$$

$$V = C_1 G\sin(\beta - \Gamma),$$

where $C_1 = S/G$ and Γ is the turning angle, both constants for given stratification and baroclinicity. The geostrophic vorticity is

$$q_g = V_{gx} - U_{gy} = (G\sin\beta)_x - (G\cos\beta)_y, \qquad (20)$$

and the total vorticity near the surface is

$$q = V_x - U_y$$

$$= C_1\{[G\sin(\beta - \Gamma)]_x - [G\cos(\beta - \Gamma)]_y\} \qquad (21)$$

$$q = C_1[(G\sin\beta\cos\Gamma)_x - (G\cos\beta\sin\Gamma)_x] - [(G\cos\beta\cos\Gamma)_y - (G\sin\beta\sin\Gamma)_y].$$

Now, let $\sin\Gamma = C_2$ and $\cos\Gamma = C_3$, then,

$$q = C_1[C_3 q_g - C_2(U_{gx} + V_{gy})]$$

$$= C_1[C_3 q_g - C_2 D_g] = C_1 C_3 q_g = K q_g \qquad (22)$$

since D_g (the divergence of the geostrophic wind) is zero. Here $C = 1/K = G/(S\cos\Gamma)$ can be evaluated from a planetary boundary layer model that predicts G/S and Γ. Clearly, in the basic state when $q = 0$ but $q_a = -2\alpha^2$, Eq. (22) breaks down. However, when q and q_g are larger, we expect the prediction of the boundary layer similarity model to be more accurate.

The Ekman/Taylor solution (e.g., see Brown, 1982) is a solution to the steady-state momentum equations that gives the velocity profile throughout the Ekman layer, assuming the wind is geostrophic at $z = h$, the top of the boundary layer, and has a finite value at the lower boundary (i.e., at the top of the surface layer). The observed wind generally deviates from this idealized profile due to its instability to perturbations and baroclinic effects (i.e., vertical shear of the pressure gradients in the boundary layer). Secondary circulations that develop as a result of these effects may transport considerable momentum vertically. The solution may be modified to include the changes to the mean flow by those effects and then matched with the stability dependent solution for the logarithmic layer flow at the lower boundary. Two similarity equations

$$U(z) - G\cos\Gamma = U_*/kA(z/h, s, V_T, V_m)$$

$$V(z) - G\sin\Gamma = U_*/kB(z/h, s, V_T, V_m) \qquad (23)$$

are formed. Here the surface layer flow is assumed to be in the positive x direction, s is a stability parameter, V_T is the thermal wind (i.e., the vertical geostrophic wind shear), and V_m is the modification to the mean flow by secondary circulations. The universal functions A and B have to be determined either empirically from observations or theoretically by the aid of closed systems of equations for the turbulent planetary boundary layer. Thus, knowing the stratification and baroclinicity, one can use a two-layer similarity model to predict G/S, Γ and C.

It is clear from Eq. (19) that the ageostrophic feedback mechanism will work as a negative feedback only if q_a is positive or $C < 1.0$, and thus we proceed now to evaluate the variation in C under conditions corresponding to a cold front. A two-layer similarity model for the planetary boundary layer as described in Brown, 1982 is used to impose an increasing west–east temperature gradient, corresponding to a two-dimensional, straight cold front. The ratio between the geostrophic and the total vorticities, C, is computed for different stratifications. Figure 1 shows the variation in C with baroclinicity for neutral, slightly unstable, slightly stable and moderately stable conditions. For all the cases, C is increasing with growing horizontal temperature gradients (stronger fronts). The values are in excess of one, and imply that the near-surface flow is subgeostrophic and the ageostrophic feedback mechanism is enhancing convergence growth.

In Fig. 2 the streamlines of the vectors dD/dt, dq/dt are plotted in the D–q plane with a constant C and a prescribed deformation imposed. They show an unstable behavior with a tendency for an unlimited growth of vorticity and convergence. Similar behavior resulted in all cases, regardless of the amount of deformation imposed, even for C smaller than one when the ageostrophic feedback mechanism works as a negative feedback. We conclude, therefore, that in the cases tested, the positive feedback mechanisms are stronger than the negative effect of the ageostrophic vorticity.

5. Gravity waves as a potential explanation

A plausible explanation to the simulated phase shift which merits further investigation is the effect of gravity waves produced during the breakdown of cross-frontal geostrophic balance. Ley and Peltier (1978) examined the importance of cross-front acceleration by using it to determine the correction to the HB solution in the form of a packet of atmospheric gravity waves. By dividing the flow into 1) the background quasi-geostrophic flow, 2) the nongeostrophic flows associated

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC

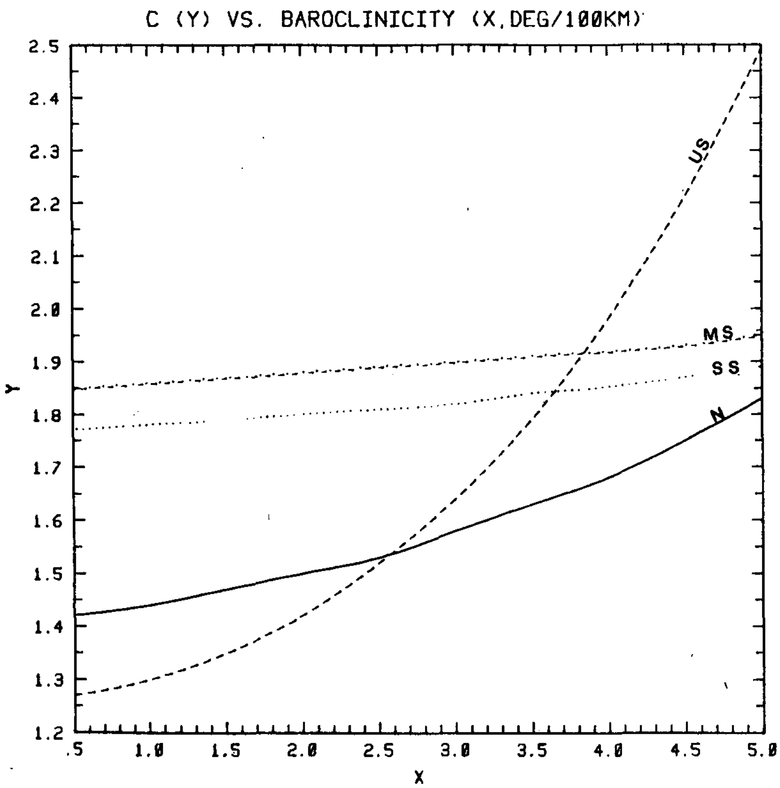

FIG. 1. Variation of the ratio of the geostrophic to total vorticities (C) with horizontal temperature gradients (°C/100 km) for neutral (N), slightly unstable (US, Ri = −0.12), slightly stable (SS, Ri = 0.09), and moderately stable (MS, Ri = 0.13) stratifications.

with the front itself, and 3) transient gravity waves excited by such nongeostrophic conditions, they have shown that the cross-front acceleration dU/dt, omitted in deriving the HB solution, can serve as a source for gravity waves before the onset of rapid mixing arising from shear instability. In their solution they use a multiple scale analysis whereby the zero order equations satisfy the quasi-geostrophic background state; the first-order corrections included in the HB system of equations are those associated with the front and do not include wave terms, while the correction to the HB equations form the residual wave equation. Such gravity waves, which are primarily irrotational, provide a very plausible explanation to OR's simulated phase shift. In a linear plane gravity wave the vorticity maximum lags the convergence maximum by 90°. Given the maximum surface divergence D_{max}, the horizontal wavelength L and an estimate of the phase speed c of the gravity wave, one can estimate the vorticity associated with such a gravity wave to be $-(D_{max}Lf)/2c$. Based on OR's simulation, we chose $D_{max} = -3.0 \times 10^{-5}$ s^{-1}, L = 100 km, c = 5 m s^{-1} and found that this vorticity amounts to only 20–30% of the vorticity associated with the front. It is plausible that the convergence maximum is associated with a gravity wave, while the vorticity maximum is predominately associated with frontogenesis. Orlanski and Ross' report that "the behavior of the convergence field is more oscillatory than is vorticity" and that "the effective phase shift of the convergence field ahead of the vorticity shows little sensitivity to the different surface dissipation conditions used", seem to support such an interpretation, suggesting that the convergence is a result of a process independent of the surface friction.

The calculation by Ley and Peltier suggests the existence of a local region of horizontal convergence at a distance of approximately 75–125 km into the warm air in advance of the front and a horizontal rate of propagation of approximately 6 m s^{-1}. A crude estimate of the rate of propagation in the case simulated by OR (from Fig. 3 in OR) suggests horizontal propagation of 5–6 m s^{-1}. Considering that the details of numerical simulations depend on the model used and that divergence fields are essentially obtained only as average values and the averaging process actually reduces the amplitude of disturbances, OR's and Ley and Peltier's simulations seem to agree with each other. Orlanski and Ross (1977) suggested that the differences between a diagnosed Sawyer-Eliassen circulation and a two-dimensional model result were caused by gravity waves. Unfortunately, initialization of numerical models can create spurious internal gravity waves with no mete-

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC

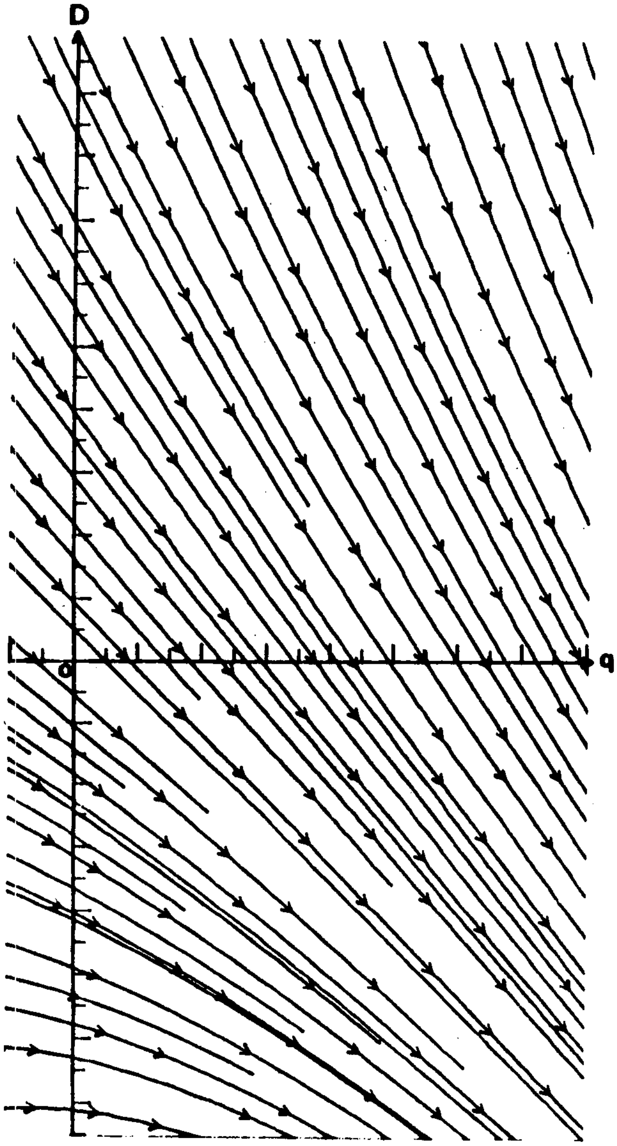

FIG. 2. Streamlines of the vectors dD/dt, dq/dt in the q–D plane (marked at intervals of $0.5f$) for a constant $C = 0.9$ and imposed background deformation of $\alpha = f$.

orological significance. This explanation of the phase shift in terms of a gravity wave remains tentative.

6. Summary and discussion

The relationship between the different feedback mechanisms in the vorticity and divergence equations in a surface cold front was investigated in light of OR's simulation of a steady-state mature cold front with a phase shift between the vorticity and convergence maxima at the surface. OR postulated that an ageostrophic feedback mechanism becomes important as the front matures, leading to a steady-state situation that requires a phase shift between vorticity and divergence. We have reexamined some assumptions implicit in their explanation in light of semigeostrophic theory and a primitive equation system. While their mechanism is plausible, it cannot be considered in isolation from the effects of the nonlinear terms and friction on the surface divergence tendency.

In the primitive equation system, when the geostrophic deformation is large, the nonlinear terms become important and act to enhance convergence and vorticity. Friction is crucial in determining both the ageostrophic and the dissipation terms in the divergence equation. Our analysis shows that the geostrophic vorticity in the boundary layer tends to exceed the vorticity and, through the ageostrophic term, enhance convergence in a cold front. Furthermore, the dissipation terms also play an important role in the divergence near the surface. They may limit frontogenesis but do not require a shift in phase between the maxima in convergence and vorticity. Williams (1974) showed that imposing friction on the HB model can lead to a steady state front. The amount of friction used by OR was similar to that used by Williams (although the boundary condition was different).

A gravity wave as proposed by Ley and Peltier (1978) provides a plausible explanation for the simulated phase shift even before the onset of rapid mixing or the breakdown of semigeostrophy. Such a gravity wave may be masked, however, by boundary layer processes not included in OR's model. More theoretical, numerical, and observational studies with detailed boundary layer physics are needed before such an explanation and its existence in nature can be established.

Acknowledgments. Support during this research was provided by NASA Grant NAGW-679.

REFERENCES

Brown, R. A., 1982: On two-layer models and the similarity functions for the PBL. *Bound.-Lay. Meteor.*, **24**, 451–463.

Eliassen, A., 1959: On the formation of fronts in the atmosphere. *The Atmosphere and The Sea in Motion,* Rockefeller Institute Press, 277–287.

Hoskins, B. J., 1971: Atmospheric frontogenesis, some solutions. *Quart. J. Roy. Meteor. Soc.*, **97**, 139–153.

——, and F. P. Bretherton, 1972: Atmospheric frontogenesis models: Mathematical formulation and solution. *J. Atmos. Sci.*, **29**, 11–37.

Ley, B. E., and W. R. Peltier, 1978: Wave generation and frontal collapse. *J. Atmos. Sci.*, **35**, 3–17.

Orlanski, I., and B. B. Ross, 1977: The circulation associated with a cold front. Part I: Dry case. *J. Atmos. Sci.*, **35**, 1619–1633.

——, and ——, 1984: The evaluation of an observed front. Part II: mesoscale dynamics. *J. Atmos. Sci.*, **41**, 1669–1703.

Ross, B. B., and I. Orlanski, 1982: The evaluation of an observed front. Part I: numerical simulation. *J. Atmos. Sci.*, **39**, 296–326.

Williams, R. T., 1974: Numerical simulation of a steady state front. *J. Atmos. Sci.*, **31**, 1286–1296.

Unauthenticated | Downloaded 11/10/24 01:28 AM UTC