A Three Parameter Representation of the Shape and Size Distributions of Hailstones—A Case Study

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ABSTRACT

The characteristics of shapes and sizes of a sample of 679 hailstones, collected on 22 June 1976 during a hailstorm at Grover, Colorado, were analyzed using a three-parameter formula developed by us previously. These parameters are a, the horizontal dimension, c, the vertical dimension, and λ, the shape parameter. Once these three parameters are specified, both the shape and size of a hailstone are fixed. It is believed that this analysis produces the most complete and quantitative information about the hailstone shape and size distributions reported so far. The dataset of the three parameters also allows the relatively good reconstruction of the sizes and shapes of the original hailstones if desired. The results for this collection of hail show that the distributions of both horizontal and vertical dimensions can be described by gamma distributions, while the shape parameter can be described by an exponential distribution. Since the shape parameter basically describes the vertical asymmetry, it may provide additional information about the physics of particles in clouds and precipitation. The distributions of axial cross-sectional areas, surface areas, and volumes are also presented. They too can be described by gamma distributions. Finally, it was found that the geometrical quantities of the hailstones are best represented by a characteristic dimension r_c, defined as the average of the horizontal and vertical dimensions.

1. Introduction

The shapes and sizes of particles play a fundamental role in the physics of clouds and precipitation. They are known to be closely related to the physical and dynamical states of cloud fields (Pruppacher and Klett, 1978). It is therefore important to study the shape and size distributions of these particles. However, at present, only the size distributions are studied in a quantitative manner. The shape distributions, on the other hand, are usually described qualitatively. Words like spheres, spheroids, columns, plates, dendrites, and cones are used to describe the shapes of clouddrops, raindrops, snow crystals, and hailstones. While these words do represent some sort of classification scheme, they are not quantitative, which is necessary for systematic calculations. Some simple mathematical expressions are needed, each containing a small number of free parameters, that can generate the particle sizes and shapes in a systematic manner. These expressions can then be used to fit the shapes and sizes of actual samples so that the resulting shape and size distributions are described quantitatively by these free parameters. This motivated Wang (1982), Wang and Denzer (1983), and Wang (1987) to develop several mathematical expressions that are capable of generating curves that simulate the shapes of conical hydrometeors (hailstones, graupel, and raindrops), plane hexagonal snow crystals, and general polygonally symmetric particles. The free parameters involved in these expressions can be used to characterize the shape and size distributions.

This paper is concerned with the shape and size distributions of hailstones. Many investigators have studied this subject before (e.g., Auer, 1972; Barge and Issac, 1973; Federer and Waldvogel, 1975; Smith et al., 1976; among others). They either treated the hailstones as spheres and therefore spoke of the "diameters" of them or treated them as spheroids and spoke of the axis ratios. While these pioneering studies contributed significantly to the study of hail physics, it is also clear that the shapes of hailstones were oversimplified. In view of the possibility that the true shape distributions may be of importance, it is desirable to describe the shape and size distributions of hail in a more quantitative way. For this purpose we used the formula developed by Wang (1982) to analyze the sizes and shapes of a sample of 679 hailstones. These hailstones were collected by Nancy Knight of the National Center for Atmospheric Research (NCAR) during the 22 June 1976 hailstorm at Grover, Colorado. Their shape and size distributions are represented by the three parameters of Wang's formula. It will be seen that these distributions reveal simple relations between the number concentrations of hailstones and each parameter. This fact is encouraging since, in our point of view, an analytical technique is useful only when it can reduce the raw data into simple relations that can be subject to physical interpretations and calculations. The raw data of these three parameters can be used to reconstruct the shapes and sizes of the original hailstones. It is believed that this analysis produces the most complete and quantitative results of the hail shape and size distributions reported to this date.

2. Methodology

The mathematical expression used in this study for fitting the shape and size of a hailstone is that given by Wang (1982):

$$x = \pm a[1-(z^2/c^2)]^{1/2}\cos^{-1}(z/\lambda c) \quad (1)$$

where x and z are horizontal and vertical coordinates, respectively, a ($=L/\pi$) is the horizontal semiaxis of the generating ellipse, L is the width along the x-axis, c is one-half of the length from the apex to the bottom along the z-axis, and λ is a dimensionless shape factor. All of these quantities except λ are also shown graphically in Fig. 1. For a more detailed description of Fig. 1, see Wang (1982).

The shape parameter λ must be equal to or larger than 1. It determines the shape asymmetry with respect to the horizontal axis. As λ approaches 1, the shape of the curve becomes pointed at the apex and flattens at the base. When λ is large, the shape is more circular or elliptical and the apex becomes round. Values of λ greater than 10 do not change the shape significantly from that at $\lambda = 10$.

The parameters a, c and λ are all that are necessary to fix the size and shape of a hailstone. The determination of these parameters for a given hailstone can be done manually by following the procedure given by Wang (1982). However, when the number of hailstones is large, manual measurements are tedious and time-consuming. Thus, in the present study we developed a semiautomatic procedure of determining these parameters.

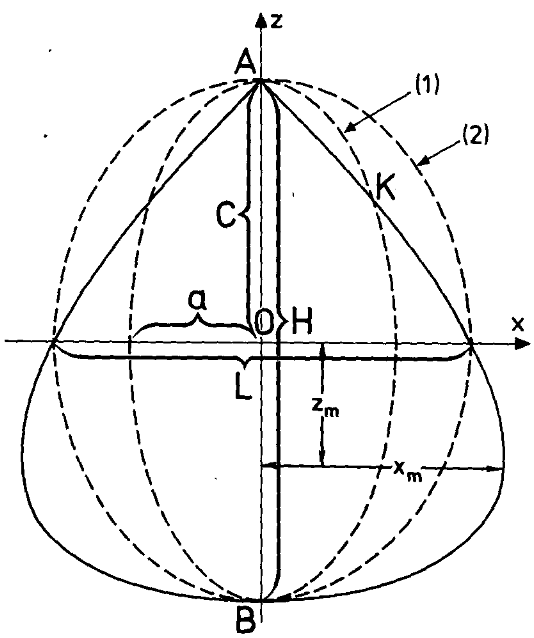

FIG. 1. Definition of the coordinate system and various quantities. The solid curve is an axial cross section of a conical body. Dashed curves (1) and (2) are the generating ellipse and limiting ellipse, respectively. x_m and z_m are the largest horizontal coordinate and the corresponding vertical coordinate, respectively.

An electronic digitizer (GTCO Model GAP-1) was used to determine the x- and z-coordinates of the points on the conical curve of a hailstone. It has a five-button cursor and 0.001 inch resolution. The digitizer was connected to an IBM Personal Computer so that the digitized points could be transferred to the computer and stored on a floppy disk.

The first step of the digitization was to determine the x- and z-axes of the hailstones. Since these stones were collected at the ground level, there is no way to be sure what their actual orientations were in the air. In the present study we assumed that spheroidal hailstones fell with their minor axes in the vertical (z) direction while conical hailstones fell with apices pointing upward. List and Schemenauer (1971) reported that a conical particle could also fall with its apex pointing downward. Fortunately, this does not alter our conclusions here as long as the z-axis is in the apex-base direction.

Once the axes were determined, 20 points were digitized for each hailstone. The sequence of digitization was designed so that the parameters a, c and λ could be calculated one by one. After the hailstone photograph was properly aligned, a point at the apex was digitized first, then a second point at the base of hailstone with the same x-coordinate as the initial point was digitized. One-half of the difference between the z-coordinate values of these two points was the parameter c.

After the second point was digitized, the origin coordinates of the hailstone's axes were displayed on the computer screen. Once this information was known, the x-axis could be defined by digitizing two points representing the width (L) of the hailstone. The parameter a was then computed by dividing L by π.

The next 16 points were digitized along the perimeter of the hailstone in a clockwise direction beginning near the apex. After the final point was digitized, the program then exited from the digitization loop to calculate the shape parameter λ.

The optimum value of the shape parameter λ was determined by the method of least-squares for a set of data points. This was found by minimizing the sum of the squared residuals (SSR):

$$s(\lambda) = \sum_{i=1}^{N}\{\pm a[1-(z_i^2/c^2)]^{1/2}\cos^{-1}(z_i/\lambda c) - x_i\}^2 \quad (2)$$

where N is the total number of data points. The minimum of $S(\lambda)$ is referred to as the sum of the squared residuals. Ideally, the λ that corresponds to minimum

$S(\lambda)$ would be obtained by differentiating $S(\lambda)$ with λ, setting it to zero, and then solving for λ. This is very difficult for the present case because the λ is contained in a transcendental function. Instead, we used a grid search method (cf. Bevington, 1969). This method involves using an initial guess for the parameter and then searching in parameter space by incrementing the parameter in the direction where $S(\lambda)$ decreases until the minimum is found. In our case the initial value of λ is 1, and it was increased in 0.1 increments (unless the optimum λ was less than 1.1). The minimum was then examined in greater detail using increments of 0.01. Once the minimum sum of the squared residuals had been found, the corresponding value of λ was computed by parabolic interpolation using the following formula:

$$\lambda(\min) = \lambda(3) - \Delta\lambda\left[\frac{S_3(\lambda) - S_2(\lambda)}{S_3(\lambda) - 2S_2(\lambda) + S_1(\lambda)} + \frac{1}{2}\right] \quad (3)$$

where $\Delta\lambda$ is the increment in λ, $\lambda(3)$ the value of λ at point 3, and $S_1(\lambda)$, $S_2(\lambda)$, $S_3(\lambda)$ are the values of $S(\lambda)$ at points 1, 2, 3 as shown in Fig. 2.

This method of interpolation works well for values of λ near 1 but is unsatisfactory when λ is large. Fortunately, however, this larger deviation of λ does not lead to large deviation of the shape when the value of λ itself is large. The digitized data points and the fitted shape [using the calculated a, c, λ and putting into Eq. (1) to generate a curve] were displayed on the screen for a visual comparison. Also displayed on the screen was other pertinent information such as the value of λ and its standard deviation and the sum of squared residuals in (2).

In addition to the sum of squared residuals, we also calculated the coefficient of determination R^2. This was chosen over various other coefficients because it can be used with linear and nonlinear regression. The coefficient of determination is defined as

$$R^2 = \frac{\text{Explained variance}}{\text{Total variance}} = 1 - \frac{S(\lambda)}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \quad (4)$$

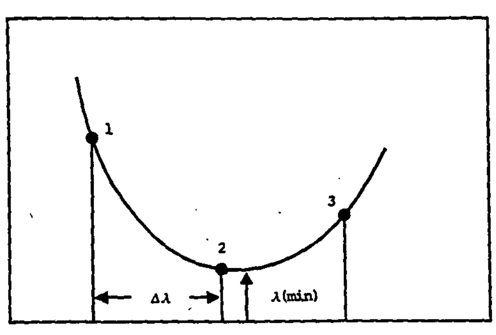

FIG. 2. Illustration of parabolic interpolation to find $\lambda(\min)$ for minimum $S(\lambda)$ using the values of S at λ = 1, 2 and 3. See Bevington (1969) for details.

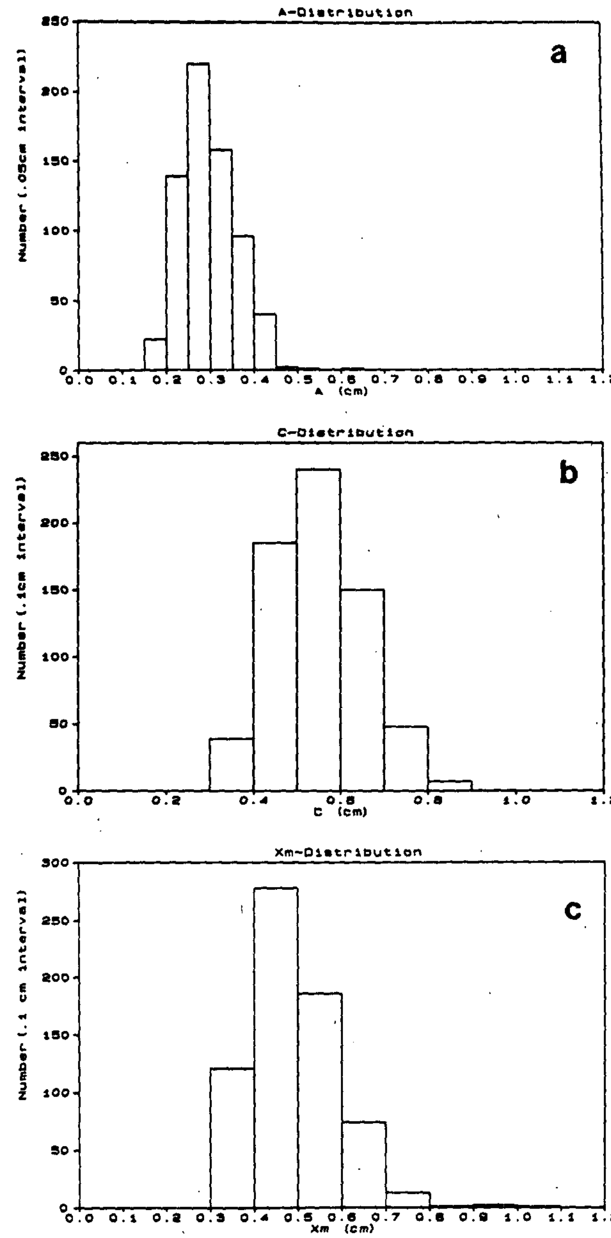

FIG. 3. (a) The a-distribution, (b) the c-distribution, and (c) the x_m-distribution.

where x_i's are the data points and $\bar{x}$ the average. A value of R^2 near 0 indicates a poor fit, while a value near 1 indicates a good fit. From the analysis of the hailstones the R^2 value were all close to 0.90, indicating that the fit was generally satisfactory.

Once the parameters a, c and λ were determined, other geometrical characteristics of the hailstones were calculated using the formulas given by Wang (1982) if we assume that the hailstones possess rotational symmetry with respect to the z-axis. Wang's formulas are as follows:

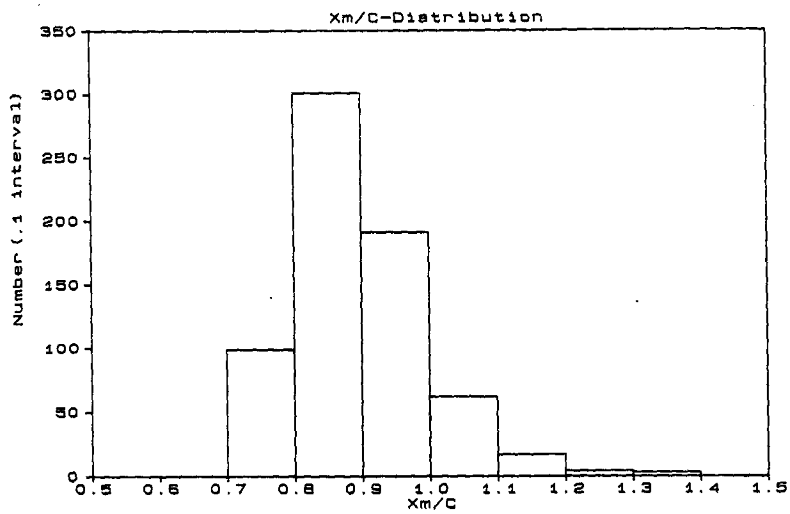

FIG. 4. The x_m/c-distribution.

axial cross-sectional area:

$$A_x = \pi^2 ac/2 \quad (5)$$

total surface area:

$$A_r \approx 2\pi^2 ac \quad (6)$$

volume:

$$V \approx \pi a^2 c\left[3.2889 + \frac{0.2667}{\lambda^2} + \frac{0.0382}{\lambda^4} + \frac{0.0113}{\lambda^6} + \frac{0.0046}{\lambda^8} + \frac{0.0024}{\lambda^{10}}\right]. \quad (7)$$

3. Hail data

The 679 hailstones analyzed here were collected during a hailstorm at Grover, Colorado, on 22 June 1976. This sample is composed of time-resolved data collected by a method detailed by Knight et al. (1982). Many more samples were collected on that particular day, but the one analyzed here was collected between 1640 and 1642 MDT. These hailstones were obtained from a mosquito-net funnel that separated the rain and directed the jar into hexane at dry ice temperature. Each hailstone was then aligned horizontally, fixed frozen on a black background, and photographed. This was done so that the maximum dimensions of the stones can be measured directly from the photographs. The stones were then measured and weighed, and some were sectioned to examine their interior structures. The size distributions of these hailstones were also reported by Knight et al. (1982), taking the largest dimension of a stone as the definition of its size. Like many previous works, this definition does not allow the quantitative description of both sizes and shapes of hailstones. On the other hand, our present method makes such a description possible. According to the calculation of Knight et al. (1982), the average concentration of the hailstones in the air was about 2.4 m^{-3}. Therefore, a sample of 679 hailstones would correspond to a total sample volume of 1629.6 m^3. This number can be used to convert the various distributions obtained in the following section into distributions per unit volume when desired.

The fact that this hail sample was collected at the same location within a relatively short period (~2 min) means that the results obtained here should be considered as representing the *instantaneous* observation. It is clear that this observation may be different in different instants. If the same analysis is applied to samples collected at other instants, then the results can be combined to show the evolution of the shape and size distributions of hailstones in the storm. If, in addition, samples from different storms at various locations are available, then we can study the hail climatology.

4. Results and discussions

The distributions of six geometrical quantities were determined for this collection of hailstones. These quantities were a, c, λ, A_x, A_r and V. The latter three quantities were calculated from a, c, and λ using Eqs. (5), (6) and (7). In each distribution, an interval width was selected so that a reasonably smooth distribution resulted.

Figure 3 shows the distributions of linear dimensions of the hailstones. The upper diagram shows the a-distribution. It is seen here that the population of hailstones initially increases with increasing size, reaching a maximum at 0.25–0.3 cm and then tailing off gradually, much like the gamma distribution common to many atmospheric particles. It is interesting to note that even if we limit ourselves to a smaller sample, as was done by Wang (1984), the result is still a gamma

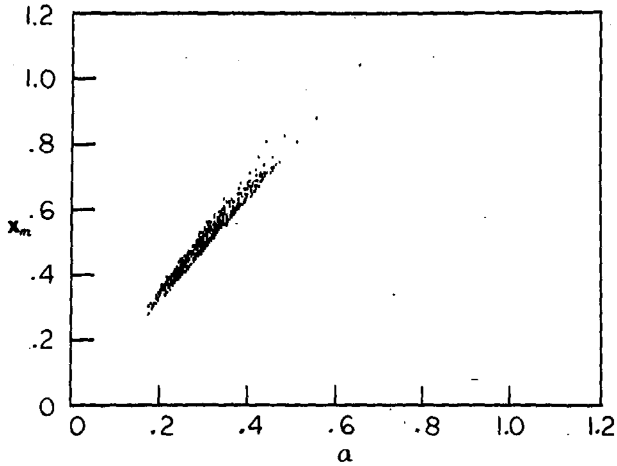

FIG. 5. Relation between a and x_m.

distribution. The minimum size cutoff is about 0.15 cm and the maximum cutoff is 0.65 cm.

Figure 3b shows the c-distribution. It is apparent that this distribution is also a gamma type. The overall magnitude of a is smaller than c, but this should *not* be taken to mean that the horizontal dimensions of the hailstones are smaller than the vertical ones. This is because a is merely the horizontal semiaxis of the generating ellipse (see Fig. 1, also see Wang, 1982). The parameter x_m, one-half of the maximum horizontal length, is more relevant for the comparison with c. Figure 3c shows the x_m-distribution. Apart from the fact that it looks like a-distribution, it is also obvious that the magnitudes of x_m are close to that of c.

To see whether the hailstones are more elongated in the vertical direction or in the horizontal direction, we plotted the distribution of x_m/c. This is shown in Fig. 4. The mode of this distribution is at $x_m/c \approx 0.85$, which indicates that these hailstones tend to be more elongated in the vertical direction. Again, the x_m/c distribution is gamma-type.

Both a and x_m can be used to represent the horizontal dimension and the related geometrical or physical properties. For example, the quantity $2x_m$ has been used as a characteristic length for describing hydrodynamic behavior of hydrometeors (List and Schemenauer, 1971; Heymsfield, 1978). One may worry here that the physical properties related to the horizontal dimension represented by a may be different from that represented by x_m. This worry may be settled by looking at Fig. 5, which shows that x_m and a have a nearly linear relation. Therefore, the physical properties represented by x_m would be equally represented by a.

The linear relationship between x_m and a can be understood in the following manner. If we approximate a hailstone by an ellipse, then L is equal to x_m. Hence x_m equals $\pi a/2$. This gives a linear relationship between x_m and a with a slope of $\pi/2$, which is comparable to that of $\sim$1.63 in Fig. 5.

Figure 6 shows the λ-distribution. To understand this diagram, it is useful to remember that λ represents the degree of asymmetry of a hailstone in the z-direction. When λ is close to 1, the hailstone has a sharp apex. The volume of the upper half is much smaller than that of the lower half (assuming rotational symmetry). In other words, the hailstone is conical. With λ increasing, the difference in volume becomes smaller and smaller. When λ approaches ∞, the shape becomes a perfect ellipsoid and the masses of the upper and lower halves are equal. The hailstone with large λ therefore appears to be round, either like a sphere or

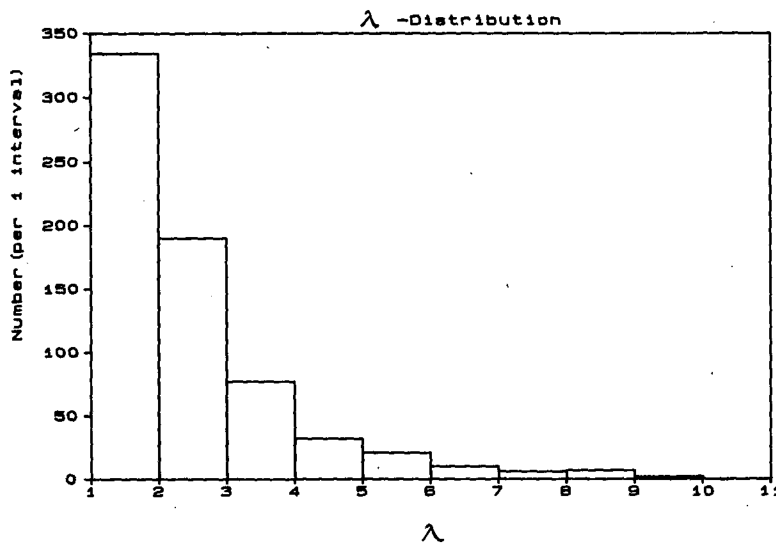

FIG. 6. The λ-distribution.

a spheroid. Figure 6 shows that, for this particular sample, most of the hailstones are conical, as indicated by the high peak of λ between 1 and 2. The number gradually tails off as λ increases. This tailing-off behavior of λ can be fitted by an exponential curve, as will be seen later.

Care should be taken when interpreting the λ-distribution. For example, in this sample we observe that the conical particles are the most numerous kind. This does not necessarily mean that the majority of these hailstones are sharp-apex particles with slender bodies, as the term "conical" may imply. In our definition, a hailstone can be conical even if its horizontal dimension is larger than its vertical dimension, as long as the volume of the upper half is smaller than the lower half as demanded by the low λ-value. For this particular sample, however, most hailstones do have horizontal dimensions smaller than vertical dimensions. Thus, they are mostly conical in the usual sense.

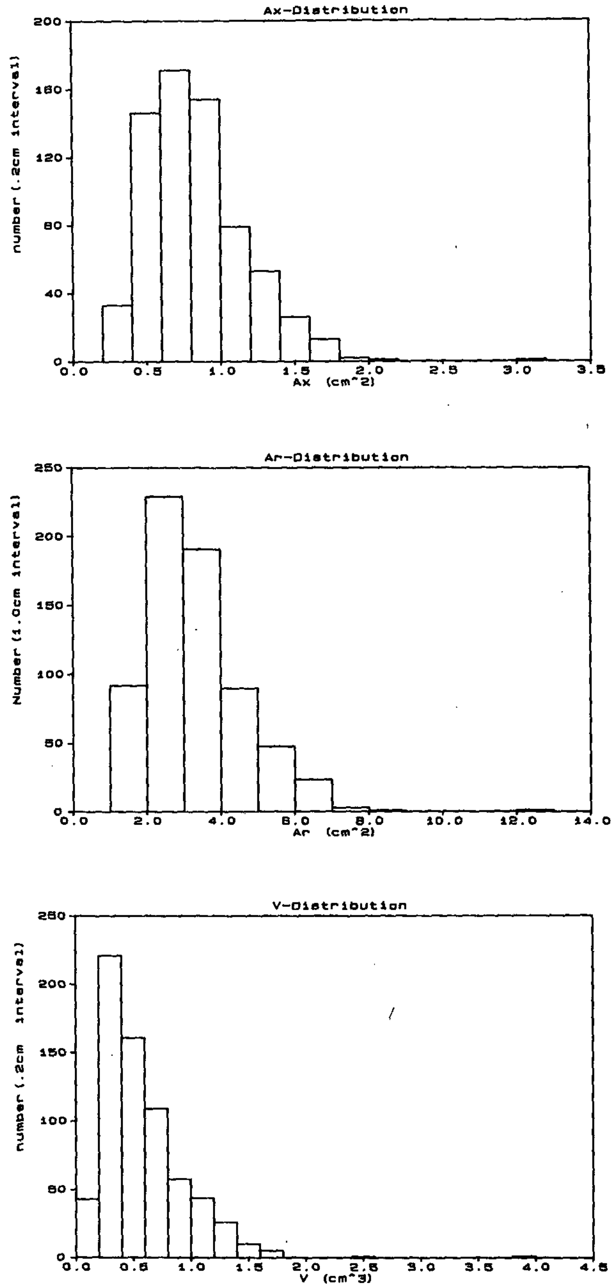

FIG. 7. The distribution of (a) the axial cross-sectional area, (b) the total surface area, and (c) the volume.

The distributions of the axial cross sections A_x, total surface area A_r, and volumes V are shown in Fig. 7. These quantities were calculated from a, c, and λ. In calculating A_r and V, the rotational symmetry of the hail with respect to the z-axis was assumed. This may not be the case, but due to the lack of detail information this assumption is the best one can do at this time. The behavior of these distributions looks very similar to that of the a- and c-distributions; namely, they also look like gamma distributions. This is due to the fact that A_x, A_r and V are basically multiplication of the powers of a and c whose distributions are also of the gamma functional forms.

The empirical relations of these distributions are given in the following. Since these relations were developed for specific parameter-size intervals, the values of the coefficients are only valid when these intervals are used. One common mistake is to take them as continuous distributions while, in fact, they are not.

The following exponential fitting function was used for the λ-distribution:

$$N(\lambda) = A \exp[-\beta(\lambda - 1)] \qquad (8)$$

where A and β are adjustable constants. The distributions of other parameters can be described by a modified gamma function:

$$N(x) = A(x-\gamma)^{\alpha} \exp[-\beta(x-\gamma)] \qquad (9)$$

where A, α, β, and γ are adjustable constants. The values of these constants for this sample of hailstones are given in Table 1. Figure 8 shows the fitted curves of Eqs. (8) and (9) compared with the data points for the parameters λ and c.

There appears to be no strong relation between the shape factor λ and the linear dimensions a and c; i.e., for any given size of a and c, there is no obvious preferred λ value. If we plot the number of hailstones of a particular λ-interval, we obtain a set of curves that more or less resembles the total concentration distributions as in Fig. 3. This again confirms that for each size category, there is no preferred λ value. Since λ is an indicator of the vertical asymmetry, this means that hailstones of different sizes have about an equal chance of vertical asymmetry.

On the other hand, if we plot λ versus $L/2c$ (not shown), we find that there is a small positive correla-

TABLE 1. Values of constants in Eqs. (8) and (9) for various distributions.

Distribution	Interval size	A	α	β	ν	R
λ	1.0	516.97	—	0.74	—	0.9986
a	0.05 cm	1.02×10^7	3.472	28.54	0.150	0.9961
C	0.1 cm	1.94×10^7	4.612	19.523	0.277	0.9930
A_x	0.2 cm^2	6598.8	2.012	4.531	0.201	0.9972
A_r	1.0 cm^2	552.26	2.194	1.226	0.854	0.9964
V	0.2 cm^3	9496.9	1.551	6.192	0.065	0.9923

tion. This means that hailstones with $L/2c$ ratios less than 1.0 are more likely to be conical in shape, while those with ratios near 1.0 are likely to be spheroidal in shape. A plot of λ versus x_m/c would show the same trend. This indicates that conical stones tend to elongate in the vertical direction while spheroidal stones have axes that are nearly equal in length. Of course, it has to be kept in mind that these conclusions are valid only for this hail sample. Other samples may behave differently.

Figure 9 shows the scatter diagrams of A_r and V versus a and c. Both diagrams show some spread with respect to either a or c. A plot of A_x versus a and c shows a similar situation. For A_x and A_r, the percentage scattering of a is only slightly less than that of c, and for practical purpose they can be considered as the same. For the parameter V the scattering of a is smaller than that of c. Thus, for the volume at least, the parameter a seems to be a better characteristic dimension than c.

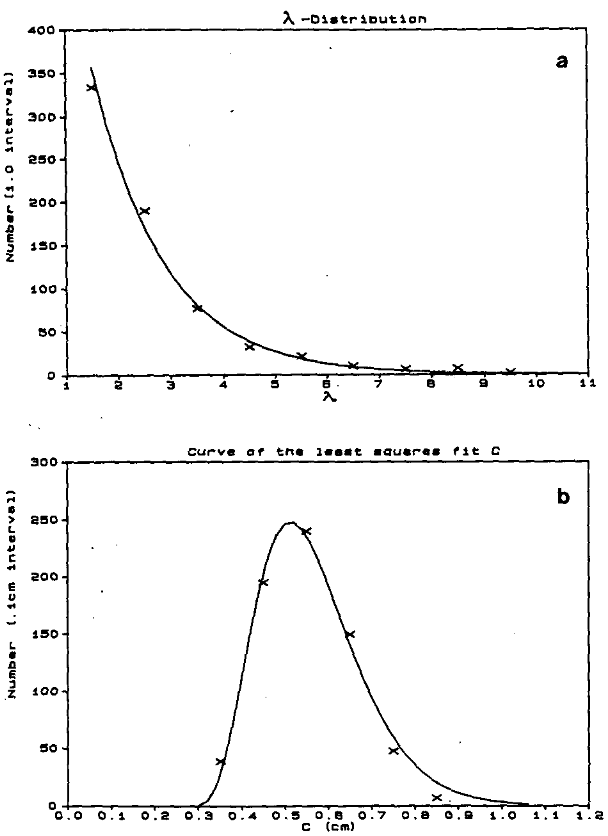

FIG. 8. Fitted curve for (a) the λ-distribution and (b) the c-distribution.

It is of interest to see if there is any other characteristic dimension that is even more representative than a or c as far as A_x, A_r and V are concerned. For this purpose we tried a characteristic radius defined as

$$r_c = f(L/2 + c)/2 \qquad (10)$$

where f is an adjustable factor. For this sample, it turned out that the best value of f is 0.98. Figure 10 shows the plots of A_x, A_r and V versus r_c. It is quite obvious here that all the data points show very coherent behavior. For example, in Fig. 10a, it is seen that all points collapse on the curve for the A_x of a sphere with radius r_c. The degree of scatter is very small. The same can be said for A_r and V. Although the scatter for V is somewhat larger, it is still smaller than that shown in Fig. 9. It is felt, therefore, that r_c may serve as the characteristic dimension as far as the geometrical quantities are concerned. There is no theoretical explanation of the factor $f = 0.98$. It may be another characteristic number that varies from sample to sample.

5. Conclusions

In the foregoing, the results of the shape and size analyses of a sample of 679 hailstones were presented. It is believed that the analytical technique used here yields the most comprehensive and quantitative information about the shape and size distributions of hailstones so far obtained. Using the raw dataset of a, c and λ, one can easily reconstruct fairly closely the shapes and sizes of the original hailstones. The distributions of a, c and λ were shown in Fig. 3. These distributions exhibit simple patterns, being gamma distributions for a and c and exponential for λ. Surface irregularities were ignored in this analysis as we merely wanted to concentrate on the major shape and size characteristics. If rotational symmetry with respect to the fall axis is assumed, then the axial cross-sectional areas, the total surface areas, and the volumes can be

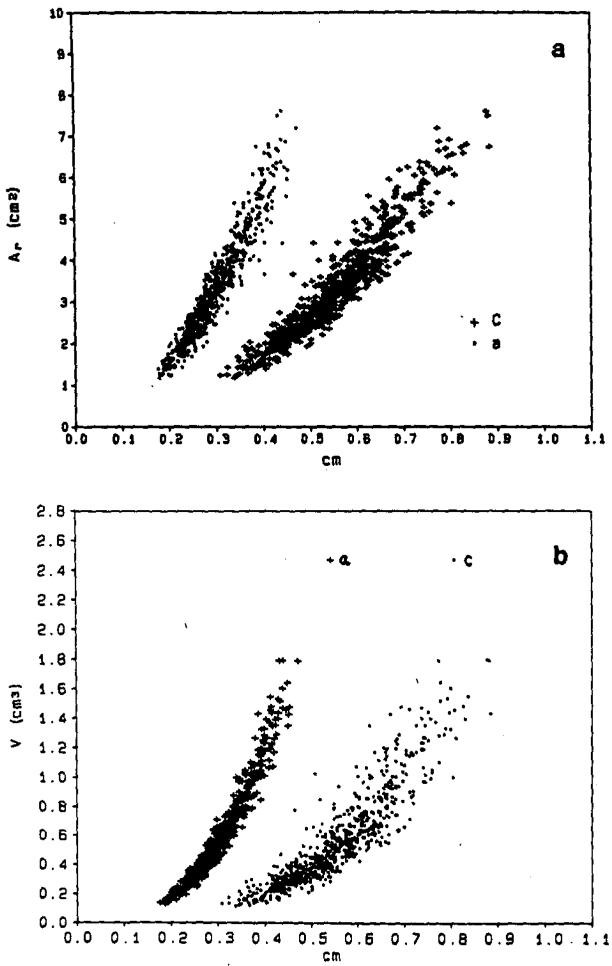

FIG. 9. (a) Relations between A_r and a and c and (b) relations between V and a and c.

calculated and their distributions would be given by Fig. 7. While this assumption is obviously not completely true, this is probably the best one can do at present. A few scatter diagrams (Fig. 9) revealed that a is probably a better representation of the dimension than c. From the pure geometrical point of view, however, the most representative quantity is probably the r_c given in Eq. (10), as revealed by Fig. 10. Despite various assumptions made in the above analysis, we feel that the major shape and size characteristics of this hailstone sample are meaningfully represented by the distributions shown. It is hoped that more hailstone samples can be analyzed so that the similarities and differences between different samples can be studied.

The present analysis should contribute to a better understanding of the basic properties of hailstones. The shapes and sizes of hailstones are connected to the microphysics of their formation and growth. Thus, by studying these distributions, it is possible to gain more insights into hail physics. In addition, by analyzing more hail samples, it may be possible to study the hail climatology. Finally, the results from such an analysis provide detailed information about the shapes and sizes

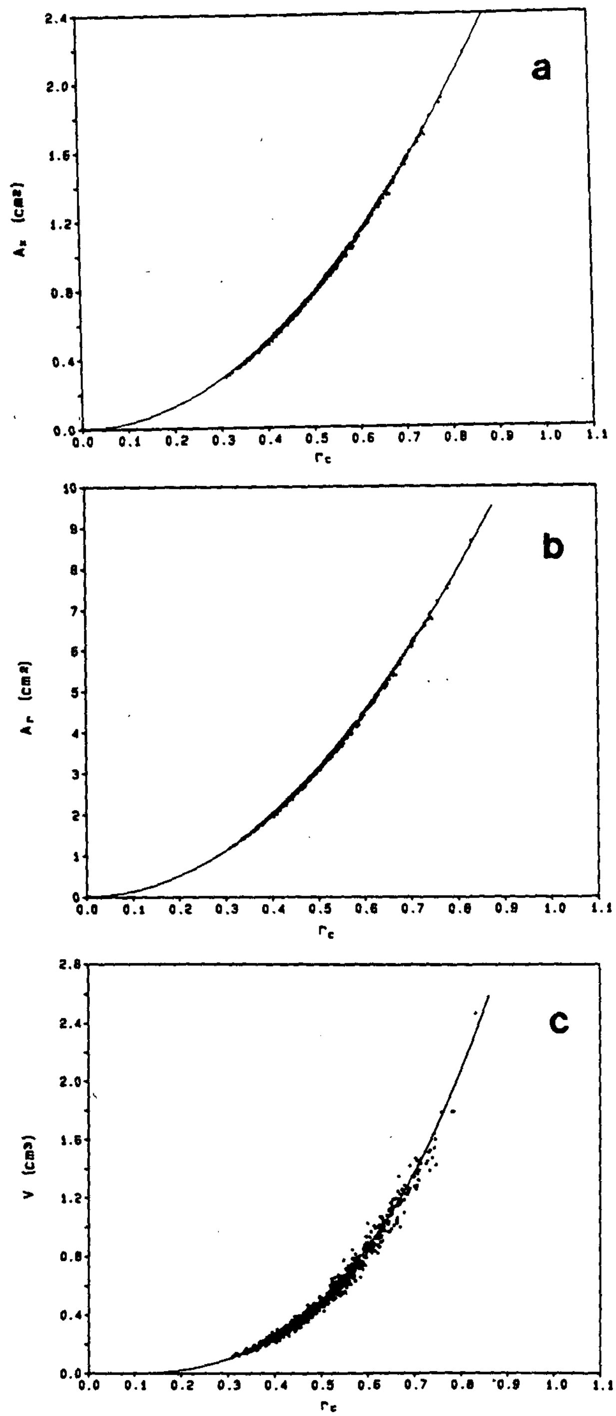

FIG. 10. (a) Plot of A_x versus r_c defined in Eq. (10), (b) plot of A_r versus r_c, and (c) plot of V versus r_c. Solid lines in (a), (b) and (c) are the A_x, A_r and V of a sphere with radius r_c.

of hydrometeors that may be useful to some multiparameter radar techniques (e.g., Seliga and Bringi, 1976; Cherry et al., 1980).

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