Turbulence Structure of the Convective Boundary Layer. Part III: The Vertical Velocity Budgets of Thermals and Their Environment

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ABSTRACT

The dynamics of thermal updrafts and compensating environmental downdrafts in the convective boundary layer are examined using observations from the Phoenix 78 field experiment. Separate vertical velocity budgets are presented for thermal updrafts and environmental downdrafts. These two budgets show the existence of qualitative differences in the forcing of the two legs of the convective circulations.

1. Introduction

The convective circulations which arise when buoyancy is supplied to the lower boundary of the atmosphere are commonly called thermals. These circulations are composed of updrafts and downdrafts with a dominant horizontal scale of approximately 1.5 times the depth of the boundary layer, z_i. The thermal updrafts and compensating environmental downdrafts span the depth of the convective boundary layer (CBL) as either bubbles or plumes extending from the surface layer to the capping inversion (Richter et al. 1974; Hall et al. 1975; Kaimal et al. 1976; Kunkel et al. 1977; Emmitt 1978; Gaynor and Mandics 1978; Caughey and Palmer 1979; Taconet and Weill 1983).

The structure of convective elements in the CBL has been discussed in a number of previous studies (Grant 1965; Manton 1977; Coulman 1978; Lamb 1978; Melling and List 1980; Lenschow and Stephens 1980 and 1982; Greenhut and Khalsa 1982; Khalsa and Greenhut 1985a,b; and others). These studies have been primarily descriptive, focusing on the structure of thermals and their effects on the budgets of turbulence kinetic energy and other second-order turbulence statistics. Conditional sampling was used to divide the CBL into two or more regions with thermal updrafts and environmental downdrafts being of primary interest. Part II of this series (Young 1988b) describes the use of conditional sampling of Phoenix 78 aircraft turbulence data to characterize the turbulence structure of the thermal updrafts and environmental downdrafts of a continental plains site in eastern Colorado.

This form of conditional sampling provides separate turbulence statistics for each of the regions in the two plume conceptual model of the CBL. This conceptual model approximates the vertical circulations of the CBL as two flows, one upward and the other downward, each of which is horizontally homogeneous but is permitted to vary in the vertical. The two plume conceptual model has traditionally been called the top hat model. This model has been popular because of its simplicity. In this study the effects of perturbations on the plume means will be taken into account, yielding a model with far fewer simplifying assumptions than the simplest top hat models which retain only the first order terms. The top hat model has been popular not only for descriptive studies of thermals but also for descriptive and diagnostic studies of convective clouds (Fraedrich 1973; Betts 1973, 1975; Lenschow and Stephens 1980).

Betts (1973, 1975, 1976) developed a top hat model for the budgets of dry and moist static energy which he eventually adapted to the study of CBL convection under fair weather cumulus. The soundings used by Betts (1976) included no vertical velocity data so it was necessary to close the model with assumptions about the heat flux and lateral mass exchange between updrafts and downdrafts in order to solve for the convective mass flux. The use of aircraft observations of turbulence permits direct measurement of the convective mass flux and other turbulence statistics which were neglected, parameterized or dependent variables in Betts' convective budget studies.

Lenschow and Stephens (1980, hereafter LS80), derived a slightly simplified form of the budget equation for plume mean vertical velocity in thermals. Using this equation, they were able to diagnose the sum of

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the lateral mass exchange and pressure effects for oceanic thermal updrafts. In the current study, a similar budget equation is used to diagnose the profiles of this sum for both thermal updrafts and environmental downdrafts at a continental site.

The budget equations for the mean vertical velocity profiles for thermal updrafts and environmental downdrafts are presented in the next section. In the following sections, the physical interpretation of the profiles of the observed and diagnosed terms in these budgets is discussed. Profiles of the measurable terms are determined from the aircraft observations made during the Phoenix 78 CBL experiment. The weather conditions, flight operations and instrumentation of the Phoenix 78 experiment are discussed in Part I of this series (Young 1988a). The conditional sampling method used and the results of a descriptive study of the characteristics of thermal updrafts and environmental downdrafts are given in Part II of this series (Young 1988b).

2. The plume mean budget equation for vertical velocity

In this section, a mathematical model of boundary layer convective motions is developed. This model is based on the two plume conceptual model described in the introduction. The model consists of separate budget equations for the profiles of plume mean vertical velocity in thermal updrafts and in environmental downdrafts. These budgets must account not only for processes occurring within each of the plumes but also for the lateral exchange between the two plumes.

The derivation of these budgets is contained in appendix A. The nomenclature used in this section is defined at the beginning of the derivation. The plume mean vertical velocity budget is derived from the vertical component of the equation of motion. This equation is averaged horizontally over the plume area. The vertical variations in this area, σ, necessitate the use of Liebnitz' rule to move the averages inside the spatial derivatives.

The terms which include the vertical variations of the plume area are called size terms following Lenschow and Stephens (1980). In this formulation, the lateral mass exchange term always acts as a loss term in the vertical velocity budget because mass is exchanged with a flow in the opposite direction. Thus, the various lateral exchange terms which arise in this equation sum to exert a mixing drag on the flow. The resulting budget equation for the profile of plume mean vertical velocity of thermal updrafts is (Eq. (A21) of appendix A)

$$\left\{\frac{z_i}{w_*^2}\right\}\frac{\partial w_*}{\partial t}\left\{\frac{([w]_T-[w])}{w_*}\right\}+\frac{\partial}{\partial z_*}\left\{\frac{([w]_T-[w])^2}{w_*^2}\right\}$$
$$+\left\{\frac{([w]_T-[w])^2}{w_*^2}\right\}\frac{1}{\sigma}\frac{\partial\sigma}{\partial z_*}+\frac{\partial}{\partial z_*}\left\{\frac{[[w]_T'^2]_T}{w_*^2}\right\}$$
$$+\left\{\frac{[[w]_T'^2]_T}{w_*^2}\right\}\frac{1}{\sigma}\frac{\partial\sigma}{\partial z_*}+\left\{\frac{z_i}{w_*^2}\right\}P_T$$
$$+\left\{\frac{z_i}{w_*^2}\right\}M_T-\left\{\frac{[\theta_v]_T-[\theta_v]}{\theta_*}\right\}=0. \quad (1)$$

The equation has been nondimensionalized by the convective scaling parameters: z_i, the mixed layer depth, and w_*, the convective velocity scale which equals $(z_i\overline{w'\theta_{v0}'}g/\overline{T})^{1/3}$. Nondimensional height, z_*, equals z/z_i. There is an analogous budget equation for the profile of plume mean vertical velocity in the environmental downdrafts.

The first term in Eq. (1) is the time rate of change of the plume mean vertical velocity. The next four terms are the components of the vertical divergence of the vertical velocity flux. These four terms arise in the following manner. For each plume, the forcing by the vertical flux of vertical velocity can be divided into two parts, one due to changes in the magnitude of the flux with height (the flux gradient part) and one due to changes in the plume area over which that flux extends (the plume area change part). Each of these two parts may be further subdivided based on the scale of motion causing the flux. This subdivision is into the contribution of plume mean motions and the contribution of subplume motions.

The next term, $(z_i/w_*^2)P_T$, represents the pressure effects. The next to last term $(z_i/w_*^2)M_T$, represents the exchange of vertical velocity between thermal updrafts and their environment due to lateral mass exchange. The last term is the buoyant forcing. The plume mean vertical velocity budget is therefore a balance of five effects: temporal change in the vertical velocity profile, vertical divergence of the vertical velocity flux, lateral exchange of vertical velocity between the thermal updrafts and environmental downdrafts, pressure effects and buoyant forcing.

The time tendency term, the four flux divergence terms and the buoyancy term can all be determined directly from the Phoenix 78 aircraft and Boulder Atmospheric Observatory, BAO, tower turbulence data. Therefore, the sum of the pressure and lateral exchange terms can be determined as a residual. The observed terms in these budgets along with the diagnosed profiles of this residual will be discussed in the next section.

3. Diagnosis of pressure and lateral exchange effects from the plume mean vertical velocity budget

The only time dependent term in the plume mean vertical velocity budgets is the term which includes the rate of change of w_*. Composite time series were used to determine typical and extreme values of this term.

The 1240 LST profiles of this term are shown in Fig. 1 for the thermal updraft and environmental downdraft regions. These profiles have the same shape as the plume mean vertical velocity perturbation profiles be-

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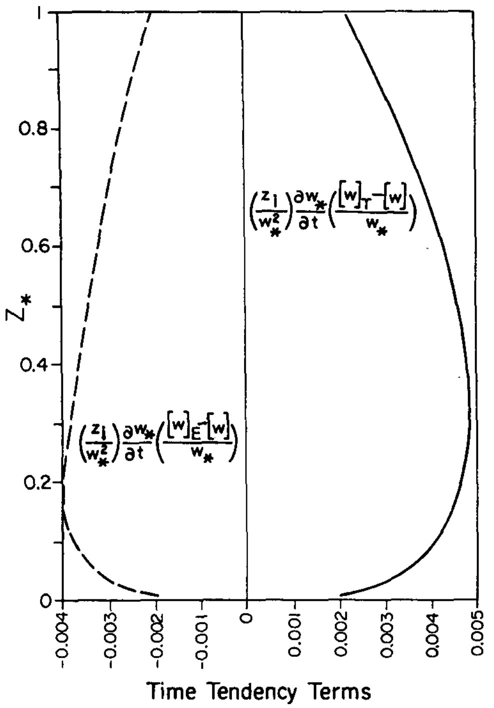

FIG. 1. The vertical profile of the time tendency term in the budgets of plume mean vertical velocity for thermals and their environment. This is the first term in Eq. (1) and the similar equation for the environment. The solid line represents the profile for thermals while the dashed line represents the profile for their environment. The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

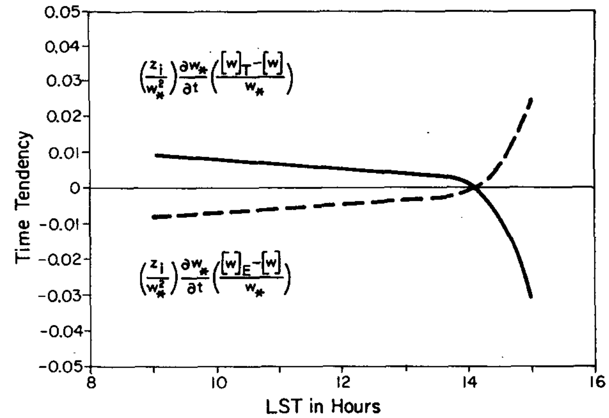

FIG. 2. The composite time series of the maximum amplitude of the time tendency term in the budgets of plume mean vertical velocity for thermals and their environment. This is the first term in equation 1 and the similar equation for the environment. The solid line represents the series for thermals while the dashed line represents the series for their environment. The horizontal coordinate is in hours after midnight local standard time. The vertical scale is nondimensional.

cause that is the only factor in the time tendency term which has height dependence. The shapes of these profiles do not change with time but their amplitudes do. As shown in Fig. 2, the amplitudes at 1240 LST are typical of morning and midday. The terms change sign, however, and increase in amplitude during the afternoon. At their largest, these time tendency terms are an order of magnitude or more smaller than the diagnosed residual in the plume mean vertical velocity budgets. Therefore, the effect of the time tendency term on the diagnosed pressure and lateral mixing effects is small and the analysis can be safely done for 1400 LST when w_* is maximum and the time tendency terms zero.

The plume area change contribution to the divergence of the subplume perturbation part of the vertical velocity flux in updrafts and downdrafts is shown in Fig. 3 while the plume area change contribution to the divergence of the vertical velocity flux by plume mean motions in updrafts and downdrafts is shown in Fig. 4. All of these terms act to accelerate the updrafts and downdrafts in the lower CBL and to decelerate them in the upper CBL. The area change terms are small throughout the CBL.

The profiles of the plume mean buoyant forcing and the two flux gradient parts of the divergence of vertical velocity flux are shown for thermal updrafts and environmental downdrafts in Figs. 5 and 6. Because of the mixed layer scaling, the plume mean buoyant forcing profiles are the same as the plume mean pertur-

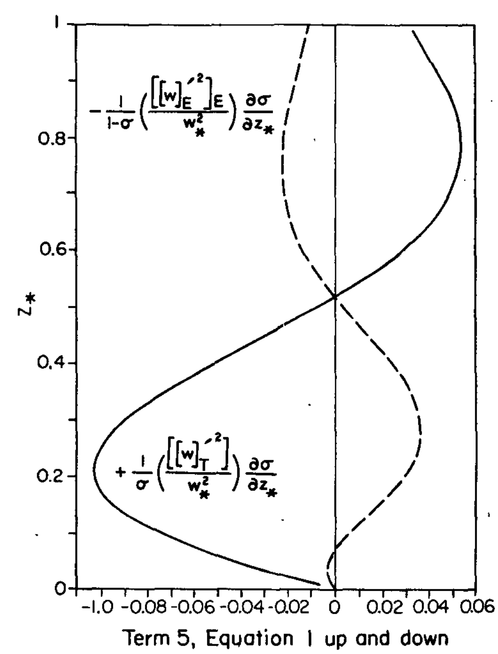

FIG. 3. The plume area change contribution to the divergence of the subplume perturbation part of the vertical velocity flux in updrafts and downdrafts, the fifth term in the plume mean vertical velocity budgets. The solid line represents the profile for thermals while the dashed line represents the profile for their environment. The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

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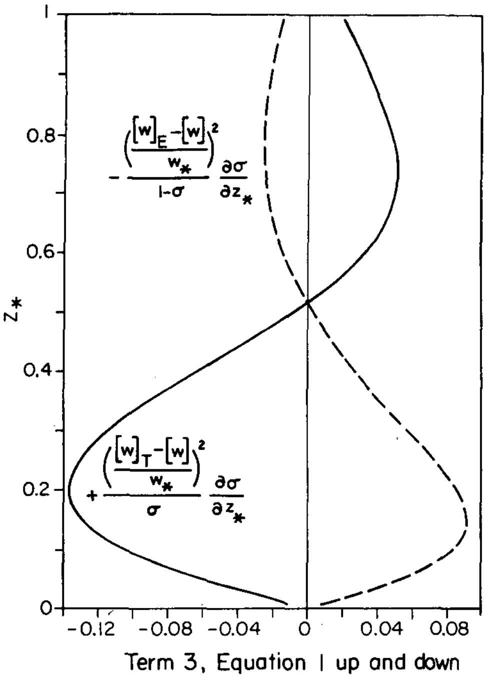

FIG. 4. The plume area change contribution to the divergence of the plume mean part of the vertical velocity flux in updrafts and downdrafts, the third term in the plume mean vertical velocity budgets. The solid line represents the profile for thermals while the dashed line represents the profile for their environment. The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

bation θ_v profiles. Buoyancy acts to accelerate both updrafts and downdrafts in the lower two thirds of the CBL but acts to decelerate both in the top third of the CBL. This lack of buoyant forcing for downdrafts in their upper CBL source region will be discussed below in relation to their initial triggering.

The flux gradient component of the divergence of the subplume perturbation part of the vertical velocity flux in updrafts is large and positive in the lower CBL. In the upper CBL, this term is negative and of the same order as the buoyant forcing. This term acts to decelerate updrafts in their lower CBL source region and accelerate them in the upper CBL. Thus, except for a region in the mid CBL, this term opposes buoyant forcing in updrafts. This term acts similarly to the buoyancy for downdrafts, however, decelerating them in their upper CBL source region and accelerating them in the lower CBL. This term acts against the observed acceleration patterns in both updrafts and downdrafts. Therefore, this term cannot provide the initial impetus for downdrafts in the CBL.

The profiles of the flux gradient component of the divergence of the plume mean part of the vertical velocity flux are of similar form for thermal updrafts and environmental downdrafts. This term is large and positive in the lower CBL. In the upper CBL, this term is negative. The sign change occurs just below the level of maximum amplitude of plume mean vertical velocity. This level is somewhat lower for downdrafts than for updrafts. This flux divergence term is also of the wrong sign to provide the initial impetus for CBL downdrafts.

The subplume perturbation and plume mean contributions to the divergence of vertical velocity flux are of similar magnitude. Thus, both scales of motion must be accounted for in diagnostic models of CBL convection. The subplume perturbation contribution is equally important in the budgets for thermal updrafts and the environmental downdrafts.

Only the unmeasured lateral mass exchange and pressure forcing terms remain to provide the initial impetus for CBL downdrafts. The lateral mass exchange term for a plume involves the loss of air which has momentum of the same sign as the plume and the incorporation of air with momentum of opposite sign into the plume. Thus, the lateral mass exchange must always exert a net drag on the plume. Therefore, the pressure forcing term must provide the initial impetus to CBL downdrafts.

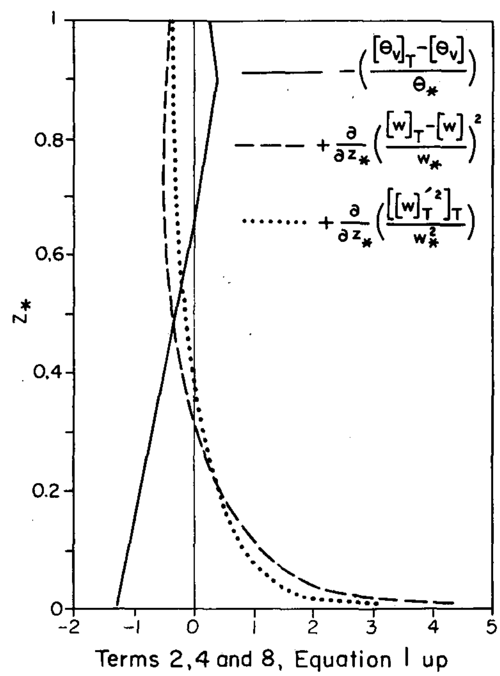

FIG. 5. The profiles of the plume mean buoyant forcing and the two flux gradient parts of the divergence of the vertical velocity flux are shown for thermal updrafts. The solid line represents the profile of the plume mean buoyant forcing of the vertical velocity budget for thermals, term 8 of Eq. (1). The dashed line represents the profile for the plume mean contribution to the flux gradient part of the divergence of vertical velocity flux, term 2 of Eq. (1). The dotted line represents the profile for the subplume perturbation contribution to the flux gradient part of the divergence of vertical velocity flux, term 4 of Eq. (1). The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

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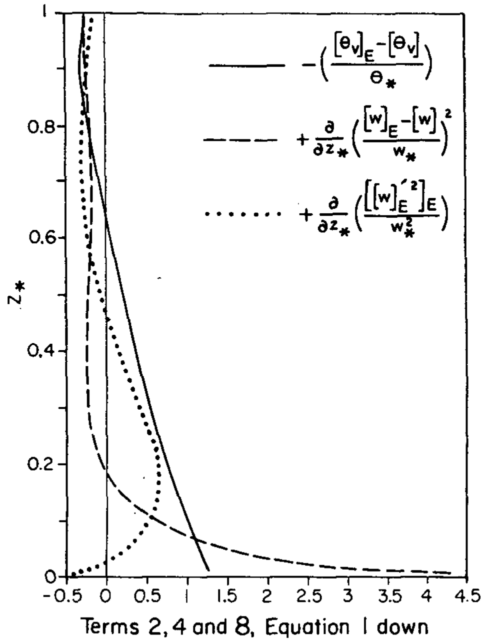

FIG. 6. The profiles of the plume mean buoyant forcing and the two flux gradient parts of the divergence of the vertical velocity flux are shown for environmental downdrafts. The solid line represents the profile of the plume mean buoyant forcing of the vertical velocity budget of the environment. The dashed (dotted) line represents the profile for the plume mean (subplume perturbation) contribution to the flux gradient part of the divergence of vertical velocity flux. The vertical coodinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

The vertical velocity budgets of thermal updrafts and environmental downdrafts are shown in Figs. 7 and 8, respectively. Following LS80 related terms in each budget have been combined. The resulting budgets each have four terms: acceleration, buoyancy, plume size and a residual. The acceleration term is the sum of the two flux divergence terms in Eq. 1. The buoyancy term is the same as in Eq. 1 and the plume size term is the sum of the two terms in Eq. 1 that depended on the vertical gradient of plume area. The residual is the sum of the lateral mass exchange and pressure effects. Lenschow and Stephens used the phrase "edge effects" when referring to the lateral mass exchange. Their formulation includes a somewhat different partitioning of the plume size and edge effects which permits the latter term to assume negative values.

The buoyancy forcing on thermal updrafts is upwards in the lower two thirds of the CBL and more weakly downwards in the upper CBL. Lenschow and Stephens (1980) observed buoyant forcing of approximately twice this magnitude in the lower CBL and near zero in the upper CBL. Their conditional sampling technique was designed to accept only the cores of thermals rather than the entire updraft area as in the current study. Therefore, the difference in the results between the two studies suggests that the cores of thermals are more buoyant than their outer reaches and may experience less buoyant deceleration in the upper CBL.

The plume size term acts to accelerate thermals in the lower CBL where the updraft area decreases with height, and to decelerate thermals in the upper CBL where the updraft area increases with height. This is generally the smallest of the terms. Lenschow and Stephens report this term as a small loss at all heights. The thermal size profile upon which the calculation of this term is based is quite sensitive to the conditional sampling criteria used so the inter-experimental differences are not surprising.

The acceleration term in the vertical velocity budget of thermal updrafts is a loss in the lower CBL and a gain in the upper CBL. The profile is similar to that observed by LS80 with a zero crossing near one third of the boundary layer depth. The acceleration term had a somewhat greater magnitude in the current study. This difference arises from the relative weakness of the thermal updrafts observed by LS80.

Both studies observed an extensive zone in the mid-CBL where thermal updrafts were decelerating despite

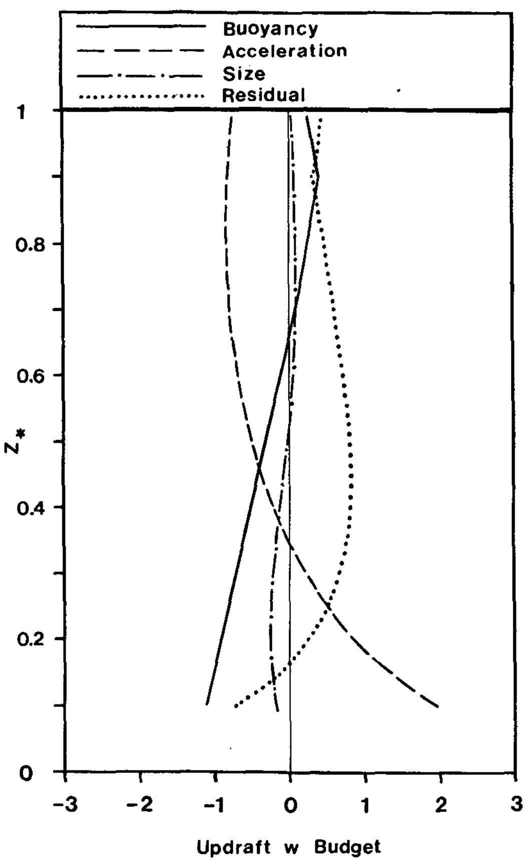

FIG. 7. The plume mean vertical velocity budget for thermal updrafts. The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

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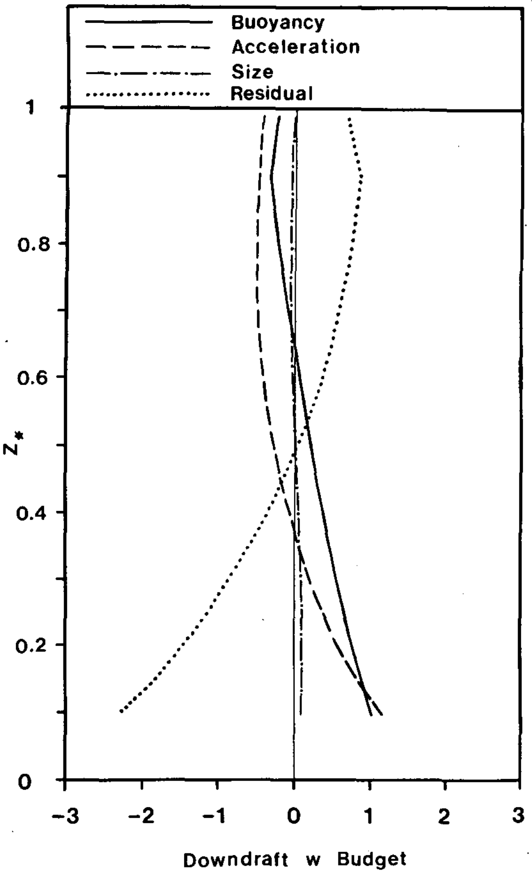

FIG. 8. The plume mean vertical velocity budget for environmental downdrafts. The vertical coordinate is z_*, the fraction of the depth of the convective boundary layer. The horizontal scale is nondimensional.

continued buoyant forcing and negligible plume size effects. The residual term, the combined effects of pressure forces and lateral mass exchange account for this deceleration. The profiles of this term found in LS80 and the current study are similar. Both profiles indicate momentum loss to this term at all levels above the surface layer. A local minimum in the magnitude of this drag is found in the upper CBL and a local maximum in the lower CBL. Neither of these extremes occur at the boundaries of the vertical domain studied. These features are located somewhat lower in the profile from LS80 than in that of the present study. Both studies also show a sharp change of sign to upwards forcing in the surface layer.

The low-level positive contribution by the residual can be interpreted in two ways. First it could be the result of errors arising from the fitting of turbulence profile formulae which approach the free convective limit as z_* approaches zero. Both studies used this formulation down to the lower limits of observation as is traditional in mixed layer studies. The second interpretation is that the pressure forces are helping to trigger thermals. This would occur if thermals were triggered by the convergence of the surface layer outflows of two or more environmental downdrafts. Radar studies of the horizontal flow patterns in the CBL strongly suggest that this may be the case (Kropfli and Hildebrand 1980).

The vertical velocity budget for environmental downdrafts is qualitatively different from that for thermal updrafts. This qualitative difference can be seen by comparing Figs. 7 and 8. The behavior of thermal updrafts can be broadly explained as acceleration and deceleration in response to buoyant forcing. Environmental downdrafts, on the other hand, accelerate downwards against buoyant forcing in the upper CBL where they form and decelerate against buoyant forcing as they approach the surface. Only in the mid-CBL do environmental downdrafts accelerate in the direction of their buoyant forcing. The size term is quite small throughout the CBL but generally acts against the observed acceleration. Therefore, the residual term acts in the direction of the observed acceleration throughout most of the depth of the CBL. In the upper CBL, the residual forcing is downwards indicating that the pressure forcing is overcoming the drag from the lateral mass exchange which always acts against the flow in this formulation. In the lower CBL, the residual acts as a drag on environmental downdrafts, acting against both buoyancy and vertical transport of momentum to produce the observed deceleration as the downdrafts approach the surface. From this formulation, it is impossible to tell whether the pressure forcing acts to aid or hinder lateral mass exchange in producing this drag. Lenschow and Stephens (1980) did not present a vertical velocity budget for environmental downdrafts so intercomparison is not possible.

4. Conclusions

Observations from the PHOENIX 78 CBL field experiment show that the fluxes by subplume perturbations are as important as the fluxes by the plume mean motions in the budgets of plume mean vertical velocity. Hence, it is necessary to include the subplume fluxes in any diagnostic study of the dynamics of thermal updrafts or environmental downdrafts.

Diagnostic use of the plume mean vertical velocity budgets for thermal updrafts and environmental downdrafts shows qualitative differences in the forcing of the two legs of the convective circulation. The acceleration of thermal updrafts is generally in phase with buoyant forcing while the acceleration of environmental downdrafts is generally out of phase with buoyant forcing. Therefore, pressure effects dominate and counteract buoyancy at most levels in forcing environmental downdrafts.

It should be noted that errors in the other terms will be reflected in the residual term, the sum of the pressure forcing and lateral mixing effects. These errors have

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the least relative effect on the residual at the levels where the residual is largest. Therefore, the conclusions are based on the results from these levels of greatest accuracy.

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APPENDIX A

Derivation of the Plume Mean Vertical Velocity Budgets for Thermal Updrafts and Environmental Downdrafts

The goal of this Appendix is to derive a form of the vertical velocity budget which applies to the plume mean for thermal updrafts and an analogous budget which applies to the plume mean for environmental downdrafts. The derivation of the budget of plume mean vertical velocity for thermal updrafts consists of averaging the vertical equation of motion over the horizontal region covered by thermal updrafts. For this study, the averaging operators must be designed for use on data series collected by aircraft during horizontal flight legs. Therefore, these are horizontal averages along the flight path rather than vertical averages or volume averages.

Following the conventional approach (Manton 1977; Greenhut and Khalsa 1982), the plume mean of a variable X for thermal updrafts is defined using an indicator function:

$$[X]_T = \frac{\sum_{t=1}^{N} I^+(t)X(t)}{\sum_{t=1}^{N} I^+(t)}. \tag{A1}$$

Here $I^+(t)$ is the function that indicates whether the data point collected at time t meets the criteria for being part of a thermal updraft as defined in Part II of this series; $I^+(t) = 1$ in thermal updrafts and $I^+(t) = 0$ in environmental downdrafts. N is the number of data points in the series. $X(t)$ is the observed value of the variable at time t.

The instantaneous local perturbations from the plume mean for thermal updrafts are computed by subtraction of that plume mean from the instantaneous local values of the variable:

$$[X]'_T = X(t) - [X]_T. \tag{A2}$$

Similarly the plume mean for environmental downdrafts is defined as:

$$[X]_E = \frac{\sum_{t=1}^{N} I^-(t)X(t)}{\sum_{t=1}^{N} I^-(t)} \tag{A3}$$

where $I^-(t)$ is the function which indicates whether the data point collected at time t meets the criteria for being part of an environmental downdraft as defined in Part II of this series. $I^-(t) = 1$ in environmental downdrafts and $I^-(t) = 0$ in thermal updrafts. Each data point is included in the plume mean for either thermal updrafts or environmental downdrafts.

The instantaneous local perturbations from the plume mean for environmental downdrafts are computed analogously to those for thermal updrafts:

$$[X]'_E = X(t) - [X]_E. \tag{A4}$$

The fractional area coverage by thermal updrafts and environmental downdrafts are

$$\sigma = \frac{\sum_{t=1}^{N} I^+(t)}{N} \tag{A5}$$

and

$$1 - \sigma = \frac{\sum_{t=1}^{N} I^-(t)}{N} \tag{A6}$$

respectively.

The overall average of a variable X is defined in the usual fashion:

$$[X] = \frac{\sum_{t=1}^{N} X(t)}{N}. \tag{A7}$$

Using the definitions above, the following equivalent expression for the overall average can be derived.

$$[X] = \sigma[X]_T + (1 - \sigma)[X]_E. \tag{A8}$$

Values of each variable were available in series of approximately 10^4 points for each of 45 horizontal flight legs. Plume means and perturbations were calculated separately for each of these horizontal flight legs using the formulae defined above. This procedure yielded one value of each plume mean for each flight leg. Because the individual flight legs were at a number of different levels, (z_* from 0.1 to 1.0) profiles of the plume means were produced. Some of these profiles were presented in Part II. Under the assumptions of mixed layer scaling, these data represent points on universal profiles. In the context of mixed layer similarity, the term universal means that a quantity is not a function of time as long as the similarity assumptions hold. Scatter in the profiles is a result of sampling errors

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(Lenschow and Stankov 1986) and departures from mixed layer similarity. The effect of these errors on the results of the Phoenix 78 experiment are discussed in more detail in Part I. Following standard practice (Caughey and Palmer 1976; Lenschow et al. 1980; and Lenschow and Stephens 1980) analytic curves were fit by eye to these profiles. It is these fitted curves rather than the raw data that are used in the budget equations because of the need to compute vertical derivatives of the profiles. These curves were subject to the consistency constraints derived below.

If the variable X in Eq. (A1) is a product, WZ, then from the definitions above:

$$[WZ]_T = \frac{\sum_{t=1}^{N} I^+(t)W(t)Z(t)}{\sum_{t=1}^{N} I^+(t)} \qquad \text{(A9)}$$

or equivalently

$$[WZ]_T = \frac{\sum_{t=1}^{N} I^+(t)\{[W]_T + [W]'_T\}\{[Z]_T + [Z]'_T\}}{\sum_{t=1}^{N} I^+(t)}. \qquad \text{(A10)}$$

Equation A10 can be simplified by taking the quantities which do not depend on t outside of the summation over t and by eliminating the terms which sum to zero to get Eq. (A11). This operation is equivalent to the standard Reynolds averaging.

$$[WZ]_T = [W]_T[Z]_T + \frac{\sum_{t=1}^{N} I^+(t)[W]'_T[Z]'_T}{\sum_{t=1}^{N} I^+(t)} \qquad \text{(A11)}$$

or equivalently

$$[WZ]_T = [W]_T[Z]_T + [[W]'_T[Z]'_T]_T. \qquad \text{(A12)}$$

Equation (A12) states that the plume mean of a product equals the sum of the contribution by the product of the plume means and the contribution by the plume mean of the product of the subplume perturbations. This equation was derived for thermal updrafts. An analogous equation can be derived for environmental downdrafts. This equation imposes a constraint on the consistency of the algebraic formulae, which can be fit to the profiles of the observed values of $[WZ]_T$, $[W]_T$, $[Z]_T$ and $[[W]'_T[Z]'_T]_T$.

The derivation of the budget equation for the plume mean vertical velocity for thermal updrafts starts from the instantaneous local w equation. Molecular diffusion is ignored because it is negligibly small on the boundary-layer convective scale. The flux form of the equation is used so that Leibnitz' rule can be used to move the averaging operator inside the vertical derivatives.

$$\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{g}{\theta_0}\{\theta_v - [\theta_v]\}. \qquad \text{(A13)}$$

Equation (A13) relates the local change in w to the three components of the flux divergence, the vertical component of the pressure gradient force and the buoyancy force. The operator defined in Eq. (A1) is applied to average Eq. (A13) over the region of thermals.

$$\left[\frac{\partial w}{\partial t}\right]_T + \left[\frac{\partial uw}{\partial x}\right]_T + \left[\frac{\partial vw}{\partial x}\right]_T + \left[\frac{\partial w^2}{\partial z}\right]_T = -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T + \left[\frac{g}{\theta_0}(\theta_v - [\theta])\right]_T. \qquad \text{(A14)}$$

Equation (A2) is used to expand each variable in Eq. (A14) into a perturbation and an average part. Equation (A15) results.

$$\left[\frac{\partial\{[w]_T + [w]'_T\}}{\partial t}\right]_T + \left[\frac{\partial\{[u]_T + [u]'_T\}\{[w]_T + [w]'_T\}}{\partial x}\right]_T + \left[\frac{\partial\{[v]_T + [v]'_T\}\{[w]_T + [w]'_T\}}{\partial y}\right]_T + \left[\frac{\partial\{[w]_T + [w]'_T\}^2}{\partial z}\right]_T = -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T + \left[\frac{g}{\theta_0}\{[\theta_v]_T + [\theta_v]'_T - [\theta_v]\}\right]_T. \qquad \text{(A14)}$$

Next the products in Eq. (A14) are expanded and the average operators are taken inside the time derivatives. The time derivative and averaging operators are commutative as long as universal profiles are used.

$$\frac{\partial[[w]_T]_T}{\partial t} + \frac{\partial[[w]'_T]_T}{\partial t} + \left[\frac{\partial\{[u]_T[w]_T + [u]'_T[w]'_T + [u]_T[w]'_T + [u]'_T[w]_T\}}{\partial x}\right]_T + \left[\frac{\partial\{[v]_T[w]_T + [v]'_T[w]'_T + [v]_T[w]'_T + [v]'_T[w]_T\}}{\partial y}\right]_T + \left[\frac{\partial\{[w]_T^2 + [w]'^2_T + 2[w]_T[w]'_T\}}{\partial z}\right]_T = -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T + \left[\frac{g}{\theta_0}\{[\theta_v]_T + [\theta_v]'_T - [\theta_v]\}\right]_T. \qquad \text{(A15)}$$

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Distribution of the derivatives over the sums transforms Eq. (A15) into Eq. (A16), which permits simplification by use of various properties of the averaging operators.

$$\underbrace{\frac{\partial[[w]_T]_T}{\partial t}}_{\substack{\text{avg. of}\\ \text{avg. = avg.}}} + \underbrace{\frac{\partial[[w]'_T]_T}{\partial t}}_{\substack{\text{0 by (A1)}\\ \text{and (A2)}}} + \underbrace{\left[\frac{\partial[u]_T[w]_T}{\partial x}\right]_T}_{\text{0 by H.H.}} + \left[\frac{\partial[u]'_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[u]_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[u]'_T[w]_T}{\partial x}\right]_T$$

$$+ \underbrace{\left[\frac{\partial[v]_T[w]_T}{\partial y}\right]_T}_{\text{0 by H.H.}} + \left[\frac{\partial[v]'_T[w]'_T}{\partial y}\right]_T + \left[\frac{\partial[v]_T[w]'_T}{\partial y}\right]_T + \left[\frac{\partial[v]'_T[w]_T}{\partial y}\right]_T + \left[\frac{\partial[w]^2_T}{\partial z}\right]_T + \left[\frac{\partial[w]'^2_T}{\partial z}\right]_T$$

$$+ 2\left[\frac{\partial[w]_T[w]'_T}{\partial z}\right]_T = -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T + \frac{g}{\theta_0}\Big\{\underbrace{[[\theta_v]_T]_T}_{\substack{\text{avg. of}\\ \text{avg. = avg.}}} + \underbrace{[[\theta_v]'_T]_T}_{\substack{\text{0 by (A1)}\\ \text{and (A2)}}} - \underbrace{[[\theta_v]]_T}_{\substack{\text{avg. of}\\ \text{avg. = avg.}}}\Big\} \quad \text{(A16)}$$

The notation 0 by (A1) and (A2) means that the definitions of means and perturbations contained in Eqs. (A1) and (A2) can be used to show that these terms are zero as in Reynolds averaging. 0 by H.H. means zero by assumption of horizontal homogeneity. Avg. of Avg. = Avg. means that equation A1 can be used to show that the outer averaging operator has no effect because its operand is constant over its domain. Thus, equation A16 simplifies to Eq. (A17).

$$\frac{\partial[w]_T}{\partial t} + \left[\frac{\partial[u]'_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[u]_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[u]'_T[w]_T}{\partial x}\right]_T + \left[\frac{\partial[v]'_T[w]'_T}{\partial y}\right]_T$$

$$+ \left[\frac{\partial[v]_T[w]'_T}{\partial y}\right]_T + \left[\frac{\partial[v]'_T[w]_T}{\partial y}\right]_T + \left[\frac{\partial[w]^2_T}{\partial z}\right]_T + \left[\frac{\partial[w]'^2_T}{\partial z}\right]_T + 2\left[\frac{\partial[w]_T[w]'_T}{\partial z}\right]_T$$

$$= -\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T + \frac{g}{\theta_0}\{[\theta_v]_T - [\theta_v]\}. \quad \text{(A17)}$$

Moving the averaging operator inside the vertical derivatives requires the use of Liebnitz' rule because the area covered by the averaging operator, σ, varies with z. Equation (A18) results.

$$\underbrace{\frac{\partial[w]_T}{\partial t}}_{\text{A}} + \underbrace{\left[\frac{\partial[u]'_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[v]'_T[w]'_T}{\partial y}\right]_T}_{\text{B}} + \underbrace{\left[\frac{\partial[u]_T[w]'_T}{\partial x}\right]_T + \left[\frac{\partial[v]_T[w]'_T}{\partial y}\right]_T}_{\text{C}}$$

$$+ \underbrace{\left[\frac{\partial[u]'_T[w]_T}{\partial x}\right]_T + \left[\frac{\partial[v]'_T[w]_T}{\partial y}\right]_T}_{\text{D}} + \underbrace{\frac{\partial[[w]^2_T]_T}{\partial z}}_{\text{E}} + \underbrace{\frac{[[w]^2_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}}_{\text{F}} + \underbrace{\frac{[[w]^2_T]_B}{\sigma}\frac{\partial\sigma}{\partial z}}_{\text{G}} + \underbrace{\frac{\partial[[w]'^2_T]_T}{\partial z}}_{\text{H}}$$

$$+ \underbrace{\frac{[[w]'^2_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}}_{\text{I}} + \underbrace{\frac{[[w]'^2_T]_B}{\sigma}\frac{\partial\sigma}{\partial z}}_{\text{J}} + 2\underbrace{\frac{\partial[[w]_T[w]'_T]_T}{\partial z}}_{\substack{\downarrow\\ \text{0 by (A1)}\\ \text{and (A2)}}} + 2\underbrace{\frac{[[w]_T[w]'_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}}_{\substack{\downarrow\\ \text{0 by (A1)}\\ \text{and (A2)}}} + 2\underbrace{\frac{[[w]_T[w]'_T]_B}{\sigma}\frac{\partial\sigma}{\partial z}}_{\text{K}}$$

$$= -\underbrace{\left[\frac{1}{\rho}\frac{\partial p}{\partial z}\right]_T}_{\text{L}} + \underbrace{\frac{g}{\theta_0}\{[\theta_v]_T - [\theta_v]\}}_{\text{M}}. \quad \text{(A18)}$$

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Where [$]_B$ indicates the average along the boundary between the updraft region and the downdraft region as defined in Liebnitz' rule. The differences that occur between the use of the Liebnitz' rule with integral operators and its use with averaging operators should be noted. When used to move an integral operator inside a derivative, Liebnitz' rule gives rise to a boundary term such as

$$\frac{[[w]'^2_T]_B}{\sigma}\frac{\partial\sigma}{\partial z}.$$

However, when Liebnitz' rule is used to move an averaging operator inside a derivative another similar term arises via the chain rule. This additional term includes an area average as in

$$\frac{[[w]'^2_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}.$$

The terms have the following physical interpretations:

A storage
B the subplume scale lateral flux into the thermal region from the environmental region.
C the advection of cross plume gradients by the plume mean horizontal flow.
D the transport of plume mean w across the boundary between the thermal region and the environmental region by subplume scale horizontal motions. The divergence theorem is used to arrive at this interpretation.
E the flux gradient part of the divergence of the flux caused by plume mean motions.
F the plume area gradient part of the divergence of the flux caused by plume mean motions.
G the transport of plume mean w across the boundary between the thermal updraft region and the environmental downdraft region by the plume mean w.
H the flux gradient part of the divergence of the flux caused by subplume scale motions.
I the plume area gradient part of the divergence of the flux caused by subplume scale motions.
J the transport of subplume scale w across the boundary between the thermal updraft region and the environmental downdraft region by subplume scale w.
K the transport of plume mean w across the boundary between the thermal region and the environmental region by subplume scale vertical motions.
L pressure effects
M buoyancy

Terms E and F can be simplified by using Eq. (A1) to show that the outer averaging operator has no effect. The terms B, C, D, G, J and K are all part of the mixing between thermal updrafts and environmental downdrafts. The individual mixing terms cannot be measured directly from the aircraft data, so they will be combined into a single unknown term, M_T, on the left-hand side of Eq. (A19). Term L, the pressure effects, cannot be measured either and so will be represented by an unknown, P_T, also on the left-hand side of Eq. (A19).

$$\frac{\partial[w]_T}{\partial t}+\frac{\partial[w]_T^2}{\partial z}+\frac{[w]_T^2}{\sigma}\frac{\partial\sigma}{\partial z}+\frac{\partial[[w]'^2_T]_T}{\partial z}+\frac{[[w]'^2_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}+P_T+M_T-\frac{g}{\theta_0}\{[\theta_v]_T-[\theta_v]\}=0. \quad \text{(A19)}$$

This result is a budget equation for the plume mean vertical velocity in thermal updrafts in terms of observed profiles of the four components of the flux divergence, the observed profile of buoyancy and the unmeasured profiles of pressure effects and mixing between thermal updrafts and environmental downdrafts.

Now noting that $[w] \ll [w]_T$ for the fair weather CBL so that $[w]_T \cong [w]_T - [w]$, Eq. (A19) becomes Eq. (A20). This transformation is necessary for use of the budget with aircraft data which contain no information on the large scale vertical motion.

$$\frac{\partial([w]_T-[w])}{\partial t}+\frac{\partial([w]_T-[w])^2}{\partial z}+\frac{([w]_T-[w])^2}{\sigma}\frac{\partial\sigma}{\partial z}+\frac{\partial[[w]'^2_T]_T}{\partial z}+\frac{[[w]'^2_T]_T}{\sigma}\frac{\partial\sigma}{\partial z}+P_T+M_T-\frac{g}{\theta_0}\{[\theta_v]_T-[\theta_v]\}=0. \quad \text{(A20)}$$

This equation can be normalized using mixed layer scaling and the fact that w_*, θ_* and z_i change only with time and not with height to get Eq. (A21). This normalization is necessary so that aircraft turbulence data collected on different days can be combined for use in the solution.

$$\left\{\frac{z_i}{w_*^2}\right\}\frac{\partial w_*}{\partial t}\left\{\frac{([w]_T-[w])}{w_*}\right\}+\frac{\partial}{\partial z_*}\left\{\frac{([w]_T-[w])^2}{w_*^2}\right\}+\left\{\frac{([w]_T-[w])^2}{w_*^2}\right\}\frac{1}{\sigma}\frac{\partial\sigma}{\partial z_*}+\frac{\partial}{\partial z_*}\left\{\frac{[[w]'^2_T]_T}{w_*^2}\right\}+\left\{\frac{[[w]'^2_T]_T}{w_*^2}\right\}\frac{1}{\sigma}\frac{\partial\sigma}{\partial z_*}+\left\{\frac{z_i}{w_*^2}\right\}P_T+\left\{\frac{z_i}{w_*^2}\right\}M_T-\left\{\frac{[\theta_v]_T-[\theta_v]}{\theta_*}\right\}=0. \quad \text{(A21)}$$

This is the final form of the budget equation for the plume mean vertical velocity in thermals. There is an

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analogous equation for the plume mean vertical velocity in environmental downdrafts, $[w]_E$.

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