Global Behavior of the Evolution of a Rossby Wave Packet in Barotropic Flows on the Earth's δ-Surface*

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ABSTRACT

A concept of the δ-approximation for the earth's surface has been introduced. Using the Rossby wave packet approximation and the WKB method, the evolution of a single geostrophic synoptic disturbance system has been further studied on the δ-surface of the earth. The global behavior of the structural changes of the wave packet due to the zonal, the meridional and the asymmetric basic currents and the variety of the topography on the δ-surface of the earth have been discussed and compared with those on the earth's β-plane by using the WKB phase plane, i.e., the wave packet's local wavenumber phase plane. The results show that the governing system on the earth's δ-surface may be dynamically different from that on the earth's β-plane. Moreover, the wave packet structural vacillation has been found on both the β-plane and the δ-surface. Wave packet structural vacillation is characterized by the time-periodic changes of the wave packet's structure. Both the tilt and the spatial scales of the packet will evolve periodic changes simultaneously. The wave packet structural vacillations are also characterized by the closed WKB trajectories on the WKB phase plane. The results show that in the presence of the asymmetric basic currents, the WKB trajectories on the WKB phase plane appear simply to be elliptical, e.g., in the case of a southwesterly jet, or hyperbolic, e.g., in the case of a southeasterly jet. The results suggest that it is possible for the packet structural vacillations to exist in the presence of some asymmetric basic currents, e.g., a southwesterly jet. The behaviors related to topography in various distributions have also been discussed. It has been demonstrated that the quadratic east–west oriented topography modifies only the δ-effect, and that with some topographies, e.g., convex topographies, the wave packet structural vacillation can also exist. In some cases, however, the behaviors of the evolution of a packet will be qualitatively different on the earth's δ-surface from those on the earth's β-plane. For example, in the meridional basic current, or on the north–south oriented topographies, only on the δ-surface of the earth do there exist such wave packet structural vacillations. On the other hand, in some cases, the wave packet solutions have been obtained on both the β-plane and the δ-surface. The wave packet vacillation suggests a possible mechanism of vacillations observed in the atmosphere.

1. Introduction

There have been many studies on the subject of Rossby waves (e.g., Phillips, 1965; Platzman, 1968; Pedlosky, 1979; Held, 1983; Karoly and Hoskins, 1982). As is well known, the propagation properties of the Rossby wave are related to the linear change of the Coriolis parameter with respect to latitude; this is called the β-effect. Attention has been focused primarily on the dynamics of the disturbance system on the so-called β-plane of the earth, which is the linear approximation of the earth's Coriolis parameter. For example, there have been numerous studies on the wave-zonal flow interaction on the β-plane. However, so far, few have considered the effect of the variation of β with respect to latitude (Yang, 1987) except on a spheric geometry (e.g., Hoskins and Karoly, 1981; Karoly and Hoskins, 1982; Karoly, 1983). Almost no one has considered the dynamics of the disturbance system on the approximate surface of the earth in which the effect of the second derivative of the Coriolis parameter with respect to latitude has been included. In what follows, we will introduce the δ-surface approximation of the earth's surface by considering the second derivative of the Coriolis parameter with respect to latitude. We propose to study the dynamics of a single geostrophic disturbance system on such a surface, comparing it with that on the β-plane.

Recently, the author (Yang, 1987) discussed the structural changes of a Rossby wave packet due to the variation of β with respect to latitude, the asymmetric basic currents and the variety of topography. The results show that the δ-effect, which is referred to the effect of the second derivative of the Coriolis parameter with respect to latitude on the structural change of the Rossby wave packet, will lengthen the packet's latitudinal scale and cause the westward tilting trough line to lean toward the Y-axis, i.e., the north. However, there are different effects on the wave packet's structure in the various asymmetric basic currents, depending

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on the different positions of the wave packet relative to the basic current and the variety of topography. The results show that when the Rossby wave packet is located on the left (right) side of a southwesterly jet its longitudinal scale (latitudinal scale) will lengthen and its latitudinal scale (longitudinal scale) will shrink, while the westward tilting trough line will tilt more westward (toward the Y-axis). Linearly sloping topographies will not affect the structure of the Rossby wave packet. However, nonlinearly distributed topographies do affect the structure of the Rossby wave packet. The results suggest that mountains, especially the Rocky Mountains, decrease (increase) the X-propagating disturbance system when it is tilted westward (eastward); this is consistent with the findings of Hayashi and Golder (1983) with the GFDL spectral general circulation models.

In the present study, we will further investigate the global behaviors of the evolution of a geostrophic synoptic disturbance system on the δ-surface of the earth. The global behaviors of the structural changes of the wave packet on the earth's δ-surface due to the zonal, meridional, and asymmetric basic currents and the variety of topography will be discussed and compared with those on the earth's β-plane by using the WKB phase plane, i.e., the wave packet's local wavenumber phase plane.

Kasahara (1980) studied the effect of zonal flows on the free oscillations of a barotropic atmosphere. Solutions of the linearized global shallow-water equation (Laplace tidal equations) including the effect of a mean zonal flow were obtained by the Galerkin-transform method and showed some solutions were significantly affected by a zonal flow different from solid rotation. Ahlquist (1982) also investigated normal modes of linearized global primitive shallow-water equations for a given basic state of latitudinally dependent steady zonal flow. Wave solutions were also obtained by Longuet-Higgins (1968). However, although the normal mode of method is a good approach to understanding the evolution of weather systems, in the present work we will use a different approach. As in the previous study, in this paper we will continue to employ the Rossby wave packet approximation and the WKB method, which has been successfully used in many studies of geophysical flows. For example, Zeng (1982, 1983a, 1983b) considered the development of an individual quasi-geostropic disturbance in a zonal mean flow by using this method. Hoskins and Karoly (1981) used a similar idea in investigation of the effect of the zonal symmetric basic current on the steady linear response to thermal and orographic forcing. They showed that linear barotropic models, baroclinic models and the WKB analysis had all given similar results, which were generally in agreement with those from observational studies and, to some extent, GCM integrations. Recent model experiments have shown that zonal variations of the basic state may be important in determining the planetary wave response (Webster and Holton, 1982). Motivated by this suggestion and encouraged by previous success, Karoly (1983) further considered the effect of the asymmetric basic flow. In the asymmetric basic current, he showed the agreement between the WKB solutions and the numerical solutions of linearized barotropic models and demonstrated that the essential characteristics of barotropic Rossby wave propagation is retained by the theory. However, these studies (Hoskins and Karoly, 1981; Karoly and Hoskins, 1982; Karoly, 1983) are mainly focused on the propagation properties of Rossby wave.

In this paper, we will show that it is possible for the wave packet structural vacillation to exist in some basic currents or topographies on both the β-plane and the δ-surface. The wave packet structural vacillation is characterized by the time-periodic changes in the structure of the wave packet. Both the tilt and spatial scales, including the X-direction scale, the Y-direction scale and the whole scale, of the packet will simultaneously vary periodically. It will be shown that the WKB trajectory of the wave packet structural vacillation is characterized by a closed curve on the WKB phase plane. The wave packet structural vacillation suggests a possible mechanism of vacillations observed in the atmosphere.

The model equations will be addressed in section 2. The main attention will be focused on the global behaviors of the structural changes of a Rossby wave packet due to the various basic currents on the earth β-plane and δ-surface in section 3 and section 4, respectively. In section 5, we will consider the effects of a variety of topography. The final section will present the conclusions.

2. Equations governing the evolution of a Rossby wave packet on the earth's δ-surface

The so-called earth δ-surface approximation is defined as

$$f \approx f_0 + \beta(y - y_0) - \frac{1}{2}\delta(y - y_0)^2 \qquad (2.1a)$$

in which the effect of the second derivative of the Coriolis parameter with respect to latitude has been included; β and δ are constants, defined by

$$\beta = \left.\frac{df}{dy}\right|_{y=y_0}, \qquad (2.1b)$$

$$\delta = -\left.\frac{d^2f}{dy^2}\right|_{y=y_0} \qquad (2.1c)$$

The main difference between the earth's δ-surface approximation and the earth's β-plane approximation is this: on the β-plane the surface of the earth is considered to be a plane by taking the Coriolis parameter as a linear function of latitude whereas on the δ-surface, the surface of the earth is considered to be a quadratic surface by taking the Coriolis parameter as a quadratic function of latitude. Since the Coriolis force is of great importance in the study of geophysical fluid dynamics, especially for large-scale motions, and the δ-surface approximation is more realistic than the β-plane approximation, we will show that the dynamics of a disturbance system on the δ-surface will be different from that on the β-plane, especially, the structural change of the system. Moreover, the difference might be of critical significance near the earth's poles in the dynamical processes of large-scale flows.

The linearized potential vorticity equation of the synoptic scale disturbance system on the earth's surface can be written in the following dimensionless form (Yang, 1987):

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y}\right)(\nabla^2\psi - F\psi) + (B_1 + \beta_1)\frac{\partial\psi}{\partial x} - (B_2 + \beta_2)\frac{\partial\psi}{\partial y} = 0, \quad (2.2)$$

where

$$B_1 = FU + \beta_0 - \epsilon\delta_0(y - y_0) - \frac{\partial^2 U}{\partial y^2}, \quad (2.3a)$$

$$B_2 = \frac{\partial^2 V}{\partial x^2} - FV, \quad (2.3b)$$

and U is the zonal component of the basic current, which is a function of latitude and time, V is the meridional component of the basic current, which is a function of longitude and time, and

$$\beta_1 = \frac{\partial\eta_B}{\partial y}, \quad (2.3c)$$

$$\beta_2 = \frac{\partial\eta_B}{\partial x}, \quad (2.3d)$$

$$F = \frac{f_0^2L^2}{gD}, \quad (2.3e)$$

where η_B is the height of the topography relative to the vertical scale of the model; β_2 and β_1 describe the east–west and north–south slopes of the topography, respectively; F is the Froude number; L and D are horizontal and vertical characteristic scales of the system, respectively; δ_0 is the second derivative of the Coriolis parameter with respect to latitude and ϵ is a small parameter defined as

$$\epsilon = O\left(\frac{L}{R}\right); \quad (2.4)$$

where R is the average radius of the earth.

As in the author's recent study (Yang, 1987), we will simplify an individual single disturbance system as a Rossby wave packet and use the WKB method to investigate the present problem. Again, we restrict our discussion to a case in which the basic currents are varying slowly in spatial variables and time, and the topographies are large scale and smooth shaped. For further discussion on the WKB method as used here, see the author's previous paper (Yang, 1987). The slowly varying time, spatial variables, and streamfunction can be introduced as follows:

$$T = \epsilon t, \quad X = \epsilon x, \quad Y = \epsilon y, \quad (2.5)$$

$$\psi = \Psi(X, Y, T)e^{i\theta(X,Y,T)/\epsilon}, \quad (2.6)$$

where

$$\Psi(X, Y, T) = \Psi_0(X, Y, T) + \epsilon\Psi_1(X, Y, T) + \cdots, \quad (2.7)$$

$$\sigma = -\frac{\partial\theta}{\partial T}, \quad m = \frac{\partial\theta}{\partial X}, \quad n = \frac{\partial\theta}{\partial Y}, \quad (2.8)$$

where σ, m and n are, respectively, the local frequency, the local wavenumber along the X-direction and the local wavenumber along the Y-direction of the Rossby wave packet.

Substituting (2.7) into (2.2), using (2.5), (2.6) and (2.8), we have the zero order approximation, or the dispersion relation:

$$(\sigma - Um - Vn)K^2 + (B_1 + \beta_1)m - (B_2 + \beta_2)n = 0 \quad (2.9)$$

where

$$K^2 = m^2 + n^2 + F. \quad (2.10)$$

The first order approximation gives us the amplitude equation, that is,

$$\left(\frac{\partial}{\partial T} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)(K^2\Psi_0) + K^{-2}\{(B_1 + \beta_1)m - (B_2 + \beta_2)n\} \times \left\{\left(\frac{\partial m}{\partial X} + \frac{\partial n}{\partial Y}\right)\Psi_0 + 2\left(m\frac{\partial}{\partial X} + n\frac{\partial}{\partial Y}\right)\Psi_0\right\} - \left\{(B_1 + \beta_1)\frac{\partial}{\partial X} - (B_2 + \beta_2)\frac{\partial}{\partial Y}\right\}\Psi_0 = 0. \quad (2.11)$$

Using (2.8) and (2.9), it can be simply proven that

$$\frac{D_g\sigma}{DT} = m\left(\frac{\partial U}{\partial T} - K^{-2}\frac{\partial B_1}{\partial T}\right) + n\left(K^{-2}\frac{\partial B_2}{\partial T} + \frac{\partial V}{\partial T}\right), \quad (2.12)$$

$$\frac{D_g m}{DT} = -\left\{n\frac{\partial V}{\partial X} - \frac{m}{K^2}\frac{\partial\beta_1}{\partial X} + \frac{n}{K^2}\left(\frac{\partial B_2}{\partial X} + \frac{\partial\beta_2}{\partial X}\right)\right\}, \quad (2.13)$$

$$\frac{D_g n}{DT} = -\left\{m\frac{\partial U}{\partial Y} - \frac{m}{K^2}\left(\frac{\partial B_1}{\partial Y} + \frac{\partial\beta_1}{\partial Y}\right) + \frac{n}{K^2}\frac{\partial\beta_2}{\partial Y}\right\}. \quad (2.14)$$

In addition, the equations governing the whole structural change of the Rossby wave packet are

$$\frac{D_g}{DT}(m^2+n^2) = -2mn\left(\frac{\partial V}{\partial X}+\frac{\partial U}{\partial Y}\right) + \frac{2m^2}{K^2}\frac{\partial \beta_1}{\partial X} - \frac{2mn}{K^2}\left(\frac{\partial B_2}{\partial X}+\frac{\partial \beta_2}{\partial X}\right) + \frac{2mn}{K^2} \times \left(\frac{\partial B_1}{\partial Y}+\frac{\partial \beta_1}{\partial Y}\right) - \frac{2n^2}{K^2}\frac{\partial \beta_2}{\partial Y}, \quad (2.15)$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right) = \frac{\partial U}{\partial Y} - \frac{n^2}{m^2}\frac{\partial V}{\partial X} - \frac{1}{K^2}\left(\frac{\partial B_1}{\partial Y}+\frac{\partial \beta_1}{\partial Y}\right) + \frac{n}{mK^2}\frac{\partial \beta_2}{\partial Y} + \frac{n}{mK^2}\frac{\partial \beta_1}{\partial X} - \frac{n^2}{m^2K^2}\left(\frac{\partial B_2}{\partial X}+\frac{\partial \beta_2}{\partial X}\right), \quad (2.16)$$

where

$$\frac{D_g}{DT} \equiv \frac{\partial}{\partial T} + \mathbf{C}_g\cdot\nabla = \frac{\partial}{\partial T} + C_{gx}\frac{\partial}{\partial X} + C_{gY}\frac{\partial}{\partial Y}. \quad (2.17)$$

Here, $\mathbf{C}_g$ is the group velocity of the Rossby wave packet; it can be shown

$$C_{gX} \equiv \frac{\partial \sigma}{\partial m} = U - K^{-4}\{(B_1+\beta_1)K^2 - 2m[(B_1+\beta_1)m - (B_2+\beta_2)n]\}, \quad (2.18)$$

$$C_{gY} \equiv \frac{\partial \sigma}{\partial n} = V + K^{-4}\{(B_2+\beta_2)K^2 + 2n[(B_1+\beta_1)m - (B_2+\beta_2)n]\}. \quad (2.19)$$

So far, all the equations governing the structural change of the Rossby wave packet [(2.12)–(2.16)] have been obtained.

If we let

$$\Psi_0 = |\Psi_0|e^{i\alpha(X,Y,T)}, \quad (2.20)$$

then (2.11) will become

$$\frac{D_g\alpha}{DT} = 0, \quad (2.21)$$

$$\left(\frac{\partial}{\partial T} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)(K^2|\Psi_0|) + K^{-2}\{(B_1+\beta_1)m - (B_2+\beta_2)n\} \times \left\{\left(\frac{\partial m}{\partial X}+\frac{\partial n}{\partial Y}\right)|\Psi_0| + 2\left(m\frac{\partial}{\partial X}+n\frac{\partial}{\partial Y}\right)|\Psi_0|\right\} - \left\{(B_1+\beta_1)\frac{\partial}{\partial X} - (B_2+\beta_2)\frac{\partial}{\partial Y}\right\}|\Psi_0| = 0. \quad (2.22)$$

3. Behaviors of structural changes due to the basic currents on the earth's β-plane

It should be pointed out that in the present model the disturbance energy is not conserved. However, it can be shown that for a supposed basic current or topography the "wave action," which in this model is the ratio of the energy density and the frequency is conserved (Whitham, 1974). On the other hand, Eq. (2.21) states that the energy of the Rossby wave packet will propagate with the group velocity.

When only the effect of the main part of the basic current is considered on the earth's β-plane, the governing equations (2.13)–(2.16) will become

$$\frac{D_g m}{DT} = -\frac{n}{K^2}(m^2+n^2)\frac{\partial V}{\partial X}, \quad (3.1)$$

$$\frac{D_g n}{DT} = -\frac{m}{K^2}(m^2+n^2)\frac{\partial U}{\partial Y}, \quad (3.2)$$

$$\frac{D_g}{DT}(m^2+n^2) = -2mn\frac{m^2+n^2}{K^2}\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right), \quad (3.3)$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right) = \left(\frac{\partial U}{\partial Y} - \frac{n^2}{m^2}\frac{\partial V}{\partial X}\right)\frac{m^2+n^2}{K^2}. \quad (3.4)$$

a. *Zonal basic current*

It can be shown from (3.1)–(3.4) that the Rossby wave packet's longitudinal scale will not change on the zonal basic current, whereas the westward tilting wave packet's latitudinal scale will increase (decrease) when the wave packet is located on the right side (left side) of the jet center. Since the energy of the Rossby wave packet will always propagate with its group velocity, we could take the group velocity as the characteristic direction and then integrate the wave packet along that characteristic line. It can be readily proven that there are simple wave packet solutions which are related by

$$m = m_0 = \text{constant}, \quad (3.5)$$

$$n + \frac{F}{m}\operatorname{arctg}\frac{n}{m} = -m\frac{\partial U}{\partial Y}T + C, \quad (3.6)$$

where C is the integration constant to be determined by the initial condition.

b. *Meridional basic current*

Meridional basic current may easily be observed in the ocean, especially near the north–south oriented coasts. From (3.1)–(3.4) it can be shown that the meridional basic current will increase (decrease) the westward tilting of Rossby wave packet's longitudinal scale when it is located on the right side (left side) of the jet center. The meridional basic current will not alter the packet's latitudinal scale. Integrating along the characteristic line, we have

$$m + \frac{F}{n} \arctan \frac{m}{n} = -n \frac{\partial V}{\partial X} T + C, \quad (3.7)$$

$$n = n_0 = \text{constant}, \quad (3.8)$$

where C is the integration constant to be determined by the initial condition.

The results show that on the earth's β-plane the property of the dynamical effect of the meridional basic current is similar to that of the zonal basic current. The only difference is that the zonal basic current changes the packet's latitudinal scale, whereas the meridional basic current alters the longitudinal scale.

c. *Asymmetric basic current*

From (3.1) and (3.2), integrating along the characteristic line of the wave packet, we can obtain the WKB trajectories of the Rossby wave packet for the supposed basic current as follows:

$$\frac{\partial U}{\partial Y} m^2 - \frac{\partial V}{\partial X} n^2 = C. \quad (3.9)$$

The wave packet solutions can also be obtained as shown in appendix A.

Figure 1 shows the global behaviors of the evolution of a Rossby wave packet by the WKB trajectories on the WKB phase plane in the asymmetric basic current in which $(\partial U/\partial Y)(\partial V/\partial X) \leq 0$. The southwesterly jet, which often occurs above southeast Asia in the atmosphere, is a typical example of this case. Figure 1a is the case in which $\partial U/\partial Y < 0$ and the two straight lines correspond to the case of the zonal basic current. Figure 1b is the case in which $\partial V/\partial X < 0$ and the two straight lines correspond to the case of the meridional basic current. The arrows in the WKB phase plane indicate the flow direction of the evolution of the wave packet. Figure 2a shows the evolution of the whole scale of the packet defined by $2\pi/(m^2 + n^2)^{1/2}$. Figure 2b, c illustrates the evolution of the packet's local wavenumbers along the X and Y-direction, respectively. Figure 3 is the evolution of the tilt of the wave packet. In Figs. 2 and 3, the wave packet is set with $m = n = 6$ to begin with, and the parameters $\partial U/\partial Y = -0.02$, $\partial V/\partial X = 0.01$ and $F = 1.0$; $\partial U/\partial Y = -0.02$ and $\partial V/\partial X = 0.01$ correspond to the basic current changes of about 2 m s^{-1} in the U component and 1 m s^{-1} in the V component, 100 km from the jet center; $F = 1.0$ corresponds to the case in which the wavelength of the disturbance system in the atmosphere is about 3000–4000 km in the middle latitude regions. The figures show that the whole scale and the tilt of the Rossby wave packet will change periodically with time. We call this phenomenon the wave packet structural vacillation. From the figures, one will find that the wave packet structural vacillation is characterized by the simultaneous periodic changes in the tilt, the scale along the X-direction, the scale along the Y-direction, and the whole scale of the wave packet. In the case of Figures 2 and 3, it can be found that the period of the wave packet structural vacillation is about 450 dimensionless time steps, which is about 11–12 days in dimensional time scale, a typical order of magnitude of time scale for weather systems. Figure 4 is an example of the evolution of a Rossby wave packet in a southwesterly jet on the earth's β-plane, starting with $m = m = 6$ and taking $\partial U/\partial Y = 0.02$, $\partial V/\partial X = -0.01$ and $F = 1$. In Fig. 4, the solid line and the broken line are an ideal streamline and the trough line of the wave packet, respectively. From these figures, it can be seen that the tilt of the packet first tilts north–west to south–east, $m = n = 6$, at $T = 0$ (Fig. 4a), then north to south, $m = 0$, $n = 10$, reaching its maximum longitudinal scale, at $T = 79$ (Fig. 4b), and later north–east to south–west, $m = -6$, $n = 6$, at $T = 150$ (Fig. 4c).

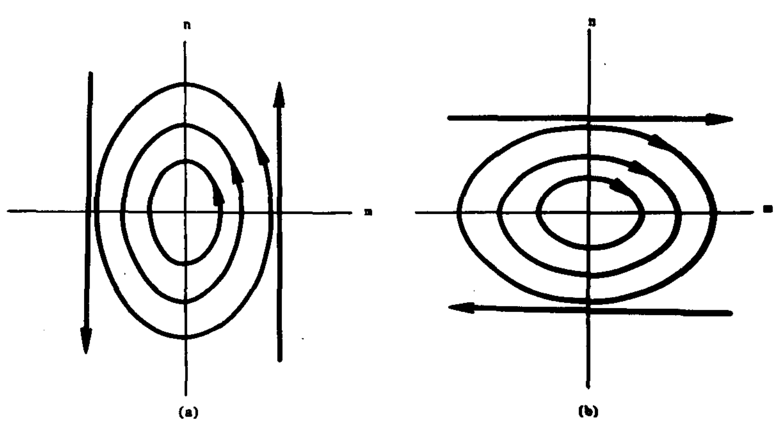


FIG. 1. The WKB trajectories on the WKB phase plane on the earth's β-plane in the presence of basic currents where $(\partial U/\partial Y)(\partial V/\partial X) \leq 0$. (a) $\partial U/\partial Y < 0$, and (b) $\partial U/\partial Y > 0$. The arrow indicates the flow direction of the evolution of the wave packet. Two straight lines correspond to the zonal basic current in panel a and the meridional basic current in panel b.

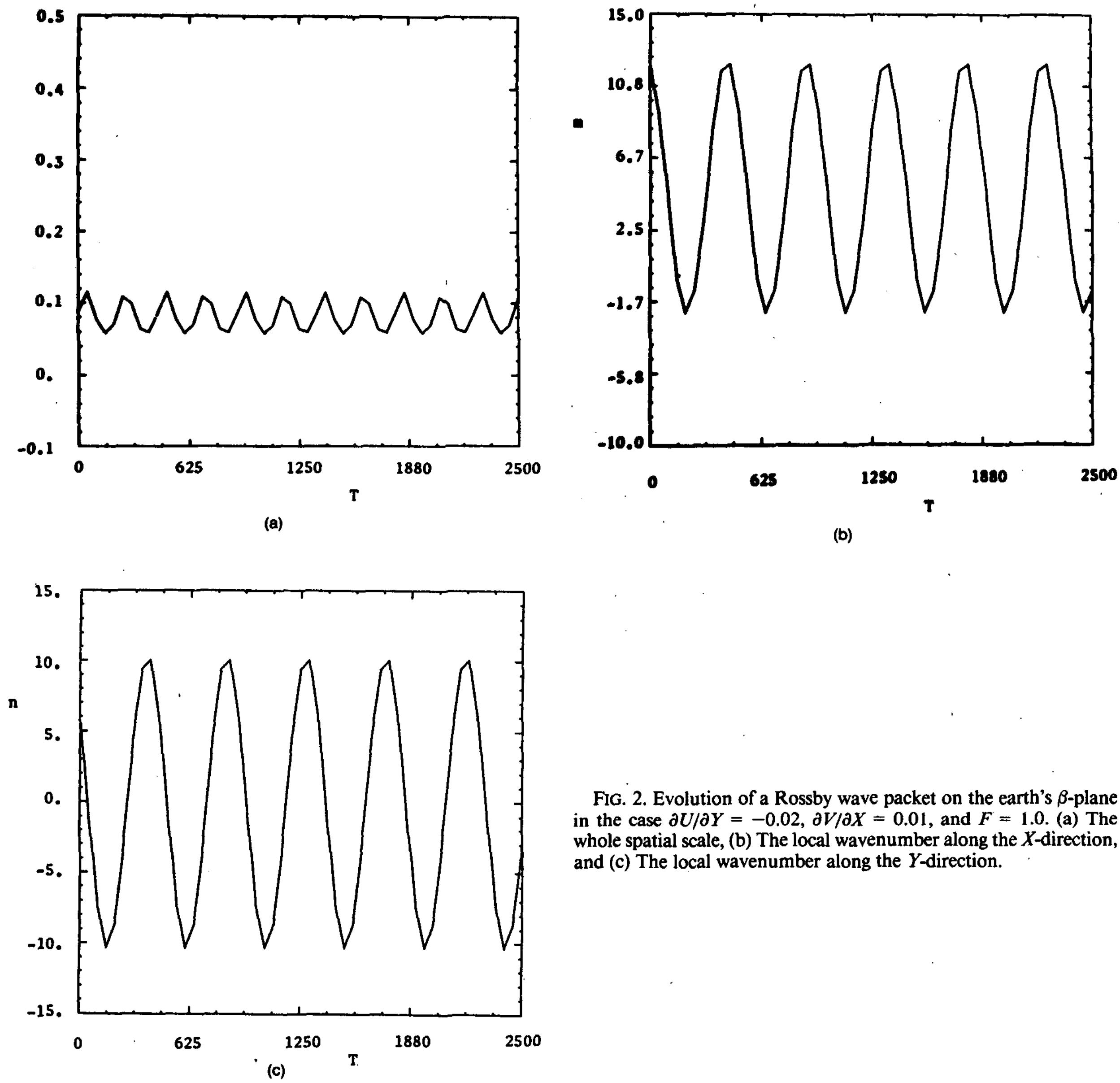


FIG. 2. Evolution of a Rossby wave packet on the earth's β-plane in the case $\partial U/\partial Y = -0.02$, $\partial V/\partial X = 0.01$, and $F = 1.0$. (a) The whole spatial scale, (b) The local wavenumber along the X-direction, and (c) The local wavenumber along the Y-direction.

This phenomenon is reminiscent of the vacillations in the atmosphere. On examining daily weather maps, one could readily find that some weather patterns will repeat in quasi-periodic manner. The index cycle is a typical example, which has about a 13-day period. It is well known that the steady basic current is mainly driven by topography and heating. This current is not zonally symmetric. From observation, we know that the basic current in the atmosphere is a wave-like jet located on the climate-average position. In addition, using 9.5 years of daily NMC gridded height and temperature field, McGuirk and Reiter (1976) found a strong, persistent and significant oscillation of about a 24-day periodicity in the energy parameter during the winter season. Gruber (1975) has observed the 14–16 day periodicity in the tropics; an 18–23 day period vacillation was also found by Webster and Keller (1975) in the EOLE data from the Southern Hemisphere. From the analysis, we find that the orders of magnitude of the time period scale and the wavelength scale in the atmosphere agree with the predictions. Therefore, the present theory suggests a possible mechanism for the vacillations so observed. The vacillation may be caused by the asymmetric wave-like jet structure of basic current driven by the topography and the heating. It will be shown that the wave packet structural vacillation could also exist under other conditions. However, in the case of $(\partial U/\partial Y)(\partial V/\partial X) > 0$, e.g., a southeasterly jet, the behavior of the evolution of the Rossby wave packet will be qualitatively different from the case

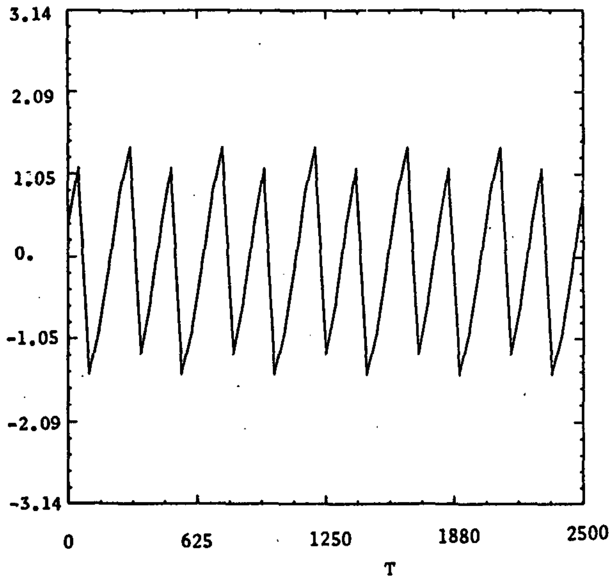


FIG. 3. The evolution of the tilt of the Rossby wave packet corresponding to Fig. 2.

of the southwesterly jet. Figure 5 illustrates the behavior of the packet in such a case on the WKB phase plane. Figure 5a shows the results in which the Rossby wave packet is located on the right side of a southeasterly jet in the case of $|\partial U/\partial Y| \leqslant |\partial V/\partial X|$ and $\partial U/\partial Y < 0$ and Fig. 5b shows results in which the packet is located on the left side of a southeasterly jet in the case of $|\partial U/\partial Y| \geqslant |\partial V/\partial X|$ and $\partial U/\partial Y > 0$. Again, the arrows in the figures indicate the flow direction of the evolution of the wave packet on the WKB phase plane.

From the WKB phase plane, we can clearly see the global behaviors of the evolution of a Rossby wave packet. The whole scale and tilt of the packet are evolved along the WKB trajectory in the arrow direction on the WKB phase plane with time. Comparing the results of Fig. 5 with those of Fig. 1, one can readily see that there is a qualitative difference between the effect of a southwesterly and a southeasterly jet. In a southwesterly jet, there is the wave packet structural vacillation, whereas in a southeasterly, there is no such structural vacillation since the longitudinal scale and latitudinal scale increases (or decreases) simultaneously. Therefore, the two systems are dynamically different since on the WKB phase plane, one is characterized by an open line, while the other is characterized by a closed line.

4. Behaviors of the structural changes due to the basic currents on the earth's δ-surface

It can be easily shown that the β-effect only affects the propagation properties of the Rossby wave packet and not the structure. On the other hand, the δ-effect, which is referred to as the effect of the second derivative of the Coriolis parameter with respect to latitude, will not only affect the propagation properties of the packet, but also will alter the structure of a Rossby wave packet. This may be of great importance near the earth's poles, where $\beta = 0$. It had been shown (Yang, 1987) that the δ-effect will result in the Rossby wave packet increasing its latitudinal scale and tending its westward tilting trough line toward the Y-axis.

By considering only the δ-effect in (2.12) and (2.13) and integrating along the characteristic line, one can readily obtain the wave packet solutions that are related by the following relation:

$$m = m_0 = \text{constant}, \tag{4.1}$$

$$(F + m^2)n + \frac{1}{3}n^3 = -\delta_0 mT + C, \tag{4.2}$$

where C in the constant of integration to be determined by the initial condition. Equations (4.1) and (4.2) describe all possibilities of the δ-effect upon the structural changes of a Rossby wave packet.

Because of the special property of the δ-effect, it is natural to consider the dynamic behaviors of a Rossby wave packet on the earth's δ-surface. In what follows, we will discuss the global behaviors of the evolution of the packet on the earth's δ-surface and compare those on the earth's β-plane in the presence of a variety of basic currents. The equations governing the evolution of a Rossby wave packet on the δ-surface in the presence

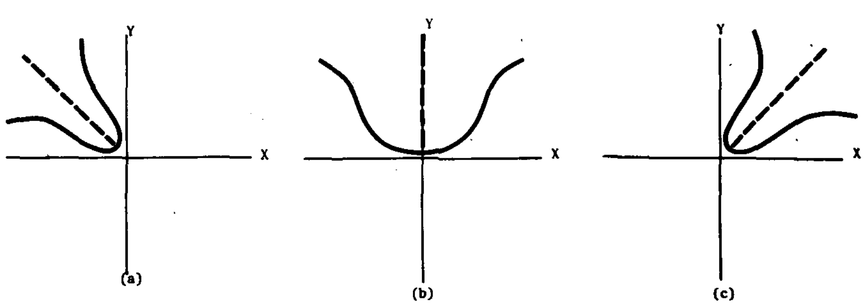


FIG. 4. The wave packet structural vacillation, starting with $m = n = 6$ and taking $\partial U/\partial Y = 0.02$, $\partial V/\partial X = -0.01$ and $F = 1.0$. (a) $T = 0$, (b) $T = 79$, and (c) $T = 150$.

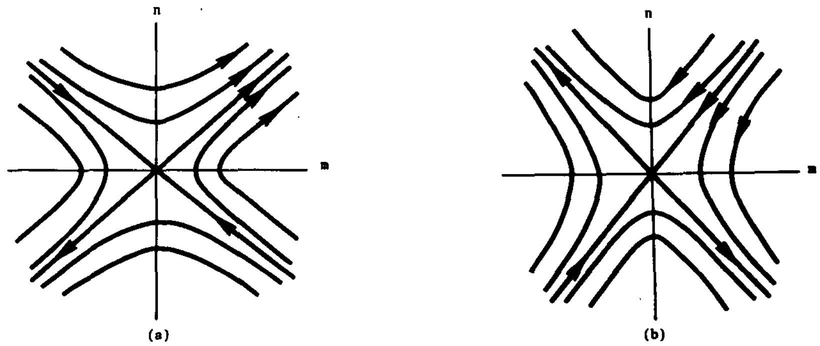


FIG. 5. The WKB phase plane on the earth's β-plane in the case of a southeasterly jet on the (a) right side and (b) left side of the jet.

of the basic current can be obtained from (2.12)–(2.15), i.e.,

$$\frac{D_g m}{DT} = -n \frac{m^2 + n^2}{K^2} \frac{\partial V}{\partial X}, \tag{4.3}$$

$$\frac{D_g n}{DT} = -\frac{m}{K^2} \left\{ \delta_0 + (m^2 + n^2) \frac{\partial U}{\partial Y} \right\}, \tag{4.4}$$

$$\frac{D_g}{DT}(m^2 + n^2) = -\frac{2mn}{K^2} \left\{ \delta_0 + (m^2 + n^2)\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \right\}, \tag{4.5}$$

$$\frac{D_g}{DT}\left(-\frac{n}{m} \right) = \frac{1}{K^2} \left\{ \delta_0 + \left(\frac{\partial U}{\partial Y} - \frac{n^2}{m^2} \frac{\partial V}{\partial X} \right)(m^2 + n^2) \right\}. \tag{4.6}$$

Zeng (1982, 1983a, 1983b) obtained similar results in the zonal symmetric basic flow. With our considerations, his results agree with ours. The asymmetric basic flow was considered by Karoly (1983). He obtained a complete set of equations for the zonally varying basic flow on a spheric geometry in a barotropic model, which is similar to the present one. In his results, if we consider that the basic state is geostrophic and the spheric geometry is substituted by the δ-surface approximation, then the present results can be derived from his, provided that only the main part of basic current has been included. Comparison of the two results reveals the advantage of the δ-surface approximation. Under such an approximation, the system of equations becomes simply an ordinary system with constant coefficients for the supposed basic current. However, in his results, it is impossible due to the variation of second derivative of mean potential vorticity in latitude even for an assumed basic current.

a. Zonal basic current

In the zonal current, (4.3) shows that the longitudinal scale of the Rossby wave packet will not change, but rather the packet's latitudinal scale. Again, integrating along the characteristic line, we have the following wave packet solutions

$$m = m_0 = \text{constant}, \tag{4.7}$$

$$n + \frac{F - \delta_0 \Big/ \frac{\partial U}{\partial Y}}{\left(\delta_0 \Big/ \frac{\partial U}{\partial Y} + m^2 \right)^{1/2}} \arctan \frac{n}{\left(\delta_0 \Big/ \frac{\partial U}{\partial Y} + m^2 \right)^{1/2}} = -m \frac{\partial U}{\partial Y} T + C, \tag{4.8}$$

when

$$\frac{\partial U}{\partial Y} > -\frac{\delta_0}{m^2}$$

or

$$m = m_0 = \text{constant}, \tag{4.9}$$

$$n + \frac{\dfrac{F - \delta_0}{\partial U/\partial Y}}{2\left| \dfrac{\delta_0}{\partial U/\partial Y} + m^2 \right|^{1/2}} \ln \frac{n + \left| m^2 + \dfrac{\delta_0}{\partial U/\partial Y} \right|^{1/2}}{n - \left| m^2 + \dfrac{\delta_0}{\partial U/\partial Y} \right|^{1/2}} = -m \frac{\partial U}{\partial Y} T + C, \tag{4.10}$$

when

$$\frac{\partial U}{\partial Y} < -\frac{\delta_0}{m^2}.$$

By comparison of the results with those on the earth's β-plane, it is shown that when $\partial U/\partial Y > -(\delta_0/m^2)$, the qualitative properties of the evolution of a Rossby wave packet on the δ-surface is similar to that on the β-plane. However, when $\partial U/\partial Y < -(\delta_0/m^2)$, the qualitative properties on the δ-surface will be different from that on the β-plane.

b. Meridional basic current

In the meridional basic current, the equations governing the evolution of a packet on the earth's δ-surface will become

$$\frac{D_g m}{DT} = -n \frac{m^2 + n^2}{K^2} \frac{\partial V}{\partial X}, \tag{4.11}$$

$$\frac{D_g n}{DT} = -\frac{m}{K^2} \delta_0, \tag{4.12}$$

$$\frac{D_g}{DT}(m^2 + n^2) = -\frac{2mn}{K^2}\left\{\delta_0 + (m^2 + n^2)\frac{\partial V}{\partial X}\right\}, \tag{4.13}$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right) = \frac{1}{K^2}\left\{\delta_0 - \frac{n^2}{m^2}(m^2 + n^2)\frac{\partial V}{\partial X}\right\}. \tag{4.14}$$

From Eqs. (4.11)–(4.14), it can be found that on the δ-surface the effect of a meridional basic current upon the evolution of a packet will be qualitatively different from the effect on the β-plane. On the δ-surface, the longitudinal scale and the latitudinal scale of the packet will change simultaneously, but on the β-plane the latitudinal scale of the packet will not change with time. Moreover, the dynamical effect of the meridional basic current on the packet is also qualitatively different from that of the zonal basic current on the δ-surface; this is also different on the β-plane.

Figure 6 shows the global behaviors of the packet in the meridional basic current on the δ-surface. The results demonstrate that in this case the governing system on the earth's δ-surface is dynamically different from the governing system on the earth's β-plane. In Fig. 6, the solid lines are the WKB trajectories, while the two straight lines parallel to the *m*-axis are the WKB trajectories on the β-plane. Figure 6a corresponds to the case on the right side of the meridional basic current, i.e., $\partial V/\partial X < 0$, while Fig. 6b corresponds to the case on the left side of the meridional basic current, i.e., $\partial V/\partial X > 0$. On the δ-surface, the effect of the meridional basic current on the left side is essentially different from that on the right. On the right side, the WKB trajectory of the wave packet on the WKB phase plane is a closed line. Therefore, the evolution of the packet is varying periodically with time. The results suggest that on the right side of the meridional basic current, the evolution of a Rossby wave packet exhibits the wave packet structural vacillation on the δ-surface of the earth, while this is impossible on the earth's β-plane. Figure 7 shows the results of such a wave packet structural vacillation in the case $V/X = -0.01$, $\delta_0 = 0.2$, $F = 1.0$, and starting from $m = n = 6$; $\partial V/\partial X = -0.01$ corresponds to the meridional basic current change of about 1 m s^{-1}, 100 km out from the jet center; $\delta_0 = 0.2$ and $F = 1.0$ correspond to the case in which the wave length of the disturbance system is about 3000–4000 km in the middle latitude regions of the atmosphere. Figure 7a is the WKB trajectory of the evolution on the WKB phase plane and Fig. 7b is the whole spatial scale of the packet changing with time. Figure 7c shows the time change of the tilt of the packet and Fig. 7d, e corresponds to the time changes of the packet's local wavenumbers in the *X* and *Y*-direction, respectively. We found in this case that the period of the packet structural vacillation is larger than the case of the asymmetric basic current on the β-plane using the same assumptions.

c. Asymmetric basic current

In this case, the governing equations are (4.3)–(4.6). From the equations, it could be found that the behavior of the evolution of a packet on the δ-surface will be different from those on the β-plane because of the presence of δ in the governing equations. Figure 8 shows the global behaviors of the evolution of a packet on the δ-surface by the WKB trajectories on the WKB phase plane, taking $\partial U/\partial Y = -0.02$, $\partial V/\partial X = 0.01$, $\delta_0 = 0.2$ and $F = 1.0$. The figure suggests that in such a case, it may still be possible for the packet structural

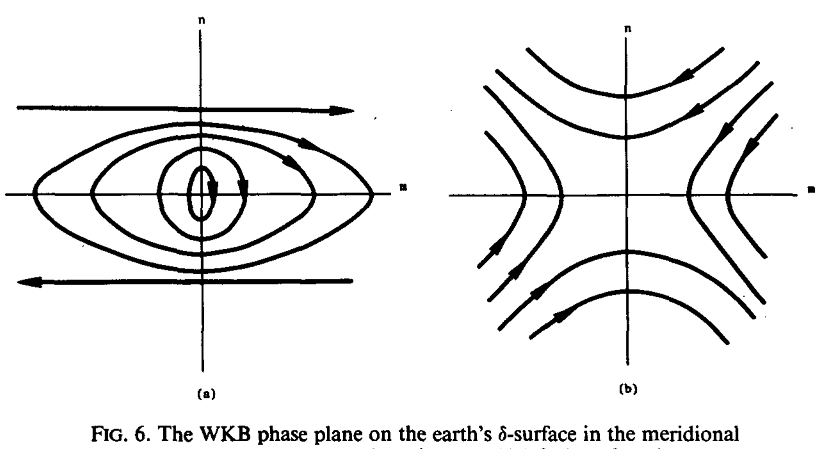
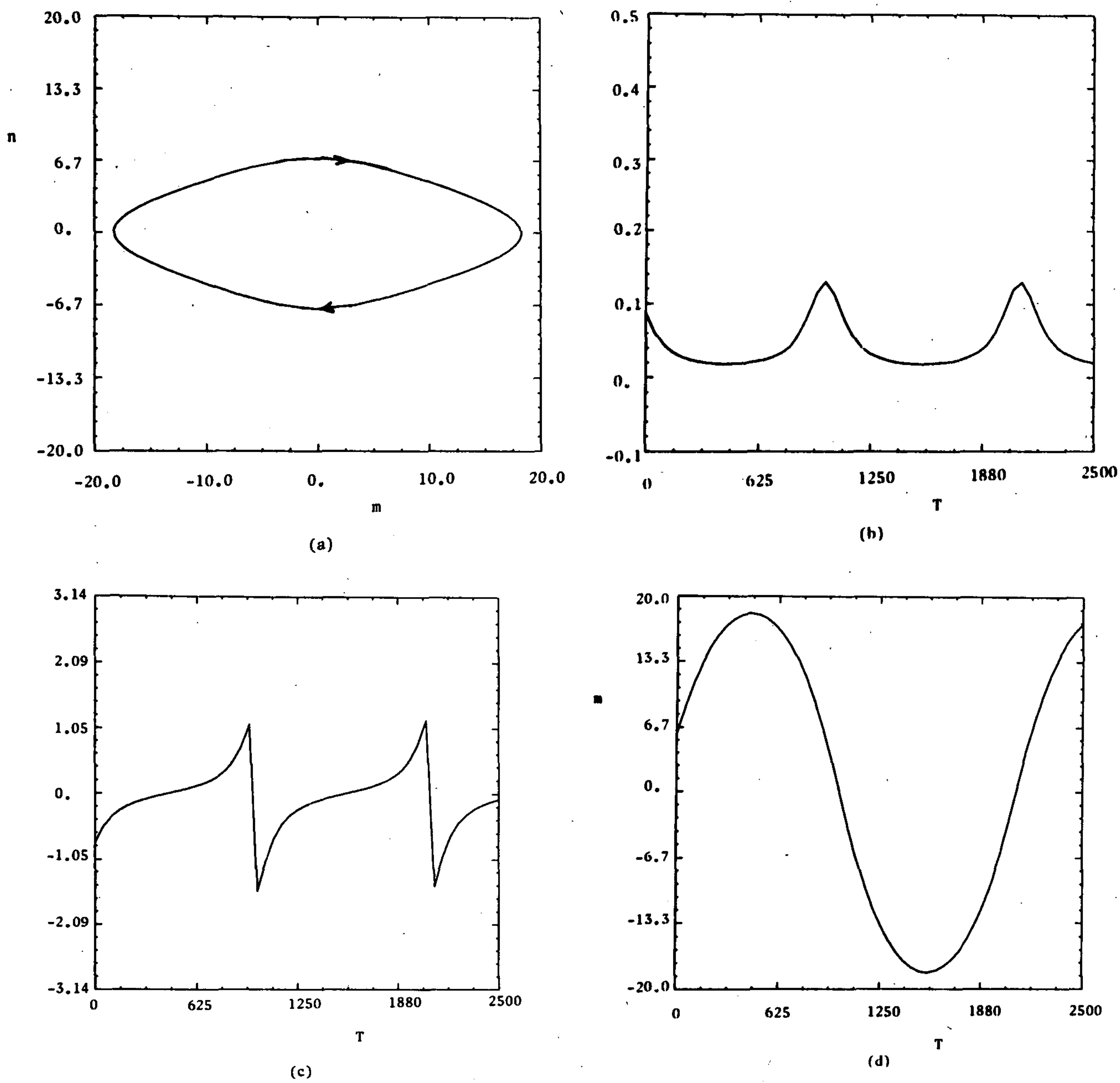


FIG. 6. The WKB phase plane on the earth's δ-surface in the meridional basic current on the (a) right side and (b) left side of the jet.



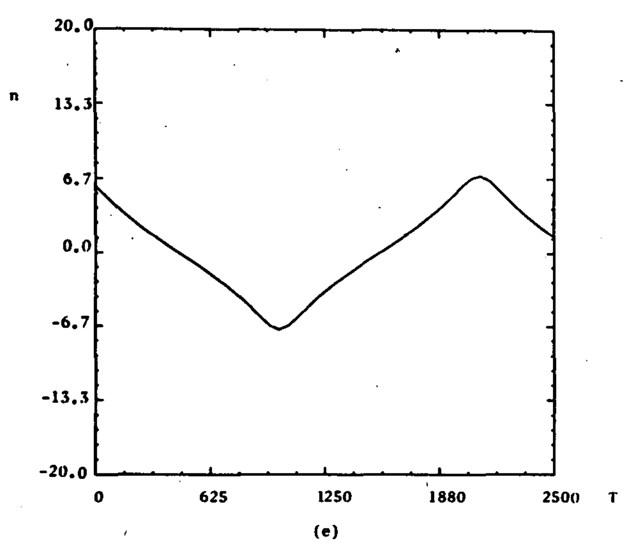


FIG. 7. Wave packet vacillation on the earth's δ-surface in the presence of a meridional basic current. (a) The WKB phase plane, (b) time change of the whole spatial scale, (c) time change of the tilt, (d) time change of the local wavenumber along the X-direction, and (e) time change of the local wavenumber along the Y-direction.

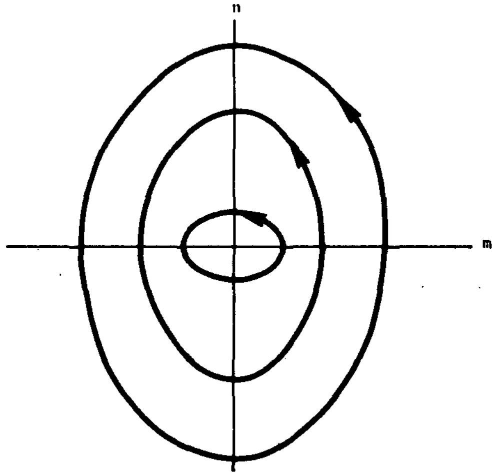


FIG. 8. The WKB phase plane on the earth's δ-surface in the presence of a southwesterly basic current.

vacillation to exist on the δ-surface. However, the period of the packet structural vacillation now will be altered because of the presence of the δ-effect. The δ-effect makes the problem rather complicated. In the present case, it is hard to obtain its complete integrals by integrating along the characteristic line, though from the equations one could show that there are still periodic solutions once the condition $(\partial U/\partial Y)(\partial V/\partial X) < 0$ is satisfied.

5. Behaviors on the topography

If we consider only the topography, the governing equations will become

$$\frac{D_g m}{DT} = \frac{1}{K^2}(mk_1 - nk_2), \tag{5.1}$$

$$\frac{D_g n}{DT} = -\frac{1}{K^2}(m\delta_0 - mk_3 + nk_1), \tag{5.2}$$

$$\frac{D_g}{DT}(m^2 + n^2) = \frac{2}{K^2}\{m^2k_1 - n^2k_1 + mn(k_3 - \delta_0 - k_2)\}, \tag{5.3}$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right) = \frac{1}{K^2}\left(\delta_0 - k_3 + \frac{2n}{m}k_1 - \frac{n^2}{m^2}k_2\right), \tag{5.4}$$

where

$$k_1 = \frac{\partial^2\eta_B}{\partial y\partial X} = \frac{\partial^2\eta_B}{\partial x\partial Y}, \quad k_2 = \frac{\partial^2\eta_B}{\partial x\partial X}, \quad k_3 = \frac{\partial^2\eta_B}{\partial y\partial Y}. \tag{5.5}$$

From Eqs. (5.1)–(5.5), it can easily be seen that the effect of a quadratic east–west oriented topography upon the structural change of the Rossby wave packet only modifies the δ-effect. This is reminiscent of the β-effect, which is related to the propagation properties of the Rossby waves. In that case the linearly sloping topography only modifies the β-effect.

When the topography is in the $k_1 = 0$ case (e.g., symmetric topography) integrating along the characteristic line, i.e., along the packet's group velocity, we can obtain the following relation on the WKB phase plane of the packet:

$$(\delta_0 - k_3)m^2 - k_2n^2 = C, \tag{5.6}$$

where C is the integration constant to be determined by the initial condition.

Figure 9 illustrates the WKB trajectories of the evolution of a wave packet on such an axisymmetric topography. Figure 9a corresponds to the WKB trajectories on the β-plane with $k_2 = -0.5$, $k_3 = -0.5$ and $F = 1.0$. Here, if we consider the topography to be an elliptic surface, then the conditions that $k_2 = -0.5$ and $k_3 = -0.5$ correspond to $\eta_B = H - 0.5xX - 0.5yY$. Figure 9b shows the results on the saddle topography on the β-plane. Figure 9a suggests that the packet in that case changes its whole spatial scale and tilts peri-

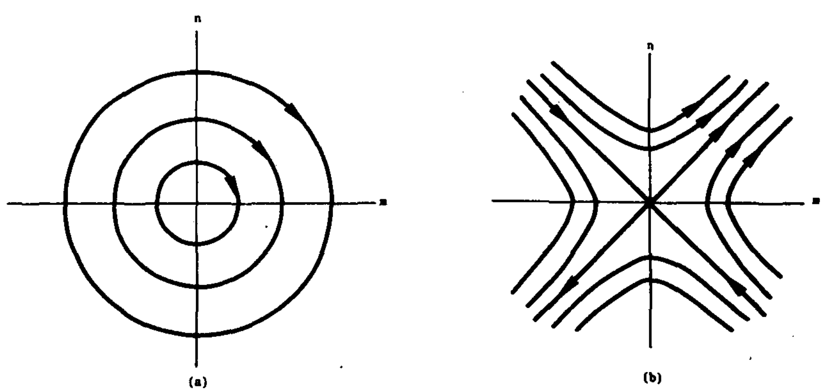


FIG. 9. The WKB phase plane on the earth's β-plane in the presence of an axisymmetric topography for (a) convex topography and (b) saddle topography. Here $k_2 = 0.05$, $k_3 = -0.5$ and $k_1 = 0$.

odically in time. This is the wave packet structural vacillation. However, in Fig. 9b, we cannot find such a vacillation, since the longitudinal and latitudinal scales are increasing (or decreasing) simultaneously. The results demonstrate that on the north–south oriented topography, the packet structural vacillation can exist only on the δ-surface. However on the β-plane, the packet structural vacillation can occur on the convex topography. Further, it was also found that on the β-plane, the packet structural vacillation could still exist on the concave topography. However, on the δ-surface the δ-effect may destroy the property of such packet structural vacillation on the concave topography once the condition $\delta_0 > k_3$ is satisfied. Moreover, in some cases on the saddle topography, the packet structural vacillation on the β-plane of the earth does not exist.

In the topography case $k_1 \neq 0$ (for instance, the asymmetric topography), we have to keep all terms in Eqs. (5.1)–(5.4). The asymmetry of the topography will alter the evolution of the wave packet. The two governing systems, however, are dynamically similar.

Figure 10 exhibits the results of the WKB trajectories of the wave packet on the WKB phase plane in the case that $k_2 = -0.5$, $k_3 = -0.5$ and $k_1 = -0.2$ on the δ-surface with $\delta_0 = 0.2$. From the figure, one can readily find that the evolution on such an asymmetric topography is different from that on the axisymmetric topography. With axisymmetric topography, the WKB trajectories of the wave packet on the WKB phase plane appear to be canonical ellipses; with the asymmetric topography, however, the trajectories are rotated ellipses on the WKB phase plane, although the evolution of the wave packet still is time periodic. The difference means that with the symmetric topography, the wave packet's whole spatial scale will be larger or smaller when the packet is tilted parallel to the Y-axis, whereas

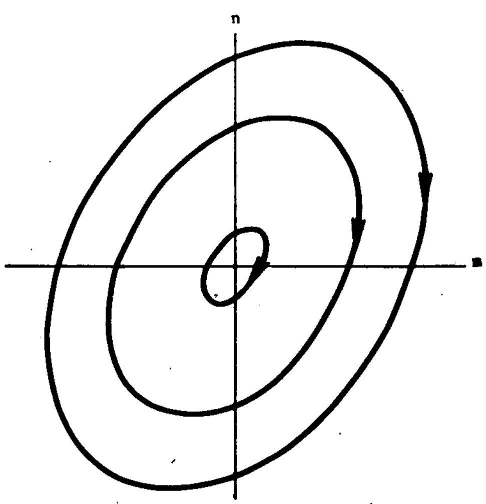


FIG. 10. The WKB phase plane on an asymmetric topography on the earth's δ-surface. Here $k_2 = -0.5$, $k_3 = -0.05$, $k_1 = -0.2$ and $\delta_0 = 0.2$.

with the asymmetric topography the wave packet's whole scale will reach its maximum or minimum when the wave packet is tilted north-west to south-east or north-east or south-west, for example, in Fig. 10. The results suggest that the evolution of a Rossby wave packet might undergo the wave packet structural vacillation on some general topography.

In addition, by integrating the packet along the characteristic line, we can obtain the complete integrals for the different cases, as shown in appendix B.

6. Conclusions

From the foregoing discussion, we can draw the following main conclusions:

1) The δ-approximation of the earth's surface is capable of describing the structural change of a synoptic disturbance system. The presence of the δ-effect is of importance in the evolution of a Rossby wave packet and in the general dynamics of large-scale geophysical flows, especially near the earth's poles. The results show that the governing system on the earth's δ-surface may be dynamically different from the system on the earth's β-plane. Therefore, on the δ-surface of the earth, the evolution of a Rossby wave packet will differ from the evolution on the β-plane of the earth, either qualitatively or quantitatively. The computation results support the theory.

2) Wave packet structural vacillation has been found on both the β-plane and the δ-surface in the presence of basic current and/or topography. The wave packet structural vacillation is characterized by time-periodic changes of the wave packet structure. Both the tilt and spatial scales (in the X-direction, the Y-direction and the whole) will evolve periodic changes simultaneously. The wave packet structural vacillation is characterized by the closed WKB trajectory on the WKB phase plane. Moreover, the wave packet vacillation suggests a possible mechanism of vacillations observed in the atmosphere.

3) In the meridional basic current or on the north–south oriented topography, there is no such packet structural vacillation on the β-plane. On the δ-surface, however, there may exist such packet structural vacillations.

4) The quadratic east–west oriented topography only modifies the δ-effect in the structural change of the wave packet. This result approximates the view that linearly sloping topography only modifies the β-effect in the propagation properties of the Rossby waves.

5) The results show that the global behaviors of the evolution of a Rossby wave packet may vary accordingly. For example, in a southwesterly jet or convex topography, the WKB trajectories of the wave packet on the WKB phase plane appear to be closed lines, e.g., ellipses, which suggest that there exist wave packet structural vacillations. In a southeasterly jet or with a saddle topography, for example, the WKB trajectories

on the WKB phase plane exhibit open lines, e.g., hyperbola or straight lines, which suggest that both spatial scales of the wave packet in the X-direction and in the Y-direction are increasing (or decreasing) simultaneously.

The wave packet structural vacillation is a little different from vacillations observed in the atmosphere, since the wave packet structural vacillation undergoes a time periodic change in both the tilt and the spatial scales (in the X-direction, in the Y-direction and the whole) simultaneously. Therefore, the process of vacillations in the real geophysical flows (e.g., Gruber, 1975; Webster and Keller, 1975; McGuirk and Reither, 1976), will be much more complicated than the packet structural vacillation found in this study. Understanding the mechanism for such vacillations in real geophysical flows is still challenging to investigators. It is surprising to notice that though only employing a simple barotropic model, one still can find such wave packet structural vacillations very easily, since, in general, the wave structural vacillations have been mainly considered baroclinic processes. Recently, the role of topography in the atmosphere has attracted increasing attention (e.g., Hart, 1979; Charney and DeVore, 1979; Hendon, 1986). The present results suggest a new role of topography in geophysical flows. Furthermore, the results strongly suggest that both the topography and basic current could result in the wave packet structural vacillation in the β-plane and in the δ-surface as well. In spite of this, one still should exercise extreme care in applying the present theory to the real geophysical flows, since the baroclinic process may dominate in some cases, and the flows will be more complicated than in the present model. Obviously, further work is needed on a comparative basis with prediction, by using real data and observation.

As the author (Yang, 1987) has already mentioned, the validity of the WKB method used here is limited. Obviously, the narrow jets and sharp topography do not suit the present method. The present model precludes investigation of three-dimensional problems, which are of great importance in real geophysical flows and studies in geophysical fluid dynamics. Therefore, further work should be focused on the three-dimensional problem in the δ-surface of the earth in order to understand better the mechanism of vacillations in the annulus and real geophysical flows, as well as the dynamical significance of the δ-effect in the structural change of the disturbance system in the real geophysical flows. In the present study, only one wave packet has been taken to represent a disturbance system. Using multiple wave packets to study nonlinear or seminonlinear phenomena in the geophysical flows will be of interest, since it would be an oversimplification to take the real disturbance system as a single isolated wave packet. Furthermore, it may be worthwhile to investigate the interaction among different disturbance systems by using the concept of multiple wave packets.

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APPENDIX A

The Wave Packet Solutions for the Asymmetric Basic Current on the Earth's β-Plane

Since there is a relationship between m and n, i.e., (3.9), we need only give the expressions for m. As an example, we consider the case of $(\partial U/\partial Y)(\partial V/\partial X) < 0$ and $\partial V/\partial X > 0$.

Define

$$k^2 = -\frac{\partial U}{\partial Y}\frac{\partial V}{\partial X} > 0. \quad \text{(A1)}$$

Then, when

$$1 - k^2 = 0,$$

$$\left(1 + \frac{F}{C_1}\right)\sin\frac{mk}{\sqrt{C}} = -\frac{\partial V}{\partial X}T + C_2, \quad \text{(A2)}$$

when

$$1 - k^2 > 0,$$

$$\arcsin\frac{mk}{\sqrt{C_3}} + \frac{F}{(1-k^2)\left(\frac{C_3}{1-k^2}\right)^{1/2}\left(\frac{C_3}{1-k^2}+\frac{C_3}{k^2}\right)^{1/2}} \times \arctan\frac{m\left(\frac{C_3}{1-k^2}+\frac{C_3}{k^2}\right)^{1/2}}{\left(\frac{C_3}{1-k^2}\right)^{1/2}\left(\frac{C_3}{k^2}-m^2\right)^{1/2}} = -\frac{\partial V}{\partial X}kT + C_4, \quad \text{(A3)}$$

when

$$k^2 > 1,$$

$$\arcsin\frac{mk}{\sqrt{C_5}} + \frac{F}{(k^2-1)\left(\frac{C_5}{k^2-1}\right)^{1/2}\left(\frac{C_5}{k^2-1}-\frac{C_5}{k^2}\right)^{1/2}} \times \arctan\frac{m\left(\frac{C_5}{k^2-1}-\frac{C_5}{k^2}\right)^{1/2}}{\left(\frac{C_5}{k^2-1}\right)^{1/2}\left(\frac{C_5}{k^2}-m^2\right)^{1/2}} = -\frac{\partial V}{\partial X}kT + C_6. \quad \text{(A4)}$$

Here, C_1, C_2, C_3, C_4, C_5 and C_6 are constants to be determined by initial conditions, and $C = C_1 k^2$.

APPENDIX B

The Expressions on the WKB Phase Plane for the Asymmetric Topography on the Earth's δ-surface

When $\delta_0 = k_3$,

$$m = -\frac{k_2}{k_1} n \ln n + C_1 n, \quad \text{(B1)}$$

or,

$$n = C'_1 e^{-(k_1/k_2)(m/n)}. \quad \text{(B1)}'$$

When $\delta_0 \neq k_3$,

$$\frac{1}{2}\ln\left(\frac{m^2}{n^2} + \frac{k_2}{\delta_0 - k_3}\right) + \frac{k_1}{\delta_0 - k_3}\arctan\left(\left(\frac{\delta_0 - k_3}{k_2}\right)^{1/2}\frac{m}{n}\right) = \ln n + C_2, \quad \text{(B2)}$$

or,

$$\left(m^2 + \frac{k_2}{\delta_0 - k_3} n^2\right)^{1/2} = C'_2 n^2 e^{-[k_1/\sqrt{k_2(\delta_0 - k_3)}]\arctan\{[\sqrt{(\delta_0 - k_3)/k_2}](m/n)\}}, \quad \text{(B2)}'$$

if

$$k_2(\delta_0 - k_3) > 0. \text{ And}$$

$$\ln\left(m^2 + \frac{k_2}{\delta_0 - k_3} n^2\right)^{1/2} + \frac{k_1}{2\sqrt{|k_2(\delta_0 - k_3)|}} \times \ln\frac{m - \left|\frac{\delta_0 - k_3}{k_2}\right|^{1/2}}{m + \left|\frac{\delta_0 - k_3}{k_2}\right|^{1/2}} = \ln n^2 + C_3, \quad \text{(B3)}$$

or,

$$\left(m^2 + \frac{k_2}{\delta_0 - k_3}\right)^{1/2}\left(\frac{m - \left|\frac{\delta_0 - k_3}{k_2}\right|^{1/2}}{m + \left|\frac{\delta_0 - k_3}{k_2}\right|^{1/2}}\right)^{k_1/[2\sqrt{|k_2(\delta_0 - k_3)|}]} = C'_3 n^2, \quad \text{(B3)}'$$

if

$$(\delta_0 - k_3)k_2 < 0.$$

Here, C_1, C_2, C_3, C'_1, C'_2 and C'_3 are integration constants to be determined by initial conditions.

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