The Generation Mechanism of Mixed Rossby–Gravity Waves in the Equatorial Troposphere

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ABSTRACT

Numerical experiments are performed to clarify the excitation mechanism of mixed Rossby–gravity waves (Yanai waves) in the tropical troposphere, as well as the selection of zonal wavenumbers 4–5 and of the five-day period. The model used is governed by the primitive equations on an equatorial β-plane. Moisture budgets are calculated explicitly.

A nonlinear wave–CISK mechanism produces Yanai waves with the same spectral peaks in wavenumber and frequency as observed. In the absence of antisymmetric lateral forcing, these peaks do not appear distinctly, because the symmetric equatorially trapped modes, i.e., Kelvin-like waves having different spectral peaks, are dominant. It is the lateral antisymmetric forcing which puts the peaks characterizing the antisymmetric Yanai waves in evidence.

It appears that Yanai waves of very small wavenumbers (1–3) cannot have large amplitudes because their frequencies are too large for moisture to be effectively supplied for the convection associated with these waves. Symmetric Kelvin modes are dominant in the absence of forcing asymmetries due at least in part to the difference in the nature of heating between symmetric and antisymmetric modes: precipitation, and hence heating, is not normally distributed. Given a strongly skewed distribution of heating, it can be shown that symmetric modes are excited more effectively. Finally, our results indicate that the vertical wavenumber, and hence the period of Yanai waves are selected by the height of cumulus convection, while the lateral forcing selects the horizontal wavenumber within a certain band provided by the nonlinear wave–CISK mechanism.

1. Introduction and motivation

Mixed Rossby–gravity waves were discovered in the tropical stratosphere by Yanai and Maruyama (1966). Shortly after their discovery, it was shown, theoretically by Lindzen and Matsuno (1968) and observationally by Yanai and Hayashi (1969), that these waves are generated in the troposphere and propagate into the stratosphere. However, the tropospheric excitation mechanism which selects the observed zonal wavenumber of 4–5 and the period of 5 days is still unclear.

A plane wave in a three-dimensional medium is characterized by four parameters: frequency $\omega$, meridional wavenumber $n$, zonal wavenumber $k$ and vertical wavenumber $m$ (or equivalent depth $h$). A mixed Rossby–gravity wave has $n = 0$ (Blandford, 1966; Matsuno, 1966). The dispersion relation then will fully characterize the Rossby–gravity wave, provided two out of $\omega$, $k$ and $h$ are determined. In constructing a theory for such a wave, it is important which parameter is determined by what mechanism.

Two important hypotheses on the generation of mixed Rossby–gravity waves, often called Yanai waves (Gill, 1982, p. 662; Pedlosky, 1979, section 8.5), have been formulated. One is the lateral forcing theory proposed by Mak (1969). In this theory, $h$ is selected by linear resonance, so that the vertical wavelength of the wave coincides with the vertical wavelength of lateral forcing. Then, $\omega$ or $k$ is determined by a resonance condition on the horizontal structure (Hayashi, 1976). Mak examined the response of the tropical atmosphere to lateral boundary forcing. In his model, westward-moving resonant waves had a large response at wavenumber 3 and period 4 days. Still, it appears unlikely that $k$ or $\omega$ of the equatorially trapped waves are directly influenced by the lateral forcing away from the equator, where the waves are exponentially evanescent. Moreover, Hayashi (1976) noted that by imposing midlatitude forcing as a lateral boundary forcing, the resonant waves are spurious “westward Kelvin waves” having a horizontal structure completely different from observations of Yanai waves.

The other hypothesis is called wave–CISK theory (Hayashi, 1970). This theory proposes that the interaction between equatorial waves and cumulus convec-

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tion is unstable and that it can produce Yanai waves as free modes. Hayashi found that these unstable waves show a realistic horizontal structure and a realistic vertical wavelength of 10–15 km in the troposphere, when using a linearized CISK parameterization to model cumulus convection. Nevertheless, he could not explain the selection of the zonal wavenumber and the period. The reason is obvious. In this theory, $h$ is determined by the vertical structure equation as a complex eigenvalue. More specifically, the height and the form of cumulus convection determine a value of $h$. Next, $\omega$ or $k$ is selected by the growth rate. If the imaginary part of $h$ is small, the growth rate is nearly proportional to the frequency. Thus, the prediction of $\omega$ and $k$ by this theory necessarily fails, because gravity wave modes with large wavenumbers have very high frequencies, and hence will grow faster than Kelvin or Yanai waves with small wavenumbers.

Although the resonant waves of Mak (1969) proved to be unrealistic, recent data analyses (Zangvil and Yanai, 1980; Yanai and Lu, 1983; M.-M. Lu, personal communication, 1986) still show that observed Yanai waves are closely related to lateral forcing, i.e., to the energy flux of zonal wavenumbers 3–6 from midlatitudes. On the other hand, Zangvil and Yanai (1981) found that the cloud cover in the tropics has the same peaks in wavenumber and frequency as the waves (see also Gruber, 1974). This suggests a strong coupling between the waves and cumulus convection. A series of experiments with the GFDL general circulation model (Hayashi and Golder, 1978) also shows that both midlatitude forcing and interaction with cumulus convection are important for the excitation of Yanai waves (see also Hayashi, 1974). However, the relation between the lateral forcing and the interaction with cumuli is not clear. A further difficulty is that the dominant period of planetary-scale waves (wavenumbers 3–6) in midlatitudes (10–20 days) is much longer than that for Yanai waves (Hayashi and Golder, 1978; Lu, 1987).

Any theory involving lateral forcing must explain the difference of dominant periods between midlatitudes and tropics.

Models with lateral forcing, as well as a cumulus effect, have been proposed by Lamb (1973) and Itoh (1978). Both authors obtained in their models a selection of wavenumbers and periods by the lateral forcing, while cumulus convection amplified the waves. Itoh also showed that the selection of vertical wavelengths is done by the cumulus convection. Lamb used a simple cumulus parameterization and obtained sharp spectral peaks in zonal wavenumber and frequency determined by the lateral forcing. Since he used the same type of lateral boundary forcing as Mak (1969), her results about the selection of wavenumbers and periods are subject to the same questions raised by Hayashi (1976).

Itoh proposed replacing lateral boundary forcing by lateral body forcing, thus eliminating spurious wave reflection from the lateral boundaries. He still found the largest response at moderate zonal wavenumbers of 3–7, although this was not as sharp a peak as obtained for less realistic waves. However, Itoh's (1978) model largely depends on a linearized version of the Arakawa-Schubert (1974) cumulus parameterization. It is not clear whether the linearization and the direct adaptation of this cumulus parameterization to large-scale phenomena are meaningful.

The studies of Lamb (1973) and Itoh (1978) started from the premise that the wave–CISK mechanism by itself would select wavenumbers and periods different from those observed. Very recently, however, it has been shown by two studies that if a nonlinear effect is taken into account in wave–CISK, results about the wavenumber and period selection change completely. In these two studies (Hayashi and Sumi, 1986; Lau and Peng, 1987), numerical experiments were performed for the purpose of reproducing the equatorially symmetric wavenumber-1 oscillation with a period of 30–50 days in the tropics. In these experiments, Kelvin waves are essentially generated by the interaction with cumulus convection, as in linear wave–CISK theory. The strikingly new result was that wavenumber 1 is dominant, in contradistinction to the results of linear wave–CISK theory. Hayashi and Sumi used a model in which water vapor budgets are explicitly calculated. They pointed out that the coupling of small cumulus scales and large wave scales leads to a selectivity different from linear wave–CISK theory.

Lau and Peng performed two experiments. One had unconditional heating like in linear wave–CISK theory (i.e., it had negative diabatic heating in the downward motion area) and the other had conditional heating (i.e., it had no diabatic heating in the downward motion area). Larger wavenumbers became dominant in the former experiment, as in the linear theory, but the latter experiment could reproduce the dominance of wavenumber 1. From these results, they concluded that the conditional heating, i.e., the nonlinearity in the thermodynamics, is essential for the dominance of wavenumber 1. Thus, nonlinear wave–CISK can select a wavenumber and period different from linear wave–CISK.

It is interesting therefore to reexamine the generation mechanism of Yanai waves from the standpoint of nonlinear wave–CISK (cf. also Hayashi and Golder, 1978). The first purpose of this paper is to investigate whether a nonlinear model with wave-cumulus interaction produces Yanai waves with the same spectral peak in wavenumber and frequency as observed. In contradistinction to the equatorially symmetric Kelvin waves, the dominant wavenumbers for the antisymmetric Yanai waves are not very low, but are intermediate (4–5). To clarify the reason for this dominance would be an important contribution to the wave-cumulus interaction problem.

If this nonlinear wave–CISK mechanism can produce the correct Yanai waves, the next problem is the
relation to lateral forcing: Why do recent data analyses show a close relation between midlatitude waves and Yanai waves, although the latter appear to be generated by wave-CISK alone? The second purpose of this paper is to clarify this relation. Furthermore, we attempt to explain the difference in dominant period between Yanai waves and midlatitude waves.

In section 2, we describe the model used in this study. The results of three basic experiments are given in section 3. These results show that: 1) the nonlinear wave-CISK mechanism can select the same wavenumber and period of Yanai waves as observed; 2) in the absence of asymmetries in lateral forcing, symmetric Kelvin modes are dominant over antisymmetric Yanai modes; 3) if lateral antisymmetric forcing with arbitrary periods is given, the correct spectral peak of Yanai waves becomes apparent. The following three sections give further insights into these main results.

In section 4 it is shown how the nonlinear wave-CISK mechanism selects the observed spectral peaks in zonal wavenumber and frequency. In section 5 we address the question of why symmetric modes dominate antisymmetric Yanai waves. In section 6 we examine why the tropical atmosphere selects the periods of Yanai waves independently of those of lateral forcing. Concluding remarks follow in section 7. Three appendices provide details on cumulus parameterization, interaction between symmetric and antisymmetric modes, and the role of baroclinic waves in the model, respectively.

2. Model description

The model used in this study is governed by the primitive equations in \( \sigma \)-coordinates on an equatorial \( \beta \)-plane, with moisture budgets treated explicitly. The basic equations are the following:

\[
\frac{\partial u}{\partial t} = \eta v - \frac{\partial}{\partial x} (K + \phi) - \frac{\partial u}{\partial \sigma} - \sigma \frac{\partial \pi}{\partial \sigma} + F_u + k_\eta \nabla^2 u, \quad (1)
\]

\[
\frac{\partial v}{\partial t} = -\eta u - \frac{\partial}{\partial y} (K + \phi) - \frac{\partial v}{\partial \sigma} - \sigma \frac{\partial \pi}{\partial \sigma} + F_v + k_\eta \nabla^2 v, \quad (2)
\]

\[
\frac{\partial \pi \theta}{\partial t} = -\nabla \cdot (\pi \nabla \theta) - \frac{\partial}{\partial \sigma} (\pi \theta) + \frac{\partial}{\partial \sigma} \left( \frac{\dot{Q}}{C_p} \right) + \pi R + \pi F_\theta + \pi k_\eta \nabla^2 \theta, \quad (3)
\]

\[
\frac{\partial \pi q}{\partial t} = -\nabla \cdot (\pi \nabla q) - \frac{\partial}{\partial \sigma} (\pi \theta) - \pi C + \pi F_q + \pi k_\eta \nabla^2 q, \quad (4)
\]

Independent variables are the time \( t \), the eastward and northward coordinates \( x \) and \( y \), and the vertical coordinate \( \sigma \). The prognostic variables are the eastward and northward wind components \( v = (u, v) \), the potential temperature \( \theta \), the water-vapor mixing ratio \( q \), and \( \pi = \pi_s - \pi_t \), where \( \pi_t \) is the surface pressure and \( \pi_s \) the pressure of the model top. Diagnostic variables are the geopotential \( \phi \) calculated from the hydrostatic equation (6), the absolute vorticity \( \eta \), the kinetic energy \( K \), the specific volume \( \alpha \) and the vertical velocity \( \dot{\sigma} \), which is estimated from the relation

\[
\pi \dot{\sigma} = -\nabla \cdot \int_0^\sigma \pi v d\sigma - \frac{\partial \pi}{\partial t}, \quad (7a)
\]

with the boundary condition

\[
\dot{\sigma} = 0 \quad \text{at} \quad \sigma = 0 \text{ and } 1. \quad (7b)
\]

Model constants are \( c_p \), the specific heat at constant pressure, \( R_0 \), the gas constant of dry air, \( p_0 = 1000 \text{ mb} \), and \( \kappa = R_0/c_P \). The horizontal boundary conditions are

\[
v = 0, \quad \frac{\partial u}{\partial y} = \frac{\partial \theta}{\partial y} = \frac{\partial p}{\partial y} = \frac{\partial \pi}{\partial y} = 0 \quad \text{at} \quad y = \pm B, \quad (8)
\]

with \( B \) corresponding to 40° latitude.

In a study geared at explaining basic mechanisms like the present one, the model should be as simple as possible while still capturing the phenomenon under investigation. Thus, the maximum linearization possible would appear desirable. But preliminary experiments without an explicit water-vapor budget did not simulate Yanai waves well. Since the distribution of water vapor is determined mainly by the advection, the nonlinear advection terms in Eq. (4) must be included. The analogous terms in the other equations were thus retained for consistency, which does not prevent us from using linearized dynamics in the interpretation of results (cf. section 6).

The \( \dot{Q} \) in Eq. (3) is the diabatic heating by cumulus and large-scale condensation, and \( C \) in Eq. (4) stands for the moisture sink. The parameterization of cumulus convection is based on moist convective adjustment (Manabe et al., 1965). When the lapse rate exceeds \( \Gamma_m \), the moist adiabatic one, and the relative humidity is over 85%, convection is assumed to occur. The lapse rate is then adjusted to \( \Gamma_m \) and the relative humidity is changed to 85%, requiring that the vertical integral of \( c_p T + q \) be conserved, where \( T \) is the temperature and \( L \) represents the latent heat.

We modified this scheme only in the lowest \( \sigma \)-layer, following the suggestion of Arakawa and Chen (1987):
The lapse rate there is adjusted to a lapse rate $\Gamma$ larger than the moist adiabatic one,

$$\Gamma = \Gamma_m + C_0(\Gamma_d - \Gamma_m),$$  \hspace{1cm} (9)

where $\Gamma_d$ stands for the dry adiabatic lapse rate, and $C_0$ is a constant, taken as 0.2 except in a special experiment in section 4. This modification has the result that penetrating convection occurs less frequently than in the conventional adjustment scheme. This scheme may be the simplest substitute for the Arakawa-Schubert parameterization over the oceans (Arakawa and Chen, 1987). Kuo's (1974) parameterization is also used for verification. The implementation of this parameterization and the corresponding results are described in appendix A.

The $R$ in Eq. (3) represents net radiation. For simplicity, Newtonian heating and cooling are assumed:

$$R = -h_0(\theta - \theta_e),$$  \hspace{1cm} (10)

where $h_0$ is the Newtonian heating/cooling rate and $\theta_e$ expresses the potential temperature at radiative equilibrium. The $F_u$, $F_v$, $F_z$ and $F_q$ represent vertical diffusion of each variable

$$F_u = \frac{\partial \tau_u}{\partial \sigma},$$  \hspace{1cm} (11)

and so on. Here $\tau_u$ is the vertical flux of the $u$-momentum and is defined within the free atmosphere as follows:

$$\tau_u = \frac{\rho g}{p_s} k_w \frac{\partial u}{\partial \sigma},$$  \hspace{1cm} (12)

where $k_w$ is the vertical diffusion coefficient for the momentum, $g$ is the gravity, $\rho$ the density and $p_s$ the surface pressure.

Surface fluxes are calculated as follows:

$$\tau_u = \rho_a C_D |v_a| u_a,$$  \hspace{1cm} (13a)

$$\tau_v = \rho_a C_D |v_a| v_a,$$  \hspace{1cm} (13b)

$$F_H = \rho_a C_D |v_a| (T_s - T_a),$$  \hspace{1cm} (13c)

$$F_q = \rho_a C_D |v_a| (q_s - q_a),$$  \hspace{1cm} (13d)

where the subscripts $a$ and $s$ denote values at the level just above the surface and values at the earth’s surface, respectively, and $C_D$ is the drag coefficient. We assume that the whole surface is ocean covered.

The vertical finite-difference scheme follows Arakawa and Suarez (1983), $\hat{\sigma}$ being staggered with respect to $v$, $\theta$, $q$ and $\phi$. The latter are defined at levels 1, 2, 3, 4 and 5, while $\hat{\sigma}$ is defined at levels $\frac{1}{2}$, $\frac{1}{2}$, ..., $\frac{5}{2}$, which correspond to $p = 80, 160, 320, 540, 900$ and 1013 mb. Boundary conditions $\hat{\sigma} = 0$ [cf. Eq. (7b)] are prescribed at $p_i = 80$ mb and $p_{i+1} = 1013$ mb.

In the horizontal, an Arakawa C-grid of $4^\circ$ lat $\times$ 6$^\circ$ long is used (Arakawa and Lamb, 1977), with $v$-points and $\eta$-points along the equator and every 4$^\circ$ lat, alternating with $\phi$-points and $u$-points at 26N and so on; $\theta$, $q$ and $\pi$ are all defined at $\phi$ points. The lateral boundaries are set at 40$^\circ$N and 40$^\circ$S, and we shall see in section 3 that the tropical motions are hardly influenced at all by waves reflected from these boundaries.

The potential-entropy-conserving scheme of Arakawa and Lamb (1981) is adopted for the horizontal advection of momentum; it is second-order accurate in space. The schemes used to advance in time potential temperature and water-vapor mixing ratio also have second-order accuracy in space. Vertical advection is done by the conventional scheme which conserves quadratic invariants, except for the water-vapor mixing ratio. For the latter we use the scheme of Arakawa and Lamb (1977) in order to avoid conditional instability of the computational kind. The complete spatial finite-differencing scheme is thus second-order accurate and conserves the integrals of $\pi$, $\theta$, $\theta^2$, total energy and potential entrophy.

Time differencing is done by the leapfrog method, with the Matsuno (Euler-backward) scheme inserted every four time steps to damp the computational mode of the leapfrog scheme. The time increment is 10 minutes. Cumulus convection and large-scale condensation are computed every 80 minutes, and are added evenly to the prognostic variables over the succeeding eight steps.

The basic initial condition is a state of rest. The vertical structure of the temperature is shown in Fig. 1. The observed values (solid line) are obtained by av-

**Fig. 1.** Vertical profiles of temperature, as a function of approximate height $z$. The solid line shows the initial profile, the dashed line indicates radiative-equilibrium temperature and the dash-dotted line gives the average temperature at 2$^\circ$N and 2$^\circ$S from day 20 through day 50 in Experiment 1. The initial profile is computed from the average of FGGE data at 2$^\circ$N and 2$^\circ$S between May and July 1979.
eraging FGGE data at 2°N and 2°S from May through July 1979. The water-vapor mixing ratio is prescribed initially so that the relative humidity is 85% at each level except the highest, where it is zero.

Small initial perturbations of the potential temperature are added only in the lowest level, as follows:

\[ I_+ = a_+ \cos \frac{2\pi}{B} y \cos kx \quad \text{for symmetric modes}, \]

\[ I_- = a_- \sin \frac{2\pi}{B} y \cos kx \quad \text{for antisymmetric modes}, \]

(14a, 14b)

where \( a \) is the amplitude, \( k \) a zonal wavenumber, and \( 2B \) is the width of the equatorial \( \beta \)-channel [cf. Eq. (8)]. With such a perturbation, it is expected that cumulus convection will occur in certain regions and motions will grow rapidly, if the system is unstable.

In some experiments, we shall also prescribe lateral forcing. This is formulated as body forcing, following Itoh (1978), because boundary forcing might lead to the growth of spurious waves, as discussed in section 1. We add the following term to Eq. (3) in a latitude band between 18° and 34° in either hemisphere:

\[ F = \frac{b}{c_p} \left( \frac{\rho_0}{\rho} \right)^\kappa \sin \pi \sigma \cos(kx - \omega t). \]

Here \( b \) denotes the amplitude at 26°N, the sign is reversed between the Northern and Southern Hemispheres, and \( \omega \) is the frequency of the forcing. This forcing is added at latitudes 22°, 26° and 30° only, the amplitude at latitudes 22° and 30° being \( \pm b/2 \), depending on the hemisphere. Although it is possible to add forcing to other equations, this appears to be the simplest and most reasonable, considering the nature of geostrophic adjustment in midlatitudes.

The numerical experiments were performed for 50 days, except as indicated. Since Yanai waves have relatively short periods and, therefore, short response times, 50 days appear to be entirely sufficient to study their growth and structure. Furthermore, the model’s “eastern hemisphere” only was used, and periodic boundary conditions imposed in longitude. The reduced domain of integration was introduced for computational economy, and results in the elimination of all odd zonal wavenumbers. No strong Yanai waves are observed at wavenumbers 1 or 3. Therefore, the use of the reduced domain of integration does not present a serious problem (compare also Ghil et al., 1981; Legras and Ghil, 1985).

The only question arising might be that we neglect the large signal of the 30–50 day wavenumber 1 oscillation. This, however, is not as serious a problem as it appears, because symmetric wavenumber 2 eastward-moving modes with shorter periods (observed by Lu and Yanai, 1987) have large variances in our model instead of wavenumber 1, as shown later (e.g., Figs. 6b and 11b). The interaction between very slow, symmetric modes, and intermediate, antisymmetric modes can still be studied in our numerical experiments.

The radiative equilibrium temperature used in this study is shown by the dash-dotted line in Fig. 1. It has no north–south gradient because the effects of baroclinicity may obscure the wave–CISK mechanism and therefore it is desirable to eliminate this effect in our study, as much as possible (but see appendix C). The \( T_e \) is calculated by extrapolating from the lowest-level temperature under the assumption of a lapse rate of 7 K km\(^{-1}\), and \( q_e = R_h(5)q^*(T_e) \), where \( R_h(5) \) stands for the relative humidity at level 5 and \( q^* \) represents the saturated value. The sea surface temperature \( T_s \) is assumed to be longitudinally uniform, and constant between 10°S and 10°N, \( T_s = 27.5°C \). It is taken to increase linearly from \( T_s = 20°C \) at the boundaries to its equatorial value at 10° lat. This is obviously an oversimplification, but a very good approximation to observed sea surface temperature in the tropics (0°–20° lat). We also assume that \( u_s \) and \( v_s \) are equal to the values at level 5 and \( q_s = q(T_s) \).

Other parameter values are as follows: \( h_0 = 1/(8 \text{ days}), C_D = 0.0025, k_{uv} = 10 \text{ m}^2 \text{ s}^{-1}, k_{vq} = k_{uq} = 0 \). Therefore, there is no vertical diffusion of potential temperature and moisture in our model. The Newtonian heating/cooling coefficient \( h_0 \) gives cooling rates of 1–1.5 K day\(^{-1}\) at levels 3 and 4 (e.g., Webster, 1983), when used in Eq. (10) with the model’s radiative equilibrium temperature, \( \theta_e \) (Fig. 1, dash-dotted). The horizontal diffusion coefficient \( k_{uv} \) is taken equal to 2 × 10\(^4\) m\(^2\) s\(^{-1}\) in the interior of the domain, up to 26°N. It is monotonically increased to 2 × 10\(^6\) m\(^2\) s\(^{-1}\) at the boundaries in order to prevent spurious wave reflection.

We shall be interested in space–time spectral analyses of model solutions. The spatial Fourier coefficients are calculated by straightforward harmonic analysis. Power or cross spectra in the frequency domain are then computed by the lag-correlation (Blackman-Tukey) method with a Hanning spectral window of suitable width (Hayashi, 1971b).

Finally we define more precisely a symmetric and an antisymmetric mode. A symmetric mode has symmetry in \( u, T, q \), heating and precipitation with respect to the equator. Such a mode is antisymmetric in the \( v \)-component. An antisymmetric mode has the reverse configuration. Of course, theoretical Yanai waves are antisymmetric, while theoretical Kelvin waves are symmetric modes. Observationally, the 30–50 day oscillation in the tropics contains a traveling wavenumber 1 structure which is nearly symmetric about a circle of latitude close to the equator, while the observations of intermediate wavenumbers with periods near 5 days indicate strong antisymmetric features.

We shall need to calculate the symmetric and antisymmetric parts of an arbitrary field \( A = A(x, y) \):

\[ A_+ = \frac{(A_N + A_S)}{2}, \]

(16a)
\[ A_\text{\#} = (A_N - A_S)/2, \]  

(16b)

where \( A_N \) is a value at a grid point in the Northern Hemisphere and \( A_S \) is the corresponding value at the same longitude and latitude but in the Southern Hemisphere (cf. Yanai and Murakami, 1970). Therefore, a symmetric mode is composed of \( v_\text{n} \) and many \( (\ldots) \) quantities and an antisymmetric mode is a combination of \( v_\text{n} \) and many \( (\ldots) \) quantities.

3. Results of basic experiments

Nine experiments altogether were performed, differing from each other in the initial data, lateral forcing and cumulus parameterization. These experiments are listed in Table 1, along with the number of the section in which each experiment is discussed.

a. Antisymmetric initial perturbation

In our first experiment, a purely antisymmetric perturbation (14b) is prescribed at the initial time, with zonal wavenumber \( k = 2 \) and amplitude \( a_\text{\#} = 0.1^\circ \text{C} \). No lateral forcing is used, i.e., \( b = 0 \) in Experiment 1.

Symmetric modes can also be produced from these initial data, even in the absence of numerical round-off errors, due to the advective nonlinearity, as described in appendix B. An even more important source of symmetric modes is the nonlinear rectification in cumulus heating, i.e., there is no negative and hence no antisymmetric cumulus heating. On the other hand, initial symmetric modes can only produce antisymmetric modes through round-off errors.

Figure 2a shows the time evolution of symmetric (dashed line) and antisymmetric (solid line) kinetic energy densities, averaged over latitudes \( 0^\circ - 18^\circ \). These quantities are defined in appendix B. For the sake of later convenience, results for days 51–90 (Experiment 1A) are also shown. Long waves, especially the antisymmetric mode of wavenumber 2 present in the initial data (solid line, Fig. 2b) and the symmetric mode of wavenumber 4 generated kinematically by it (not shown) develop strongly in the early stages of the time integration, as the water-vapor supply is plentiful.

After day 20, the kinetic energy densities and all other flow quantities reach a nearly stationary regime, with relatively small fluctuations about the mean. This flow regime changes again gradually at about day 50. Therefore, we shall concentrate on studying the results for days 20–50, except when stated otherwise (Experiment 1A). In Experiment 1, most of the symmetric part of kinetic energy is contained in the longest waves (not shown), while the antisymmetric modes have kinetic energies a little larger in wavenumbers 6 and 8 than in wavenumbers 2 and 4 (Fig. 2b).

We examine first the time-averaged features of the results from day 20 to day 50. Figure 3 shows the zonal mean wind so averaged. The zonal mean wind is very nearly symmetric and has the maximum value of 16.5 m/s at latitude 22°. This feature is of course different from observations, because our mean zonal flow is generated only by the latitudinal difference in cumulus heating. The mean zonal wind velocity is weak, so baroclinic waves are also relatively weak. Therefore, the intrinsic lateral forcing in the model is rather small, but nonvanishing (cf. appendix C).

Figure 1 shows also the average zonal mean temperature at the equator in this period. Compared with observations, the model temperature is high by about 3–6°C in the lowest three levels, in spite of the large Newtonian cooling rate. This is the most serious deficiency in the present results. Such a profile does not favor the development of cumulus convection. Therefore, wave amplitudes in all experiments are smaller than observed. Together with the modified convective

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</tbody>
</table>
Fig. 2. Time evolution of the kinetic energy densities in Experiments 1 and 1A. (a) The total energy is shown for antisymmetric modes (solid line) and symmetric modes (dashed line). (b) The antisymmetric energy of each wavenumber (WN in the figure) is shown for wavenumber 2 (solid line), wavenumber 4 (dashed line), wavenumber 6 (dash-dotted line) and wavenumber 8 (dashes and two dots). These values are averaged in whole layers and over latitudes $0^\circ$–$18^\circ$. The temporal and zonal mean is subtracted. Note that the period of the time mean here is 90 days and differs from later analyses, in which periods are days 20–50 and days 51–90.

adjustment scheme, this also hampers the occurrence of penetrating convection. As a result, some model waves have longer periods than observed, because the vertical wavelength is regulated by the height of cumulus convection, as will be seen later.

The time-mean precipitation is shown in Fig. 4. We can see that the intertropical convergence zone (ITCZ) is clearly formed, although its position is somewhat further poleward in either hemisphere than observed. The figure also suggests that persistent stationary waves are present in the model, although their amplitude is small, and we shall not discuss them further.

The features of zonal mean winds and temperatures are similar in all experiments to those shown in Figs. 1 and 3. The detailed patterns of time-mean precipitation are different from case to case, but the presence of an ITCZ is common to all. We shall mostly concentrate hereafter on the fields at level 2 (approximately 240 mb), because it is at about this level that Yanai waves are most apparent in the real troposphere, and because this level is not too strongly influenced by the top boundary conditions in the present model. Indeed, the vertical energy flux is upward in the upper troposphere (cf. Fig. 8), so the influence of the top boundary on Yanai waves is weak in the model.

We consider now the time-varying perturbation fields over the period of time over which the averages were taken, i.e., the perturbation fields are just the instantaneous fields from which the mean in time and in longitude was subtracted. Figure 5 illustrates the evolution in longitude and time (Hovmöller diagram) of the $v$-component associated with wavenumber 4 at

Fig. 3. Experiment 1: zonal mean flow averaged from day 20 through day 50. All subsequent time means are taken over the same period, except for Experiment 1A. The shaded areas indicate easterly flow, and units are m s$^{-1}$.

Fig. 4. Horizontal structure of the time-mean precipitation. Units are mm/12 h.
level 2 along the equator. It shows the westward progression of the waves with a period of about 5 days, interrupted by two breaks at day 32–35 and day 43–46.

Figure 6 shows the longitude–time power spectra of the horizontal velocity components at level 2 over the equator, with the component averaged between $2^\circ$S and $2^\circ$N. In Fig. 6a, we can see the clear peak at zonal wavenumber 4 and period 5 days in the component, as expected from Fig. 5. The wave associated with this peak can be identified by its horizontal structure, shown in Fig. 7, as a Yanai wave. The component has its peak at wavenumber 2 and longer periods. In this experiment, it should be noted that eastward-moving modes have comparable total power to the westward-moving modes.

Next, we inspect the vertical structure of the wave with $k = 4$ and period 5 days. Figure 8 illustrates the phases and the coherences of the component over the equator and of the antisymmetric temperature at $10^\circ$ lat. The phase varies smoothly with height, increasing in both $v$ and $T$ from the ground to about 7 km and decreasing from there to 15 km. The coherence between level 2 and the other levels is excellent in $v$ (above 0.9) and quite good in $T$ (above 0.75). This structure is similar to observations (e.g., Nitta, 1972) and to the results from general circulation models (Hayashi, 1974; Tsay, 1974).

This wave does not appear to be influenced by the lateral boundaries. Figure 9 shows the latitudinal profiles of the meridional wave energy flux, $\Phi v$ (heavy solid line), and the kinetic energy for wavenumber 4 (light solid). Within the latitude band $20^\circ$S to $20^\circ$N the fluxes are equatorward and outside this band they are poleward. The kinetic energy also has large values inside this band and much smaller ones outside it. These facts suggest that the waves are generated by cumulus convection within this band and are not influenced by reflected waves from the lateral boundaries.

Thus our model produces Yanai waves peaking spectrally at wavenumber 4 and period 5 days without lateral forcing. Last, we show that the waves are indeed generated by the wave–CISK mechanism. The most
FIG. 8. Vertical structure of the westward-moving wave with zonal wavenumber 4 and period of 5 days. (a) Phases of the $v$-component over the equator (solid line) and the antisymmetric temperature component at $10^\circ$ lat (dashed line) are shown; (b) coherences.

direct way may be to compare wave fields with the heating, but there are certain difficulties in analyzing this quantity. First, the heating is very height dependent, so it is difficult to compare it with the height-coherent wave fields (see Fig. 8). Second, the time scale of the heating is much shorter than that of other quantities. We use, therefore, the antisymmetric perturbation fields of precipitation accumulated over 12 h periods as a substitute for heating. This field does not suffer from the above defects.

Figure 10a shows the longitude-time section of the antisymmetric precipitation perturbation associated with wavenumber 4, averaged over latitudes $2^\circ$–$14^\circ$, i.e., over roughly the model's most active precipitation band. At first glance, the precipitation in this figure does not seem to be correlated with the $v$-component in Fig. 5. Actually, the precipitation does not show a clear spectral peak at the period of 5 days in westward-moving modes.

A more careful look reveals though the following relations between Figs. 5 and 10:

1) The coherence and the phase difference are 0.591 and 173 degrees, respectively, with the precipitation leading the $v$-component; the coherence and phase difference being the absolute value and phase, respectively, of the complex correlation coefficient between the two time series. The coherence is acceptable and the phase difference seems reasonable, considering the known phase relations for Yanai waves in wave–CISK (e.g., Fig. 10 in Hayashi, 1970, or Fig. 5 in Itoh, 1978, where the $v$-component near the tropopause is nearly out of phase with the vertically integrated heating). This phase opposition is also consistent with Fig. 8 because heating
is generated over low-level convergence zones and convergence occurs in areas where the \( v \)-component is positive in the lower layer.

2) Although the precipitation does not show sustained, rapid westward movement for long intervals, it does show frequent episodes of westward movement for short intervals of time and with somewhat slower speeds than the velocity field (days 24–26, 35–38, 41–44 and 46–48). The \( v \)-component during these intervals also moves to the west, tending to maintain the approximate opposition of phase. Even more interestingly, except for the episode of very low precipitation intensity at days 26–31, the intervals when precipitation is stationary or moves slightly to the east coincide with stationary or eastward moving episodes for the velocity field (days 20–23, 32–35, 44–46).

3) When relatively heavy precipitation occurs, the amplitudes of the \( v \)-component also increase (days 24, 35, 41 and 47).

4) Even during the interval at days 26–31 when the two fields move in opposite directions, the \( v \)-component has large amplitudes only when it is out of phase with the precipitation.

To bring out in a more objective fashion the features outlined in the paragraphs above, we have applied to the precipitation field given in Fig. 10a the band-pass filter of Murakami (1979). This filter has maximum response at 5 days and the half-response periods are 6 days and 4.17 days. The filtered result is shown in Fig. 10b and indicates clearly the general westward movement, with occasional stationary episodes, apparent for the velocity field in Fig. 5.

Space–time cross spectra between \( v \)-component and precipitation (not shown) also give a marked joint peak at wavenumber 4 and period 5, with a coherency of 0.925. Thus we can say that the two fields illustrate behavior consistent with the wave being excited by and interacting with cumulus convection.

b. Other initial perturbations

The Yanai waves obtained in section 3a have a clear spectral peak at wavenumber 4 and period 5 days, but the initial data used in Experiment 1 were rather special. In Experiment 2 we add a symmetric mode of wavenumber 2 to the initial perturbation. Both amplitudes are \( a_x = a_z = 0.1^\circ C. \)

Results are shown in Fig. 11. The \( v \)-component does not have a sharp spectral peak corresponding to a Yanai wave: the largest peak in westward-moving modes is at wavenumber \( k = 8 \) and period 10 days, with a smaller peak at \( k = 4 \) and period 4–5 days, while in the eastward-moving modes the largest peak is at \( k = 2 \) and period 2.86 days. The \( u \)-component has an even larger peak at \( k = 2 \) than in Fig. 6b, with eastward-moving long periods. The horizontal structure of this mode (not shown) clearly indicates that it is a Kelvin-like mode. The symmetric precipitation perturbation of wavenumber 2 (Fig. 12) also shows predominantly eastward movement with long periods, and is closely related to the movement of the \( u \)-component. It is most likely that if wavenumber 1 were present in the model, this mode would correspond to a 30–50 day period and have an even larger fraction of spectral power.

Next, we extend the time integration in Experiment 1 to 90 days to check to which extent the results depend on the special period analyzed so far, from day 20 to 50. We call this experiment, starting with initial data on day 50 of Experiment 1, our Experiment 1A. The time evolution of kinetic energy is shown in Fig. 2. Symmetric energy is clearly dominant over the antisymmetric energy during this experiment (days 51–90), as it is in Experiment 2. More specifically, the symmetric eastward-moving mode with wavenumber 2 and a large \( u \)-component becomes dominant over the equator (not shown), and the sharp spectral peak associated with the \( v \)-component of the westward-moving Yanai wave of wavenumber 4 disappears.

We summarize in Table 2 the relative power in the westward-moving \( v \)-component of wavenumber 4 and the eastward-moving \( u \)-component of wavenumber 2, for various experiments, one of which is yet to be de-
ward. This period is longer than the dominant period of 5 days for Yanai waves, but is realistic for midlatitude planetary wave activity.

The power spectra of the $v$-component are shown in Fig. 13. There is again a peak at wavenumber 4 and period 5 days in westward-propagating waves, although the peak in wavenumber 8, with a period of 6.67 days, is dominant. Both of these waves have the horizontal structure of Yanai waves (not shown). The latter peak is not observed in the real atmosphere; it is due to the spurious strength of baroclinic waves of wavenumber 8 in the model’s “subtropics,” cf. appendix C. In this appendix, an additional experiment, labeled 3B, with equal symmetric and antisymmetric lateral forcings, is also discussed.

Figure 14 illustrates the latitude-period section of the power spectra for wavenumber 4 in the $v$-component of antisymmetric modes. The subtropics show a strong spectral peak at 10 days, in response to the forcing, but the spectral peak in the tropics is at 5 days.

The lateral energy flux, $\delta v$, at wavenumber 4 and period 10 days is equatorward at latitude 20° and flux convergence occurs at lower latitudes (not shown), in accordance with observations (Lu, 1987). It is interesting that while the latitudinal profile of total energy flux, integrated over frequency and wavenumber, is almost the same as in Experiment 1 (Fig. 9) without lateral forcing. In fact the coherency between Yanai-wave activity and meridional energy-flux convergence is quite low, even when considering convergence for antisymmetric modes only. We shall see in section 6 that lateral forcing acts on the waves mostly through low-level wind-field convergence and ensuing precipitation.

Similar results were obtained in experiment 3' with forcing of period 15 days. Thus, the dominant period in the tropics is independent of the midlatitude forcing period.

---

c. Lateral forcing

We are thus led to conduct an experiment with antisymmetric lateral forcing, cf. Eq. (15), using the same initial data as in Experiment 2. Equatorially antisymmetric components of midlatitude motions are in fact an important part of the global atmosphere’s natural variability, justifying this Experiment 3. The wavenumber of the forcing in this experiment is $k = 4$ and the amplitude at 26°N is $b = 1^\circ C$ day$^{-1}$. The period of the forcing is $2\pi/\omega = 10$ days and its motion is westward. These two fully developed, dominant waves seem to compete for the same supply of energy from convection, the sum of their powers being nearly constant and approximately equal to 1.1 m$^2$ s$^{-2}$. From this table and Figs. 2 and 12 it appears that ultralong symmetric modes are more efficient in feeding on cumulus convection than intermediate-wavenumber antisymmetric modes. Moreover, the mutual reinforcement between the former modes and cumulus convection lasts longer. Thus, for a fully established tropical flow regime with symmetric lateral boundary conditions, Kelvin waves dominate Yanai waves, although the wave–CISK mechanism has the potential of generating both types of waves, each with the observed spectral peaks.

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**Table 2. Maximal values of the spectral power multiplied by frequency in the $v$-component of wavenumber 4 and the $u$-component of wavenumber 2; units are m$^2$ s$^{-3}$. Numbers in parentheses show the period at which the spectra have the maximum, in days.$^1$**

<table>
<thead>
<tr>
<th>Component</th>
<th>Experiment</th>
<th>1</th>
<th>2</th>
<th>1A</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td></td>
<td>.26 (5.0)</td>
<td>.14 (4.0)</td>
<td>.14 (6.67)</td>
<td>.64 (3.33)</td>
</tr>
<tr>
<td>$u$</td>
<td></td>
<td>.80 (20.0)</td>
<td>1.17 (20.0)</td>
<td>.93 (10.0)</td>
<td>.44 (10.0)</td>
</tr>
</tbody>
</table>

$^1$ The interval over which the spectral analysis was performed is days 20–50, except for Experiment 1A, where it is days 1–40 (i.e., days 51–90 of Experiment 1).

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**Fig. 13. Power spectra of the $v$-component (only) in Experiment 3; see Fig. 6 for details and units.**
The next problem is what role the wavenumbers of lateral forcing play in determining the dominant zonal wavenumber of Yanai waves. Another experiment, 3A, was performed with forcing of wavenumber 6 and period 10 days. In this experiment, the dominant wavenumber in the \( v \)-component over the equator is 6, with a period of 6.67 days, and the mode has the horizontal structure of a Yanai wave (not shown). Except for wavenumber 8 appearing in Experiment 3, the wavenumbers of lateral forcing thus seem to select the dominant wavenumber of Yanai waves, as long as they are within the broad band of intermediate wavenumbers excited by the nonlinear wave–CISK mechanism.

From the experiments discussed so far, it follows that Yanai waves with spectral peaks at intermediate wavenumbers and periods between 4 and 7 days are selectively generated by nonlinear wave–CISK and are resonantly reinforced by antisymmetric lateral forcing with the same wavenumber and an arbitrary period. In the absence of antisymmetric forcing, symmetric Kelvin-like waves are more efficient at extracting their energy from the wave–CISK instability, and dominate the flow.

In the subsequent three sections we shall try to answer the following questions: 1) Why does nonlinear wave–CISK select the Yanai wave of wavenumber 4 and period 5 days? 2) Why do symmetric modes normally dominate antisymmetric ones? 3) How can the response frequency in the tropics be independent of the frequency of lateral forcing, while the zonal wavenumbers are the same?

4. Wave-parameter selection for Yanai waves

We study in this section why wavenumber 4 and the period of 5 days are selected for Yanai waves. In our model, using a nonlinear wave–CISK mechanism, growth rates given by the linear theory are not helpful, especially since they are incorrect (cf. Introduction). Instead, our results are consistent with those of Hayashi and Sumi (1986) and of Lau and Peng (1987), the low-wavenumber, long-period symmetric modes are dominant in the presence of nonlinear wave–CISK and in the absence of antisymmetric lateral forcing. In the presence of lateral forcing, we have seen that intermediate-wavenumber, intermediate-period antisymmetric modes become strongly noticeable.

We are thus led to conjecture that frequencies of Yanai waves with very low wavenumbers are too high for the water vapor to be supplied for convection effectively. A similar idea was proposed by Kuo (1975) (see also Hayashi, 1971a), who hypothesized that the evaporation rate of the water over the tropical ocean and the convergence of the water vapor are too small for waves with periods of less than about 3 days to be effectively excited by the wave–CISK mechanism. In the present study, we can ascertain the validity of this hypothesis, because the water vapor budget is explicitly calculated in our model.

First, let us compare wave fields of wavenumber 2 with the precipitation, in order to see why antisymmetric wavenumber 2 does not exhibit a large spectral peak in Experiment 1. Figures 15 and 16 show the longitude–time sections of the \( v \)-component at level 2 along the equator, and the antisymmetric precipitation perturbation averaged over latitudes \( 2^\circ \sim 14^\circ \). Comparison with Figs. 5 and 10, respectively, indicates that the amplitudes of both these fields are comparable in magnitude to those in wavenumber 4.

The most prominent difference in the relative behavior of the precipitation and the \( v \)-component for wavenumber 2 from that for wavenumber 4 is in the phase speeds: The precipitation exhibits slow eastward movement, while the \( v \)-component is characterized by rapid motions, westward-moving modes being Yanai modes and eastward-moving modes being pure gravity modes. Even when the precipitation and the \( v \)-component are in exact opposition of phase at some instant, they lose the correct phase relationship very rapidly, and the release of latent heat is not sufficient to enhance the waves. Hence the Yanai wave of wavenumber 2 cannot exhibit a large spectral peak.

The initially strong development of wavenumber 2 (solid line) in Fig. 2b argues in favor of this hypothesis. At the initial stage of Experiment 1, water vapor was plentiful, relative humidity being 85% everywhere. Thus, even the Yanai wave of wavenumber 2 can develop fully, given a plentiful water vapor supply.

Next, we examine the validity of the Kuo hypothesis by performing an additional experiment. We wish to check that in a situation in which frequencies of very low wavenumbers are not high, ultralong waves will be dominant. Since the frequency of equatorial waves decreases with vertical wavelength, we modify the model to have only shallow convection. To achieve
this, $C_0$ in the convective adjustment scheme (9) is set at levels 4 and 5 to have values 0.1 and 0.3, respectively. These unrealistic values prohibit penetrating convection, and in this Experiment 4 convection occurs indeed almost only in the lowest layer. All other conditions are the same as in Experiment 1.

As we shall see in section 6, Fig. 18, the height of the convection does indeed determine the equivalent depth of Yanai waves. Figure 17 shows the power spectra for the $v$-component over the equator in Experiment 4. Wavenumber 2 now does show large peaks at periods of 4 days in westward-moving modes and 2.86 days in eastward-moving modes, while wavenumber 6 also exhibits a peak at period 10 days in westward-moving modes. These periods are considerably longer than those given by the dispersion relation for ultralong Yanai waves having an equivalent depth equal to that of the observed, intermediate-wavenumber waves. Hence it appears that the shortness of periods is at least one reason why ultralong Yanai waves are not very strongly excited by wave–CISK.

5. Symmetric and antisymmetric modes

We consider in this section the reasons for the dominance of symmetric modes over antisymmetric ones, when forcing antisymmetries are absent. To start with, kinetic energy exchanges between symmetric and antisymmetric modes were calculated. This exchange term is formulated in appendix B, Eq. (B7). Table 3 shows the values of the energy exchange, as well as the temperature and precipitation associated with the symmetric and antisymmetric part of the perturbation, as discussed below. Since the energy flows from symmetric to antisymmetric modes in all experiments except Experiment 1, we cannot attribute the dominance of symmetric modes only to the kinematics of the waves.

Next, the thermodynamics is a possible candidate. We recall the difference in heating between the two types of modes as stated in section 3a: antisymmetric modes can produce symmetric modes, but the reverse is not true. We examine further this preference of heating for symmetry. If we consider total fields, rather than perturbations, heating is always positive, and it follows from Eqs. (16a, b) that the symmetric part of the heating is necessarily larger than the antisymmetric part. But the relevant field is really the perturbation, which does contain "negative" heating. At this point, the crucial observation is that heating is not normally distributed about its mean; the "negative" part consists in frequent occurrences with small absolute values and the positive part in fewer occurrences with large values (see Fig. B1). In such a skewed distribution, the decomposition (16) yields necessarily larger symmetric modes, as shown in appendix B.

We calculated antisymmetric and symmetric modes of the perturbation precipitation and temperature fields, defined as follows:

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**Fig. 15.** Hovmöller diagram for the $v$-component of wavenumber 2 in Experiment 1; compare Fig. 5 for details and units.

**Fig. 16.** Hovmöller diagram for the precipitation of wavenumber 2 in Experiment 1; see Fig. 10 for details and units.

**Fig. 17.** Power spectra of the $v$-component in Experiment 4; see Fig. 6 for details and units.
Table 3. Energy conversion, temperature and precipitation of symmetric and antisymmetric modes in various experiments. The averages and 99% confidence intervals are shown for each quantity. These are calculated over latitudes 0°-18°.

<table>
<thead>
<tr>
<th>Exp.</th>
<th>Conversion (m^2 s^{-1}/day)</th>
<th>Temperature (°C)</th>
<th>Precipitation (mm/12 h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>symmetric: 0.047 ± 0.083</td>
<td>0.220 ± 0.007</td>
<td>1.356 ± 0.039</td>
</tr>
<tr>
<td></td>
<td>anti:           -0.087 ± 0.055</td>
<td>0.215 ± 0.005</td>
<td>1.287 ± 0.005</td>
</tr>
<tr>
<td>1A</td>
<td>symmetric: 0.246 ± 0.008</td>
<td>0.209 ± 0.004</td>
<td>1.403 ± 0.039</td>
</tr>
<tr>
<td></td>
<td>anti:            -0.096 ± 0.059</td>
<td>0.231 ± 0.005</td>
<td>1.316 ± 0.005</td>
</tr>
<tr>
<td>2</td>
<td>symmetric: 0.006 ± 0.059</td>
<td>0.197 ± 0.009</td>
<td>1.403 ± 0.028</td>
</tr>
<tr>
<td></td>
<td>anti:               -0.051 ± 0.054</td>
<td>0.196 ± 0.005</td>
<td>1.344 ± 0.004</td>
</tr>
<tr>
<td>3</td>
<td>symmetric: 0.228 ± 0.009</td>
<td>0.228 ± 0.009</td>
<td>1.393 ± 0.051</td>
</tr>
<tr>
<td></td>
<td>anti:                -0.051 ± 0.054</td>
<td>0.196 ± 0.005</td>
<td>1.347 ± 0.012</td>
</tr>
</tbody>
</table>

\[
\frac{1}{N} \sum_{i=1}^{N} |P_-|, \quad \frac{1}{N} \sum_{i=1}^{N} |P_+|, \quad \frac{1}{N} \sum_{i=1}^{N} \int_0^1 |T_-|d\sigma
\]

and

\[
\frac{1}{N} \sum_{i=1}^{N} \int_0^1 |T_+|d\sigma
\]

where \( P \) stands for the precipitation perturbation and the summation is over grid points in a tropical latitude band up to 18°. From Table 3 it is clear that symmetric values are significantly larger than antisymmetric values for both precipitation and temperature in Experiments 1A and 2. From the twice-daily model data, one can further find that the symmetric part of the precipitation perturbation is greater than the antisymmetric part at all times in these two experiments. Only in Experiment 3 is the difference between the two modes in precipitation not significant at the 99% level. It seems that the antisymmetric forcing has the effect of counterbalancing the preference of heating for symmetry, although the reason why this is not reflected in the temperature fields of experiment 3 is unclear.

It follows that in nonlinear wave–CISK the equatorially symmetric part of the heating perturbation tends to be significantly larger than the antisymmetric part, thus favoring the extraction of energy by symmetric modes.

6. Periods of Yanai waves and the lateral forcing

To investigate the mechanism for the independence of the period of Yanai waves from that of lateral forcing, it suffices to examine in further detail our earlier results. First, these results show that the vertical wavelengths of Yanai waves are completely determined by the height of cumulus convection. Figure 18 illustrates the theoretical dispersion relation of Yanai waves for a few equivalent depths.

Let us draw a least-squares straight line connecting the maxima with respect to frequency of the power spectra for every wavenumber in the westward-moving \( \psi \)-component, in each experiment so far. Every such line should coincide in principle with one of the nearly straight lines in Fig. 18. Indeed, Fig. 13 shows that the equivalent depth for the waves in Experiment 3 is about 20 m, while Fig. A1 (Kuo’s parameterization) gives 200 m. That is, each wavenumber in these two experiments has the inherent frequency determined by the dispersion relation, independent of the frequency of lateral forcing. In some of the other experiments, this coincidence between theoretical and computed dispersion relations is less obvious, due to the much larger change in slope of a least-squares curve, which we attribute to nonlinear effects.

The role of lateral forcing is to selectively reinforce Yanai waves, enabling them to partially suppress the symmetric modes. The detailed comparison of Figs. 5 and 10a made at the end of section 3a, although Experiment 1 does not include lateral forcing, suggests the following picture of the role of lateral forcing in reinforcing Yanai waves: Antisymmetric cumulus convection is excited by lateral forcing with a given frequency or sporadically. This convection then generates Yanai waves with a period of 5 days, determined by the height of the cumulus convection. Yanai waves and antisymmetric cumulus convection now mutually enhance each other for a while, starting, however, also

![Fig. 18. Theoretical dispersion relations of Yanai waves for equivalent depths of 5, 15, 50 and 200 m.](image-url)
to reinforce symmetric fields. The symmetric modes then proceed to grow and to gradually "starve out" the antisymmetric fields because of the limited supply of total water vapor; as a result, the antisymmetric modes become weaker, without disappearing entirely. The next episode of antisymmetric, laterally forced convection makes cooperative interaction between Yanai waves and cumulus heating possible again. Therefore, it is quite plausible that the frequency of Yanai waves be independent of the frequency of lateral forcing, especially when the latter is lower.

We can illustrate the scenario above more quantitatively in Experiment 3. In order to do so, we define the quantity:

\[ V = 2v_E - (v_N + v_S), \tag{17} \]

where subscript \( E \) expresses values at the equator. Thus, \( V \) is the difference between twice the \( v \) components of antisymmetric modes at the equator and off the equator. The former is an indicator of the amplitude of Yanai waves or, strictly speaking, of antisymmetric modes. The latter is an indicator of the lateral forcing if the latitude off the equator is taken near the forcing latitudes. Hence \( V \) is proportional to the convergence of antisymmetric modes into an equatorial belt. We calculate the coherences between \((v_N + v_S)\), \( V \) and \( v_E \) for wavenumber 4. Data at level 5 are used, because these three quantities are likely to be most closely correlated at level 5 through low-level convergence and convection. We calculated \( v_N \) and \( v_S \) at 20°.

Results are shown in Table 4, including coherences with the precipitation. Results from Experiments 1 and 2 are also shown for comparison. As seen already in section 3c, the coherence between the "lateral forcing" \((v_N + v_S)\) and the "Yanai-wave indicator" \( v_E \) is very low. But, the coherences between the "lateral forcing" and the low-level convergence \( V \), on the one hand, and between the "Yanai wave" and the convergence on the other, are very high. These values are clearly larger in Experiment 3 than in Experiments 1 and 2. We can also see that the low-level convergence and the precipitation have a high correlation. Therefore, lateral forcing appears to influence Yanai waves via low-level convergence and ensuing precipitation, but exhibits only a low direct correlation with the waves.

This causal chain of phenomena helps explain the difference in frequencies between Yanai waves and the lateral forcing, while the zonal wavenumbers are the same.

### 7. Concluding remarks

The vanishing of planetary vorticity at the equator leads to the existence of two types of planetary waves unique to the tropical atmosphere and oceans, and exponentially trapped in an equatorial latitude belt: Kelvin waves and Yanai waves (Gill, 1982, Ch. 11; Pedlosky, §8.5). Kelvin waves are symmetric about the equator, Yanai waves are antisymmetric. In the troposphere, observed structures resembling Kelvin waves have zonal wavenumber 1 and periods of the order of one month, while observed Yanai waves have typically wavenumbers 4–5 and periods of the order of one week. Renewed interest in these waves has been caused by recent efforts to understand low-frequency atmospheric variability and by the availability of global datasets.

We studied the generation mechanism of Yanai waves in the tropical troposphere. The model used in this study is governed by the primitive equations on an equatorial \( \beta \)-plane, with 5 levels in the vertical and a 4° lat \( \times \) 6° long grid in the horizontal. Moisture budgets are calculated explicitly.

When only antisymmetric perturbations were present at the initial time, Yanai waves with a spectral peak at zonal wavenumber 4 and period 5 days, as observed, were dominant between days 20 and 50 of the simulation, until submerged by the growing symmetric modes. In the presence of significant symmetric components in the thermodynamic or kinematic fields, symmetric long-period modes, of Kelvin-wave type, became dominant. Introducing antisymmetric lateral forcing, Yanai waves with the correct spectral properties were strongly noticeable again, given arbitrary initial data.

Our study, along with the earlier one of Hayashi and Golder (1978), and with those of Hayashi and Sumi (1986) and of Lau and Peng (1987) dedicated to Kelvin waves, indicates that the basic mechanism for the generation of both types of equatorial waves in the troposphere is nonlinear wave–CISK. This mechanism essentially produces Kelvin waves and Yanai waves with the proper spectral peaks.

The natural dominance of the symmetric modes in the dynamics is due to the greater strength of low-frequency symmetric modes in the non-Gaussian distribution of precipitation. To become noticeable, Yanai waves need either a temporary suppression of symmetric perturbations about a temporarily and zonally averaged basic state which is itself symmetric about the equator, or the presence of antisymmetric lateral forc-

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**Table 4. Coherences between "lateral forcing" (LF, i.e., \( v_N + v_S \)), convergence of antisymmetric modes onto the equator (\( V \)), Yanai-wave indicator (YW, i.e., \( v_E \)) and precipitation (\( P \)).**

<table>
<thead>
<tr>
<th>Experiment 1</th>
<th>Experiment 2</th>
<th>Experiment 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( YW )</td>
<td>( P )</td>
</tr>
<tr>
<td>LF</td>
<td>.665</td>
<td>.121</td>
</tr>
<tr>
<td>( V )</td>
<td>.767</td>
<td>.395</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>.745</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( V )</th>
<th>( YW )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LF</td>
<td>.832</td>
<td>.395</td>
<td>.458</td>
</tr>
<tr>
<td>( V )</td>
<td>1.000</td>
<td>.811</td>
<td>.795</td>
</tr>
</tbody>
</table>
ing from midlatitudes. The former is fortuitous and cannot easily be checked from existing observations; the latter should be the case during the seasons of largest difference between hemispheres.

Thus, we are led to suspect that Yanai waves should be stronger on the average in the troposphere during summer and winter, rather than spring and fall. This is at least consistent with the evidence from the one-year experiment with the GFDL general circulation model of Hayashi and Golder (1980, Fig. 3.3). It is also possible, although less easily checked from observations, that episodes of Yanai-wave activity may be triggered by the development of a cyclone–anticyclone pair in one hemisphere only, during any season, or of such developments simultaneously in both hemispheres, but in different longitudinal sectors.

Our study strongly suggests the following selection mechanism for the observed characteristic parameters of Yanai waves: First, the equivalent depth, and hence vertical wavelength, are selected by cumulus convection interacting with the wave. Next, the smallest zonal wavenumber possible is selected by the nonlinear wave–CISK mechanism, in a manner to be elucidated further by work in progress. This minimal wavenumber available is \( k = 4 \), since lower wavenumbers have periods too short to receive an adequate moisture supply from the associated convection. Finally, the frequency is determined by the dispersion relation from the vertical and zonal wavenumbers, the meridional one being zero.

Within a certain band of intermediate zonal wavenumbers, the antisymmetric midlatitude forcing will also help select the dominant one for the Yanai wave. But the period of the latter will be determined to a much greater extent by the vertical wavenumber, and hence by the depth of cumulus convection. Therefore, the frequencies of midlatitude forcing and of Yanai waves are mutually independent.

The picture of symmetric and antisymmetric tropical waves generated by closely related instabilities of nonlinear wave–CISK type, and competing for the same source of energy, is very intriguing and appears ideally suited for the application of complementary ideas from nonlinear fluid dynamics: bifurcation theory and the statistical theory of turbulence (Ghil et al., 1985; Itoh, 1985). We plan to pursue further studies of finite-amplitude tropical waves, and their interaction with the midlatitude atmosphere, along these lines.

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The main computations were carried out on the IBM 4361 computer of the Department of Atmospheric Sciences and on the IBM 3090 of the Office of Academic Computing at UCLA. Additional computations were performed on the FACOM VP-200 of the Data Processing Center at Kyoto University. The manuscript was typed cheerfully and accurately by D. Deutsch and B. Gola, and figures were drawn by K. Martelli.

APPENDIX A

Kuo's Parameterization and Results

The conditions for convection in Kuo's parameterization are that 1) the stratification be conditionally unstable, 2) the moisture convergence in the cloud layer be positive, and 3) the mean relative humidity in the cloud layer be greater than some critical value. In our experiment, this critical value is taken to be 80%. The cloud base is assumed to be the lowest level. The cloud top is defined as the level where the moist adiabat from the cloud base intersects the model temperature. Therefore, this scheme guarantees that convection is always deep, because the stratification in our model is conditionally unstable in a deep layer.

Once convection does set in, cumulus heating is taken to be proportional to the difference between model temperature and the temperature in the cloud, whose lapse rate is assumed to be the moist adiabatic one. In the present version of Kuo's parameterization net moistening by convective activity is neglected.

In the process of testing this parameterization in our model, we noticed that typhoon-like disturbances appeared in which low-level convergence became extremely large at one grid point. To avoid spuriously excessive moistening, an upper limit of \( q = 0.023 \) for the water vapor mixing ratio was set. This value corresponds to the saturated value at about 26.3°C at level 5. If the air temperature at the equator is the same as the sea surface temperature, level-5 temperature there should be about 24.0°C. Therefore, this upper bound should not affect large-scale fields. In all other respects this Experiment 5 is similar to Experiment 1.

Figure A1 shows the power spectra of the perturbation velocities in this experiment. We can see two peaks in each component. The peak of westward-moving modes in the \( v \)-component corresponds to the Yanai wave, with wavenumber 4 dominant. The periods, however, are shorter than those in Experiment 1. We can safely say that this difference is caused by the difference of the heights in the convection, cf. Fig. 18 and discussion thereof. The eastward-moving mode in \( v \) at wavenumber 2 can be identified as a gravity mode from its horizontal structure (not shown). This spurious mode appeared also weakly in Experiments 1–4 (see Figs. 6, 11, 13 and 17) and is most conspicuous in the present experiment. The much stronger peak of
eastward-moving modes appearing in the \( u \)-component corresponds to a Kelvin wave. The weaker peak in \( u \) for westward-moving modes may be a Rossby wave.

Of all these peaks, the most unambiguously consistent one appears to be in the westward-moving Yanai waves. Hence our results concerning the generation and wave parameter selection of Yanai waves by interaction between the velocity field and convection appear to be somewhat independent of the detailed parameterization of the latter.

APPENDIX B

Symmetric and Antisymmetric Modes

The equations in pressure coordinates for the symmetric parts of the horizontal velocity components \( u_+ \) and \( v_- \), and for the antisymmetric parts, \( u_- \) and \( v_+ \), are:

\[
\frac{\partial u_+}{\partial t} = -\left( u_+ \frac{\partial u_+}{\partial x} + u_- \frac{\partial u_-}{\partial x} + v_+ \frac{\partial u_+}{\partial y} + v_- \frac{\partial u_-}{\partial y} + \omega_- \frac{\partial u_+}{\partial \rho} + \omega_+ \frac{\partial u_-}{\partial \rho} \right),
\]

\[
\frac{\partial u_-}{\partial t} = -\left( u_- \frac{\partial u_+}{\partial x} + u_+ \frac{\partial u_-}{\partial x} + v_- \frac{\partial u_+}{\partial y} + v_+ \frac{\partial u_-}{\partial y} + \omega_+ \frac{\partial u_+}{\partial \rho} + \omega_- \frac{\partial u_-}{\partial \rho} \right),
\]

\[
\frac{\partial v_+}{\partial t} = -\left( u_+ \frac{\partial v_+}{\partial x} + u_- \frac{\partial v_-}{\partial x} + v_+ \frac{\partial v_+}{\partial y} + v_- \frac{\partial v_-}{\partial y} + \omega_+ \frac{\partial v_+}{\partial \rho} + \omega_- \frac{\partial v_-}{\partial \rho} \right),
\]

\[
\frac{\partial v_-}{\partial t} = -\left( u_- \frac{\partial v_+}{\partial x} + u_+ \frac{\partial v_-}{\partial x} + v_- \frac{\partial v_+}{\partial y} + v_+ \frac{\partial v_-}{\partial y} + \omega_- \frac{\partial v_+}{\partial \rho} + \omega_+ \frac{\partial v_-}{\partial \rho} \right); \tag{B1}
\]

linear terms are omitted for simplicity. It is clear that two antisymmetric modes can produce a symmetric mode through the advective nonlinearity, e.g., \( \partial u_+/\partial y \) in (B1) is antisymmetric, but the opposite does not hold.

The symmetric and antisymmetric kinetic energy densities are defined to be \( K_S = u_+^2 + v_-^2 \) and \( K_A = u_-^2 + v_+^2 \), respectively. Therefore, multiplying (B1) by \( 2u_+ \) and (B2) by \( 2v_- \), and adding, we get the symmetric energy equation

\[
\frac{\partial}{\partial t} K_S = 2A - \frac{\partial}{\partial x} K_S u_+ - \frac{\partial}{\partial y} K_S v_- - \frac{\partial}{\partial \rho} K_S \omega_+ - \frac{\partial}{\partial \rho} K_S \omega_-
\]

\[
- 2 \left\{ \frac{\partial}{\partial x} u_- u_+ u_- + \frac{\partial}{\partial y} v_- u_+ u_- + \frac{\partial}{\partial \rho} \omega_- u_+ u_- \right\}. \tag{B5}
\]

Similarly, the antisymmetric energy equation is

\[
\frac{\partial}{\partial t} K_A = \frac{\partial}{\partial x} K_A u_+ - \frac{\partial}{\partial y} K_A v_- - \frac{\partial}{\partial \rho} K_A \omega_+ - \frac{\partial}{\partial \rho} K_A \omega_-
\]

\[
- 2 \left\{ \frac{\partial}{\partial x} u_- v_+ v_- + \frac{\partial}{\partial y} v_- v_+ v_- + \frac{\partial}{\partial \rho} \omega_- v_+ v_- \right\}. \tag{B6}
\]

where

\[
A = u_+^2 \frac{\partial u_+}{\partial x} + u_- v_- \frac{\partial u_-}{\partial y} + u_- \omega_- \frac{\partial u_+}{\partial \rho}
\]

\[
- u_- v_- \frac{\partial v_-}{\partial x} - v_-^2 \frac{\partial v_-}{\partial y} - v_- \omega_- \frac{\partial v_+}{\partial \rho}. \tag{B7}
\]

The term defined by (B7) is exactly the conversion term from the antisymmetric to the symmetric kinetic energy density. An alternative way of defining the conversion term is

\[
A' = -u_+ \frac{\partial}{\partial x} u_+ - u_+ \frac{\partial}{\partial y} u_- v_- - u_- \frac{\partial}{\partial \rho} u_- u_- + v_+ \frac{\partial}{\partial y} v_+ v_+ - v_+ \frac{\partial}{\partial \rho} v_+ v_. \tag{B8}
\]
in which case the last three terms on the right-hand side of Eqs. (B5) and (B6) would also be different, as a matter of course.

In Table 3, formula (B7) was used. The second and fourth terms in (B7) were calculated for simplicity at \( \eta \) points and the first and fifth terms at \( \phi \) points. This scheme does not conserve the total energy, but is accurate enough for our purposes. The third and sixth terms were neglected.

To consider the difference between symmetric and antisymmetric modes in thermodynamic quantities like heating or precipitation, we define a transient perturbation, \( p' \), by

\[
p' = p - [p].
\]

where \( p \) is heating or precipitation at a given instant and grid point and \([p]\) stands for the zonal and time mean of \( p \). Generally speaking, \([p]\) is small, and negative values of \( p' \) are also small in absolute value.

The histogram for precipitation from Experiment 2 is given in Fig. B1. We want to show that \( \sum |p_+| > \sum |p_-| \), where \( \sum \) represents summation over grid points. This is equivalent to showing that the expected value of \( |p_1 + p_2| \) is larger than that of \( |p_1 - p_2| \) when we take two arbitrary samples of \( p \), \( p_1 \) and \( p_2 \), from a distribution function like Fig. B1. In a distribution symmetric about zero these expected values are the same. In a skewed distribution they may be different, although not necessarily. In the sample distribution of Fig. B1, the mean value of \( |p_1 + p_2| \) is 1.403, which is significantly larger than that of \( |p_1 - p_2| \), namely 1.334 (see Table 2 for standard deviations, and hence significance levels). In general, we expect that this inequality holds whenever, like in Fig. B1, the mean of the distribution is zero, but the median is positive.

**APPENDIX C**

**The Peak at Wavenumber 8 and Single-Hemisphere Lateral Forcing**

The longitude–time spectral peak at wavenumber 8 and period of 6.67 days is probably due to the large model amplitude of wavenumber 8 in the subtropics. Figure C1 shows the lateral energy flux, \( \phi v \), of wavenumber 8 in Experiment 3. The flux associated with eastward-moving modes, especially at a period of 10 days, is dominant, even when compared with that of wavenumber 4 (not shown), whose maximum value is about 120 m\(^3\) s\(^{-3}\)/day. The phase velocity, of about 6 m s\(^{-1}\), and the vertical structure (not shown) of the wavenumber 8 waves indicate clearly that they are baroclinic. Wavenumber 8 appears to be the most baroclinically unstable one in our model.

In Experiment 1 wavenumber 8 has almost the same amplitude in the *subtropics*; but in this case the equatorially trapped antisymmetric modes are excited by pure wave–cumulus interaction, so the motions in the subtropics have no role. In Experiment 3, on the other hand, subtropical motions have the important role of preventing the dominance of symmetric modes and of enhancing the antisymmetric modes in the tropics. This is the likely cause of wavenumber 8 having large amplitudes in the *tropics* in Experiment 3, but not in Experiment 1.

In the real atmosphere, baroclinic waves would influence tropical motions but little, because the most intense baroclinic zone is at about 30° latitude, far from the equator, and baroclinic waves can hardly propagate into the tropics. Only waves with low wave-numbers can propagate in reality into the tropics, and influence greatly tropical motions. The present model, however, is laterally confined: its baroclinic zone is 20°

![Fig. C1. Latitude–frequency section of space–time spectra for meridional energy flux of wavenumber 8 at level 2 in Experiment 3. Positive values mean northward fluxes. Units are m^3 s^{-3}/day.](image-url)
latitude, and baroclinic waves are free to propagate into the model’s equatorial belt.

We have demonstrated that antisymmetric lateral forcing enhances the mixed Rossby–gravity mode. However, pure antisymmetric forcing can never be seen in the real atmosphere and lateral forcing has in general both symmetric and antisymmetric parts. The question arises therefore whether the symmetric part of lateral forcing may not enhance the symmetric modes, to the point of inhibiting the antisymmetric forcing component from effectively enhancing antisymmetric modes.

In order to answer this question, we performed yet another experiment, Experiment 3B. Forcing is added only in the Southern Hemisphere, with an amplitude of 1.0°C at 22° latitude.

This single-hemisphere forcing has symmetric and antisymmetric components which are exactly equal to each other. All other conditions are the same as in Experiment 3. Results are shown in Fig. C2. The spectral peak at wavenumber 4 and period 5 days is dominant, with another peak visible at wavenumber 8. It follows that lateral forcing with both symmetric and antisymmetric components enhances preferentially the antisymmetric modes in the tropics.

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