A Parameterization of Eddy Transfer Coefficients for Two-Level Seasonal Statistical Dynamical Zonally Averaged Models

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ABSTRACT

A parameterization of quasi-geostrophic eddy transport that takes into account the time variation of the eddy transfer coefficients according to Green's theory is studied. A relation proposed by Green connects the vertical integral of the meridional heat flux at 50°N with the second power of the 500 mb temperature difference between the boundaries of baroclinic activity. It is found that the fourth power in this relation, rather than the original second power, is obtained from analysis of zonal/monthly-mean observational data at 500 mb. For the temperature difference at 1000 mb, however, the same analysis yields a power of 1.5.

The differences in the seasonal simulation of different powers in the eddy transfer relation are explored in a two-level statistical dynamical zonally averaged model (SDZAM), and it is found that an appropriate choice of power may be of special importance if the model is devised to simulate the seasonal climate cycle, or to test astronomical changes inducing different seasonalities. With the second power in 500 mb, the particular SDZAM being tested simulates an oversensitivity in the high latitude temperature response to the seasonal cycle/astronomical changes, due to its undersensitivity in the simulation of changes in the meridional eddy heat flux. A comparison of the results of a second power at the surface level vs a fourth power at 500 mb is difficult due to the need to retune the model, but a certain advantage to the latter model is detected.

1. Introduction

In statistical dynamical zonally averaged models (SDZAMs), the need to parameterize eddy-scale motions in the atmosphere in terms of large scale motions has led to the application of conservative properties such as the potential vorticity. Being a conservative property, its flux is directed down the gradient of its zonally averaged field, a feature which is generally not true for nonconservative properties such as momentum.

The employment of the tool of potential vorticity is done either directly, by use of the quasi-geostrophic potential vorticity system (e.g., Sela and Wiin-Nielsen 1971; Wiin-Nielsen and Fuenzalida 1975; Ohring and Adler 1978 and Neeman et al. 1988c), or indirectly, by using the primitive equations and connecting the momentum flux to the heat and potential vorticity fluxes, thus utilizing the conservative properties of the latter (e.g., Vallis 1982; Taylor 1980). See also general reviews on diffuse eddy transfer parameterizations appearing in Schneider and Dickinson (1974) and Saltzman (1978).

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The diffusion-type relation between the eddy flux of potential vorticity and the meridional gradient of its zonal value is a highly nonlinear one, in which the eddy transfer coefficient itself may be formulated as a power function of the above meridional gradient. SDZAMs which do not take this nonlinearity into account cannot accurately simulate the variations in the eddy fluxes occurring during seasonal or astronomical changes. The same conclusion also applies, of course, to simpler climate models, such as the energy balance models (EBMs), that use diffusion-type eddy transfer parameterizations which are based solely on the eddy flux of heat.

The present study is concerned principally with the power law in the aforementioned diffusion-type relations. A relation that takes into account the nonlinear time variation of the eddy transfer coefficients is sought based on Green's (1970) eddy transfer theory and on observational verification. The relation is specifically developed for seasonal two-level SDZAMs, where eddy scales are not explicitly resolved. This result is presented in section 3 after a short background (section 2). The eddy transfer parameterization is applied to the SDZAM in section 4. An example of the SDZAM simulation of the seasonal cycle for present and different astronomical conditions is presented in section 5.

2. Background

The main advantage of using the quasi-geostrophic (QG) system of equations is that under adiabatic frictionless conditions, the potential vorticity $Q = f(\phi) + \zeta - \partial(\gamma T)/\partial p$ is a conserved property along isobaric surfaces (Green 1970). Here, $f(\phi)$ is the vertical component of the earth's vorticity (Coriolis parameter), ϕ is the latitude, ζ is the vorticity, p is the pressure, T the temperature, and $\gamma = f_0 R/\sigma p$, where f_0 is a global-mean value of $f(\phi)$, R is the gas constant and σ is the static stability.

Therefore, since Q is a conserved property along isobaric surfaces, $\overline{v'Q'}$, the zonal average of the eddy flux of potential vorticity, can be written in terms of the meridional gradient of the zonal average of Q, through a diffusion type relation,

$$\overline{v'Q'} = -K(p, \phi, t)\frac{\partial \bar{Q}}{a\partial\phi}, \tag{1}$$

where the overbar indicates zonal average and also time average on the eddy lifetime, primes indicate deviations from it, a is the radius of the earth, v the meridional wind and $K(p, \phi, t)$ the transfer coefficient, which is a function of pressure p, latitude ϕ and time t. Equation (1) has been the basic assumption of Sela and Wiin-Nielsen (1971) concerning the large scale horizontal eddy fluxes.

The above assumption is common not only to models in which the potential vorticity equations are used. It is a key assumption in the transfer theory of Green (1970), in which the eddy fluxes of momentum or vorticity are related to those of potential vorticity and potential temperature, or temperature along an isobaric surface, by a relation of the form

$$F = -\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}(\overline{v'u'}a\cos^2\phi)$$
$$= \overline{v'\zeta'} = \overline{v'Q'} + \frac{\partial}{\partial p}(\gamma\overline{v'T'}). \tag{2}$$

Equation (2) was derived in the present form in detail in Wiin-Nielsen and Sela (1971), making use of the nondivergent property of the quasi-geostrophic wind.

Green (1970) formulated a diffusion-type relation for the eddy flux of entropy S (a conserved property in the absence of diabatic heating), which in y–z (meridional–vertical) coordinates has the form $\overline{v'S'} = -K_{vy}\partial S/\partial y - K_{vz}\partial S/\partial z$. Noting that along an isobaric surface $dS/c_p = d\ln\theta = d\ln T = dT/T$ and that $\sigma = (\rho\theta)^{-1}(\partial\theta/\partial p)$, where θ is the potential temperature and ρ the density, it is possible to obtain an approximate analog to the latter equation for $\overline{v'T'}$ in spherical meridional–pressure coordinates:

$$\overline{v'T'} = -K_y(p, \phi, t)\frac{\partial\bar{T}}{a\partial\phi} - K_p(p, \phi, t)\sigma. \tag{3}$$

Furthermore, Green has assumed that $K_y(p, \phi, t)$ was equivalent to $K(p, \phi, t)$ of Eq. (1).

3. The relation between the meridional heat flux and temperature gradient

From theoretical considerations, Green (1970) has obtained [his transformed Eq. (5)]:

$$\frac{1}{p_1 - p_2}\int_{p_1}^{p_2}\overline{v'T'}dp = A(\Delta T)^n, \tag{4}$$

where $n = 2$ in Green's treatment and the left-hand-side of (4) is the mass-weighted vertical integral of the eddy heat flux, p_1 and p_2 bound tropospheric motions (see discussion below), ΔT is a characteristic difference in 500 mb temperature between the boundaries of baroclinic activity (see further discussion below) and where:

$$A = \frac{\alpha}{T_0}\left(\frac{gT_0}{\partial\theta/\partial z}\right)^{1/2}, \tag{5}$$

where g is the acceleration of gravity, T_0 a characteristic temperature, θ the potential temperature and α a phenomenological coefficient. Green neglected changes in the static stability in (4), i.e., he assumed A = const. (see also justification below). In an alternative derivation, Stone (1972) has obtained a similar expression:

$$\overline{v'\theta'} = A\left(\frac{\partial\bar{\theta}}{\partial y}\right)^2. \tag{6}$$

While Green treated ΔT (or ΔS) across a fixed scale of meridional width, Stone worked with a local temperature gradient and assumed a meridional scale that is proportional to the deformation radius of an Eady model, which is itself proportional to $(\partial\bar{\theta}/\partial z)^{1/2}$. Because of this assumption, A in Stone's parameterization is proportional to a ½ power of $\partial\bar{\theta}/\partial z$ rather than −½ power in Green's case (see Branscome 1983).

Green and Stone have both assumed a fixed vertical scale for the eddy motion. Held (1978) has looked at the ratio of

$$h = -\frac{f}{\beta}\frac{\partial\bar{\theta}/\partial y}{\partial\bar{\theta}/\partial z}$$

to H, the scale height of the atmosphere (and where β is the latitudinal gradient of the Coriolis parameter). When $h \ll H$, the eddy fluxes are confined to a region above the surface of depth proportional to the meridional temperature gradient. In this limit, Held's scaling analysis has resulted in

$$\overline{v'\theta'} = A\left(\frac{\partial\bar{\theta}}{\partial y}\right)^5 \tag{7}$$

(where A is proportional to $(\partial\bar{\theta}/\partial z)^{-5/2}$). For $h \gg H$, the vertical extent of the eddy motions is fixed (and confined to H), resulting in the Green-Stone f-plane model with the square law formulation.

For a more detailed discussion see Branscome (1983), who has developed continuous expressions involving the parameter h/H for the vertical structure of the eddy heat fluxes, from baroclinic wave behavior. The expressions reduce to the second- and fifth-power laws, respectively, when $h \gg H$ and $h \ll H$.

Stone and Miller (1980) have carried out an empirical study focusing on the relation (4) and its exponent n. For $\Delta T(\phi)$ they took $\bar{T}_{1000}(\phi - 15°) - \bar{T}_{1000}(\phi + 15°)$, where ϕ is the latitude at which $\overline{v'T'}$ is taken, and $\bar{T}_{1000}$ is the zonal temperature at 1000 mb. For p_1 and p_2 they took 1000 and 75 mb, respectively. Their Fig. 5 has demonstrated that strong variations in static stability are confined to latitudes north of 55°N, thus justifying the A = const. assumption for latitudes up to 55°N. For comparing their results with theoretical studies, they took for $\overline{v'T'}$ in (4) the sum of the transient and stationary sensible heat fluxes, since several studies (including Green 1970) implicated baroclinic instability in the generation of stationary as well as transient eddy fluxes. At $\phi = 50°$N, where the eddy sensible heat flux peaks all year long, they obtain $n = 1.68$, while at $\phi = 30°$N they obtain $n = 3.43$. They show that at 30°N the heat flux has a much greater vertical extent in January [when ΔT is large] than in July, while at 50°N the vertical extent in January is less than in July, limiting the applicability of Held's proposed relation with a higher exponent to 30°N.

The present study in its initial form has been influenced by the concept of Green (1970), that ΔT should be taken at 500 mb, rather than at 1000 mb. Green (1970) has shown (in his Fig. 3) that the mean horizontal temperature gradient is roughly constant between 2 and 8 km, above which the gradient changes sign and below which the gradient increases, presumably because of cold polar surface air, particularly in winter. This observation is also supported by data of Oort and Rasmusson (1971), which show that as the level is decreased from 850 mb to 1000 mb, ΔT in January increases at a fast rate while the lapse rate at 65°N is zero inside this layer. Green wrote: *"Since we are here concerned with tropospheric motion and do not believe that the extreme conditions in the polar layer are important we select the 500 mb temperature difference to give a representative value for* $\Delta\phi$*."* [$=\Delta S$]

Branscome (personal communication) and an anonymous reviewer, however, favor using the 1000 mb meridional temperature difference, despite Green's observation, since linear instability theory (e.g., Staley and Gall 1977) tends to show that low level temperature gradients are more important for growth rates. Our study in its present form therefore includes both 500 and 1000 mb formulations.

Starting with Green's approach, the calculations of Stone and Miller (1980) were repeated with ΔT at 500 mb (instead of 1000 mb), and p_2 was set as 250 mb (instead of 75 mb), in order to exclude the stratosphere, where the temperature gradient changes sign, as mentioned before. Except for these two changes, the calculations were similar to Stone and Miller, performing a linear regression between $\log \int \overline{v'T'}\, dp$ and $\log\Delta T$ using the database of Oort and Rasmusson (1971). For the sum of the transient and stationary sensible heat flux at $\phi = 50°$N, the result was $n = 3.84$ with a correlation coefficient of $r = 0.89$. This result is shown in Fig. 1, where the solid line represents the regression best fit. The dotted and dashed lines show the best fit curves with $n = 4$ and $n = 2$, respectively. When p_2, the upper pressure integration boundary was set at 50 mb instead of 250 mb, the resulting n decreased to 3.53 and r to 0.87. When in addition, ΔT was taken at 1000 mb instead of 500 mb, the result was $n = 1.54$, with $r = 0.98$ (shown in Fig. 2), in agreement with Stone and Miller (1980).

Actually, Green (1970) worked with the entropy S rather than the temperature T. It was therefore considered interesting to repeat the above regression for Green's original relation (4), which had the form

$$\frac{1}{p_1 - p_2} \int_{p_1}^{p_2} \overline{v'S'}\, dp = A'(\Delta S)^n. \qquad (8)$$

Taking $\overline{v'S'}(p) = \overline{v'T'}(p)/T(p)$ and $\Delta S(\phi) = \Delta T(\phi)/T(\phi)$ at 500 mb (with $p_2 = 250$ mb), and performing a regression on (8) has yielded $n = 3.60$ with a slightly better correlation coefficient of $r = 0.92$, reassuring that (4) is a good approximation to Eq. (8).

Looking at Fig. 1, one observes that the magnitudes of the 500 mb $\overline{\Delta T}$ in January and February are actually the smallest for the whole period of September to April,

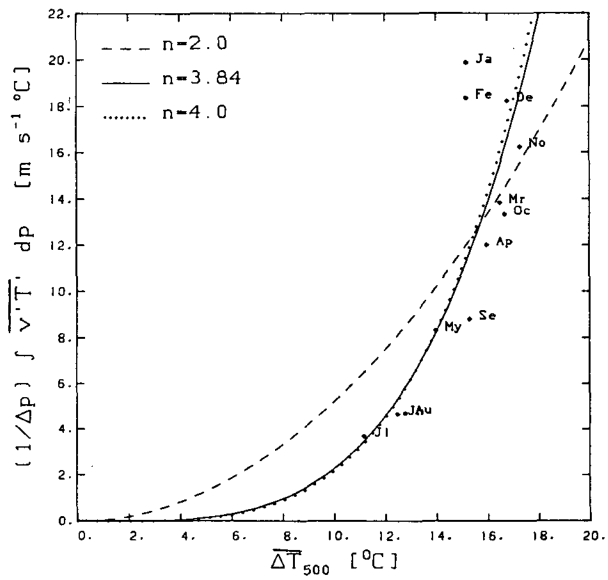

FIG. 1. The mass-weighted vertically integrated northward zonal transport of sensible heat by stationary and transient eddies at latitude 50°N vs the meridional zonal temperature difference at 500 mb. The solid line denotes the curve obtained by regression, where an exponent of $n = 3.84$ was found. The dotted and dashed lines show the best fit curves with $n = 4$ and $n = 2$, respectively.

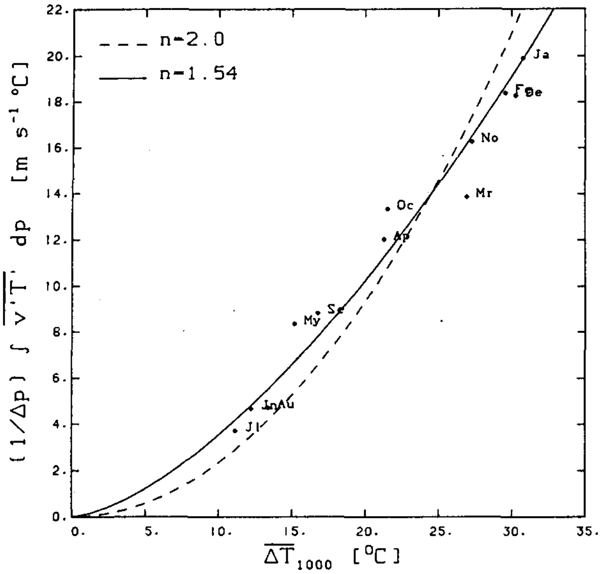

FIG. 2. As in Fig.1, except for the meridional zonal temperature difference at 1000 mb. The solid line denotes the curve obtained by regression, where an exponent of $n = 1.54$ was found. The dashed line show the best fit curves with $n = 2$.

while on the other hand, the heat fluxes are largest for this period. Some "hysteresis" was also observed by Stone and Miller (1980) for the 1000 mb temperature differences at some latitudes (e.g., their Fig. 7 for 60°N), and they noted that the flux component of the stationary eddies was responsible for it. Large-scale eddies create large fluxes, but they also have the effect of reducing the average ΔT. If the large-scale eddies of mid-winter also have a larger spatial and temporal variability, and since the available potential energy is proportional to ΔT^2, it may be more accurate to look at $\overline{\Delta T^2}$ rather than $\overline{\Delta T}^2$, as noted by Green (1970).[1]

4. Application to the SDZAM

Multiplying F (Eq. 2) by $a\cos^2\phi$ and integrating with latitude over the hemisphere, and assuming no momentum flux across the boundaries, yields an important integral constraint on the meridional eddy vorticity flux:

$$\int_0^{\pi/2} Fa\cos^2\phi d\phi = 0 \qquad (9)$$

which has been used in several studies (Marshall 1981; Vallis 1982; White and Green 1982, 1984) to get information on the vertical variation of the K's and also on the relation between K_y and K_p.

We have tried to apply some of the methods suggested by these studies in the present seasonal two-level QG system, which consists of the QG potential vorticity equations for levels 1 (250 mb) and 3 (750 mb), and the thermal streamfunction equation for level 2 (500 mb) (see Ohring and Adler 1978). The QG potential vorticity system was obtained by finite-difference manipulations, from the QG vorticity equation with friction (at levels 1 and 3) and the thermodynamic energy equation with diabatic heating (at level 2) (e.g., Sela and Wiin-Nielsen 1971). However, some difficulties have arisen: A negative K_{y1}/K_{y3} was predicted in summer months demanding that K_{y1} and K_{y3} be held at zero (see Marshall 1981, where the link with Pedlosky's necessary condition for instability is discussed). The discontinuity which resulted from setting the K_{yi}'s to zero has produced a numerical instability. Also, as the ratio K_{y1}/K_{y3} approached zero, unrealistically large values of K_{y3} were predicted. Consequently, we have decided to fix K_{y1}/K_{y3} from annual observations, as described below.

As in the studies of Marshall (1981), Vallis (1982), and in scheme 1 of White and Green (1984), K_p is set to zero in the present study. A separation of variables of the form $K_y(p, \phi, t) = \hat{K}_y(p, t)\cdot Y(p, \phi)$ is assumed. The ratio $\hat{K}_{y1}/\hat{K}_{y3}$ is fixed to a value of 0.19, as determined from an annual average observational analysis. Inserting (3) into (4) yields

$$\left[-\frac{\hat{K}_{y1}Y_1 + \hat{K}_{y3}Y_3}{2}\frac{\partial\bar{T}_2}{a\partial\phi}\right]_{\phi=50°} = A(\overline{\Delta T}_i)^n, \qquad (10)$$

from which $\hat{K}_{y1}(t)$ and $\hat{K}_{y3}(t)$ are calculated at each time step from the $\bar{T}_2$ field. Here $\overline{\Delta T}_i$ is the difference in $\bar{T}_i$ ($i = 2$ or $i = 4$ in different experiments) between latitude 35° and 55°N, as described in section 3, and $Y_1(\phi)$ and $Y_3(\phi)$ are the fixed normalized latitudinal dependencies of the eddy transfer coefficients for the two levels, given in Table 1 (see discussion later). Tests with $n = 2$ and $n = 4$ were performed, as described below.

The philosophy of the present study has been to use mean-annual rather than seasonally varying values for the prescribed fields,[2] since one of its aims is conducting experiments with climates of different seasonal contrasts, induced by orbital changes (section 5). It would be inconsistent to apply seasonal variations that are prescribed from the present observed seasonal cycle, rather than calculated from physical relations.

The values of $Y_i(\phi)$ (Table 1) are based on the annually averaged observational analysis of the present study (Fig. 3), with the following modifications. At latitude 20°N, Y_3 is set to zero to avoid negative values. Similarly, Y_1 and Y_3 are set to zero at 0° and 90°N in order not to violate the global conservation of potential

[1] See Table 1 and lines 7–8 on page 164 of Green (1970).

[2] An experiment was performed with the ratio $\hat{K}_{y1}/\hat{K}_{y3}$ prescribed to a seasonally varying set of values, and the results have shown some small changes in the zonal wind fields.

TABLE 1. The normalized latitudinal dependence of the Y_i for $i = 1$ (250 mb) and $i = 3$ (750 mb).

	Latitude (°N)							
	10	20	30	40	50	60	70	80
Y_1	.03	.05	.04	.05	.17	.31	.31	.16
Y_3	.02	0	.04	.13	.28	.28	.15	.07

vorticity by the eddies. To understand this, note that the eddy term

$$\frac{1}{a^2 \cos\phi} \frac{\partial}{\partial\phi} \left(\bar{K}_i \cos\phi \frac{\partial(\bar{Q}_i)}{\partial\phi} \right)$$

in the QG potential vorticity equation [Eqs. (1) and (2) in Ohring and Adler 1978] should only redistribute potential vorticity. When the latter term is summed up over all grid points at each level, all contributions cancel out, except for the boundaries at the equator and pole. Therefore, to avoid potential vorticity generation, it is important to set the coefficients to zero at these boundaries (or, alternatively, the same effect can be achieved by setting $\partial\bar{Q}_i/\partial\phi$ to zero at these boundaries).

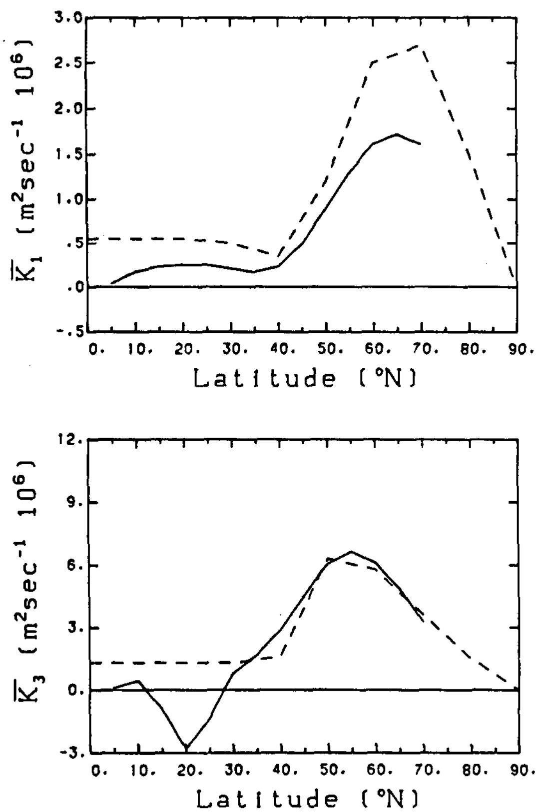

FIG. 3. The latitudinal dependence of the annually averaged $\bar{K}_i$ from the observational analysis of the present study (solid lines) and from Ohring and Adler (1978), based on Wiin-Nielsen and Sela (1971) (dashed lines), for $i = 1$ (250 mb, top) and $i = 3$ (750 mb, bottom).

Since the observational analysis of the present study is limited to 70°N, values of Y_i for 80°N were interpolated linearly. The latitudinal dependence of the Y_i's used in the present study is nearly equal to the latitudinal dependence of the eddy transfer coefficients previously used in Ohring and Adler (1978) (see Fig. 3).

The proportionality constant A of Eq. (10) was tuned while the model was run with the seasonal cycle inhibited. According to the practice of tuning, A was varied until a compromise was found, such that the simulated temperature gradient at 500 mb compared well with observations, while keeping the resulting K_i within the range of observations. By keeping the seasonal cycle inhibited, one can isolate the role of A as an annually averaged proportionality constant in the eddy transfer relation [Eq. (10)]. The constant A was multiplied by $(\Delta T_{\text{obs}})^2/(\Delta T_{\text{obs}})^n$, where ΔT_{obs} is the annually averaged observed ΔT at 500 mb and n is 2 or 4, according to the experiment. Thus, the need to tune A separately in each experiment was avoided, a possibility which would have introduced a certain inconsistency into the comparison of the results. Similarly, for ΔT at 1000 mb, A was multiplied (in some of the experiments, discussed later) by $(\Delta T_{\text{obs},500\text{ mb}})^2/(\Delta T_{\text{obs},1000\text{ mb}})^2$.

5. Tests with a seasonal SDZAM

The climate model utilized in testing the eddy transfer parameterization developed in the previous sections is a seasonal version of the mean-annual hemispheric SDZAM of Ohring and Adler (1978). The dynamics of the model consist of the potential vorticity version of the standard two-level quasi-geostrophic (QG) system (including diabatic heating and frictional dissipation) in zonally averaged form and in spherical pressure coordinates (see also section 4). The seasonal version follows Ohring and Gruber (1984).

The model has topography, expanded longitudinal separation and ice/snow schemes at the surface level, as described in detail in Neeman et al. (1988a,b,c). Although in the atmosphere it is zonally averaged, at the surface level the heat budget equation is applied separately to five different land types. Each land type has different heat storage characteristics and has a specified topography and fraction at each latitude belt (10° of latitude). A zonally averaged surface temperature, $\bar{T}_4$, is computed at each time step by averaging the temperatures of each land type according to its fraction. The atmospheric heating term is the weighted average of the contributions from each land type according to its fraction and is used to calculate the zonally averaged temperature at 500 mb, $\bar{T}_2$.

Surface and atmospheric heating processes included are solar radiation (absorption by water vapor, ozone and cloud particles), longwave radiation (a modified longwave scheme was introduced in Neeman et al.

1987), convection, evaporation at the surface, condensation in the atmosphere, subsurface heating and ocean currents.

An extensive analysis of the simulation of the present climate for four versions of the current seasonal SDZAM, each tuned differently, appears in Neeman et al. (1988c).

Three separate climate model runs will now be examined. Two of the runs are identical except for the exponent n in the relation between the eddy flux and the zonal meridional temperature gradient at 500 mb. This exponent is varied between the traditional value of $n = 2$ and the value of $n = 4$ suggested in the present study. The third run uses $n = 2$, but the temperature gradient is computed at the surface level (using $\bar{T}_4$).

Note that the surface temperature in the model, $\bar{T}_4$, is not directly comparable to the 1000 mb temperature, $\bar{T}_{1000}$, discussed in section 3. In the model, the surface temperatures of land types possessing nonzero altitudes are averaged to obtain $\bar{T}_4$, while $\bar{T}_{1000}$ was averaged only over regions where the 1000 mb exists. Therefore, between 10% and 30% of the surface latitude belt at different latitudes were excluded in the computation of $\bar{T}_{1000}$ (see Table 2 of Oort and Rasmusson 1971).

For the present climate, the seasonal cycle of the zonally averaged surface temperature $\bar{T}_4$ and 500 mb

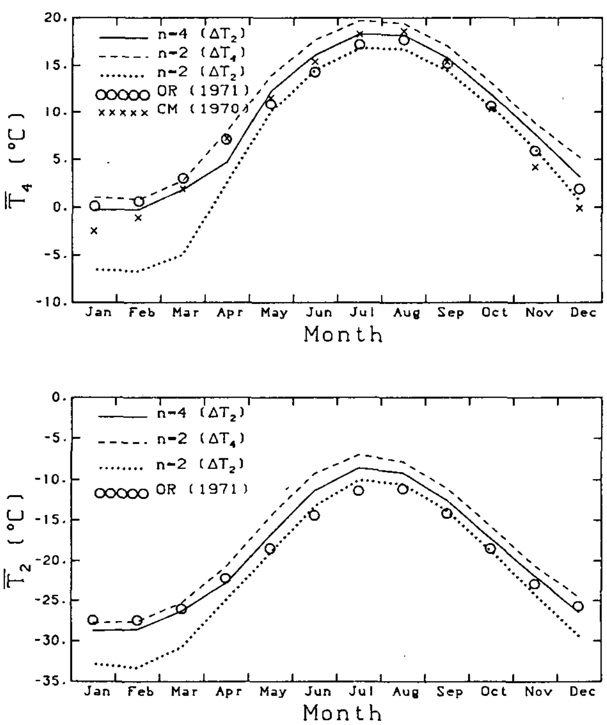

FIG. 4. The seasonal cycle of the zonally averaged surface temperature $\bar{T}_4$ (top) and 500 mb temperature $\bar{T}_2$ (bottom) for latitude 75°N, as simulated by the three model runs, compared with observations of Oort and Rasmusson (1971) (denoted by circles) and of Crutcher and Meserve (1970), with zonal averages tabulated in Warren and Schneider (1979) (denoted by crosses).

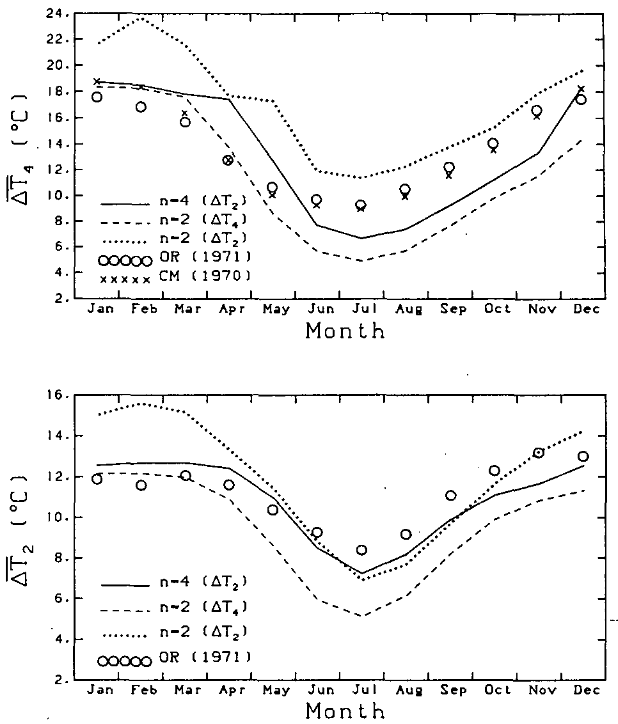

FIG. 5. The seasonal cycle of the latitudinal difference [35°N minus 55°N] of $\bar{T}_4$ (top) and $\bar{T}_2$ (bottom), as simulated by the three model runs, compared with observations (as in Fig. 4).

temperature $\bar{T}_2$ at latitude 75°N is illustrated in the top and bottom portions, respectively, of Fig. 4. The results of the three model runs are compared with observations of Oort and Rasmusson (1971) (denoted by circles) and of Crutcher and Meserve (1970) with zonal averages (denoted by crosses) tabulated in Warren and Schneider (1979). The model run with $n = 2$, ΔT_2, exhibits a poorer simulation of the seasonal cycle compared to the corresponding model with $n = 4$, ΔT_2, showing a significant negative departure of ~5°C during winter months.

The seasonal cycle of the latitudinal gradient of $\bar{T}_4$ and $\bar{T}_2$ across latitudes 35°–55°N is studied in Fig. 5, showing that the model with $n = 2$, ΔT_2, exaggerates the midlatitudinal temperature gradient in winter. The reason for this is that the low exponent in the relation between the eddy flux and the zonal meridional temperature gradient in the atmosphere results in an underestimate of changes in the meridional eddy heat transfer. The disagreements between the model versions are more pronounced in winter, due to the high nonlinearity in Eq. (10), therefore explaining the negative winter temperature departures of the $n = 2$ model discussed in the previous paragraph.

The low exponent in the 500 mb formulation therefore produces an *undersensitive* simulation of seasonal changes in the meridional eddy heat transfer. Consequently, since this heat flux acts to moderate the meridional temperature gradient across midlatitudes, the result is an *oversensitive* simulation of seasonal changes in the latter temperature gradient.

Looking at the simulation of the $n = 2$, ΔT_4 run presented in Fig. 4 (dashed lines), one notes an agreement with observations at latitude 75°N, except for somewhat larger simulated temperatures, which are caused by the fact that the heat flux is now proportional to $\Delta T_4{}^2$, which is larger in winter than $\Delta T_2{}^2$. Figure 5 (dashed lines) shows that the simulated seasonal cycle of the latitudinal gradient across latitudes 35°–55°N agrees with observations at the surface level ($\Delta \bar{T}_4$), but at 500 mb, $\Delta \bar{T}_2$ exhibits an exaggerated seasonal contrast.

In order to test if this behavior is connected with the fact that the $n = 2$, ΔT_4 model was incorrectly tuned, we retuned this model by multiplying the constant A by $(\Delta T_{\mathrm{obs,500\ mb}})^2/(\Delta T_{\mathrm{obs,1000\ mb}})^2$, where ΔT_{obs} is the annually averaged observed ΔT at 500 mb or at 1000 mb. The results of this run are presented in Figs. 6–7 (dashed lines), along with the previously described experiments and observations. Now, the mean annual global surface temperature (15.5°C) agrees better with observations, but the simulated $\bar{T}_4$ and $\bar{T}_2$ is poorer. Again, as in Fig. 5, Fig. 7 shows that the simulated seasonal cycle of $\Delta \bar{T}_4$ agrees with observations, while at 500 mb, $\Delta \bar{T}_2$ exhibits an exaggerated seasonal contrast. Note that since A was retuned to be smaller, $\Delta \bar{T}_2$ is now larger on the average [but its seasonal contrast, as in the previous tuning (Fig. 5) is not better simulated].

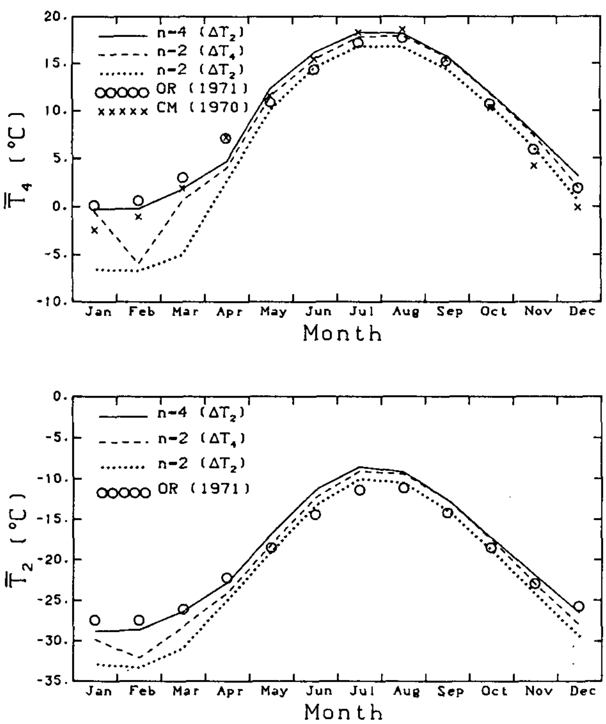

FIG. 6. As in Fig. 4, except that the $n = 2$, ΔT_4 model (dashed line) was retuned (see text).

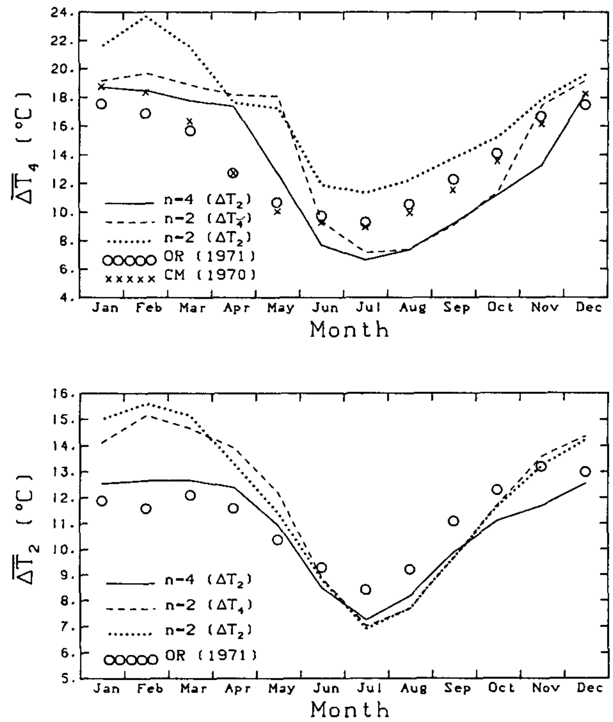

FIG. 7. As in Fig. 5, except that the $n = 2$, ΔT_4 model (dashed line) was retuned (see text).

It is equally interesting in this context to test the effect of varying the exponent n and the level in experiments of response to astronomical changes between extreme low and extreme high summer insolation orbits, referred to as LSIO and HSIO, respectively. In the former, the winter insolation is *high* and the seasonal cycle small, and in the latter the winter insolation is *low* and the seasonal cycle large. The actual orbital elements used for the LSIO and HSIO are given in Table 2. A more detailed description of the insolation anomalies associated with the above extreme orbital conditions appears in Neeman et al. (1988b).

Figure 8 shows the temperature (zonally averaged) response of the two model runs to an overall change from HSIO to LSIO conditions (top – surface, bottom – 500 mb), for latitude 75°N, as a function of time of year. It is evident from this figure that the $n = 2$ model, for both ΔT_4 and ΔT_2, is more sensitive to the

TABLE 2. Present-day and two extreme orbital element configurations.

Configuration	Obliquity (deg)	Eccentricity	Longitude of the perihelion (deg)
Present	23.45	0.0168	102.2
LSIO	22.00	0.0400	90
HSIO	24.50	0.0400	270

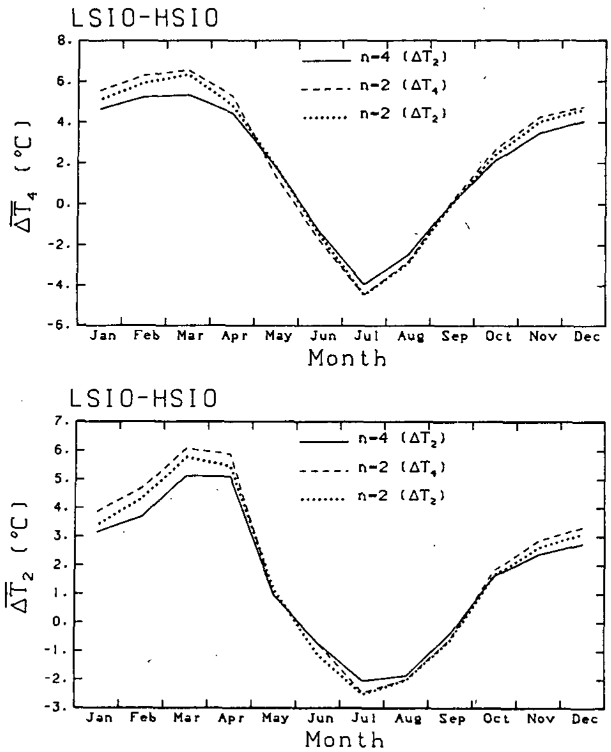

FIG. 8. The seasonal cycle of the difference [LSIO minus HSIO] in the zonally averaged surface temperature, $\overline{\Delta T_4}$ (top), and 500 mb temperature, $\overline{\Delta T_2}$ (bottom), as simulated by the two model runs, for latitude 75°N.

orbitally induced seasonality changes. This is in agreement with the above discussion on the undersensitive simulation of the $n = 2$ model for changes in the meridional eddy heat transfer. Consequently, since this heat flux acts to moderate the meridional midlatitudinal temperature gradient, the end result is the oversensitive temperature response of the $n = 2$ model to orbital changes in the seasonal contrast.

6. Concluding remarks

This study deals with a parameterization of quasi-geostrophic eddy transport that accounts for the time variation of the eddy transfer coefficients, according to a relation connecting the vertically integrated atmospheric eddy flux and the meridional gradient of zonal temperature in the midlatitude atmosphere or surface. The original relation, connecting the integral of the northward eddy entropy flux through midlatitudes with the *second* power of the difference in 500 mb entropy across the region of baroclinic activity, was derived by Green (1970). It was found in the present study that an exponent n approaching the value of 4 is obtained in this relation from a regression analysis of *zonally and monthly averaged* observational data, when the temperature gradients at 500 mb are taken. When the gradients at 1000 mb are used, however, an exponent of 1.5 is obtained.

This difference in exponent between the 500 and 1000 mb is clearly connected with the fact that the meridional temperature gradients in winter are considerably larger at 1000 mb than at 500 mb, while in summer they are about the same magnitude. Green (1970) suggested that this was due to polar-night contamination at 1000 mb, and he advocated the use of the 500 mb temperature gradient. On the other hand, Branscome (personal communication) and an anonymous reviewer argue in favor of using the 1000 mb meridional temperature difference, despite Green's observation, since linear instability theory (e.g., Staley and Gall 1977) tends to show that low level temperature gradients are more important for growth rates.

Experiments were conducted with a two-level statistical dynamical zonally averaged model (SDZAM), and it was verified that an appropriate choice of power in the eddy transfer relation may be of special importance if the model is devised to simulate the seasonal climate cycle, or to test astronomical changes inducing different seasonalities. With the second power in 500 mb, the particular SDZAM under test simulated an oversensitivity in the high latitude temperature response to the seasonal cycle/astronomical changes, due to its undersensitivity in the simulation of changes in the meridional eddy heat flux. A comparison of the results of a second power, surface level model with the results of the other model versions is difficult due to the need to retune this model, which has caused a deterioration in its simulation of the seasonal cycle of temperature at high latitudes. With or without retuning, however, the seasonal cycle of the simulated surface temperature of the second power, surface level model agreed with observations, while the seasonal cycle of its simulated atmospheric temperature was exaggerated.

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