MST Radar Observations of a Saturated Gravity Wave Spectrum

T. Tsuda,* T. Inoue,* D. C. Friggs,** T. E. VanZandt,† S. Kato,* T. Sato* and S. Fukao*

*Radio Atmospheric Science Center, Kyoto University, Uji, Kyoto, Japan
**Geophysical Institute and Department of Physics, University of Alaska, Fairbanks, Alaska
†Aeronomy Laboratory, National Oceanic and Atmospheric Administration, Boulder, Colorado

(Manuscript received 29 August 1988, in final form 27 February 1989)

ABSTRACT

We present vertical wavenumber spectra of mesoscale wind fluctuations using data observed in the troposphere, lower stratosphere and mesosphere by the MU radar at 35°N in Japan in October 1986 and June 1987, as well as lower stratospheric spectra obtained by the Arecibo UHF radar at 18°N in Puerto Rico in June 1983. These spectra are much more homogeneous than previously available spectra since all of the data were observed by the same radar technique, the data in the different atmospheric regions were taken essentially simultaneously, and all of the spectra were analyzed using very similar methods. In the large-wavenumber ranges of the observed spectra, the asymptotic slopes and amplitudes agree well with the saturated gravity wave spectral model developed by Dewan and Good (1986) and Smith et al. (1987), which has a slope of −3 and a spectral amplitude proportional to the buoyancy frequency squared. The good agreement between the model spectrum and the observed spectra from different altitudes, different seasons, and two different stations located at 35° and 18°N suggests that the model is essentially correct, in spite of the heuristic nature of some of its assumptions.

The spectral densities of the zonal and meridional components are similar at large wavenumbers, while the meridional spectrum has larger energy density at small wavenumbers where the spectrum is not saturated. The dominant vertical scales of the gravity wave field in the mesosphere, lower stratosphere, and troposphere are estimated to be >10 km, 2.2 to 3.3 km, and >3.3 km in October and >4.5 km in June, respectively, consistent with determinations from previous studies.

1. Introduction

Recent theoretical studies have strongly suggested that the midlatitude mesosphere is decelerated by the dissipation of gravity waves, with major effects on the general circulation (Houghton 1978; Lindzen 1981; Holton 1982; Matsuno 1982). The scenario is that gravity waves are generated in the lower atmosphere, grow in amplitude as they propagate upwards until they become unstable, then dissipate and transfer the horizontal momentum they carry to the background flow. The theoretical studies have considered, for simplicity, one or a few gravity waves that become unstable only in the mesosphere.

On the other hand, Dewan and Good (1986) and Smith et al. (1987) developed a theoretical model based on the concept that the short vertical wavelength part of the gravity wave spectrum is saturated at all altitudes, with the longest wavelength to which the spectrum is saturated growing with height. Their model predicts that the saturated gravity spectrum of horizontal velocity varies as $N^2/6m^3$, where $N$ is the buoyancy (Brunt–Väisälä) frequency and $m$ is the vertical wavenumber. In spite of the rather heuristic nature of their model, it has been found to agree rather well in both shape and amplitude with power spectra of horizontal velocity versus $m$ observed in the troposphere, stratosphere, mesosphere, and lower thermosphere (Smith et al. 1987) and with simultaneous spectra of horizontal velocity and temperature in the troposphere and lower stratosphere (Friggs et al. 1988).

The fact that the spectra collected by Smith et al. were taken at different locations, at different seasons, using different techniques of observation (balloon ascents, smoke trails, and radar), and with different methods of data analysis, however, somewhat reduces the significance of the agreement. One of the main purposes of the present paper is to compare the model with spectra in the troposphere, stratosphere, and mesosphere that were taken essentially simultaneously, with the same measurement and data analysis techniques. These spectra were obtained by the MST radar technique using the MU radar in Shigaraki, Japan. An additional lower stratospheric spectrum taken by the Arecibo radar in Puerto Rico is also presented.

Sections 2 through 4 are concerned with the process by which the spectra are obtained, section 2 with experimental techniques, section 3 with data reduction, and section 4 with spectral analysis. The results are presented and discussed in section 5. Concluding remarks are made in section 6.

---

Corresponding author address: Dr. T. E. VanZandt, NOAA/ERL, Aeronomy Laboratory, 325 Broadway, Boulder, CO 80303.

2. Experimental techniques

In this study we have used data observed by the MU radar (35°N, 136°E) in Shigaraki, Japan, and the Arecibo radar (18°N, 67°W), in Puerto Rico, which operate at frequencies of 46.5 MHz (wavelength 6.45 m) and 430 MHz (wavelength 70 cm), respectively. These radars are described in more detail by Fukao et al. (1985a,b) and Woodman (1980).

a. Observations in the troposphere and lower stratosphere

The MU radar observations in the troposphere and lower stratosphere were taken for 16 hours each day from 1600 to 0800 LT on 17–24 October 1986 and for 24 hours each day from 25 to 29 June 1987. (In October during the remaining hours the radar was occupied making the mesospheric wind measurements described in section 2b.) In every interpulse period the radar antenna beam was steered sequentially toward the vertical and oblique directions at a zenith angle of 20°. In October there were two oblique directions in the north and east azimuths and in June there were four directions in the cardinal azimuths. In October the radial wind velocity in each direction was sampled every 60 s, and in June it was sampled every 60 s at night and every 150 s during daylight hours. The range resolution was 150 m. The signal was sampled from 5 to 25 km altitude, but because of poor signal-to-noise ratio at the higher altitudes the effective altitude limit for velocity measurements was reduced to 21 km in October and 19 km in June.

The Arecibo radar observations used here were taken for 5 h 45 min on 11 June 1983, with the radar beam directed at a zenith angle of 15°. The data used here were taken in the cardinal azimuths. The signal was sampled from 13 to 22 km altitude with a range resolution of 150 m. The observational campaign is described in more detail by Tsuda et al. (1985).

Since saturated gravity wave theory (Dewan and Good 1986; Smith et al. 1987) predicts that the spectral density varies as \( N^2 \), it is important to separate the observed altitude range into subranges with roughly constant values of \( N^2 \). Profiles of temperature \( T \) and hence \( N^2 \) were obtained from radiosonde ascents. For the MU radar campaigns, special ascents were launched from the MU radar site. For the Arecibo campaign we used the routine National Weather Service radiosonde ascents from San Juan, Puerto Rico, 80 km east of the Arecibo Observatory.

The mean \( T \) and \( N^2 \) profiles at the MU radar for the October 1986 campaign are shown in Figs. 1 and 2 of Fritts et al. (1989). Based on the \( N^2 \) structure in these figures, we have analyzed the data into subranges from 5 to 9 km and from 12 to 19 km, with average \( N^2 \) values of \( 1.1 \times 10^{-4} \) and \( 4.1 \times 10^{-4} \) (rad s\(^{-1}\))^2, respectively. These values are characteristic of the troposphere and lower stratosphere.

![Fig. 1. Profiles of \( N^2 \) derived from radiosonde ascents from the MU radar site in June 1987 (solid line) and from San Juan, Puerto Rico, 80 km east of the Arecibo radar, in June 1983 (dashed line). The vertical bars with a circle indicate the mean \( N^2 \) in the troposphere and stratosphere for the MU radar observations, while the bar with a square indicates \( N^2 \) in the stratosphere for the Arecibo observations.](image)

b. Observations in the mesosphere

The MU radar observed in the mesosphere for eight hours each day from 0800 to 1600 LT from 13 to 31 October 1986 and from 6 to 29 June 1987. The observations were confined to daytime because the echoes were too weak at night. In this experiment the radar antenna beam was steered sequentially toward the vertical and four oblique directions at a zenith angle of 10° in the cardinal azimuths. The radial wind velocity in each direction was sampled every 145 s on 13–17 October 1986 and 20–29 June 1987 and every 60 s on 18–31 October 1986 and 6–19 June 1987. The range resolution was 600 m, but the data were oversampled...
every 300 m. The signal was sampled from 60 to 98 km, but because of poor signal-to-noise ratios at the lower heights and meteor echoes at the higher heights only the data collected from 65 to 85 km were used in this study.

Since concurrent temperature measurements in the mesosphere were not available, we have taken $N^2$ to be $5.1 \times 10^{-4}$, based on the CIRA (1972) model atmosphere in the altitude range of the observations.

3. Data reduction

In order to make reliable estimates of vertical wavenumber spectra of the fluctuating wind fields, the background wind field must first be removed. But because of the different experimental conditions in the several campaigns, we have had to use different methods to remove the background wind for the several campaigns. These methods are described in this section.

In the troposphere and lower stratosphere the radial wind velocities measured by the MU radar were averaged for one hour at each height. For the June 1987 campaign, the background wind profile for each hourly profile was computed by further averaging the hourly profiles for 24 hours centered on the particular time, in order to remove components with periods longer than the inertial period, about 21 hours at 35°N. This background wind was then subtracted from the hourly profile at the center time.

Because the October 1986 observations in the lower atmosphere extended over only 16 hours each day, from 1600 to 0800 LT, we have defined a background wind for each run by averaging over the entire 16-hour observation period and applying a 3 km smoothing as described in detail by Fritts et al. (1988). The background wind thus defined may not be suitable near the beginning and end of each run, however, since frequency components longer than the inertial period are not necessarily stationary. Therefore, fluctuating wind profiles are obtained only in the middle ten hours, from 2100 to 0300 LT.

The 5 h 45 min duration of the Arecibo radar observations was much shorter than the inertial period, about 39 hours at 18°N, which makes definition of the background wind by time averaging very difficult. The San Juan radiosonde wind profiles (Tsuda et al. 1985), however, show that the background wind profiles can be approximated by a linear profile.

Since we have used a different definition of the background and fluctuating winds for each of the three campaigns in the lower atmosphere, we need to clarify their effects on the resulting spectra. For comparison, all three methods described above have been applied to the June 1987 MU radar data. We found that the spectra obtained using the three different methods agree very well at high wavenumbers, with small discrepancies near the smallest wavenumbers of less than 10% in spectral amplitude.

For the mesospheric observations, the wind velocities are averaged for two hours centered one hour apart in order to increase confidence in the profiles, so that only seven wind profiles over the 65–85 km altitude range are normally constructed each day. Since the mesospheric echoes are sometimes vertically discontinuous, however, it is difficult to construct profiles over the entire range of observation. Therefore, in each profile, we have selected a 12 km height range (40 data points) without major gaps for spectral analysis. Before computing the spectra, a linear trend was subtracted from each profile.

4. Spectral analysis

In this section, we describe our procedure for spectral analysis and comment on the approach most appropriate for our data. Two techniques for calculation of spectra are equivalent: direct Fourier transform of the data series and Fourier transform of the autocorrelation function of the original data. The former method is straightforward, but missing data points must be interpolated before the Fourier analysis. On the other hand, the latter method can be applied even for a dataset including data gaps. Because wind profiles detected by MST radars are sometimes discontinuous, depending on the vertical distribution of radar reflectivity, the latter method is more appropriate here.

The autocorrelation functions are calculated from the fluctuating wind profiles deduced in the preceding section after removal of a linear trend. The functions are then multiplied by a lag window such as a Hanning window in order to suppress end effects due to the finite data series. They are then transformed into a spectrum using a Fast Fourier Transform.

When the spectral density varies over a large range, it is advantageous to prewhiten the data before calculating the autocorrelation function. The original data series can be prewhitened by using the formula

$$y_t = x_t - \beta x_{t-1}$$  \hspace{1cm} (1)

where $x_t$ and $y_t$ are the data series before and after prewhitening, and $\beta$ is the degree of prewhitening, which is chosen according to the spectral shape. Here we take $\beta$ to be 0.95. After the Fourier transform the true spectrum is determined by compensating for prewhitening by using a recoloring.

We now use numerical simulation to investigate distortion of the wavenumber spectrum by the Fourier analysis, including the effect of prewhitening. We assume a vertical wavenumber spectrum of the form used by Desaubies (1976) and Smith et al. (1987) to fit observed spectra of horizontal velocity in the ocean and the atmosphere, respectively:

$$F(m) = C/[1 + (m/m_*)^t],$$  \hspace{1cm} (2)

where $m_*$ is the characteristic vertical wavenumber, $t$ is the asymptotic slope of the spectrum, and $C$ is a
normalization constant. Here we use $t = 3$, which is, as we will show in the next section, the approximate asymptotic slope of the observed spectra.

For the purpose of illustration, we have used a spectrum with 20 Fourier components. Fifty wind profiles were numerically simulated. For each profile the amplitude of each Fourier component was determined by (2) with $C = 1$ and the phase was specified by a random number generator (e.g., Owens 1978). A wavenumber spectrum was calculated from each simulated profile and the 50 spectra were then averaged. The resulting spectra are plotted in Fig. 2 for two cases, with $m_* = 1$ and 4, together with the model spectrum indicated by the dashed curve. The spectra are plotted as log-log graphs in the upper panels and as area-preserving graphs in the lower panels.

With $m_* = 4$ (left panels), for $m \gtrsim 4$ the spectra determined with and without prewhitening agree fairly well with the model. On the other hand, for $m \lesssim 2$ the analysis without prewhitening considerably underestimates the spectral density, while the analysis with prewhitening agrees fairly well.

With $m_* = 1$ (right panels), the model is approximately linear with a slope of $-3$ in the entire range. For $m \gtrsim 3$ the slopes of the simulated spectra agree fairly well with the model, although the energy density seems to be overestimated by about 30%. At $m = 1$ and 2 the energy density is generally reduced from the model, but the analysis with prewhitening again gives a better result.

Now we examine the accuracy with which $m_*$ can be estimated from observed spectra. The wavenumber $m_p$ of the peak of the model spectrum given by (2) lies at

$$ m_p = 2^{-1/3} m_* = (0.79) m_* . $$

When $m_* = 4$, the model $m_p$ is 3.17 and the $m_p$ of the simulated spectra in the area-preserving graph are smaller by about 10%, so a similar error would be made in estimates of $m_*$. On the other hand, when $m_* = 1$, it is impossible to estimate $m_p$ because it is comparable to the smallest wavenumber.

In summary, spectral analysis using only a Hanning window gives a reasonable agreement with the given model for $m > 4$. When prewhitening is also used, it is especially effective in preserving the spectral density at small wavenumbers. The spectral analyses in the next section include prewhitening.

![Fig. 2. Numerical simulations of wavenumber spectra for $m_* = 4$ (left panels) and $m_* = 1$ (right panels) plotted on log-log (top) and area-preserving (bottom) graphs. The dashed curve shows the model spectrum, while the solid and dotted curves show the simulations with and without prewhitening, respectively.](image-url)
5. Results

Figures 3–5 show spectra of the zonal and meridional components of horizontal velocity in the troposphere (T), lower stratosphere (S), and mesosphere (M) obtained using the MU and Arecibo radars. In each case the spectral density has been divided by \( \sin^2 \chi \), where \( \chi \) is the zenith angle of the radar beam, in order to normalize spectra taken with different \( \chi \).

According to the theory of Dewan and Good (1986) and Smith et al. (1987), the saturated spectral density is given by

\[
F_s^c(m) = \frac{N^2}{6m^3}.
\]

The coefficient 1/6 estimated by Smith et al. is precise to within a factor of about 1.5, given the assumptions of the theory. The model spectra are shown as dashed or dot-dashed straight lines in Figs. 3–5, with values of \( N^2 \) appropriate to each campaign, as described in section 2.

Figures 3–5 will be discussed in sections 5a through 5c. The spectra are replotted in Fig. 6 in order to facilitate a more detailed discussion in section 5d.

a. MU radar observations in October 1986

Figure 3 shows the vertical wavenumber spectra computed for the MU radar observations in October 1986. In each region the shapes of the zonal and meridional spectra agree very well from the smallest wavenumber up to about \( 10^{-3} \) (c m\(^{-1}\)). (In this paper wavenumbers are always expressed in units of c m\(^{-1}\).) The difference between the shapes at larger wavenumbers is attributed to contamination of the zonal spectra by the finite-range volume effect, first reported by Fukao et al. (1988). In this effect thin scattering layers unequally distributed in height in the radar sampling volume produce spurious fine structure in the wind profile and thus a contamination of the power spectra at large \( m \). The error is more serious when the wind profiles are measured in the azimuth of the background wind, which was approximately zonal in October 1986 (Fritts et al. 1988). The dominant wavenumber of the contamination, which depends on both the radar beamwidth and the zenith angle, ranged from \( 10^{-3} \) to \( 3 \times 10^{-3} \) for the present MU radar campaigns. Thus, for \( m > 10^{-3} \) the meridional spectra are a more accurate measure of the spectral amplitude of the gravity wave field. The sudden increase in the slopes of the tropospheric and stratospheric spectra at \( m \approx 3 \times 10^{-3} \) is probably due to measurement errors.

The small differences in the amplitudes of the zonal and meridional spectra for \( m < 10^{-3} \) are evidence of azimuthal anisotropy of the wave field, which has been discussed in the stratosphere by Dewan et al. (1988) and VanZandt et al. (1989) and in the mesosphere by, e.g., Vincent and Fritts (1987) and Manson and Meek (1988).

In the large-wavenumber ranges of the observed spectra both the asymptotic slopes and the amplitudes agree rather well with the model saturated spectra given by Eq. (4). In the mesosphere and stratosphere the amplitudes agree very well with the model, except for some contamination of the stratospheric spectrum by the finite-range volume effect. In the troposphere the amplitudes are about a factor of two larger than the model; this may be due to contamination by other processes in the troposphere, such as convection.

The peak and characteristic wavenumbers, \( m_p \) and \( m_\ast \), can be estimated by reference to the simulated spectra in Fig. 2. In the mesosphere \( m_p \) and \( m_\ast \) must be smaller than the smallest \( m \), \( 1 \times 10^{-4} \). In the stratosphere \( 3.0 \times 10^{-4} \leq m_p \leq 4.5 \times 10^{-4} \) so that \( 3.8 \times 10^{-4} \leq m_\ast \leq 5.6 \times 10^{-4} \) using (3). In the troposphere only an upper limit can be put \( m_p \) since the meridional area-preserving curve is almost flat for \( m \leq 3.0 \times 10^{-4} \), so that \( m_\ast \leq 3.8 \times 10^{-4} \).
b. MU radar observations in June 1987

Figure 4 shows the vertical wavenumber spectra obtained from the MU radar observations in June 1987. The remarks concerning the October 1986 spectra generally apply here. In the mesosphere the agreement between the observed and model spectra is similar to October 1986. In the stratosphere and troposphere the agreement in amplitude is better because contamination by the finite-range volume effect is smaller since the jet stream in June was weaker than in October. In the stratosphere the wavenumber range is too narrow to compare the slopes, because the tropopause was high in June (see Fig. 1), but the amplitudes agree well. As before, the sudden increase in slope of some of the spectra at their largest wavenumbers is probably due to measurement error.

The values of $m_n$ and $m_s$ in the mesosphere are again $<10^{-4}$ and $m_s$ in the troposphere is $\approx 2.2 \times 10^{-4}$, so that $m_s < 2.8 \times 10^{-4}$. The break in the stratospheric spectrum at $\approx 10^{-3}$ is probably an artifact of the data analysis due to the limited altitude interval available.

c. Arecibo radar observation in June 1983

Figure 5 shows the vertical wavenumber spectra obtained from the Arecibo radar observations in June 1983. Because the antenna beamwidth of the Arecibo UHF radar is very narrow, the finite-range volume effect should be negligible in these observations. The zonal and meridional stratospheric spectra in the large-wavenumber range are both in excellent quantitative agreement with the model of Smith et al. (1987), with slopes very near $-3$ for $m$ between 0.7 and $2.0 \times 10^{-3}$ (c m$^{-1}$). Measurement error appears to become important at about $2 \times 10^{-3}$ (c m$^{-1}$). Again, only an upper limit can be put on $m_n$ and $m_s$ because the area-preserving graphs are almost flat for $m < 0.7 \times 10^{-3}$.

d. Comparison of normalized spectra

In Figs. 3–5 the significance of the comparison between the observed and model spectra is difficult to assess because of the steep slopes, the consequent large ranges of the ordinate, and the superposition of up to nine observed and model spectra. These difficulties are obviated in Fig. 6 by plotting the ratio of the observed meridional spectra divided by the model spectra (as well as by $\sin^2 \chi$). Only the meridional component was plotted because it is usually less contaminated by the finite-range volume effect.

In these plots the degree of agreement between the observed and model spectra is determined by the closeness to unity of the asymptotic ratio. Let us first
The asymptotic tropospheric ratios lie between about 1.5 and 2.0. This small discrepancy might be due to contamination by the finite-range volume effect from the small meridional background wind or to tropospheric motions in addition to gravity waves.

Blumen (1987, 1988) considered the saturation of gravity waves using the theory of nonlinear advection over topography. He found that at the onset of convective instability the spectrum has a slope of $-8/3$. Because the conditions under which Blumen's theory is valid are not satisfied by the observed spectra, his slope cannot be directly compared with the asymptotic slope of the observed spectra, but it may correspond to a limiting case.

6. Discussion and conclusions

The comparisons shown in the previous section show that the saturated gravity wave spectral model given by Eq. (4) agrees well with the observed spectra of horizontal velocity presented in this paper. These spectra are much more homogeneous than those that Smith et al. (1987) compared with the model, since all of the present data were taken by the same radar technique, the data in the different atmospheric regions were taken essentially simultaneously, and all of the spectra, including the stratospheric spectra taken at the Arecibo Observatory, were analyzed using very similar methods.

Nevertheless, the spectra cover a wide range of altitudes, two different seasons, and two different geographical locations, at 35° and 18° latitude. Thus, the agreement shown in this paper reinforces and extends the agreement found by Smith et al. using disparate spectra and the agreement found by Fritts et al. (1988) using simultaneous stratospheric and tropospheric spectra of horizontal velocity and temperature. As far as we know, the model does not disagree significantly with any spectra of good quality.

The fact that the agreement of the saturated gravity wave spectral model of Dewan and Good (1986) and Smith et al. improves as the observed spectra are improved suggests that the model is essentially correct, in spite of the heuristic nature of some of its assumptions. It is obviously desirable, however, that the theory be put on a more fundamental basis by justifying its assumptions.

In the saturated wavenumber range the amplitudes of the zonal and meridional spectra are, of course, about equal. But in the small wavenumber range, where the spectra are not saturated, the energy density varies with altitude and from campaign to campaign. In the present campaigns the meridional spectra are always larger than the zonal spectra at small wavenumbers.

The estimates of and limits on $m_v$ from the different campaigns are not inconsistent. For the mesosphere, stratosphere, and troposphere, the best estimates are $<10^{-4}$, 3.0 to $4.5 \times 10^{-4}$, and $<3 \times 10^{-4}$ in October and $<2.2 \times 10^{-4}$ in June, respectively. These corre-
spond to dominant vertical scales (\(=1/m_p\)) of \(>10\) km, 2.2 to 3.3 km, and \(\geq 3.3\) km in October and \(\geq 4.5\) km in June, respectively, consistent with determinations from other studies.

Acknowledgments. This paper was written while one of the authors (TT) was visiting the Aeronomy Laboratory, NOAA, supported by the Japanese Ministry of Education, Science and Culture. The MU radar is operated by the Radio Atmospheric Science Center, Kyoto University. The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, operated by Cornell University under contract with the National Science Foundation.

REFERENCES