Comments on "Delayed Albedo Effects in a Zero-Dimensional Climate Model"

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Andersson and Lundberg (1988) showed some very intriguing behavior resulted when a time delay was introduced into the shortwave albedo response in a simple energy-balance climate equation akin to that of Sellers (1969) or of Budyko (1969). One must query the relevance of their equation to any real climate system, however, seeing that their Eq. (2.4) implies a response that is delayed by a time τ, but not blurred or smudged in any way: For $\tau = 325$ years, for example, the treatment would imply that the shortwave albedo in (say) 1988 was decided only by the temperature in the year 1663, while completely unaffected by temperatures in any other year after 1663 or before it. Granted, deep ocean storage might well provide the climate system with a memory, but a memory so highly selective seems rather implausible.

If one carries out a stability analysis similar to the one employed by the authors, but replaces their selective memory $[a]_{T(t-\tau)}$ by an exponential "forgetting" described by

$$\tau^{-1} \int_0^\infty e^{-(t-t')/\tau} [a]_{T(t-t')} dt'$$

a kind of behavior that is easier to find in physical systems with memories (e.g., a capacitor in an electrical circuit), then the resulting equation for $\lambda\tau$ becomes

$$\lambda\tau = -a(\theta)(1 + \lambda\tau)^{-1} - b(\theta) \qquad (2a)$$

in contrast with the original equation (3.1 in their article)

$$\lambda\tau = -a(\theta)e^{-\lambda\tau} - b(\theta). \qquad (2b)$$

Here θ denotes a fixed point (at which $dT/dt = 0$). Finding the roots of Eq. (2a) is trivial; they are

$$-\frac{1}{2}(b + 1)\{1 \pm [1 - 4(a + b)/(b + 1)^2]^{1/2}\}. \qquad (3)$$

[For simplicity, a and b are written for $a(\theta)$ and $b(\theta)$.] Since b is unambiguously positive (being essentially the derivative with respect to temperature of the longwave emission), all *complex* roots of (2) will have negative real parts, implying a stable system in which oscillations could only be transient. A positive *real* root will occur only if $-a$ exceeds b, but then the climate equation—with no delay or memory—would already be unstable.

One must conclude that attributing a memory to the shortwave albedo is not enough to change stability into instability: To do that, it must be a geriatric kind of memory, with an acute recall of the distant past, while totally forgetting more recent times. The nature of the memory, as well as length of delay, appear to be important.

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REFERENCES

Andersson, L. S., and P. A. Lundberg, 1988: Delayed albedo effects in zero-dimensional climate model. *J. Atmos. Sci.*, **45**, 2294–2305.

Budyko, M. I., 1969: The effect of solar radiation variations on the climate of the earth. *Tellus*, **21**, 611–619.

Sellers, W. D., 1969: A global climatic model based on the energy balance of the earth–atmosphere system. *J. Appl. Meteor.*, **8**, 392–400.