Low-Frequency Variability in the Northern Hemisphere Winter: Geographical Distribution, Structure and Time-Scale Dependence

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ABSTRACT

Low-frequency variability in wintertime 500 mb height is examined, with emphasis on its structure, geographical distribution, and frequency dependence. A 39-year record of 500 mb geopotential height fields from the NMC analyses is time filtered to partition the fluctuations into frequency bands corresponding to periods of 10–60 days, 60–180 days and >180 days. Winter is defined as the six month period November through April. Variance, teleconnectivity, and anisotropy fields, and selected loading vectors derived from orthogonal and oblique rotations of the eigenvectors of the temporal correlation matrix for each band are shown and discussed.

The variability in all frequency bands exhibits substantial anisotropy, with meridionally elongated features arranged as zonally oriented wave trains prevailing over the continents and zonally elongated features organized in the form of north-south oriented dipole patterns prevailing over the oceanic sectors of the hemisphere. The wave trains are most pronounced in the 10–60 day variability, while the dipoles are most pronounced at lower frequencies. Eastward energy dispersion is apparent in the wave trains, but there is no evidence of phase propagation.

Most of the “teleconnection patterns” identified in previous studies appear among the more prominent loading vectors. However, in most cases the loading vectors occur in pairs, in which the two patterns are in spatial quadrature with one another and account for comparable fractions of the hemispherically integrated variance. It is argued that such patterns should be interpreted as basis functions that can be linearly combined to form a continuum of anisotropic structures. Evidence of the existence of discrete “modal structures” is found only in the interannual (>180-day period) variability, where two patterns stand out clearly above the background continuum: the Pacific–North American (PNA) pattern and the North Atlantic Oscillation (NAO). These patterns leave clear imprints upon the climatological mean variance of the 500 mb height field and the anisotropy tensor of the 500 mb wind field. The western Atlantic (WA) pattern stands out somewhat above the background continuum in the month-to-month (60–180 day period) variability.

1. Introduction

In contrast to the distinctive shapes associated with baroclinic instability that we are accustomed to seeing on daily sea-level pressure or upper level geopotential height charts, the spatial patterns associated with weekly, monthly, or seasonal mean geopotential height anomalies tend to be rather amorphous. It is only when ensembles of anomaly fields are statistically analyzed that we begin to see evidence of recurrent spatial patterns.

On the basis of their subjective analysis of linear temporal correlations in station data, Walker and Bliss (1932) identified two north-south “seesaw” or “dipole” patterns in sea-level pressure anomalies over the ocean basins that they referred to as the North Atlantic Oscillation (NAO) and the North Pacific Oscillation (NPO). Van Loon and Rogers (1978), Rogers and van Loon (1979), Meehl and van Loon (1979), and Rogers (1981) have documented the anomaly patterns in the midtropospheric circulation, sea-surface temperature, sea level, sea ice drift, and tropical wind and precipitation that typically occur in association with these patterns. Because such patterns are associated with strong simultaneous correlations between climatic anomalies observed over widely separated regions of the globe, they are sometimes referred to as “teleconnection patterns.”

More recent studies have tended to emphasize the structures at midtropospheric levels, which tend to be coherent over larger regions of the hemisphere and are amenable to a more direct dynamical interpretation. On the basis of a systematic examination of one-point correlation patterns derived from a 15-year record of wintertime, monthly averaged 500 mb height fields, Wallace and Gutzler (1981; hereafter referred to as WG) identified five prominent structures that they referred to on the basis of their geographical location as

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the western Pacific (WP), Pacific–North American (PNA), western Atlantic (WA), eastern Atlantic (EA), and Eurasian (EU) patterns. The first four of these patterns include north–south dipoles over the ocean basins, while the EU pattern corresponds to a large-scale wave train stretching eastward across the Eurasian continent from Scandinavia to Japan. The PNA pattern figures prominently in the earlier literature on climate variability over North America; the WA pattern contains elements in common with the NAO (though its northern center-of-action is located about 25° to the west and 10° to the south of its counterpart in the NAO). Wallace and Gutzler found very similar, but slightly less clearly defined patterns at the 700 mb level, and we have subsequently verified in unpublished results that similar patterns are also evident in the 200 mb height field. Hence it is evident that in the middle and upper troposphere, the dominant spatial patterns in the low frequency variability display an equivalent barotropic vertical structure.

Wallace and Gutzler also found that the leading eigenvectors of the correlation matrix of the monthly mean 500 mb height field contain elements of the regional patterns revealed in the one-point correlation analysis, but they tend to be more complex, and extend over larger portions of the hemisphere. Horel (1981) demonstrated that when these eigenvectors are linearly combined, using a technique known as rotated principal component analysis (RPCA), faithful replicas of WG’s “teleconnection patterns” appear among the leading modes.

Esbensen (1984) identified the prominent one-point correlation patterns in a 30-year record of 700 mb height data in which he distinguished between the interannual and intraseasonal variability. In the intraseasonal band he found patterns resembling those in WG and a new pattern over Asia, which he referred to as the northern Asian (NA) pattern. In the interannual band, represented by the low-pass filtered data, he identified two patterns, which he referred to as the zonally symmetric seesaw (ZS) and the North Pacific (NP) patterns. The Atlantic segment of the ZS pattern resembles the NAO signal in the 500 mb field as documented by Rogers (1984). He reported that the PNA shows up strongly in both bands.

Barnston and Livezey (1987; hereafter referred to as BL) performed RPCA on a 34-year record of 700 mb height data. They based the primary part of their analysis on 12 separate subsets of the data, each consisting of the monthly means for a single calendar month (e.g., the 34 Januaries from 1950 through 1983). Their analysis was designed to emphasize the interannual variability, but because they used a relatively short (1 month) averaging interval, it also picks up some of the intraseasonal variability. The NAO and the PNA pattern appear among their dominant modes of variability for the winter months. Among the other wintertime patterns that they identified are analogues to WP and EA, and two new patterns over the Pacific, which they referred to as the eastern Pacific (EP) and North Pacific (NP) patterns, both of which are north–south “seesaws” between the high and midlatitudes. They also defined a new pattern extending over North America, which they referred to as the tropical–Northern Hemisphere (TNH) pattern.

While Esbensen’s analysis suggests a distinction between interannual variability of the wintertime data and the month-to-month variability within the individual winters, a study by Blackmon et al. (1984a, b; hereafter referred to as B1 and B2 respectively) pointed out significant distinctions between week-to-week and month-to-month variability. These studies were based on three sets of filtered data derived from 18-years of wintertime, twice-daily 500 mb heights over the Northern Hemisphere. The first set was 10–30 day band-pass filtered, the second was comprised of running 30-day averages at 10-day intervals, and the third of 90-day averages for the entire winter season (December–February). Their conclusions were based on an inspection of one-point correlation maps for a selection of reference grid points designed to emphasize the dominant patterns identified by WG. Blackmon et al. (1984a) showed that the very low frequencies resolved by monthly mean data are dominated by north–south dipole patterns anchored in preferred geographical regions (primarily over the ocean sectors) which correspond to the primary centers of action of the teleconnection patterns identified in previous studies. These patterns are suggestive of discrete standing oscillations with geographically fixed nodes and antinodes. In contrast, the 10–30 day variability exhibits zonally oriented wavelike patterns confined within continental “waveguides.” The centers of action of these patterns are elongated in the meridional direction and they do not exhibit a preferred longitudinal phase. Blackmon et al. (1984b) showed that these continental wave trains do not propagate zonally, but they exhibit systematic eastward energy dispersion, manifested in the decay of upstream centers and the development and intensification of downstream centers.

Similar structural differences between 10–30 day fluctuations and those with longer time scales were noted by Wallace and Lau (1985) who documented the anisotropy of the geopotential height perturbations as reflected in the covariance tensor of the horizontal wind vector (see also section 3 below). Their analysis revealed strong contrasts in the anisotropy of the low-frequency transients over the oceans versus those over the continents. Wallace and Lau also found that low-frequency perturbations in the height field become increasingly zonally elongated and that the continental features weaken as the period of the fluctuations increases. In agreement with these results, Schubert (1986) found that the leading eigenvector patterns of the band-pass filtered 500 mb height field change in character from meridionally elongated wave trains to
zonally elongated dipoles as period of the fluctuations increases.

In the present study, we will consider, in greater depth, the some of the issues raised in the papers cited above; in particular:

- How consistent and robust are the spatial patterns derived from the various types of analysis schemes? Can the differences be reconciled?
- Is it more appropriate to view the various "teleconnection patterns" as discrete standing oscillations with geographically fixed nodes and antinodes, or merely as samples from a continuum of anisotropic structures?
- Are the patterns associated with intraseasonal and interannual variability substantially different?
- How important are the continental wave trains identified by B1 in the 10–30 day variability? Are they evident in other frequency bands as well?

In order to obtain an objective representation of the patterns of variability in the data, we have used rotated principal component analysis, rather than one-point correlation maps based on a prescribed set of reference grid points as in B1. We have divided the spectrum of variability into three frequency bands comprising of fluctuations with periods of:

- weeks, as represented by 10–60 day period band-pass filtered data;
- months, as represented by 60–180 day band-pass filtered data;
- seasons and longer, as represented by 180-day low-pass filtered data;

using recursive filters applied to daily data to obtain a distinct separation between the fluctuations in the three frequency bands in a 38 year record.

We have investigated the robustness of our rotated principal components (RPCs) by comparing them with the distribution of teleconnectivity as defined in WG, and the RPCs derived from

- data processed with a different set of digital filters in the frequency domain;
- the data for the early, middle, and late winter months;
- the first and second halves of the data record and the odd- and even-numbered winter seasons in the record;
- the covariance matrix; and
- an oblique rotation of the eigenvectors of the correlation matrix.

In the following section we describe the dataset and the analysis techniques used in this study. In section 3 we examine the structure and geographical distribution of the variability and the general characteristics of the variance and correlation fields. In section 4 we show some of the more prominent patterns of variability derived from the rotated principal component analysis of the filtered data for the three frequency bands defined above. In section 5 we describe the results of the different tests applied to the data to test the robustness of the results. In section 6 we offer a dynamical interpretation of the results and discuss their implications for our understanding of low-frequency variability.

2. Method of analysis

a. Data and filtering procedure

The data used in this study are derived from a 39-year record of NMC 500 mb height analyses, on a 1977-point mesh, superimposed on a polar stereographic projection of the Northern Hemisphere, extending from \( \sim 20^\circ \) to the pole. The data sampling rate is once daily in the earlier part of the record, from January of 1946 to March 1955, and twice daily from April 1955 onward. There are some gaps in the record, most of which are shorter than three days. Only 9 longer gaps were found, of which three were shorter than a week, four were 7–10 days long, and two (during November 1962 and April 1984) were \( \sim 20 \)-days long.

A two-step quality check was applied to the data: height deviations larger than three standard deviations from the long-term monthly means at individual grid points were flagged for further inspection, and time series of domain averaged 500 mb height, domain averaged rms 500 mb height deviation from the hemispheric mean, and domain averaged, 24-hour, rms 500 mb height difference were inspected for prominent spikes. On the basis of these checks, 14 grids were flagged and subsequently treated as missing.

Periods of missing data were filled in by linear interpolation. Gaps shorter than 3 days were filled by interpolating between the height fields just before and just after them. Longer periods of missing data were filled by interpolating between the averages over two periods of length equal to the data gap, situated immediately before and after the gap, in order to avoid aliasing. The data were then slightly smoothed using a recursive 2.5 day low-pass filter\(^1\) and decimated to a uniform sampling rate of once-per-day.

To model the climatological mean annual cycle, a 365-day time series consisting of the 39-year averages for each calendar day was calculated from the data and fitted with its mean and its first three harmonics. By removing the modeled annual cycle from every year in the data, day by day, an anomaly time series was generated that contains both intraseasonal and inter-

\(^1\) All filters used in this study were 4-pole, tangent-Butterworth recursive filters, applied to achieve symmetric results. For further technical details the reader is referred to Kaylor, R. E., 1977: Filtering and decimation of digital time series. Technical Note BN850, Institute of Physics Science and Technology, University of Maryland, College Park, 42 pp.
annual fluctuations in the height field. The anomaly time series was then transformed from the full NMC grid to a 445-point half-resolution grid, using a Gaussian interpolation scheme to avoid aliasing. The reduced resolution is adequate for capturing the essential features of low-frequency variability and reduces the amount of computation required for the statistical analysis of the data. We will hereafter refer to the coarse resolution, 2.5 day low-pass filtered series of height anomalies as the "reference" time series.

In order to study the frequency dependence of the height variability, we filtered the reference time series to isolate fluctuations in three ranges of period: 10–60 days, 60–180 days and >180 days, by consecutive application of low-pass recursive filters and subtraction of the corresponding time series from one another to generate the band-pass data. The choice of the latter two cutoff periods was based on their proximity to the half-power points of running 30-day (monthly) means and 90-day (seasonal) means. Figure 1a shows the power response functions of the 60- and 180-day low-pass filters used in the present study. The response functions of 30 and 90 day running means are shown for comparison. Figure 1b shows how the low-frequency variance is partitioned as a result of our filtering procedure. The recursive filters were applied to the continuous data record; i.e., all 12 calendar months of daily gridded fields for 39 years. In this way the filter "spinup" and "spin down" effects were confined to the first and last few months of the record, which were discarded, leaving 38 full winter seasons.

b. Statistical analysis

Richman (1986) recently reviewed the theory and application of RPCA in the diagnosis of meteorological fields. The analysis involves the production of a set of spatial patterns called loading vectors, which are linear combinations of the leading eigenvectors of the temporal correlation matrix. The linear transformation is designed to identify "simple" (i.e., relatively localized) patterns. As with the original eigenvectors, the loading vector defines the spatial distribution of weights associated with a new set of time dependent "rotated" principal components (the RPCs). The original eigenvectors depict the patterns that are most efficient at explaining the temporal variance integrated over the entire analysis domain. They tend to increase in complexity with decreasing amount of variance explained. In contrast, the rotated loading vectors depict spatial structures that exhibit large amplitude over only relatively small sections of the domain and small values elsewhere. Hence the RPCs express the variability in the data in terms of geographically local components in a manner that appears to be consistent with our dynamical perception of atmospheric variability.

In the present study, the spatial correlation matrices were calculated for each of the three frequency bands defined above, using wintertime data only. Here winter is defined as the 181-day period (182 days in leap years) extending from 1 November of one year through 30 April of the following year. The eigenvectors of the correlation matrix were evaluated using International Mathematical and Statistical Library (IMSL 1979) computer routines. Following Horel (1981) and BL, we used the VARIMAX rotation procedure to obtain the loading vectors. VARIMAX is an iterative method in which the spatial variance of the squared loadings is maximized. The method retains the temporal orthogonality between the RPCs but relaxes the requirement for orthogonality between the spatial patterns that is inherent in eigenvector analysis. In section 5 we will
discuss the effect of a more complete relaxation of the orthogonality constraint through the use of an oblique rotation method.

Prior to the eigenvector analysis, the normalized height anomalies for each grid box were weighted by the square root of its fractional area in our analysis domain. However, when presenting the results in the following sections, the area weighting is removed from the loading vectors so that the patterns can be viewed as the temporal correlation coefficients between the time dependent RPCs and the height fluctuations at each grid point. Thus, at any specified grid point the squared loadings represent the fraction of the variance explained by the respective RPCs.

Only a subset of the total number of eigenvectors derived in the analysis of the correlation matrix (the leading ones) is generally used in the rotation. The “eigenvalue-one” criterion proposed by Guttman (1954) has been applied in a number of previous studies (e.g., Horel 1981; Hsu and Wallace 1985) as a basis for determining the number of eigenvectors to be included. When we apply this criterion to our relatively high-resolution data [the number of grid points in our representation of the height field is four times as large as in Horel (1981) or Hsu and Wallace (1985)], the loading vectors become excessively simple, i.e., most of them exhibit monopolar patterns rather than the multiple “centers-of-action” characteristic of the one-point correlation maps that these patterns are designed to summarize (see also O’Lenic and Livezey 1988). These monopolar structures are a reflection of the high temporal correlations between the height fluctuations at neighboring grid points. When the grid resolution is increased to the point where the real dynamical structures in the data are, in effect, “oversolved” (i.e., number of grid points exceeds number of spatial “degrees of freedom” in the data), these local correlations begin to influence the structure of the loading vectors. Beyond this point, further increases in resolution yield increasingly monopolar patterns. In order to eliminate this spurious resolution dependence and recover the more complex patterns characteristic of the one-point correlation maps, it is necessary to rotate a smaller number eigenvectors that is determined by the number of “independent grid points” or “degrees of freedom” in the dataset, rather than by the number of points in the grid. Lacking a formal criterion, we proceeded empirically, by rotating a subset of eigenvectors having an eigenvalue \( \geq 10 \) (rather than 1). We then repeated the rotation several times, increasing the size of the subset by small increments until we were satisfied that we had rotated enough eigenvectors to ensure results that are robust in the sense that the loading vectors do not change substantially when a new EOF is added to the subset. The robustness of the loading vectors with respect to the size of the eigenvector subset and their agreement with one-point correlation fields for reference grid points at their centers-of-action were considered in determining the final subset for the presentation in section 4. The numbers of eigenvectors included in this rotation are listed at the beginning of that section.

3. Geographical distribution of variance, teleconnectivity, and anisotropy

For convenience we will refer to the period ranges of 10–60 days and 60–180 days, as the intermediate and intermonthly frequency bands, respectively, and the range of periods longer than 180 days as the interannual band. Strictly speaking, the interannual band includes fluctuations with time scales comparable to or longer than the semiannual cycle, including year-to-year fluctuations in the amplitude and phase of the annual cycle.

The geographical distribution of the rms geopotential height fluctuations in the three frequency bands is shown in Fig. 2. Consistent with results of B1 (their Fig. 2) and Esbensen (1984, Fig. 2), the frequency dependence is subtle. We find a westward and southward shift in the position of the intermonthly and interannual rms height maxima over the Pacific and Atlantic oceans, relative to their intermediate time-scale counterparts. The secondary maximum over Northern Europe is separate from the maximum over the Atlantic only for the intermonthly and interannual bands. The zonal asymmetries in the rms height field over the higher latitudes become increasingly apparent as one examines progressively longer time scales. For example, the ratio in rms amplitude between the Pacific maximum to the south of Alaska and the minimum over central Canada is \( \sim 1.5 \) to 1 in the intermediate band, almost 2 to 1 in the intermonthly band, and \( \sim 2.5 \) to 1 in the interannual band. Similar differences are evident in the rms height variability for early winter (November/December), midwinter (January/February) and late winter (March/April), not shown here.

Wallace and Gutzler (1981), B1, and Esbensen (1984) used teleconnectivity maps to assess the presence of dipole patterns and other coherent, wavelike structures in the geopotential height field. Teleconnectivity is defined as the geographical distribution of the absolute value of the strongest negative correlation in each of the columns of the temporal correlation matrix of the geopotential height field, plotted in the position corresponding to the column index. Thus the teleconnectivity reveals the existence and location of the prominent one-point correlation patterns in the height field. The teleconnectivity distributions for the three frequency bands are shown in Fig. 3. Panels (a) and (b) can be compared with Fig. 12 in B1 and Fig. 3a in Esbensen (1984) and panel (c) with Fig. 3b in Esbensen (1984). The long arrows drawn in these panels connect the teleconnectivity maxima with the grid points with which they are the most strongly negatively correlated, as determined from an examination of the corresponding one-point correlation fields.
Fig. 2. Root-mean-square of the 500 mb geopotential height fluctuations over the Northern Hemisphere during winter (1 November–30 April), based on NMC analyses from 1946/47 to 1983/84. Shown are the distributions for fluctuations with (a) periods of 10–60 days, (b) periods of 60–180 days, (c) periods longer than 180 days. Contour interval 20 m in panel (a) and 10 m in panels (b), (c).

Several general features such as the geographical location of the largest values, are common to all three teleconnectivity maps shown in Fig. 3. Over the oceans, belts of high teleconnectivity in the 50°–65°N latitude belt and the subtropics are separated by pronounced bands of lower teleconnectivity over intermediate latitudes. An examination of individual one-point correlation fields reveals that this configuration is associated with north–south seesaw patterns. High values of teleconnectivity are also found over North America and Eurasia, corresponding to northwest–southeast oriented wave trains. Compared to the fields for the intermonthly and interannual bands, the field for the intermediate band looks rather featureless, with relatively weak maxima and weak spatial gradients. Over the Atlantic the features in the teleconnectivity field become progressively more zonally elongated and narrower in the meridional extent as the period of the fluctuations increases. These results are generally consistent with the studies of B1 and Esbensen (1984) and with the patterns of variability shown in section 4.

Corresponding results for the three separate subseasons (not shown) indicate that teleconnectivity tends to be strongest in midwinter, particularly in the interannual band. The structure of the teleconnectivity field over the Pacific is similar for all three sets of months,
Fig. 3. Teleconnectivity of wintertime 500 mb height fluctuations with periods of (a) 10–60 days, (b) 60–180 days, (c) longer than 180 days. Contour interval 0.1; bold contours indicate values larger than 0.4 in panel (a) and larger than 0.5 in (b), (c). Decimal points are omitted on labels for extrema.

whereas over the Atlantic more substantial changes are observed.

The characteristic shape and orientation of the transients in various frequency bands can be inferred from the asymmetric part of the local temporal covariance tensor of the horizontal wind components. This information can be represented as line segments drawn in the direction of the local major axis of the anisotropy (Hoskins et al. 1983; Figs. 2, 3) or in the direction of the minor axis of the anisotropy (Wallace and Lau 1985; Fig. 1). In this study we use the former convention. The length of the line segments is drawn proportional to the coefficient of anisotropy, which ranges from zero for isotropic fluctuations to unity for fluctuations polarized along a single axis as in Kelvin waves (see also the schematic diagrams in the above two references). In the present study, the anisotropy has been calculated from the geostrophic wind components derived from the height anomalies in each frequency band on the half-resolution NMC grid. Results for the three frequency bands are presented in Fig. 4, superimposed upon contours of the zonal component of the coefficient of anisotropy, which is equal to the normalized difference between the transient kinetic energy associated with the zonal and meridional wind components (M/K in the notation of Hoskins et al. 1983). In all
three frequency bands the height anomalies tend to be zonally elongated, particularly over the subtropics, but limited regions exist in which the height anomalies tend to be elongated in the meridional direction. As in the case of the teleconnectivity, the anisotropy becomes stronger and its field acquires more structure as the time scale of the transients increases. We will comment further on these fields in section 4.

4. Frequency dependence of the dominant patterns of variability

In the following three subsections we will examine the loading vectors associated with some of the more prominent RPCs of the filtered data. The rotations are based on 18 eigenvectors accounting for 65% of the hemispherically integrated normalized variance in the intermediate band, 15 eigenvectors accounting for 76% of the variance in the intermonthly band, and 15 eigenvectors accounting for 82% of the variance in the interannual band. The fraction of the domain integrated normalized variance explained by the loading vectors is given on the diagrams.

We estimate that the time series for the intermediate frequency band contain $\sim 200$ degrees-of-freedom (six independent realizations per season), those in the intermonthly band contain $\sim 75$ degrees-of-freedom (two
per season) and those in the interannual band contain $\sim 35$ degrees-of-freedom (slightly less than one per season). The formal, a priori 99% significance level for temporal correlation coefficients corresponds to $\sim 0.2$ for the intermediate band, $\sim 0.3$ for the intermonthly band, and $\sim 0.4$ for the interannual band. A complete set of hemispheric maps of the loading vectors is available upon request from the first author.

### a. Intermediate frequency variability

Figure 5 shows loading vectors for the intermediate frequency band which resemble the one-point correlation patterns shown in Fig. 10b, d, f of B1. They are characterized by multiple centers-of-action arranged along a west–northwest/east–southeast axis almost parallel to a latitude circle. The wavelike pattern has a wavelength of $\sim 60^\circ$ longitude, about 50% longer than that of baroclinic waves (Blackmon et al. 1984a, Fig. 15; Wallace et al. 1988). The individual centers of action in the wave are elongated in the meridional direction, consistent with the north–south oriented line segments in the anisotropy field (Fig. 4a). The two patterns in Fig. 5 explain comparable fractions of the normalized hemispheric variance in this band and are in quadrature with one another (i.e., the north–south oriented nodal lines of one pattern coincide with the "centers of action" of the other). By linearly superimposing such patterns one could construct a wave train with the same wavelength and any specified longitudinal phase that would explain about as much variance as the patterns from which it was derived. These patterns coincide with a broad, zonally elongated band of high teleconnectivity in Fig. 3a. Hence, in a long-term statistical sense, there exists a continuum of wavelike patterns over this region, which exhibit a common zonal wavelength and meridional extent, but no preferred longitudinal phase.

Lest Fig. 5 be interpreted as the signature of zonally propagating waves analogous to baroclinic waves, it is worth recalling that the results of the lag-correlation statistics presented in B2 indicate that fluctuations with periods longer than 10 days exhibit little, if any, tendency for systematic eastward or westward propagation. In support of their interpretation, we found that the coherence square between the temporal expansion coefficients of the RPC associated with the pattern in Fig. 5a and the hemispheric 500 mb height field 3 days earlier [panel (a)] and 3 days later [panel (b)]. In agreement with results presented in B2 (their Fig. 8), we find that the evolution of these wave trains is characterized by a downstream dispersion of energy through a quasi-sta-

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1 In the present study, the time coefficients were calculated by projecting the reference time series (i.e., the daily time series of 2.5 day low-pass height anomalies) onto each of the loading vectors, day by day.

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Fig. 5. Loading vectors of the leading principal components of the intermediate frequency (10–60 day period) variability over the North American sector of the Northern Hemisphere. Contour interval 0.1; negative contours are dashed. Decimal points are omitted on labels for extrema. The percentage at the top left corner of each panel refers to the normalized hemispheric variance in this band explained by the respective loading vector.

The pair of patterns in Fig. 5 accounts for only $\sim 8\%$ of the total hemispheric variance in this frequency band. However, they explain $\sim 60\%$ of the intermediate time-scale variance over the sector of the Northern Hemisphere that includes the United States and the

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Figs. 5 and 7 correspond to the bands of high teleconnectivity in Fig. 3a stretching across North America and the Mediterranean. A third pair of patterns (not shown) is observed in association with B1's Eurasian waveguide, which also corresponds to a band of high teleconnectivity in Fig. 3a.

Over the oceans, the intermediate frequency loading vectors are characterized by zonally elongated centers of action, which often assume the form of north–south dipoles. These structures, illustrated by the ensemble of Pacific patterns shown in Fig. 8, are consistent with the sense of the anisotropy over most of western and central Pacific, as depicted in Fig. 4a. They tend to

adjacent ocean areas. This region was identified as a "waveguide" for the intermediate frequency fluctuations by B1, based upon their subjective examination of one-point correlation fields. Figure 7 shows an analogous pair of loading vectors that accounts for much of the intermediate frequency variance over the Mediterranean waveguide defined in Fig. 13 of B1. The paths traced by the centers-of-action of the patterns in

FIG. 6. Temporal correlation coefficients between the rotated principal component corresponding to the loading vector in Fig. 5a and the unfiltered height fluctuations over the entire analysis domain. (a) Principal component lagging heights by 3 days, (b) principal component leading heights by 3 days. Contour interval 0.1; the zero contour is bold and negative contours are dashed. Decimal points are omitted on labels for extrema.

FIG. 7. As in Fig. 5 but for the Mediterranean sector.
occur in pairs that are quasi-orthogonal in terms of their meridional structure. For example, the east–west oriented nodal line separating the two centers-of-action of the pattern in panel (a) coincides with the major axis of the dominant center of action in panel (b) and vice versa. A similar pair of orthogonal loading vectors is observed over eastern North America, extending into the western Atlantic (not shown).

The centers-of-action of the various patterns in Fig. 8 do not correspond well with the maxima in the teleconnectivity field (Fig. 3a). For example, the primary center-of-action of the pattern in Fig. 8b is located on the northern flank of a band of low teleconnectivity. This pattern is not reflected in the teleconnectivity field because of the weakness of its secondary centers-of-action. Had the analysis domain extended far enough south to include the center that probably exists to the south of the Hawaiian Islands, it is conceivable that the band of low teleconnectivity along 35°N would have disappeared. Because the patterns in Figs. 8a and 8b are of roughly similar strength (in terms of explained variance) and nearly in quadrature, we again interpret them as samples from a continuum of patterns (in this case north–south dipoles) over the western Pacific, as opposed to discrete, geographically fixed “standing oscillations.”

The pattern in Fig. 8c dominates the intermediate frequency variability in the eastern Pacific and is similar...
to the leading mode of the complex eigenvector analysis of 10-day low-pass filtered 500 mb heights in Kushnir (1987, Fig. 1b), where it was identified with northwestward moving disturbances associated with blocking over the Gulf of Alaska. Consistent with that analysis, it has a nearly orthogonal counterpart with centers to the northwest and southeast of its primary center over the Gulf of Alaska (not shown).

b. Intermonthly variability

Figure 9 shows a pair of intermonthly loading vectors that resemble the intermediate-frequency patterns in Fig. 5. A weaker pair of patterns (not shown) is observed over the Mediterranean waveguide. As in the intermediate frequency band, the patterns can be grouped in quasi-orthogonal pairs that explain comparable amounts of hemispherically integrated variance. The stronger of the two patterns tends to be somewhat more prominent and its centers-of-action correspond quite well to the local maxima in the teleconnectivity field over the waveguide regions (Fig. 3b). Most of the intermonthly patterns exhibit hints of north–south dipole configurations at their upstream and downstream ends that are not apparent in the intermediate frequency patterns.

The patterns in Fig. 9 are reminiscent of some of the loading vectors presented in BL, specifically those in their Fig. 3a, b, c, f, g, which they regarded as variants of the PNA pattern and in their Fig. 5, which they labeled the tropical Northern Hemisphere (TNH) pattern. Since the patterns in BL are based on monthly mean data for individual calendar months, they reflect both the intermonthly and interannual variability. Some of them resemble the intermonthly patterns discussed in this subsection, while others resemble the interannual patterns discussed in the following subsection.

Figure 10 shows the three dominant Pacific patterns for the intermonthly frequency band. Consistent with the anisotropy field for this frequency band (Fig. 4b), they are zonally elongated. They resemble the WP and PNA patterns in WG but the latter is shifted northwestward. The two patterns are nearly orthogonal, as evidenced by the fact that the primary centers-of-action of one fall on the nodal lines of the other and the pattern correlation between them is close to zero. Since they explain similar fractions of the normalized Northern Hemisphere variance, they could be interpreted as representative of a continuum of dipole-like perturbations over the central and western sector of the North Pacific basin, rather than as two discrete modes of variability.

Of the two Atlantic patterns shown in Fig. 11, the WA-like pattern (a) is the stronger one. Its signature is reflected in the anisotropy field for the intermonthly band (Fig. 4b): its nodal lines along 50° and 25°N correspond to maxima in the zonal component of the anisotropy, indicative of strong zonal wind perturbations. (There is some indication of an analogous feature in the anisotropy field over the western Pacific ocean along the nodal lines of the WP pattern.) The weaker pattern in Fig. 11b, which resembles the EA pattern, is nearly orthogonal to the WA pattern.

Consistent with the anisotropy fields, the zonal scale of the dipole-like features in Figs. 10 and 11 is longer.
than that of their intermediate band counterparts (e.g., compare Fig. 8a with 10b).

c. Interannual variability

The two most prominent patterns in the interannual frequency band, are shown in Fig. 12. (The next strongest pattern explains only 5.7% of the hemispherically integrated, normalized variance.) One is the counterpart of the PNA pattern in WG, though its strong positive center of action over the North Pacific is located ~5° farther to the north and 5° farther to the west. The other resembles the NAO at the 500 mb level as shown in Rogers (1984, Fig. 10).3 The fact that the NAO emerges as a dominant pattern in the interannual

3 When the RPC of the pattern in Fig. 12c is correlated with the sea-level pressure field, the resulting pattern resembles the NAO, as defined in van Loon and Rogers (1978). The correlation coefficients between the RPC and the Icelandic and Azores centers of the NAO in the sea-level pressure field are both on the order of 0.7 in absolute value.
stronger. The significance of the two patterns in Fig. 12 is underscored by the fact that their primary centers of action coincide with the strongest maxima in the variance and teleconnectivity fields (Figs. 2c and 3c), and the line segments representing the orientation of the major axis of the velocity covariance tensor (Fig. 4c) line up with their nodal lines.

5. Seasonal dependence and robustness

The November–April winter season used in our analysis is considerably longer than the winter seasons used as a basis for most previous studies. In order to determine the sensitivity of the results to the length of the averaging interval, the foregoing analysis was repeated for the early, middle, and late winter months separately. We found that the dominant loading vectors in the intermediate frequency band undergo only subtle changes from one subseason to another. In the intermonthly frequency band the changes are sometimes more noticeable. The midwinter patterns tend to be stronger and they resemble the interannual patterns (in Fig. 12) more closely than their counterparts based on the 6-month winter season.

The loading vectors for the interannual band are the most sensitive to the definition of the winter season, as illustrated in Figs. 13 and 14. The former shows the PNA pattern, which is the most reproducible pattern in the results for the three subseasons. The seasonal changes are much less dramatic than those inferred by BL based on their analysis of monthly mean data for individual calendar months. As noted in the previous subsections, their correlation matrix contains a substantial contribution from the month-to-month variability within individual winters. We suspect that during the midwinter months (January/February), when the interannual variability is largest and the PNA pattern in the interannual variability is much more spatially coherent than the shorter zonal wavelength “PNA-like” mode in the intermonthly variability (Fig. 9a), our loading vectors and BL’s are much the same. However, during the transition months (November/December and March/April) when the PNA pattern is less dominant, the analysis in BL picks up more of the shorter wavelength patterns in the intermonthly variability. Hence the dramatic lengthening of the PNA pattern in BL with the approach of midwinter and the shortening of the wavelength with the approach of spring may be overemphasized. In contrast to the PNA pattern, the NAO changes much more noticeably from one subseason to the other, as seen in Fig. 14. It is only in midwinter that a faithful replica of the pattern in Rogers (1984, Fig. 10) and the one in our own Fig. 12c is obtained.

Lest we overemphasize the sensitivity of the results of the length of the averaging period, it should be noted that none of the major findings reported in the text of

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**Fig. 12.** Loading vectors of the leading principal components of the interannual (>180-day period) variability over the North Hemisphere. Contour interval 0.1; negative contours are dashed. Decimal points are omitted on labels for extrema. The percentage at the top left corner of each panel refers to the normalized hemispheric variance in this band explained by the respective loading vector.
Fig. 13. Loading vectors of the rotated principal components of the interannual (>180-day period) variability that correspond to the Pacific—North American (PNA) pattern. Based on the data for: (a) early winter (November/December), (b) midwinter (January/February), and (c) late winter (March/April). Contour interval 0.1, negative contours are dashed. Decimal points are omitted on labels for extrema.

the previous section would need to be altered if we chose to present our results based on a two-month (January–February) winter season in place of those displayed in Figs. 5–12.

We tested the robustness of the results of the RPCA by repeating the analysis for the 6-month winter season for two 19-year subsets of the 38 year record. In the first test we analyzed the first (1946/47–1964/65) and second (1965/66–1984/85) halves of the record separately and in the second test we analyzed the odd and even years separately. All the patterns shown and discussed in section 4 are reproducible in the four subsets of the data, at least to some degree. Figure 15 shows the PNA pattern in the interannual variability as derived from the first and second halves of the record. The primary center-of-action over the North Pacific shifted ~15° eastward and ~5° southward from the first half of the record to the second half of the record and the downstream wave train over North America is stronger. The pattern for the second half of the record is in closer agreement with the results of WG, who analyzed the period 1962–63 through 1976–77.
The pattern correlation between the patterns shown in Fig. 15 is 0.57, and the corresponding statistic for the odd versus even years (whose patterns are not shown) is 0.78. The corresponding correlations for the NAO are 0.78 and 0.62, respectively. The sampling variability in these results is evidently quite large. These examples are typical of the results for all three frequency bands. It is apparent from Fig. 15 that the modest correlations between matching patterns generally reflect different phases rather than differences in scale or shape. They are due, in part, to sampling fluctuations, but they may also be indicative of a genuine lack of uniqueness of the phase, particularly when the patterns occur in orthogonal pairs with comparable amplitude.

The near orthogonality between patterns derived from the orthogonal rotation of the principal components motivated us to examine the impact of removing the constraint that the time coefficients of the patterns (i.e., the principal components) be temporally uncorrelated with one another. To this end we applied an oblique rotation [the Harris–Keizer method (e.g., Richman 1981, 1986) as coded in IMSL was used] to the same subsets of the eigenvectors that we had subjected to the VARIMAX rotation to produce the results.
those obtained in the orthogonal rotation. The two dominant patterns over the Pacific sector are shown in Fig. 16. They each explain slightly more than 9% of the normalized, Northern Hemisphere variance, as compared with ~6% for the strongest of the neighboring patterns (not shown). These two PNA-like pat-

FIG. 15. Loading vectors of the rotated principal components of the interannual (>180-day period) variability that correspond to the Pacific–North American (PNA) pattern. (a) Based on the 19 six-
month winter seasons for the first half of the record, 1946/47–1964/65. (b) Based on the 19 six-month winter seasons for the second half of the record 1965/67–1983/84. Contour interval 0.1, negative contours are dashed. Decimal points are omitted on labels for extrema.

described in section 4. In the intermediate and inter-
monthly frequency band (not shown) the resulting loading vectors are almost exact replicas of those shown in section 4. The spatial correlation coefficients between matching oblique and orthogonal patterns are higher than 0.9 in most cases and the degree of orthogonality between neighboring patterns is roughly comparable.

In the interannual frequency band, however, the oblique rotation yields results somewhat different from

FIG. 16. Loading vectors of the two leading rotated principal com-
ponents of the interannual (>180-day period) variability as obtained
from an oblique rotation of the first 15 eigenvectors of the correlation
matrix. Contour interval 0.1, negative contours are dashed. Decimal
points are omitted on labels for extrema.
terns are nearly in quadrature with one another (the spatial correlation coefficient $r = 0.32$). They may be regarded as variants of a single pattern which explains about 18% of the normalized variance of the hemispheric 500 mb height field and almost half of the variance over the Pacific–North American sector.

Over the Atlantic sector of the hemisphere the oblique rotation yields four loading vectors, the two strongest of which are shown in Fig. 17. Both patterns contain elements of the NAO (Fig. 12b) but neither resembles it closely. However, the oblique rotation for the midwinter months (not shown) yields a pattern much more like the one in Fig. 12b. It explains 9.6% of the normalized Northern Hemisphere variance, a substantially larger fraction than the neighboring patterns. We have also examined patterns derived from a VARIMAX rotation of the covariance matrix of the 500 mb height field, multiplied by the ratio: $\sin(45^\circ)/\sin(\text{latitude})$ at each grid point in order to account for the variation in the Coriolis parameter with latitude. All the patterns of variability discussed in section 4 are reproduced in this analysis. In the intermonthly and interannual frequency bands the oceanic patterns explain slightly more variance than their normalized counterparts and the continental patterns explain slightly less, in accordance with the longitudinal contrast in the variance field, as discussed in section 3. In the interannual frequency band the VARIMAX rotation of the covariance matrix yields two PNA-like patterns similar to those derived from the oblique rotation of the correlation matrix (Fig. 16).

The results are not sensitive to the exact choice of high and low-frequency cutoffs of the filters, as evidenced by the fact that selected calculations for the 10–40, 40–160, and >160 and >540 day frequency bands yielded results (not shown) very similar to those presented here.

6. Discussion

The results presented in the previous sections lend support to B1’s tentative categorization of low-frequency perturbations in the 500 mb height field in terms of

- meridionally elongated anomalies of alternating polarity, arranged in zonally oriented wavetrains with wavelengths on the order of 6000 km, prevalent over the continents, upstream of the climatological mean jet streams, and
- zonally elongated anomalies arranged to form north–south oriented dipoles, prevalent over the oceans near and downstream of the climatological mean jet streams.

This categorization has been objectively verified in our study. In addition, we have shown that these two contrasting modes of organization coexist within a wider range of frequencies than was suggested by the results of B1 and Schubert (1986).

In both the jet-entrance and jet-exit regions, the anisotropy is in the proper sense to effect a conversion of kinetic energy from the climatological mean flow into the low-frequency transients. Wallace and Lau (1985) showed that such a conversion is, in fact, observed, with prominent maxima not only in the jet-exit regions over the oceans, but also over the continental waveguides in the jet-entrance regions over the
United States and the Mediterranean (see their Fig. 4c). Even if the forcing of the low-frequency flow was isotropic, it is not difficult to imagine how even a relatively weak barotropic interaction with the climatological mean background flow would tend to create a certain amount of anisotropy in the observed sense. For example, in the jet-entrance regions, meridionally elongated perturbations would tend to gain kinetic energy from the mean flow while zonally elongated perturbations would lose energy to it. Only a modest rate of energy conversion would be required to account for the observed level of anisotropy in the height field.

The signatures of other processes may also be apparent in the anisotropy field. Orographic forcing impacts structure to the low-frequency transients, particularly in the lower troposphere, as evidenced by the correlation patterns presented in Hsu and Wallace (1985). Shutts (1983, 1986), Mullen (1987) and others have demonstrated that the baroclinic wave activity concentrated in the oceanic stormtracks is capable of generating modon-like “blocking patterns” downstream. Webster and Chang (1988) and Sardeshmukh and Hoskins (1988) have shown that tropical forcing may also be partially responsible for the dipole structures over the oceans.

In the intermediate frequency (10–60 day period) band the characteristics of the distribution of variance, the teleconnectivity and anisotropy fields, as well as the results of the orthogonal and oblique rotations of the principal components of the correlation matrix, indicate a lack of uniqueness with respect to the spatial phase of the patterns; i.e., the wave trains and dipole patterns do not exhibit geographically fixed nodes and antinodes. Rather than interpreting the loading vectors as discrete modes of variability, it is more appropriate to view them as a set of basis functions that describe a continuum of anisotropic perturbations in the height field.

In the intermonthly frequency (60–180 day period) band the loading vectors associated with the leading rotated principal components bear a somewhat closer resemblance to the teleconnection patterns identified by WG and other investigators. Here again, however, the tendency for these patterns to occur in orthogonal pairs, in which neither pattern clearly dominates, could be interpreted as indicative of a continuum of anisotropic patterns. A possible exception is the western Atlantic (WA) pattern, which is considerably stronger than the pattern with which it is paired and also leaves a distinct imprint upon the anisotropy field.

In the interannual frequency band, the Pacific–North American (PNA) pattern and the North Atlantic Oscillation (NAO) stand out clearly above the background continuum of anisotropic fluctuations in the 500 mb height field. They account for a substantial fraction of the variance over the Pacific and Atlantic sectors, and their signatures are evident in the variance, anisotropy, and teleconnectivity fields. Even these two patterns, which behave the most like “modal structures” of any of the loading vectors examined in this study, exhibit a range of phases, as evidenced by the emergence of two, nearly orthogonal, “PNA modes” in the oblique rotation.

The concept of a continuum of low-frequency structures is consistent with B1 and B2’s interpretation of the intermediate frequency (10–30 day) variability, but it appears, at first sight, to be at variance with their (and WG’s) use of the terminology “standing oscillations with geographically fixed nodes and antinodes” in reference to the dominant spatial structures in the month-to-month and interannual variability. Yet upon reflection, these two interpretations may be viewed as complementary.

Consider, for example a continuum of transient, wavelike fluctuations within a one-dimensional domain (which we will refer to, for convenience, as longitude x). We will assume that the disturbances have a preferred range of wavelengths (e.g., zonal wavenumbers 4–6) but that they are not periodic in time or longitude and that they have no preferred phase. If there exists a zonally uniform continuum of wavelike disturbances, the one point correlation function \( \phi(x, x') \) will be the same for all longitudes and it will resemble a damped cosine wave. For this geometry, we have verified that rotated principal component analysis of the longitudinal structure of the transient fluctuations yields one-dimensional analogues of the wave trains in Figs. 5 and 7. Each loading vector extends over only a segment of the latitude circle: the less peaked the zonal wavenumber spectrum of the disturbances, the shorter the zonal extent of the wave trains in the individual loading vectors. The loading vectors are arranged symmetrically around the latitude circle in orthogonal pairs. The longitudes at which they happen to be centered is determined entirely by the sampling variability.

Even a modest zonal asymmetry in the strength of the waveguide on a scale longer than the waves will tend to “anchor” the loading vectors at preferred longitudes. For example, Wallace et al. (1988) showed that baroclinic waves exhibit distinct and reproducible teleconnectivity maxima located one quarter of a wavelength to the east and west of the midpoint of the two major “baroclinic waveguides” or “stormtracks” [see their Fig. 6], even though the waves themselves have no preferred longitudinal phase. Fluctuations at these two locations (one half-wavelength apart, centered on the midpoint of the waveguide where the disturbances are most wavelike) exhibit stronger negative correlations with one another than fluctuations at any point along the waveguide exhibit with any other point. Loading vectors with centers of action coincident with these teleconnectivity maxima should tend to recur in rotated principal component analyses and they should explain a larger fraction of the domain integrated vari-
ance than other loading vectors whose centers of action lie farther away from the midpoint of the waveguide. Yet despite the existence of reproducible "modal structures" in a rotated principal component analysis, this situation is still best described in terms of a continuum of wave-like perturbations.

Similar arguments apply to the continental waveguides apparent in Figs. 5 and 7 of this paper, which are geometrically analogous to the baroclinic waveguides considered by Wallace et al. In the case of the zonally elongated dipole patterns over the oceanic sectors one should perhaps view the waveguides as extending meridionally rather than the zonally. The fact that these dipole patterns appear to be confined within the "westerly waveguide" i.e., the belt of strong upper level westerlies extending from the subtropics to about 60°N, is consistent with our understanding of Rossby wave dispersion. It is notable that in this case, the (meridional) extent of the waveguides happens to be roughly comparable to the wavelength of the dominant fluctuations embedded within it; i.e., the waveguides are just wide enough (in latitude) to contain one dipole pattern, with little space left over. A close match between the length of the waveguide and the wavelength of the disturbances embedded within it is conducive to the existence of modal structures characterized by strong, paired teleconnectivity maxima and prominent loading vectors in the rotated principal component analysis. In such situations, the distinction between a locally confined continuum of anisotropic perturbations and a blurry standing wave pattern superimposed upon a background continuum of anisotropic perturbations is largely semantic.

The three frequency bands considered in this study are all sufficiently close to zero frequency that time dependence should have little bearing upon the governing equation for barotropic wave dispersion, which presumably determines the dominant horizontal structures. Therefore it is not obvious, from the standpoint of the dynamics alone, why the PNA pattern and the NAO should emerge as the dominant modes of variability in the interannual frequency band, but not in the intra-seasonal variability.

It occurred to us that for fluctuations with periods longer than 10 days, the degree to which such modal structures appear to stand out above the more bland background continuum might not be an intrinsic property of frequency. Low-frequency variability isolated through the use of a low-pass filter (e.g., the interannual variability in the present study, the month-to-month variability in WG) can be viewed as being superimposed upon a unique basic state (usually the climatological mean flow), for which it should be possible, in principle, to define the waveguides unambiguously. In contrast, fluctuations isolated through the use of a band-pass filter (e.g., the 10–60 day and 60–180 day variability in the present study) are superimposed upon a time varying basic state for which the waveguides can be defined only in a probabilistic sense. For example, the 10–60 day variability during December of a particular year might be superimposed upon a different basic state than the 10–60 day variability during February of the same winter so that the positions of the waveguides and the modal structures embedded within them would be somewhat different in the two months.4 This lack of uniqueness of the basic state might be a factor in accounting for the relative absence of well-defined modal structures in our results for the 60–180 day band and particularly for the 10–60 day band. Modal structures suggestive of "teleconnection patterns" might perhaps be more evident in the intra-seasonal variability if one could artificially constrain the seasonal mean background flow to be the same every winter, and they might be apparent even in the 10–60 day variability if fluctuations with periods longer than 60 days could be suppressed.

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