NOTES AND CORRESPONDENCE

A Climatological Parameterization for Cumulus Clouds

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ABSTRACT

A research airplane was used to study the microphysical characteristics of ice-free, nonprecipitating summertime cumulus clouds in Montana. Each cloud was penetrated at a multiplicity of levels encompassing, in general, a large fraction of the cloud depth. Similar studies covering a more limited altitude range were made in New Mexico.

The clouds were substantially diluted by entrainment of environmental air, which produced great variability—at all levels and on all scales of measurement—in the liquid water content, L, and droplet number concentration, N.

The effective radius, r_{eff}, at any particular level was found to be essentially independent of L or N. Consideration of this result leads to the prediction that the parameter $A = r_{\mathrm{eff}}/r_{\mathrm{ad}} \approx 1$ everywhere within these clouds, where $r_{\mathrm{ad}} = (3/4\pi\rho w)^{1/3}(L_{\mathrm{ad}}/N_{\mathrm{ad}})^{1/3}$, N_{ad} and L_{ad} are the "adiabatic" values of N and L, and ρw is the density of water.

Analysis of the airborne data (35 cloud penetrations) for the Montana cumuli reveals that $A = 0.83 \pm 0.07$, while for the New Mexico study (25 penetrations) $A = 0.93 \pm 0.05$. Thus, the foregoing prediction is confirmed to a reasonable degree of accuracy. Model calculations for both Montana and Hawaii cumulus consistently yield values of A close to 1.0.

It is considered that the parameter A should be useful in climate modeling.

Recent studies (e.g., Hill and Choularton 1985; Blyth and Latham 1985, 1991), in which high-frequency measurements have been made of the microphysical characteristics of ice-free, nonprecipitating cumulus clouds, have revealed, in general, large variability (at all levels within the clouds, and at all distances from their vertical boundaries) in liquid water content L and droplet concentration N. A characteristic example of fluctuations in L is illustrated in Fig. 1, where it is seen that at a given level within a cumulus cloud, during a single horizontal traverse by a research airplane, L can vary from close to zero to almost the adiabatic value, L_{ad}. Such variability, which is found on all spatial scales down to the smallest detectable (usually about 10 m), is attributable to turbulent mixing between the cloud and entrained undersaturated environmental air, which causes evaporation of cloud droplets. In addition to revealing the existence of such structure at all measurement levels, the studies show that the altitudinal dependence of the penetration-averaged values of L can be highly variable from cloud to cloud, in a manner greatly different from that occurring in an adiabatic cloud.

Since this variability is so high, and the process and influence of turbulent entrainment is poorly understood, difficulties arise in attempts to predict and parameterize the cloud droplet effective radius, r_{eff}, a parameter of crucial importance in assessments of the role of clouds in climate (see Stephens 1978a, b; Slingo 1990). The purpose of this note is to present some preliminary results, emanating from field experiments and cloud modeling studies, which appear to alleviate this problem.

Figures 2 and 3 present measured high-frequency (10 Hz, ~10 m) values of r_{eff} and L in each of two horizontal penetrations of cumulus clouds examined in the CCOPE experiment, conducted in Montana in the summer of 1981. It is seen in each case that—to a good approximation—r_{eff} is independent of L, which varies very widely (from close to zero to about 1.3 g m^{-3}) in each case.

If, at a given level, r_{eff} is unaffected by entrainment, and is, therefore, independent of the value of L, it seems possible that, in attempting to parameterize r_{eff}, complexities associated with the microphysical variability produced by entrainment may be circumvented by addressing the adiabatic value of L, (L_{ad}), which is readily deduced, at any level in a cloud, from the local meteorological sounding. In other words, if N is constant or, more precisely, if we can define an adiabatic droplet

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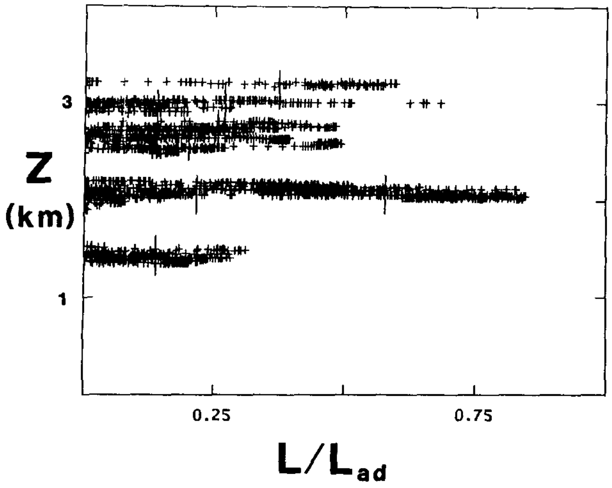

FIG. 1. The observed variations of 10 Hz (~10 m) values of normalized liquid water content L/L_A during penetrations of a cumulus cloud at six altitudes Z(km) on 23 June 1981. The vertical bars are averages.

concentration N_{ad} that is uninfluenced by entrainment and might generally be found, at a given level, in the regions of maximum liquid water content, then the effective radius should be roughly equal to the "adiabatic radius," i.e.,

$$r_{eff} = r_{ad} = \left(\frac{3}{4}\pi\rho_w\right)^{1/3}(L_{ad}/N_{ad})^{1/3} \qquad (1)$$

for *all* precipitation-free water clouds and at all levels in these clouds; ρ_w is the density of water.

Table 1 presents values of the parameter $A = r_{eff}/r_{ad}$ measured in 35 penetrations through six cumulus clouds in CCOPE. The range of altitudes covered in these flights is seen, in general, to encompass a large fraction of the depth of these clouds. The values of L_A (g m^{-3}) were determined from the meteorological soundings, r_{eff} (μm) was calculated from the measured droplet size distributions, and N_{ad} (cm^{-3}) was taken as the maximum droplet concentration measured in each cloud. We see that the measured values of L vary from about 0.05 to 1.6 g m^{-3}, L_{ad} varies from about 0.2 to 3.2 g m^{-3}, L/L_{ad} from about 0.02 to 0.6, r_{eff} from about 3 to 11 μm, and the maximum droplet number concentration in the cloud, N_{ad}, from about 230 to 840 cm^{-3}. The parameter A is seen to vary from about 0.70 to 0.97 with a mean value of 0.83 and a standard deviation of 0.07. Given the technological difficulties involved in making the relevant microphysical measurements, together with the simplistic nature of the foregoing arguments and strong likelihood that the measured maximum droplet concentration will be less than the true N_{ad} for these clouds, the prediction of constancy of A appears reasonably well vindicated; and we might expect our estimated values, A, to be less than the true ones, as observed.

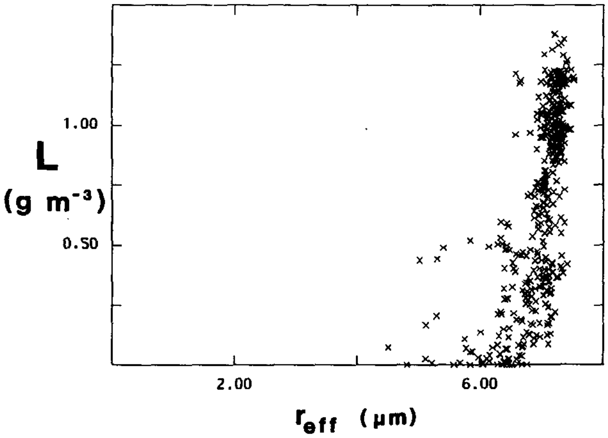

FIG. 2. The observed variations of 10z (~10 m) values of effective radius r_{eff} (μm) and liquid water content L (g m^{-3}) for a penetration of a cumulus cloud on 23 June 1981.

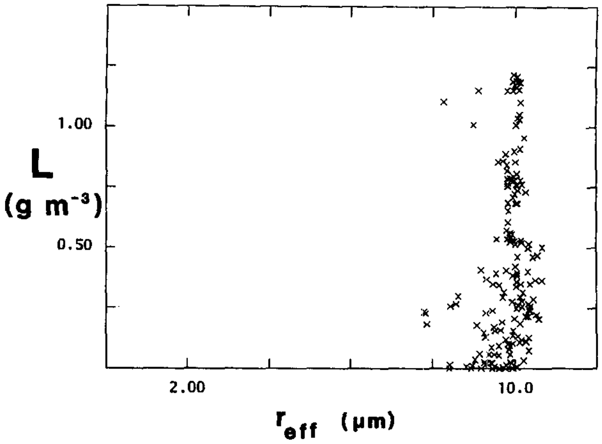

FIG. 3. The observed variations of 10 Hz (~10 m) values of effective radius r_{eff} (μm) and liquid water content L (g m^{-3}) for a penetration of a cumulus cloud on 27 July 1981.

Table 2 presents similar information obtained in one of a number of flights through subadiabatic ice-free and precipitation-free cumuli in New Mexico in the summer of 1987. In these studies the general procedure was to make multiple penetrations through each of several clouds, generally close to their summits. Information emanating from 25 penetrations is presented. The results displayed are characteristic of the total set in revealing substantial variations in L, L_{ad}, L/L_{ad}, N, and r_{eff}, together with approximate constancy in A. For these 25 penetrations $A = 0.93 \pm 0.05$, in excellent agreement with the results presented earlier for the Montana clouds.

Calculations have been made of the microphysical development of entraining summertime cumulus clouds both in Montana and in Hawaii, using observed meteorological soundings for those locations, a simple one-dimensional model of cloud evolution, and the

TABLE 1. Average values of pressure, (P, mb); altitude above cloud base (Z, m); temperature (T, °C); liquid water content (L, g m^{-3}); adiabatic liquid water content (L_{ad}, g m^{-3}); normalized liquid water content (L/L_{ad}); droplet number concentration (N, cm^{-3}); effective radius (r_{eff}, μm); and the climatological parameter A, for 34 penetrations of six cumulus clouds studied in CCOPE, Montana, 1981. Clouds A and B were studied on 23 June, cloud C on 11 July, and clouds D, E, F on 27 July. The cloud-base temperature, cloud-base pressure, and estimated cloud-top pressure for those clouds were A: 5.4°C, 746 mb, 500 mb; B: 5.4°C, 746 mb, 450 mb; C: −3.1°C, 564 mb, 450 mb; D: 8.3°C, 783 mb, 580 mb; E: 8.3°C, 783 mb, 550 mb; F: 8.3°C, 783 mb, 600 mb.

Cloud	P	Z	G	L	L_{ad}	L/L_{ad}	N	r_{eff}	$100A$
A	516	2840	−13.3	0.63	3.09	0.18	398	8.3	94
	538	2520	−11.6	0.97	2.91	0.30	449	8.0	93
	553	2700	−10.5	0.61	2.78	0.20	359	7.5	88
	563	2100	−9.8	0.82	2.68	0.27	418	7.6	90
	580	1965	−8.4	0.62	2.49	0.22	424	6.8	83
	585	1890	−7.7	0.64	2.44	0.24	346	6.8	83
B	495	3150	−15.9	0.43	3.24	0.12	841	6.3	76
	509	2940	−14.6	0.26	3.15	0.07	481	6.1	74
	504	3020	−14.9	0.24	3.18	0.07	757	6.6	80
	524	2780	−12.6	0.43	3.03	0.13	565	6.6	81
	531	2620	−12.4	0.43	2.97	0.13	561	6.6	82
	572	2070	−7.5	1.58	2.58	0.55	863	6.7	87
	572	2060	−8.4	0.61	2.58	0.21	566	6.1	80
	621	1440	−4.1	0.24	1.99	0.11	437	4.9	69
C	464	1440	−13.5	0.11	1.36	0.07	173	6.5	83
	490	1050	−11.2	0.09	1.06	0.07	105	5.9	81
	510	760	−8.7	0.11	0.81	0.12	113	5.3	79
	524	560	−7.2	0.09	0.62	0.13	165	5.0	81
	551	190	−4.6	0.08	0.21	0.33	283	3.2	76
D	592	2190	−5.8	0.25	3.00	0.07	199	10.6	97
	631	1690	−3.0	0.27	2.51	0.10	159	9.4	91
	660	1350	−1.3	0.26	2.11	0.11	154	8.7	90
	689	1020	1.0	0.32	1.67	0.17	227	8.2	91
	739	460	4.6	0.13	0.83	0.14	147	6.3	88
	769	150	6.9	0.05	0.27	0.18	128	4.0	81
E	594	2160	−5.0	0.40	2.90	0.13	301	9.2	88
	562	2580	−7.8	0.18	3.32	0.05	179	8.9	82
	604	2040	−4.4	0.34	2.86	0.11	294	9.1	88
	637	1620	−2.9	0.13	3.45	0.05	155	0.2	84
	676	1160	0.0	0.04	1.87	0.02	70	6.2	70
F	620	1830	−3.0	0.60	2.65	0.20	247	10.4	92
	652	1450	−0.9	0.40	2.23	0.16	219	9.4	88
	685	1006	0.7	0.29	1.73	0.15	196	7.8	80
	726	590	4.0	0.12	1.05	0.10	176	6.7	81
	750	340	5.8	0.22	0.63	0.32	264	6.3	91

extreme inhomogeneous model of the mixing/evaporation process (see Baker and Latham 1979; Baker et al. 1980). Values of L, L_A, r_{eff}, A, droplet size distribution $n(r)$, and other parameters were computed at a variety of levels in each model cloud for a wide range of values of entrainment rate π and droplet concentration N. Results from these calculations, for Hawaiian and CCOPE clouds, respectively, are presented in Tables 3 and 4. It is seen that in all situations $A \approx 1$, the idealized theoretical estimate.

The observed virtual independence of r_{eff} and L at all levels of measurement, together with the finding (both from the measurements and the computations) that $A \approx 1$, are consistent with the inhomogeneous mixing theory, in the extreme form of which clouds in the vicinity of entrained undersaturated environmental air are either totally removed (by evaporation or dilution) or unaffected. However, Jensen (private communication) has pointed out that for a wide range of subadiabatic conditions L/L_{ad}, classical (homogeneous) mixing theory predicts that r_{eff} is essentially independent of L/L_{ad}; so we cannot use the existing study to discriminate unequivocally between these two mixing processes.

It appears, therefore, that a reasonably accurate expression for the effective droplet radius in nonprecipitating, ice-free cumulus clouds is

$$r_{eff} \approx 3/4\pi\rho_w(L_{ad}/N_{ad})^{1/3} \qquad (2)$$

regardless of how much the clouds are diluted and rendered inhomogeneous by entrainment. This expression should prove useful in climate modeling if the question of how to establish an accurate value of N_{ad} can be

TABLE 2. Average values of pressure, (P, mb); altitude above cloud base (Z, m); temperature (T, °C); liquid water content (L, g m^{-3}); adiabatic liquid water content (L_{ad}, g m^{-3}); normalized liquid water content (L/L_{ad}); droplet number concentration (N, cm^{-3}); effective radius (r_{eff}, μm); and the climatological parameter A for 25 penetrations through New Mexico cumuli on 9 August 1987. Cloud-base temperature and pressure were 5.5°C, 630 mb.

P	Z	T	L	L_{ad}	L/L_{ad}	N	r_{eff}	$100A$
617	410	3.6	0.45	1.64	0.27	227	8.0	96
608	450	2.8	0.44	1.79	0.40	209	8.1	94
581	590	0.3	0.47	2.24	0.39	176	8.8	94
536	850	−3.2	0.28	2.92	0.38	100	9.2	91
525	910	−4.3	0.32	3.07	0.14	116	9.2	89
517	960	−4.8	0.38	3.17	0.27	137	8.9	84
529	890	−3.7	0.35	3.02	0.21	110	9.1	89
522	930	−3.8	0.21	3.10	0.45	78	9.3	90
555	740	−1.2	0.50	2.65	1.29	183	9.1	92
561	710	−0.9	0.20	2.56	0.57	161	7.6	78
575	630	0.3	0.64	2.35	0.70	244	8.9	96
548	780	−1.7	0.79	2.75	0.61	253	9.8	98
557	739	−0.7	0.98	2.62	0.90	284	9.9	101
551	760	−1.5	0.65	2.72	0.58	176	9.7	98
554	750	−1.1	0.43	2.67	1.27	143	8.8	89
556	736	−1.1	0.45	2.63	0.74	189	8.9	90
552	758	−1.0	1.03	2.69	0.73	289	9.9	100
551	763	−1.4	0.88	2.70	0.48	263	9.6	97
489	1132	−6.8	0.88	3.50	0.30	215	10.1	93
499	1073	−5.1	1.52	3.39	1.01	332	11.0	102
520	945	−3.6	1.04	3.13	0.35	258	10.5	100
524	919	−3.1	1.32	3.08	0.44	331	10.5	101
519	950	−3.9	0.49	3.15	0.51	219	8.8	85
530	880	−3.0	0.61	3.00	0.29	202	9.5	92
493	110	−6.1	0.74	3.46	0.23	168	10.5	97

TABLE 3. Calculated values of effective radius (r_{eff}, μm); adiabatic liquid water content (L_{ad}, g m^{3}); normalized liquid water content (L/L_{ad}); and climatological parameter A at various pressure levels (P, mb) as a function of entrainment rate in Hawaiian summertime cumulus.

	Entrainment rate, π (m^{-1})								
	∞	100	200	300	200	200	200	200	200
P	850	850	850	850	930	910	870	830	810
L_A	2.02	2.0	2.0	2.0	0.40	0.80	1.61	2.44	2.86
r_{eff}	13.4	13.9	13.9	13.8	7.8	10.0	12.8	14.9	15.7
L/L_{ad}	0	0.70	0.67	0.76	0.76	0.74	0.69	0.65	0.61
$100A$	103	107	107	105	102	103	105	107	108

resolved: characterization of the air mass and associated aerosol could be useful in this regard.

Future work in this area will include the subjection of the ideas presented herein to other datasets (including different cloud types) and extending preliminary work that indicates the development of the glaciation

TABLE 4. Calculated values of effective radius (r_{eff}, μm); adiabatic liquid water content (L_{ad}, g m^{-3}); and climatological parameter A at various pressure levels (P, mb) as a function of entrainment rate and droplet number concentration (N, cm^{-3}) in Montana summertime cumulus (CCOPE).

	Entrainment rate, π (m^{-1})									
	200	300	200	300	∞	∞	200	200	200	200
P	655	655	655	655	655	655	672	637	602	585
N	200	200	800	800	200	800	200	200	200	200
r_{eff}	9.82	10.0	6.19	6.32	9.97	6.28	7.76	11.3	14.2	15.3
L_A	0.83	0.83	0.83	0.83	0.83	0.83	0.41	1.25	2.09	1.51
$100A$	100	102	100	102	102	101	100	101	107	108

of cumuli (i.e., when a mixed-phase microphysical system exists) is accompanied by a systematic increase in the value of the climatological parameter A.

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