Terrestrial Superrotation: A Bifurcation of the General Circulation

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ABSTRACT

When sufficiently large zonally asymmetric tropical heating is introduced in a two-level model of global atmospheric flow, its general circulation becomes strongly superrotating. The nature of the superrotating solutions is studied by examining momentum and heat budgets for a range of values of thermal forcing. Changes in the transport of zonal momentum by transient eddies appear to play the key role in the transition to superrotation. The dramatic bifurcation of the solutions of this model may help explain the maintenance and variability of the zonal mean flow in the tropics.

1. Introduction

One of the fundamental problems in meteorology has been the development of a theory of the general circulation. Any successful theory must explain the maintenance of the time-mean circulation; the observed transports of momentum, heat, and moisture by transients; and the interplay between the two. Further, such a theory should predict the sensitivity of the circulation to changes in external parameters and address the questions of stability and uniqueness of the solutions.

Although we observe a particular general circulation, it is possible that for different values of the external parameters we might obtain a very different circulation. Lorenz (1970) referred to those systems that exhibit multiple statistically steady solutions for the same set of external parameter values as intransitive. Examples of intransitivity have been documented in replicas of atmospheric flows such as the rotating annulus (see Fultz 1959) and mathematical models (see Charney and DeVore 1979). Places in parameter space where multiple solutions arise are referred to as bifurcations.

In this paper we present a bifurcation that occurs in a two-level model when the tropical heating rate is varied. Although the model is quite idealized, we use terrestrial values of planetary radius, rotation rate, and radiative equilibrium pole-to-equator temperature difference. This bifurcation was found while using this model to study the behavior of stationary waves forced by an “eddy” heating term in the tropics. For sufficiently small values of heating rate, the model produced a flow with a three-cell mean meridional circulation, surface easterlies in the tropics and westerlies in middle- and upper-level, subtropical westerly jets. Because this circulation is similar to the observed atmospheric general circulation, we will refer to this branch of solutions as the conventional general circulation. When the amplitude of the tropical eddy heating was increased, however, this conventional circulation gave way to a new circulation with weak surface westerlies at low-latitudes, easterlies in middle and high latitudes, and a strong upper-level westerly jet over the equator. Because most of the flow is westerly, we shall refer to this branch of solutions as the superrotating general circulation.

This transition from a conventional to a strongly superrotating general circulation was obtained years ago in the simple, two-level climate model of Held and Suarez (1978). They observed that in most situations the model would generate a “realistic” climate, but occasionally solutions would drift slowly, and after several hundred days of integration reach a superrotating state. In these cases, the eddy kinetic energy would decrease, tropical temperatures would rise toward radiative equilibrium, surface winds would weaken, and the subtropical jets would move toward the equator. Held and Suarez suggested that this drift could be related to the severe spatial truncation used in their model.

To investigate our superrotating solution, we will compare its momentum and heat balances with those of the conventional circulation. The application of momentum budgets to explain superrotation is not unique to our study. Leovy (1973) and Gierasch (1975) have investigated the role that vertical, as well as horizontal, transport of angular momentum plays in the maintenance of superrotation on Venus.

In section 2 we present an overview of the model. In section 3 we present the main results and compare...
the conventional and superrotating circulations, with the emphasis on the difference in their zonal momentum budgets. In section 4 we further explore the superrotating solution with additional calculations using weak radiative forcing and adding vertical mixing of momentum. In section 5 we present our conclusions.

2. The model

The model is based on the one used in Held and Suarez (1978). It is a two-level, dry, primitive equation model on a sphere with a homogeneous surface and with \( \omega = 0 \) along the top and bottom surfaces. Winds, geopotential, and potential temperature are retained at both levels, but the vertically averaged flow is assumed to be nondivergent (Fig. 1). The atmosphere is driven by relaxing the mean atmospheric temperature toward a zonally symmetric state with a large equator-to-pole temperature difference and by adding a tropical eddy heating, which we vary. For any variable \( A \) we define the operators

\[
\tilde{A} = \frac{1}{2} (A_1 + A_2), \quad \hat{A} = \frac{1}{2} (A_1 - A_2),
\]

with the subscripts 1 and 2 denoting values at the upper \( (p_1 = 250 \text{ mb}) \) and lower \( (p_2 = 750 \text{ mb}) \) levels. The hydrostatic, momentum, thermodynamic, and continuity equations may be written as

\[
\tilde{\Phi} = -C_p \hat{\Phi},
\]

\[
\frac{\partial \tilde{\Phi}}{\partial t} = \left[ f \tilde{v} + \hat{\Phi} \right] - \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \lambda} \left[ \hat{\Phi} + \hat{F}_u \right],
\]

\[
\frac{\partial \tilde{\theta}}{\partial t} = -\left[ f \tilde{\theta} + \hat{\Phi} \right] - \frac{1}{a \phi} \frac{\partial}{\partial \phi} \left[ \hat{\Phi} + \hat{F}_u \right],
\]

\[
\frac{\partial \tilde{u}}{\partial t} = \left[ f \tilde{u} + \hat{\Phi} \right] - \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \lambda} \left[ \hat{\Phi} + \hat{F}_u \right] + \omega \tilde{u} + \hat{F}_u,
\]

\[
\frac{\partial \tilde{v}}{\partial t} = \left[ f \tilde{v} + \hat{\Phi} \right] - \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \lambda} \left[ \hat{\Phi} + \hat{F}_u \right] + \omega \tilde{v} + \hat{F}_u,
\]

\[
\frac{\partial \hat{\Phi}}{\partial t} = -\nabla \cdot \left( \tilde{\Phi} \right) - \omega \hat{\Phi} + \hat{Q},
\]

\[
\frac{\partial \hat{\theta}}{\partial t} = -\nabla \cdot \left( \tilde{\theta} \right) - \omega \hat{\theta} + \hat{Q},
\]

where \( f = 2 \Omega \sin(\phi) \) is the Coriolis parameter, \( a \) is the radius of the earth, \( \phi \) and \( \lambda \) are the latitude and longitude, respectively,

\[
\tilde{\zeta} = \frac{1}{a \cos(\phi)} \left\{ \frac{\partial \tilde{v}}{\partial \lambda} - \frac{\partial \tilde{u}}{\partial \phi} \left( u \cos(\phi) \right) \right\}
\]

is the relative vorticity, and

\[
K = \frac{1}{2} (u^2 + v^2)
\]

is the kinetic energy. The dependent variables \( v, \Phi, \theta \), and \( \omega \) denote the horizontal velocity vector, geopotential, potential temperature, and vertical pressure velocity. Finally, \( P = (p/p_0)^* \), where \( p \) is the pressure, \( p_0 = 1000 \text{ mb} \), and \( \kappa = R/C_p \). The quantities \( F \) and \( Q \) denote frictional and diabatic effects. The gas constant is denoted by \( R \), and the heat capacity of air at constant pressure by \( C_p \).

Although (2)–(11) constitute a complete mathematical description of the model atmosphere, they are not convenient for computational purposes because there is no simple prognostic or diagnostic equation for evaluating \( \tilde{\Phi} \). The computation of \( \tilde{\Phi} \) may be avoided by integrating the vorticity equation, rather than (5) and (6), for the vertical mean flow:

\[
\frac{\partial \tilde{\zeta}}{\partial t} = \frac{-2 \Omega \cos(\phi)}{a} \tilde{v} + \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \lambda} \left\{ (-\tilde{\Phi} + \hat{\Phi} + \hat{F}_u) - \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \phi} \left\{ (\cos(\phi)(\tilde{\Phi} + \hat{\Phi} + \hat{F}_u) - \frac{1}{a \cos(\phi)} \frac{\partial}{\partial \phi} \left\{ (\cos(\phi)(\tilde{\Phi} + \hat{\Phi} + \hat{F}_u) \right\} \right\} \right\}.
\]

At each time step, the eddy contribution from \( \tilde{u} \) and \( \tilde{v} \) is obtained from the eddy contribution of \( \tilde{\Phi} \) by solving \( \nabla^2 \tilde{v} = \tilde{\zeta} \). Because the zonal part of \( \tilde{\Phi} \) is zero from (9b), we need only integrate the zonal part of (5):

\[
\frac{\partial [\tilde{u}]}{\partial t} = [\tilde{v}] + [\omega \tilde{u}] + [\hat{F}_u],
\]

where the square brackets denote a zonal mean. The total \( \tilde{u} \) and \( \tilde{v} \) are obtained by summing their eddy and zonal mean parts.

The horizontal derivatives in (3), (4), (7)–(10), (12), and (13) are replaced with simple, second-order finite differences. Because of the unusual nature of the results, we have given complete details of the finite differencing in the Appendix. The model is run at \( 4^\circ \) latitude by \( 5^\circ \) longitude resolution on a staggered C-grid (Mesinger and Arakawa 1976). As in Held and

FIG. 1. Schematic of the two-level model.
Suarez (1978), a semi-implicit time-differencing scheme is used to integrate the model. Every 20th time step a Matsuno (Euler backward) time step was employed to prevent the separation of solutions that occurs with a leapfrog scheme.

The friction and heating parameterizations are as follows:

\[ F_1 = \frac{F_{\text{Shapiro}}}{\tau_{\text{filter}}} \]
\[ F_2 = -\frac{1}{\tau_{\text{drag}}} v_2 + \frac{F_{\text{Shapiro}}}{\tau_{\text{filter}}} \tag{15} \]

Here \( F_{\text{Shapiro}} \) represents the effect due to an eighth-order Shapiro filter. This filter is only applied to the \( \hat{u} \) and \( \hat{\theta} \) components and to the eddy part of \( \theta \), but not to the zonal mean \( \bar{u} \) or the temperature. The first term on the right side of (15) is meant to represent surface friction. Note that it depends only on the lower-level wind. In all experiments we use \( \tau_{\text{drag}} = 5 \) days and \( \tau_{\text{filter}} = 2 \) h. This \( \tau_{\text{drag}} \) corresponds to a drag coefficient \( C_D \) \( \sim 10^{-3} \) for a wind speed of \( \sim 10 \) m s\(^{-1} \).

The model is forced by introducing a Newtonian term that relaxes the temperature to an equilibrium distribution:

\[ \dot{\theta} = -k_{\text{rad}}(\hat{\theta} - \bar{\theta}) + \dot{\theta}_{\text{CA}} \tag{16} \]

and

\[ \dot{\theta} = -k_{\text{rad}}(\hat{\theta} - \bar{\theta}) + \dot{\theta}_{\text{tropics}} + \dot{\theta}_{\text{CA}}, \tag{17} \]

where

\[ \bar{\theta}_{\text{tropics}}(\phi) = 302 \text{ K} + \frac{\Delta \theta}{2} \cos(2\phi), \tag{18} \]

\[ \frac{\dot{\theta}_{\text{tropics}}}{\bar{P}} = \frac{Q_0}{\bar{P}} \exp\left(-\frac{\phi}{\Delta \phi}\right) \sin(m \lambda), \tag{19} \]

\( \hat{\theta} = 15 \) K, \( \Delta \theta \) is the radiative equilibrium equator-to-pole temperature difference, and \( \bar{P} = (\bar{P}^2 - \bar{P}^3)/\bar{P} \) so that the \( \dot{\theta}_{\text{tropics}} \) may be interpreted as the vertically integrated heating. Note that \( \dot{\theta}_{\text{tropics}} \) is the only zonally asymmetric forcing, since \( \bar{\theta} \) depends on latitude only. In the following calculations we have taken \( m = 2 \) and \( \Delta \phi = 15^\circ \). These values were chosen to mimic the observed distribution of heating in the tropics. Finally, \( \dot{\theta}_{\text{CA}} \) represents dry convective adjustment, which prevents \( \hat{\theta} \) from becoming less than 3 K, while conserving the mean temperature \( \bar{\theta} \). The particular value of 3 K is not crucial; we wanted \( \hat{\theta} \) to be always slightly positive.

3. Results

The model was run for 100 days starting from an isothermal state with \( \hat{\theta} = 290 \) K, \( \hat{\theta} = 15 \) K, \( Q_0 = 0 \) in (19), and \( \Delta \theta = 80 \) K in (18). At day 100, vigorous midlatitude, baroclinic disturbances were present and the atmosphere had reached an “equilibrium” circulation. Using values at day 100 as initial conditions, we performed seven 800-day simulations, with \( Q_0 = 0-6 \) K d\(^{-1} \). In this section we describe the results from these experiments.

The time evolution of the zonal mean \( \bar{u} \) at the equator for the seven runs (Fig. 2) shows a marked change in the model’s behavior between \( Q_0 = 3 \) K d\(^{-1} \) and \( Q_0 = 4 \) K d\(^{-1} \). For \( Q_0 = 0-3 \) K d\(^{-1} \), the tropical zonal wind is easterly, as observed in the atmosphere; while for \( Q_0 = 4-6 \) K d\(^{-1} \), the solution changes to a very unusual one with strong westerlies at the equator. In additional experiments varying initial conditions (not shown in Fig. 2), we found that for \( Q_0 = 3 \) K d\(^{-1} \) the model produced both the conventional and the superrotating solutions. We varied the initial conditions by extending the unforced run (\( Q_0 = 0 \)) for various intervals beyond the initial 100 days and then setting \( Q_0 \) to 3 K d\(^{-1} \). Some of the runs, like the one shown in Fig. 2, remained in the conventional state for at least 800 days; but others flipped suddenly to the superrotating solutions after several hundred days of integration. We did not observe any spontaneous transition from the superrotating to the conventional circulation, nor did we observe this behavior at any other value of \( Q_0 \).

The effect of increasing tropical eddy heating on the model’s general circulation is made clear by examining the meridional distribution of various zonal mean quantities averaged over the last 400 days of the runs (Fig. 3). Because the statistically steady states with these forcings are symmetric about the equator, we show only the average of the two hemispheres, with \( v > 0 \) for poleward flow. The seven cases (only the conventional solution for \( Q_0 = 3 \) K d\(^{-1} \) is shown) are clearly separated into two sets, with very little difference between solutions on the same set. For small \( Q_0 \), the vertically averaged zonal flow, [\( \bar{u} \)] (Fig. 3a), has the conventional pattern of tropical and high-latitude easterlies and midlatitude westerlies, while for strong tropical eddy forcing, it has a broad tropical westerly jet of about 30 m s\(^{-1} \) and weak easterlies poleward of 45°. Figure 3b shows the vertical shear of [\( \bar{u} \)]. Differences in [\( \bar{u} \)] between the two branches are largest in low latitudes, where the superrotating branch has much larger vertical shears and meridional temperature gradients (Fig. 3c). In this region, [\( \bar{u} \)] and [\( \bar{u} \)] are nearly equal, indicating a weak (but westerly) lower-level flow ([\( \bar{u}_2 \]) = [\( \bar{u} \)] - [\( \bar{u} \)]) and a strong (60 m s\(^{-1} \)) jet in the upper layer ([\( \bar{u}_1 \]) = [\( \bar{u} \)] + [\( \bar{u} \)]. The superrotating branch has weaker vertical shears between 30° and 55° latitude, but poleward of 55° there is little difference between the two branches.

Figure 3c shows the mean potential temperature, [\( \bar{T} \)], together with the imposed “radiative equilibrium” distribution. The meridional temperature gradients in lower latitudes are much smaller in the conventional than in the superrotating branch. Solutions on the superrotating branch are within a few degrees of radiative
equilibrium throughout the tropics, while the conventional circulation has nearly constant tropical temperatures—presumably because the Hadley cells are different in the two solutions. Remarkably, poleward of \( \sim 55^\circ \) all solutions have about the same meridional temperature gradients, in agreement with the vertical shears shown in Fig. 3b. High-latitude temperatures themselves are also unchanged. This suggests that in spite of the dramatic differences between the two branches in low latitudes, heat transports by midlatitude baroclinic waves, which maintain the departures from radiative equilibrium at high latitudes, are unchanged.

The two solutions also produce significant differences in static stability, \([\bar{\theta}]\) (not shown). Near the equator both have \([\bar{\theta}] \sim 16\) to 18 K ("radiative" equilibrium is 15 K). For the superrotating solution, \(\bar{\theta}\) increase to 20 K at 20° latitude and are constant beyond there. For the conventional solution, they increase to 24 K at 40° latitude and decrease slightly from there to the pole.

Finally, Fig. 3d shows the mean meridional flow \([\bar{\theta}]\). For the conventional circulation, we find a strong Hadley cell (\(\bar{\theta} > 0\)) in the tropics, a Ferrel cell (\(\bar{\theta} < 0\)) between 30° and 50°, and another, weaker direct cell in high latitudes. However, when \(Q_{\text{trop}}\) is increased to 4 K d\(^{-1}\), we have much weaker meridional circulation in the lower latitudes. Over the hemisphere, the meridional circulation contains a weak direct cell near the equator, a weak indirect circulation between \(\sim 10^\circ\) and 30°, and a stronger direct cell poleward of \(\sim 30^\circ\).

In summary, we have shown that for sufficiently small tropical heating the two-level model produces a general circulation similar to that observed in the atmosphere. In this circulation, transient disturbances in the midlatitude reduce the north–south temperature gradient, increase the static stability, and drive a midlatitude Ferrel cell. However, when sufficiently strong zonally asymmetric tropical heating is introduced, a new circulation occurs with superrotating tropical westerlies and weak polar easterlies.

Recently, Saravanan (1990) found that a spectral two-level model reproduced our results. We conclude that these results are not sensitive to the nature of the horizontal discretization, although they may be a peculiarity of two-level models. Furthermore, Saravanan found that when the model was initialized with a superrotating state, it was able to maintain the superrotation as \(Q_0\) was reduced to zero. Because there is no standing wave in this case, the superrotating solution is able to maintain itself with only the transients. This suggests that it is changes in the transients—not in the standing eddies—that play the critical role in maintaining the superrotating solution.

We verified Saravanan's results by making six runs with \(Q_0 = 0 - 5\) K d\(^{-1}\) and initializing the model from day 800 of the \(Q_0 = 6\) K d\(^{-1}\). In all cases the solution remained on the superrotation branch, with solutions...
for $Q_0 = 3$–$5$ K d$^{-1}$ matching closely the solutions obtained earlier and those for $Q_0 = 0$–$2$ K d$^{-1}$ being nearly insensitive to $Q_0$ and very similar to the $Q_0 = 4$ K d$^{-1}$ solutions shown in Fig. 3. We conclude that for the range, $Q_0 = 0$–$2$ K d$^{-1}$, the model is intransitive and, as far as we could tell, incapable of producing spontaneous transitions between the two solutions once equilibrated. For $Q_0$ greater than $\sim 4$ K d$^{-1}$, the superrotation solution appears to be unique. For $Q_0 \approx 3$ K d$^{-1}$, the model can remain in the conventional state for long periods when initialized from a lower branch solution, but eventually flips to the superrotating branch. The superrotation branch appears to be a stable solution for all values of $Q_0$.

If we denote deviations from a zonal average by $A^*$ = $A - \langle A \rangle$ and deviations from a 400-day average by $A' = A - \langle A \rangle$, the total momentum flux is given by

$$\langle uv \rangle \cos(\phi) = \langle \langle u \rangle \langle v \rangle \rangle \cos(\phi) + [\langle u \rangle^* \langle v \rangle^* \rangle \cos(\phi) + [\langle u'v' \rangle] \cos(\phi),$$

where the first term on the right side of (20) is the contribution of the mean meridional circulation (MMC); the second term, the transport due to the standing eddies; and the third term, the transport due to the transient eddies. Figure 4 shows each of the contributions from (20) computed from once daily histories of the runs. For small $Q_0$, there is a poleward flux transporting momentum from the tropical surface easterlies to the extratropical surface westerlies. However, when $Q_0 > 4$ K d$^{-1}$ and superrotation is established, we have a complete reversal, with the eddies transporting momentum from the polar easterlies to the tropical westerlies. As the figure shows, most of the
transport is done by the transients, which are presumably baroclinic waves. Furthermore, for large $Q_0$, transients near the equator become significant. From the MMC transports, we see that at large $Q_0$ the conventional meridional circulation of Hadley/Ferrel cell breaks down into a hemispheric-wide Hadley cell with only slight vestiges of the Ferrel cell left. Finally, in the superrotating cases the standing eddy contribution has increased, adding to the total equatorward transport of westerly momentum.

To gain further insight into the momentum balance, we computed the convergence of the momentum fluxes. The time-averaged angular momentum balance is

$$\frac{1}{a \cos(\phi)} \frac{\partial}{\partial \phi} \left[ \bar{u} \bar{v} \cos^2(\phi) \right] = \left[ \bar{F}_u \right] \cos(\phi),$$

which expresses the balance between the convergence of momentum fluxes and the frictional torque. This can be obtained from (5) by using the relation:

$$\frac{1}{a \cos(\phi)} \frac{\partial}{\partial \phi} \left[ \bar{u} \bar{v} \cos^2(\phi) \right] = \left[ \langle \bar{v} \bar{f} \rangle + \langle \bar{u} \bar{f} \rangle \right] + \langle \omega \bar{u} \rangle \cos(\phi).$$

The right side of (22) was used in the convergence calculations. In addition to the total flux convergence, we will also partition the convergence into its mean meridional, standing eddy, and transient components. The MMC contribution is given by

$$\left[ \langle \bar{v} \bar{f} \rangle + \langle \bar{u} \bar{f} \rangle + \langle \omega \bar{u} \rangle \right] \cos(\phi),$$

while the standing eddy component is

$$[\left[ \langle \bar{v} \rangle \langle \bar{f} \rangle \right] + [\langle \bar{u} \rangle \langle \bar{f} \rangle \rangle] + \langle \omega \rangle \langle \bar{u} \rangle \rangle] \times \cos(\phi).$$

FIG. 4. The flux of relative zonal momentum as a function of various $Q_0$ when the radiative equilibrium equator-to-pole temperature difference is 80 K. Panels (b)-(d) give the portion of the fluxes due to the mean meridional circulation, standing eddies, and the transients.
The transients' contribution was computed by subtracting the MMC and standing eddies from the total.

Figure 5a shows the various flux convergences for $Q_0 = 0$. The transients are transporting momentum poleward from the tropical easterlies to the extratropical westerlies. The meridional circulation shows the conventional direct circulation in the tropical and polar regions and an indirect (Ferrel) cell in the midlatitudes. The standing contribution should be zero for this case, the small values shown are a measure of the sampling error in the 400-day average. The various convergences for the case of strong tropical heating ($Q_0 = 6 \text{ K d}^{-1}$) are shown in Fig. 5b. We first note that peak accelerations due to both transients and MMC are much smaller than for $Q_0 = 0$ and that the standing eddies now play a significant role. The strong westerly acceleration produced by transients in midlatitudes in the conventional circulation is much diminished and shifted to the subtropics, while the region of high-latitude easterly acceleration is greatly expanded. The equatorward shift of the region of westerly acceleration suggests a similar shift in the region of generation of transient disturbances. In the subtropics, the normal easterly acceleration practically disappears. In fact, near the equator, transients are producing westerly accelerations, implying generation of transient eddies in this region, perhaps as a result of the breakdown of the large forced stationary wave. This equatorial acceleration by the transients is balanced by the MMC. The MMC contribution must be negative in that region because the vertical shear of the zonal flow is positive (superrotation) and the vertical velocity is upward in the rising branch of the Hadley cell.

Comparing Figs. 6a and 6b, it appears that the dramatic changes in the transients, more than the effects...
of the forced stationary wave, are responsible for the differences in the zonal mean flows in the two cases. The changes in the transient contributions to the momentum budget are also consistent with the changes in the zonal wind between the conventional and the superrotation case: a decrease in acceleration of tropical easterlies, a decrease in acceleration of midlatitude westerlies, and an increase in both the magnitude and extent of the acceleration of high-latitude easterlies. In contrast, the contribution of the standing eddies is much smaller than the change in the transients. Although the standing eddies tend to produce westerlies at very low latitudes, averaged from the equator to 30°—the region of strong superrotation (Fig. 3a)—their net contribution is small.

The importance of the transients is consistent with Saravanan’s “hysteresis” result that a superrotating solution, once in place, can maintain itself without any stationary eddy forcing. This does not mean, however, that the stationary waves are playing a trivial role. In the conventional circulation, near the bifurcation point ($Q_0 \sim 3$ K d$^{-1}$), their effect on the zonal flow must be such that it modifies the propagating properties of the transients and thus their momentum transports. The simplest view of the effect of the tropical stationary waves is that they will produce some westerly acceleration where they are forced—near the equator—and easterly accelerations in those regions into which they propagate, which will not be too far from the equator as long as the wave is trapped in the tropics. Such accelerations can be enough to affect the tenuous momentum budget near the equator, and perhaps, near the bifurcation point, change the sign of the upper branch zonal flow, which is easterly only within $\sim 15^\circ$ of the equator. With westerly upper-level flow in the tropics, the absorption of transient disturbances incident from the midlatitudes will be diminished and so will their deposition of easterly momentum in low latitudes. The changes in the transients between our conventional and superrotating states were analyzed by Saravanan (1990), who discussed in detail how their propagation is affected by the very different zonal flows occurring in the two states. However, it is still not clear to us how the smaller changes in the zonal flow occurring in the conventional circulation near the bifurcation point act on the transients. Whatever the details, some positive feedback mechanism must exist between the zonal flow and the transient momentum transports. In the tropical heating experiments that we have been describing, the presence of the tropical stationary waves must strengthen this feedback to the point that, for strong enough forcing, the conventional circulation becomes unstable.

For completeness, we show the heat fluxes for the $Q_0 = 0$ and $6$ K d$^{-1}$. In both cases, the total flux is poleward at all latitudes. For the conventional circulation (Fig. 6a), the Hadley cell is responsible for all of the transports in the tropics, while in midlatitudes the total flux is dominated by the transients, with some compensation from the equatorward flux by the Ferrel cell. Figure 6b shows the heat fluxes for strong tropical heating ($Q_0 = 6$ K d$^{-1}$). Poleward of 60° the fluxes are essentially the same as in the $Q_0 = 0$ case. The transient heat flux has increased in the tropics and decreased in midlatitudes. The contribution from the mean meridional circulation is now entirely poleward, reflecting the hemispheric Hadley cell. Finally, we note that the transport of heat by the standing waves is equatorward and confined to low latitudes.

In summary, we showed that for small $Q_0$ the model captures the qualitative nature of the momentum and heat budgets of the observed general circulation [see Lorenz (1967) for a discussion of the observed fluxes of heat and momentum]. However, for sufficiently strong tropical heating, it produces very different momentum and heat budgets. Transients still arise due to baroclinic instability and perhaps an instability of the standing wave. Nevertheless, because we have surface easterlies in the extratropics and surface westerlies in the tropics, there must be a net equatorward transport of westerly momentum. In the next section we will examine the sensitivity of the superrotating solutions to variations in other parameters.

4. Further experiments

In the previous section, we showed that a conventional general circulation would give way to a super-
rotating circulation if we introduced a tropical eddy heating of 3 to 4 K d\(^{-1}\). This was done for a radiative equilibrium equator-to-pole temperature difference \(\Delta \theta = 80\) K. Since, as argued above, the behavior of the midlatitude transients seems to play a key role in the maintenance of the superrotating state, we experimented in altering the transients by varying the pole-to-equator temperature difference. Thus, we repeated the experiments using \(\Delta \theta = 20\) K, 40 K, and 60 K. The results of these calculations are summarized in Fig. 7, which shows the time-averaged zonal wind \([\bar{u}]\) at the equator for various values of \(Q_0\) and \(\Delta \theta\). In all cases we again found superrotation; however, as \(\Delta \theta\) is decreased, the transition to superrotation occurs at lower values of \(Q_0\). The hysteresis effect also occurred at all values of \(\Delta \theta\).

Figure 8 shows the meridional distribution of various zonal mean quantities averaged over the last 400 days for the \(\Delta \theta = 40\) K case. These results are qualitatively the same as those in the previous section. However, the peak speed of the superrotating jet, the mean zonal velocity \([\bar{u}]\), and the vertical shear \([\bar{v}]\) are larger than their counterparts shown in the previous section. Finally, we note that near the equator \([\bar{v}]\) exceeds \([\bar{u}]\), implying an equatorward heat transport at low latitudes.

Figure 9 shows the convergence of the momentum fluxes for the \(Q_0 = 0\) and 4 K d\(^{-1}\) cases, respectively. For \(Q_0 = 0\), our results are very similar to those with the larger temperature difference, only the magnitude has been decreased substantially. Similar results hold for \(Q_0 = 4\) K d\(^{-1}\) when compared to the results from the previous section. Although we reduced the equator-to-pole difference by a factor of 2, the convergences are reduced by a larger portion due to the nonlinearity in the model. Finally, we note that the transients in the superrotating case are stronger near the equator. The tendency for the transition point to move to lower

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**Fig. 8.** The 400-day averages of zonal mean quantities (a) \([\bar{u}]\), (b) \([\bar{v}]\), (c) \([\bar{\theta}]\), (d) \([\bar{\phi}]\) for five values of \(Q_0\) and a radiative equilibrium equator-to-pole temperature difference of 40 K.
\( Q_0 \) as \( \Delta \theta \) is decreased is consistent with the view that the stationary waves are affecting the positive feedback between the transients and the zonal flow to produce the instability of the conventional circulation. As \( \Delta \theta \) is decreased at small \( Q_0 \) on the conventional branch, the transients, which are produced primarily by baroclinic processes in midlatitudes, are weakened and their contribution to the maintenance of low-latitude easterlies is decreased (compare Figs. 5a and 9a). With smaller easterly acceleration by transients in low latitudes the westerly acceleration produced by the stationary waves will have a proportionally greater effect and thus produce the transition at a lower value of the forcing.

Another set of sensitivity experiments was conducted after a Newtonian drag was added to the \( \bar{u} \) and \( \bar{\theta} \) in (3) and (4). The purpose of adding this friction was to couple the two vertical levels more strongly—an effect that might prevent the strong superrotation of the upper-level flow. The relaxation time used was 20 days. Six experiments were run with \( Q_0 = 0-5 \text{K d}^{-1} \) for 800 days with a radiative equilibrium equator-to-pole temperature difference of 80 K. The evolution of [\( \bar{u} \)] at the equator is shown in Fig. 10. Comparing with Fig. 2 we see that the transition to a superrotating state appears more gradually.

Although Fig. 10 appears to show a smooth transition, an examination of the other fields shows that this actually applies only to the area of \( 0^\circ-20^\circ \). Figure 11 shows meridional distributions of [\( \bar{u} \)], [\( \bar{\theta} \)], and [\( \bar{\phi} \)] averaged over the last 400 days. First, we note that the conventional circulations with and without internal friction are very similar. Furthermore, in the superrotating solutions poleward of \( 40^\circ \) we have a sudden transition between the heating rates of 2 and 3 K d\(^{-1}\). Equatorward of \( 30^\circ \), however, the transition is much smoother. Recall that, in the case of superrotation with no damping on the vertical shear, the zonal vertical shear was a maximum at the equator (see Fig. 3b). In spite of these differences near the equator, changes in the heat and momentum budgets are similar to those for the case with no internal friction.

5. Concluding remarks

In this paper we showed that a two-level primitive equation model may exhibit a general circulation that is radically different from the conventional one. This other general circulation is superrotating, with surface westerlies in the tropics and surface easterlies in the extratropics. Although the heat transport is still poleward, the momentum transport is equatorward. The mean meridional circulation is a hemispheric Hadley cell, and tropical temperatures are near radiative equilibrium, with large vertical shears in low latitudes resulting in strong upper-level westerlies throughout the tropics.

The superrotating solution was first obtained by increasing tropical eddy heating beyond a critical value at which the conventional circulation became unstable. It was found, however, that once established the superrotation circulation would maintain itself without tropical heating, resulting in a hysteresis effect and multiple statistically stable states for small values of tropical forcing. Analysis of the momentum budget of the two circulations revealed large differences in transient momentum transports due to changes in both the region of generation of transient disturbances, which is shifted equatorward in the superrotating solution, and in their propagation properties.

We also performed additional experiments varying the radiative equilibrium equator-to-pole temperature difference and adding a frictional damping to the vertical shear. We found that when the equator-to-pole temperature difference is decreased the transition to a superrotating solution occurs at a lower value of the
Fig. 10. Same as Fig. 2 except that a Newtonian drag has been added to Eqs. (3)-(4).

Fig. 11. Four-hundred-day averages of zonal mean quantities (a) $[\bar{u}]$, (b) $[\bar{v}]$, (c) $[\bar{\theta}]$, and (d) $[\bar{q}]$ for six values of $Q_0$, a radiative equilibrium equator-to-pole temperature difference of 80 K and internal friction.
tropical forcing. This is presumably due to the weakening of the transients and of their tendency to produce easterly flow in low latitudes. Experiments damping the vertical shear also resulted in superrotating solutions for large values of tropical heating, but the transition, at least in low latitudes, was more gradual.

Finally, the task remains to explain the instability of the conventional circulation that leads to the transition to the superrotating solution. We suggested that this instability must be related to the positive feedbacks in wave-mean flow interactions. In the conventional circulation, the maintenance of the tropical easterlies can be explained in terms of the meridional propagation of transient disturbances (Simmons and Hoskins 1978; Edmon et al. 1980) and their subsequent absorption at the waves' critical latitude on the equatorward flank of the subtropical jet where they deposit their easterly momentum. When we force a stationary wave in the tropics, it appears that we change the propagation properties of the transients in such a way as to diminish their easterly contribution to the low-latitude momentum budget. However, the details of how this occurs and the nature of the instability remain unknown.

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APPENDIX

Horizontal Finite Differencing

The horizontal differencing is patterned after Arakawa and Lamb (1981) but uses a somewhat simpler enstrophy conserving scheme (Sadourny 1975). The differencing is done on a staggered (C grid) latitude-longitude grid, with J latitude bands. The potential temperature \( \theta \), geopotential \( \Phi \), vertical pressure velocity \( \omega \), and the zonal velocity component \( u \) are defined at the \( J-1 \) latitudes, \( \phi_j = -\pi/2 + j\Delta \phi \), for \( j = 1, 2, \ldots, J-1 \), where \( \Delta \phi = \pi/J \). Variables at these \( \theta - u \) latitudes will be indexed by integer \( j \) subscripts. The meridional velocity component \( v \), the absolute vorticity \( q \), and streamfunction \( \psi \) lie at the \( J-2 \) intervening latitudes and at the poles. Variables at these \( q - v \) latitudes will be indexed with half-integer \( j \) subscripts, with \( j = 1/2 \) denoting the south pole and \( j = J-1/2 \) the north pole. In this arrangement, the grid is slightly stretched at the poles, with the meridional distance between the pole and the next \( q - v \) latitude being \( 3\Delta \phi/2 \).

In the zonal direction, the absolute vorticity and streamfunction are defined at \( I \) meridians, half of a grid interval to the east or west of \( v \). The variable \( u \) is defined on the same meridians, but half of a grid interval to the east or west of the \( \theta, \Phi, \) and \( \omega \). The \( \theta - v \) meridians will be indexed by integer \( i \) subscripts, and the \( u - q \) meridians by half-integer subscripts.

For any variable \( \chi \), let

\[
(\bar{\chi}^n)_{n+1/2} = \frac{1}{2} \left( \chi_n + \chi_{n+1} \right),
\]

\[
(\delta_n \chi)_{n+1/2} = (\chi_{n+1} - \chi_n),
\]

and let

\[
\Delta \tilde{\chi}^j = a \cos(\phi_j) \Delta \lambda,
\]

\[
\Delta \chi^j_{n+1/2} = \Delta \tilde{\chi}^j_{n+1/2}, \quad j = 1, \ldots, J-1,
\]

\[
\Delta \chi^j_i = \Delta \tilde{\chi}^j_i = 0, \quad j = 1, \ldots, J, \quad i = 1, \ldots, J-2,
\]

\[
\Delta \chi^j_i = a \Delta \phi_i, \quad j = 2, \ldots, J-2, \quad i = J-2,
\]

\[
\Delta \chi^j_i = 3a \Delta \phi_i/2, \quad j = 2, \ldots, J-2, \quad i = J-1,
\]

where \( \Delta \lambda = 2\pi/I \). With this notation, the grid area at \( \theta - u \) (integer) latitudes is defined as

\[
\Delta \tilde{\chi}^j = (\Delta \tilde{\chi}^j_i) \Delta \tilde{\chi}^j_j, \quad j = 2, \ldots, J-2,
\]

\[
\Delta \chi^j_i = \Delta \tilde{\chi}^j_i \Delta \tilde{\chi}^j_i,
\]

\[
\Delta \chi^j_i = \Delta \tilde{\chi}^j_i \Delta \tilde{\chi}^j_i.
\]

It is also useful to define

\[
\tilde{U}_{i+1/2,j} = \Delta \chi^j_i \tilde{U}_{i+1/2,j}, \quad \tilde{U}_{i+1/2,j} = \Delta \chi^j_i \tilde{U}_{i+1/2,j},
\]

\[
j = 1, \ldots, J, \quad j = 1, \ldots, J-1.
\]

\[
\tilde{V}_{i,j-1/2} = \Delta \chi^j_i \tilde{V}_{i,j-1/2}, \quad \tilde{V}_{i,j-1/2} = \Delta \chi^j_i \tilde{V}_{i,j-1/2},
\]

\[
j = 1, \ldots, J, \quad j = 1, \ldots, J-1.
\]

The finite-difference analogs of (7)-(9a) are then

\[
\omega_{ij} = -\frac{(\delta_i \tilde{U} + \delta_j \tilde{V})_{ij}}{\Delta \tilde{\chi}^j_i}, \quad j = 1, \ldots, J-1,
\]

\[
\frac{\partial \tilde{\theta}}{\partial t} = -\frac{(\delta_i \tilde{U} \tilde{\theta}^j + \delta_j \tilde{V} \tilde{\theta}^i)_{ij}}{\Delta \tilde{\chi}^j_i},
\]

\[
j = 1, \ldots, J-1,
\]

and

\[
\frac{\partial \tilde{\phi}_{ij}}{\partial t} = -\frac{(\delta_i \tilde{U} \tilde{\phi}^j + \delta_j \tilde{V} \tilde{\phi}^i)_{ij} - \omega_{ij} \tilde{\phi}_{ij}}{\Delta \tilde{\chi}^j_i},
\]

\[
j = 1, \ldots, J-1.
\]

Three diagnostic quantities are used: the relative vorticity, \( \tilde{\chi}_{i+1/2,j-1/2} \), kinetic energy, \( K_{ij} \), and the streamfunction, \( \tilde{\psi}_{i+1/2,j-1/2} \). The finite-difference form of \( \tilde{\chi}_{i+1/2,j-1/2} \), away from the poles, is
\[ \xi_{i+1/2,j-1/2}^{*} = \frac{\Big( \Delta^y (\delta_i v) - \delta_j (\Delta^x u) \Big)_{i+1/2,j-1/2}}{\Delta_j^{-1/2}}, \]

\[ j = 2, \ldots, J - 1, \] (A13)

while at the poles

\[ \xi_{sp} = \frac{1}{2} \Delta_1^{2} [u]_1, \]

and

\[ \xi_{np} = \frac{1}{2} \Delta_2^{2} [u]_{J-1}, \]

where the square brackets denote a zonal mean. The absolute vorticities are

\[ \overline{q}_{i+1/2,j-1/2} = f_{i-1/2} + \xi_{i+1/2,j-1/2}, \quad j = 1, \ldots, J, \] (A14a)

and

\[ \psi_{i+1/2,j-1/2} = \xi_{i+1/2,j-1/2}, \quad j = 1, \ldots, J, \] (A14b)

where

\[ f_{i-1/2} = -\frac{\Omega}{\Delta \lambda} \left( \frac{\delta_i ((\Delta^y)^2)}{(\Delta_j^y)^2} \right)_{j-1/2}, \]

\[ j = 2, \ldots, J - 1, \]

\[ f_{i-1/2} = \frac{2\Omega}{\Delta \lambda} \left( \frac{(\Delta_j^{y-1})^2}{\Delta_j^{y-1}} \right)_{j-2} \]

\[ f_{i-1/2} = \frac{2\Omega}{\Delta \lambda} \left( \frac{(\Delta_j^{y-2})^2}{\Delta_j^{y-1}} \right)_{j-3}. \] (A15)

The discrete form of the kinetic energy is

\[ K_j = \frac{1}{2} \left( \Delta^x \Delta^y \overline{u}^2 + \Delta^x \Delta^y \overline{v}^2 \right)_{j}, \quad j = 1, \ldots, J - 1. \] (A16)

For the vertical shear momentum equations are

\[ \frac{\partial \overline{u}^j}{\partial t}_{i+1/2,j} = \frac{\overline{q}^j \overline{\psi}^j}{\Delta_j^{y-1/2}} - \delta_j (\overline{\Phi} + \overline{K}) + (\overline{\omega}^j \overline{u})_{i+1/2,j}, \]

\[ j = 1, \ldots, J - 1, \] (A17)

\[ \frac{\partial \overline{v}^j}{\partial t}_{i,j-1/2} = \frac{\overline{q}^j \overline{\psi}^j}{\Delta_j^{y-2/2}} + \delta_{i} (\overline{\Phi} + \overline{K}) + (\overline{\omega}^j \overline{v})_{i,j-1/2}, \]

\[ j = 2, \ldots, J - 1, \] (A18)

and for the vertical shear

\[ \frac{\partial \overline{u}^j}{\partial t}_{i+1/2,j} = \frac{\overline{q}^j \overline{\psi}^j}{\Delta_j^{y-1/2}} - \delta_j (\overline{\Phi} + \overline{K}), \]

\[ j = 1, \ldots, J - 1, \] (A19)

\[ \frac{\partial \overline{v}^j}{\partial t}_{i,j-1/2} = -\frac{\overline{q}^j \overline{\psi}^j}{\Delta_j^{y-2/2}} + \delta_{i} (\overline{\Phi} + \overline{K}), \]

\[ j = 2, \ldots, J - 1. \] (A20)

Because the vertical mean flow is required to be non-divergent, rather than using (A17) and (A18) directly, we use the zonal mean of (A17),

\[ \frac{\partial \overline{u}^j}{\partial t}_{i,j-1/2} = \frac{1}{\Delta_j^{y-1/2}} \left( \overline{q}^j \overline{\psi}^j \right)_{i,j} + \left( \overline{\omega}^j \overline{u} \right)_{j}, \]

\[ j = 1, \ldots, J - 1, \] (A21)

and the eddy vorticity equation,

\[ \frac{\partial \overline{\omega}^j}{\partial t}_{i+1/2,j-1/2} = \frac{1}{\Delta_j^{y-1/2}} \left( \overline{q}^j \overline{\psi}^j \right)_{i+1/2,j-1} - \delta_{j} \left( \overline{\Delta^x} \overline{\omega}^j \overline{u} \right)_{i+1/2,j-1/2}, \]

\[ j = 2, \ldots, J - 1, \] (A22)

obtained by differentiating (A13).

Using this method, a Poisson equation must be solved at each time step for the eddy streamfunction of the vertical mean flow, \( \overline{\psi}_{i+1/2,j-1/2} \). Substituting the relations

\[ \overline{v}_{i,j-1/2} = \frac{\delta_i \overline{\psi}^*}{\Delta_j^{y-1/2}}, \quad j = 1, \ldots, J, \]

\[ \overline{u}_{i+1/2,j} = -\frac{\delta_j \overline{\psi}^*}{\Delta_j^{y-2/2}}, \quad j = 1, \ldots, J - 1, \]

obtained by differentiating (A13), we obtain the following finite-difference form of the Poisson equation:

\[ \xi_{i+1/2,j-1/2} = \frac{1}{\Delta_j^{y-1/2}} \left( \Delta_j^{y-1/2} \delta_{j} \left( \overline{\delta_i \overline{\psi}^*} \right) + \delta_{j} \left( \overline{\Delta^x} \overline{\delta_j \overline{\psi}^*} \right) \right)_{i+1/2,j-1/2}, \]

\[ j = 2, \ldots, J - 1, \] (A25)

with \( \xi_{i,1/2} = \xi_{i,J-1/2} = 0 \). Because of the periodicity in the \( i \) direction, (A25) can be solved by first taking its Fourier transform in the zonal direction. For each har-
monic, we then have a tridiagonal system of order $J$, which can be solved directly subject to the boundary condition, $\tilde{\psi}_{m,1/2} = \tilde{\psi}_{m,J-1/2} = 0$ for all wavenumbers $m$. Finally, the solution, $\psi_{1/2,J-1/2}$, is found by taking the inverse Fourier transform.

REFERENCES


