SHORTER CONTRIBUTIONS

THE DISTRIBUTION OF RAINDROPS WITH SIZE

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Measurements of raindrop records on dyed filter papers were made for correlation with radar echoes (Marshall, Langille, and Palmer, 1947). These measurements have been analyzed to give the distribution of drops with size (fig. 1). The distributions are in fair agreement with those of Laws and Parsons (1943).

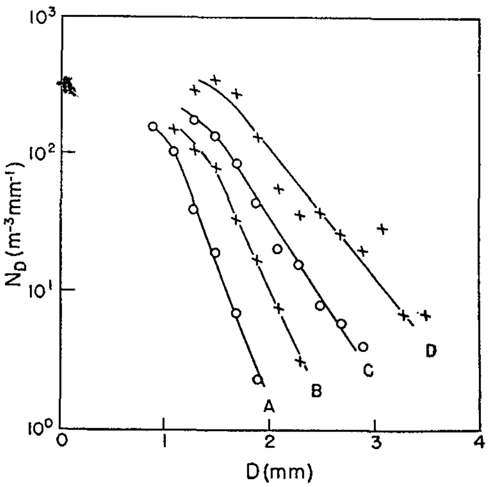

FIG. 1. Distribution of number versus diameter for raindrops recorded at Ottawa, summer 1946. Curve A is for rate of rainfall 1.0 mm hr^{-1}, curves B, C, D, for 2.8, 6.3, 23.0 mm hr^{-1}. $N_D\delta D$ is the number of drops per cubic meter, of diameter between D and $D + \delta D$ mm. Multiplication by 10^{-5} will convert N_D to the units of equation (2).

Except at small diameters, both sets of experimental observations can be fitted (fig. 2) by a general relation,

$$N_D = N_0 e^{-\Lambda D}, \qquad (1)$$

where D is the diameter, $N_D\delta D$ is the number of drops of diameter between D and $D + \delta D$ in unit volume of space, and N_0 is the value of N_D for $D = 0$.

It is found that

$$N_0 = 0.08 \text{ cm}^{-4} \qquad (2)$$

for any intensity of rainfall, and that

$$\Lambda = 41\, R^{-0.21} \text{ cm}^{-1}, \qquad (3)$$

where R is the rate of rainfall in mm hr^{-1}.

For diameters less than about 1.5 mm, both sets of observations fall short of the value for N_D given by equation (1), and they disagree slightly with each other. Laws and Parsons' observations are better in this region, and tend toward a common value of N_0 for all rates of rainfall.

The mass of rain water M per unit volume of space, and the sum Z of sixth powers of drop diameters in unit volume (a radar quantity), can be calculated as functions of Λ from equation (1), and so correlated with the rate of rainfall R by equation (3). It is of interest to compare these correlations with those obtained when M, Z, and R are determined more directly from the experimental records (table 1). The deficit of small drops in the observations, as compared with equation (1), should make the observed value of M, and to a lesser extent that of Z, smaller than those derived from the equations.

TABLE 1. $M = \frac{1}{6}\pi\Sigma N_D D^3 \delta D$ and $Z = \Sigma N_D D^6 \delta D$ as functions of the rate of rainfall R.

Reference	M mgm m^{-3}	Z mm^6 m^{-3}
Marshall, Langille and Palmer (1947)	80 $R^{0.83}$	190 $R^{1.72}$
Revision of the above	72 $R^{0.88}$	220 $R^{1.60}$
Z/R correlation by Wexler (1947) (data of Laws and Parsons, 1943)	68 $R^{0.88}$	320 $R^{1.44}$
From equations (1) and (3)	89 $R^{0.84}$	296 $R^{1.47}$

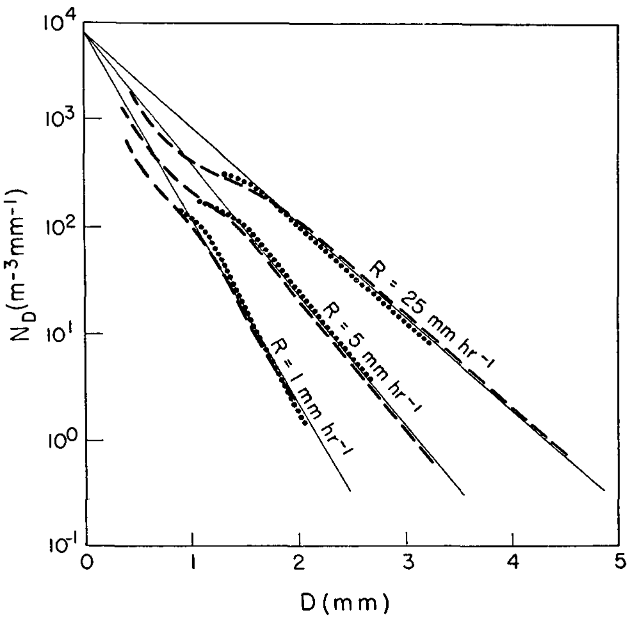

FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

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The exponential distribution of equation (1) is the type that would obtain if growing drops were in continual danger of disintegrating, the likelihood of disintegration being proportional to the increment in diameter or in distance of fall through cloud. Such behavior might be explained by the random accumulation by each drop of electrical charge as more and more randomly charged cloud drops or smaller raindrops are acquired by coalescence, and the resultant disintegration of overcharged drops. Relevant calculations and experiments on coalescence are in progress.

Part of the work reported here was done during summer employment in the Radar Meteorology Section of the Defense Research Board's Radio Propagation Laboratory at Ottawa.

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THE PROPAGATION OF PERMANENT-TYPE WAVES IN HORIZONTAL FLOW

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1. Introduction.—The propagation of atmospheric waves in horizontal flow was first considered by Rossby [5], who showed that waves in an infinitely broad westerly current travel with speeds less than that of the current itself.

$$c = U - \frac{\beta L^2}{4\pi^2}.$$

In Rossby's notation, c is the phase speed relative to the earth; U, the speed of the current; β, latitudinal variation of the Coriolis parameter; and L is the wave length of the oscillation.

Although this result is valid only under special conditions, later generalizations of the theory of atmospheric waves have led to similar conclusions. By linearizing the vorticity equation, Haurwitz [3] has derived the phase speed of infinitesmal wave disturbances in flows of finite lateral extent, verifying Rossby's formula as a special case. The theory has been extended to waves of finite amplitude through the solutions of the nonlinear vorticity equation recently given by Craig [2] and Neamtan [4].

In his discussion of harmonic waves, the latter has at least qualitatively confirmed the results of Haurwitz' perturbation analysis, but failed to recognize in the unidentified terms of his solution a fundamental parameter mentioned earlier, though rather summarily, by Rossby. The following investigation of waves of permanent type reveals that the phase speed depends not only on those parameters previously found by Haurwitz, but also on the variation of wind shear from one lateral boundary to the other.

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2. An integral of the vorticity equation.—The system of equations governing the two-dimensional motion of a homogeneous fluid may be reduced to a single differential equation which involves only one scalar unknown. This equation, a variant of the vorticity theorem due to Helmholtz, simply states that the absolute vorticity of any particular fluid element remains forever the same.

$$\frac{\partial}{\partial t}\nabla^2\psi - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\nabla^2\psi + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\nabla^2\psi + \frac{\partial\psi}{\partial x}\frac{\partial\lambda}{\partial y} = 0. \quad (1)$$

The locally Cartesian plane of motion (x,y) is tangent to the earth in a point fixed on its surface; the x-axis is directed eastward. The Coriolis parameter λ, which expresses absolute rotation of the coordinate frame, consequently depends on y alone. The stream function ψ is defined by a pair of relations which, taken together, satisfy the condition for incompressibility: $u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$, where u is the eastward component of velocity, and v the northward component. Thus velocity and all other kinematic variables may be represented by operations on the stream function; vorticity, for instance, is the Laplacian derivative of the stream function.

The remainder of this discussion will deal with ψ-fields which, traveling with constant speed through the (x,y) plane, suffer no change of shape—*i.e.*, with waves of permanent type. To simplify matters considerably, we shall suppose that the ψ-field is propagated at speed c toward the east or x-direction. In that case, $\partial/\partial t = -c\,\partial/\partial x$, so that the vorticity equa-

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