MICROTEMPERATURE FLUCTUATIONS

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ABSTRACT

Selected portions of microtemperature data obtained continuously and with near simultaneity at several levels up to six feet over a desert surface are plotted on expanded height–time coordinates. The resulting isotherm patterns are shown to be strikingly consistent at all levels and are qualitatively analyzed in relation to the turbulence field present. Correlation coefficients between temperature fluctuations simultaneously at two levels and at a point for various time intervals are evaluated and their variation with separation, time, wind speed, and thermal stability is discussed. Tentative intensity and scale measures of turbulence derived from temperature data are presented.

1. Introduction

Under the auspices of a contract from the Office of Naval Research to study micrometeorology, the Electrical Engineering Research Laboratory of the University of Texas established a research station near Gila Bend, Arizona, during April 1947. Measurements were made by means of four separate installations as follows: (*1*) a single aerated psychrograph[1] carried aloft by means of a barrage balloon up to 500 ft, (*2*) a string of 18 aerated psychrographs arranged in an approximate logarithmic distribution on a 200-ft tower, (*3*) a series of 12 thermistors set in a vertical line from 18 inches below the surface to 72 inches above the surface, and (*4*) a set of three cup-type anemometers (U. S. Army model ML-203) at 10, 100, and 200 ft. This paper will be concerned primarily with the air temperature measurements taken up to six feet with the exposed thermistors.

All temperature measuring elements were 2-inch ceramic thermistor rods having a lag coefficient of about 2 seconds and an absolute accuracy of ±0.2C. The exposed elements, coated with white and aluminum paints to reduce radiation effects to a minimum, were mounted at the surface and at intervals of 1, 6, 12, 24, 48, and 72 inches above the surface over a representative desert-type terrain about 18 inches from a small supporting post. A switching arrangement made it possible to take a complete sounding in about 75 seconds by recording a 5-second reading for each of the 12 elements in succession. Wind speed and direction recordings were made for 5-minute periods at each of the three levels in succession. This type of step recording combined with a chart speed of 3 inches per hour made effective correlation of wind speed with temperature fluctuations admittedly difficult.

2. Preparation of Data

Representative portions of the data were selected for detailed analysis based on the criteria of temperature lapse rate and wind speed. Sample analyses were made under each of the following thermodynamic specifications in order best to include the effects of both convective and frictional turbulence:

Stability	Wind	Turbulence
Thermally stable	Calm to light winds	Little or no turbulence
Thermally stable	Moderate to strong winds	Predominantly frictional turbulence
Thermally unstable	Calm to light winds	Predominantly convectional turbulence
Thermally unstable	Moderate to strong winds	Combined frictional and convectional turbulence

Representative samples of microtemperature data taken over an approximate 30-minute period for each of the above cases were examined in order to study the variations in the individual readings. The illustrations of this report are the results of such studies and are presented here as striking indications of the degree to which temperature fluctuations can depict low-level atmospheric turbulence. Little more than a qualitative analysis can be made of these data now, but with the correspondingly greater flexibility and detail of both wind and temperature measurements to be made at a new installation this year,[2] it is expected that a much more complete analysis can be carried out later on more comprehensive data.

3. Presentation of Data

A. General.—The general method of data presentation as shown in fig. 1 is the same for all figures where the average 5-second air temperature readings

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[1] A Radiation Laboratory instrument [5] which measures wet- and dry-bulb temperatures as functions of the resistance of ceramic elements.

[2] A 300-ft tower has recently been erected near Austin, Texas, for the purpose of studying heat and moisture transfer and the structure of atmospheric turbulence near the earth's surface. A complete description is given by Jehn [4].

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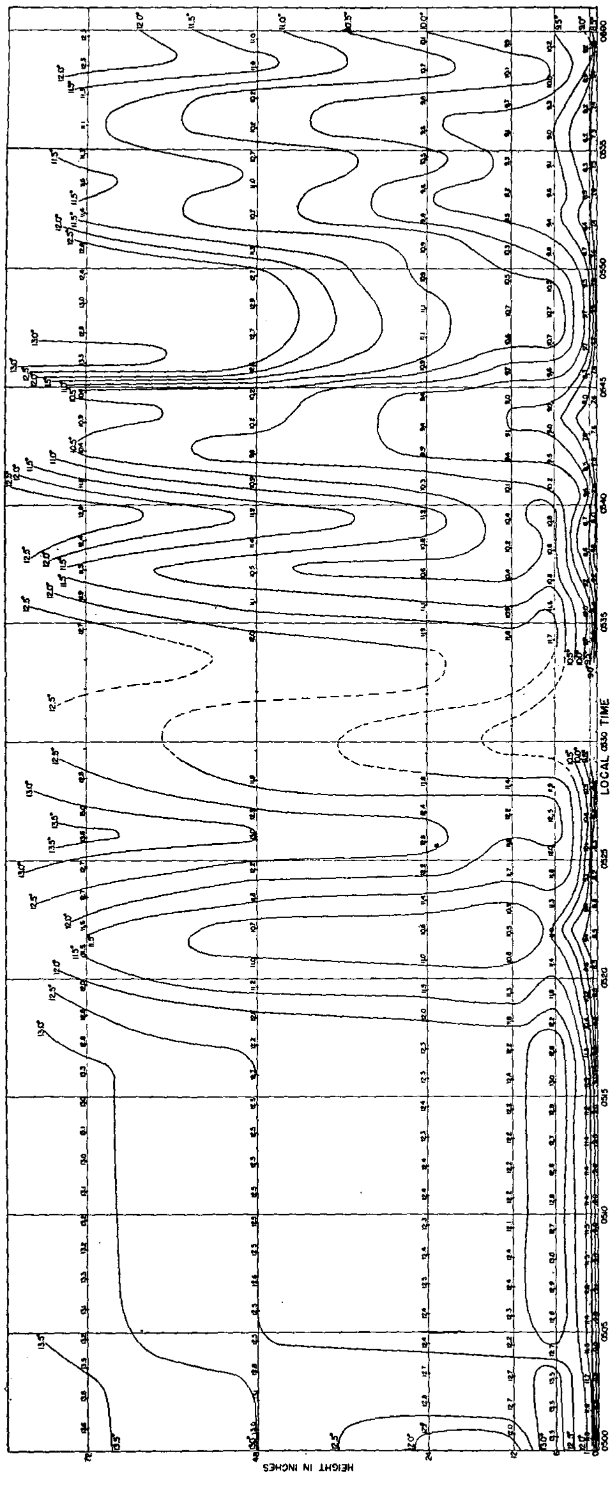

FIG. 1a. Thermal stability under light variable winds. Gila Bend, Arizona, 8 April 1947, 05^h00^m–06^h00^m local time. Isotherms in degrees C; interval, 0.5C.

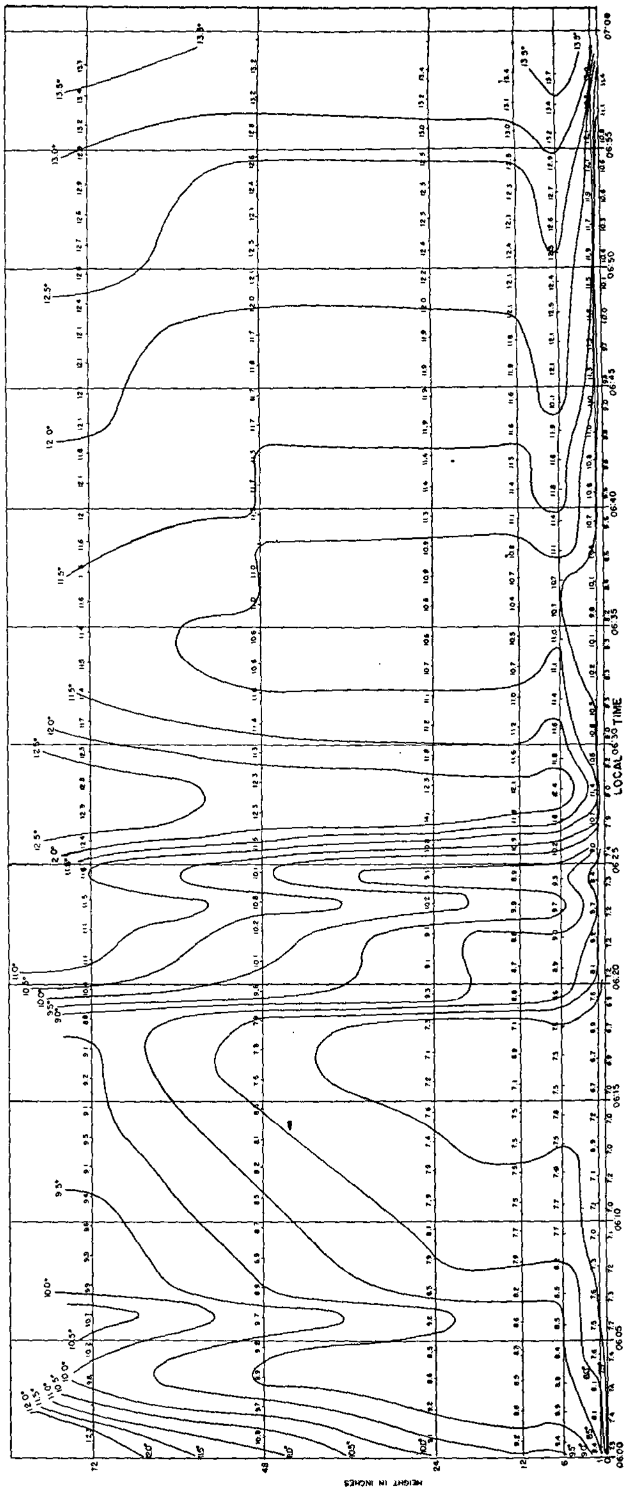

FIG. 1b. Continuation of fig. 1a; 06^h00^m–07^h00^m local time.

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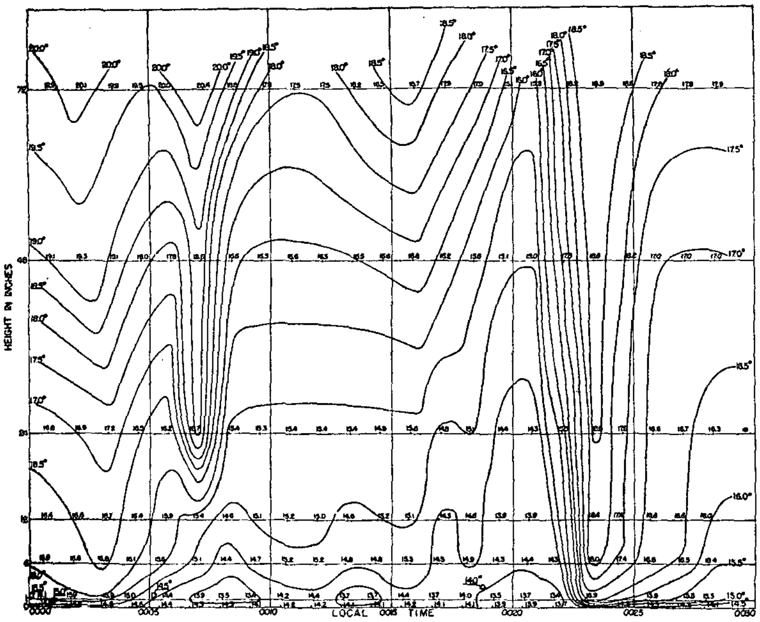

FIG. 2. Thermal stability under near calm conditions. Gila Bend, Arizona, 26 April 1947, 00^h00^m–00^h30^m local time. Isotherms in degrees C; interval, 0.5C.

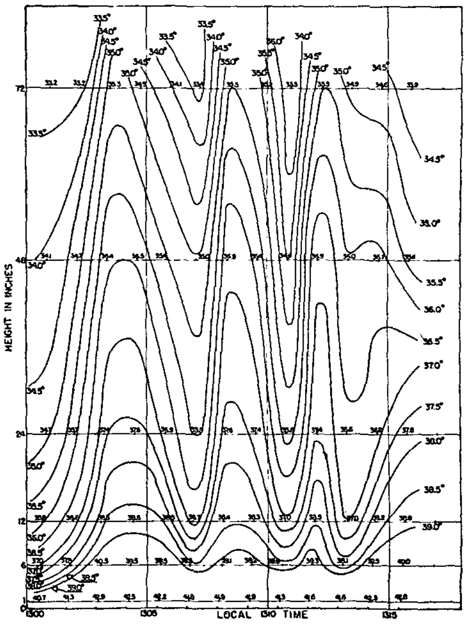

FIG. 4. Thermal instability under moderate variable winds. Gila Bend, Arizona, 29 April 1947, 13^h00^m–13^h15^m local time. Isotherms in degrees C; interval, 0.5C.

are plotted on height–time coordinates. Isotherms, smoothed for irregularities $\leqq \pm 0.2$C, were drawn for every 0.5C by using linear interpolation between the height and time scales. The one most remarkable feature common to all the presentations is the consistency with which individual temperature variations fit into an overall turbulence pattern, for almost without exception the recorded temperature fluctuations at a given height had a high correlation with those at all other heights. Thus the pattern of these fluctuations, although essentially random at a given point, must represent variations, with periods equal to or greater than approximately 40 seconds,[3] in horizontal and vertical temperature gradients due to the action of convectional and frictional turbulence. Indications of the superposition of fluctuations of two or more periods may be observed throughout the data.

B. Temperatures of exposed thermistors.—(*i*) *Thermally stable.* The data for fig. 1 were taken during a period of clear skies beginning about an hour and a half prior to sunrise where radiational cooling had created a temperature inversion of approximately 4C between soil and the six-foot level. Although the available recording of wind speed is inadequate to be able to make exact time correlations with the isotherm pattern, it is possible to make a qualitative analysis of some of the salient features of each pattern. The wind speed at the 10-ft level for the two hours prior to 0500 (local time) on April 8 had been very uniform with fluctuations of less than ± 1 mi hr^{-1} about an average 4–5 mi hr^{-1}, giving rise to the uniform horizontal temperature gradient depicted from 0500 to 0515 where 75 per cent or more of the inversion was concentrated in the first 6 inches. Beginning around 0515 the average wind speed at all levels began to fall slowly, with however, a corresponding increase in fluctuations from the mean to as much as ± 3 mi hr^{-1} at the low level. Wind speeds returned to initial values by 0615. Thus the increased gustiness of the wind and resultant variations in frictional turbulence during the period 0515–0615 reduced the vertical temperature gradient to about 2C in 6 ft and to less than 0.5C in the first 6 inches by the end of the period. The average period of the temperature fluctuations, or the time interval between successive crests or troughs in the isotherm field, during this stable period is of the order of 5–10 minutes. Since the time coordinate can be converted to a distance coordinate by assuming a mean wind speed at a level, it is interesting to note that these periods can be considered as equivalent to a turbulence "wave length" of 800–1800 ft for a 2-mi hr^{-1} mean wind. Except for a short period of possible gustiness between 0620–0630, the wind and temperature fluctuations had now returned to minimum values and we obtain a uniform transition from cooling to warming at all levels beginning about 0620 due to initial insolational heating.

Fig. 2 is a plot of the temperature data from 0000–0030 on April 26 and illustrates the type of extreme temperature fluctuations possible under stability conditions. With clear skies, the wind had fallen to a near calm at the 10-ft level and to 2–3 mi hr^{-1} at the 100- and 200-ft levels by 0000, giving rise to an unusually uniform temperature inversion of over 5C to the 6-ft level. However, the mixing produced by two isolated troughs in the isotherm pattern, the latter

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[3] A lower limit of periods due to instrumental sounding procedure.

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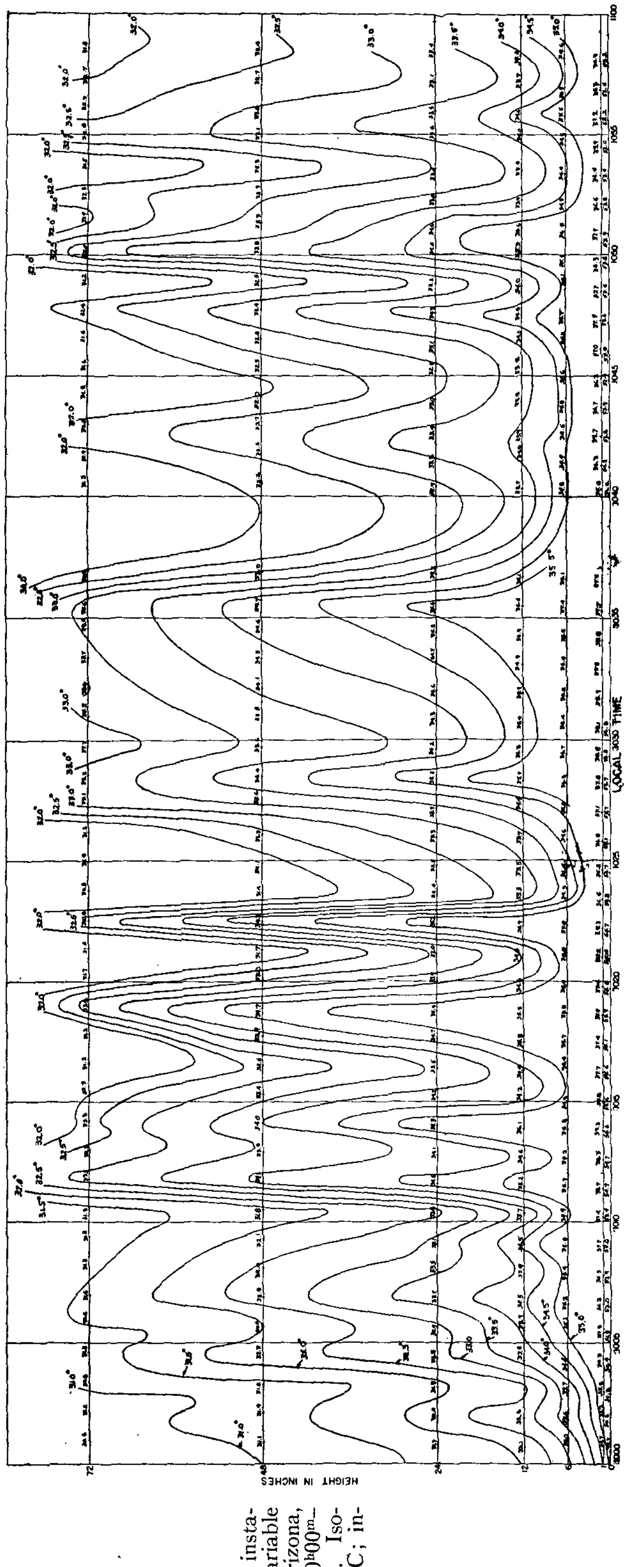

FIG. 3. Thermal instability under light variable winds. Gila Bend, Arizona, 29 April 1947, 10h00m–11h00m local time. Isotherms in degrees C; interval, 0.5C.

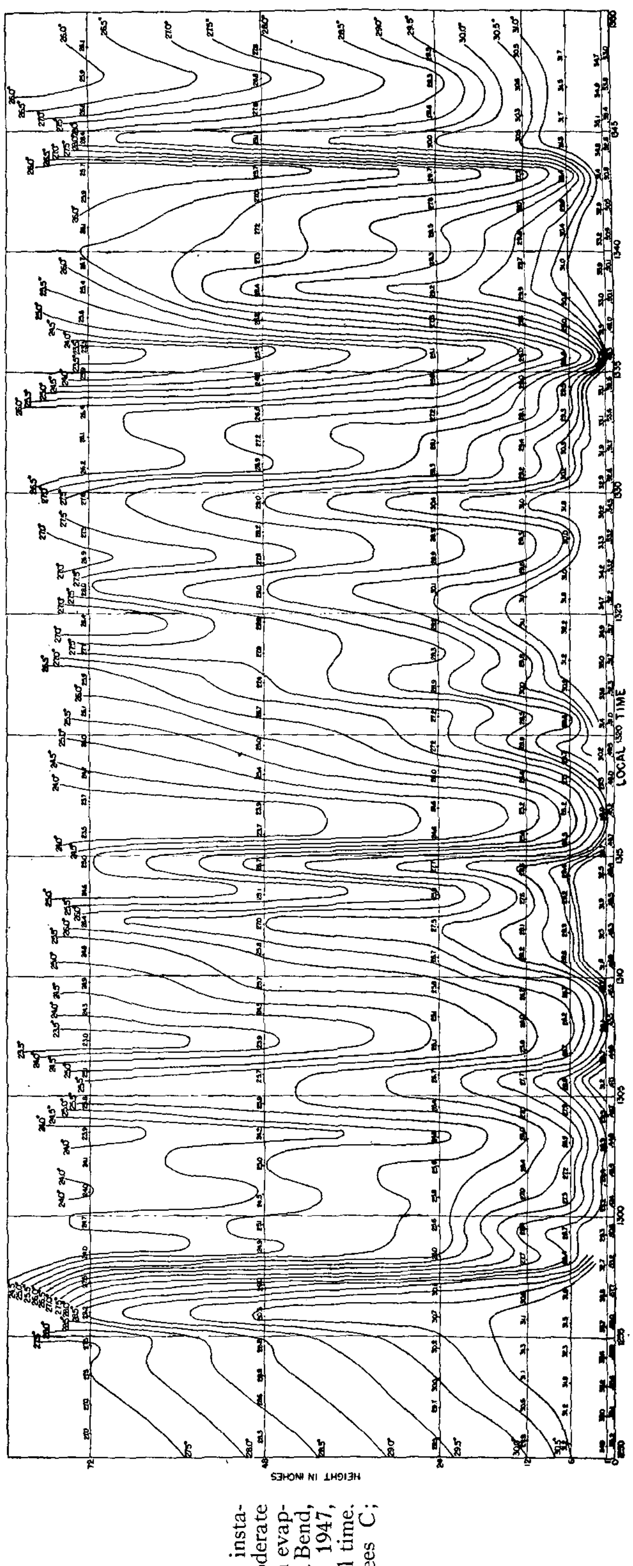

FIG. 5. Thermal instability under moderate variable winds with evaporation cooling, Gila Bend, Arizona, 23 April 1947, 12h50m–13h50m local time. Isotherms in degrees C; interval, 0.5C.

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of which correlated with a jump in wind speed from 0 to 3 mi hr^{-1} at the low level at approximately 0025, was sufficient to reduce the inversion to 3.5C by the end of the period. The air brought down from aloft by the second vertical current was sufficient to cause a warming of over 4C at levels from 6 inches to 24 inches in a period of less than 2 minutes.

An observation of the temperature distribution for thermal stability under strong winds is not presented here but measurements for scattered periods indicate an isothermal lapse rate over an inversion concentrated in the first inch above the surface. Such a plot could not illustrate frictional turbulence directly due to the uniformity of the temperature field.

(*ii*) *Thermally unstable.* The data for fig. 3 were taken on April 29 between 1000–1100 during a period of scattered high, thin cloudiness where intense surface heating had already created a temperature difference of 15–18C between the surface and one inch above the surface. The wind speed at the 10-ft level was recorded as an average 2–3 mi hr^{-1} with variation of $\pm$1–3 mi hr^{-1}. With the exception of the first 10 minutes of the period where the wind fluctuations were lower than average, no detailed correspondence between the isotherm pattern and wind speed is possible. However, it is worthy of note that with a wind pattern somewhat similar to that existing for fig. 1, the time intervals between successive crests or troughs has been reduced from 5–10 minutes for the stable case to 1–5 minutes for the unstable case. Thus, while it is not yet possible to make a quantitative separation of the effects of frictional and convectional turbulence on such a temperature-fluctuation diagram, the net effect of instability is equivalent to a reduction in the "wave length" of the turbulence pattern. Another effect which is evident in the case of superadiabatic lapse rates as contrasted with the stable case is the apparent interference of the two types of turbulence giving rise to localized horizontal divergence and convergence. Thus the upward thrust of the isotherms at 1022 on fig. 3 is very sharp and quite intense at all levels, while a thrust which starts out with the same magnitude at 1014 is almost entirely damped out at the 6-ft level.

A much more striking example of this effect is indicated during the last three minutes of fig. 4, where due to simultaneous ascending and descending isotherms at 1318, a vertical gradient of 2.5C per ft is concentrated temporarily between 3 and 4 ft. It is unfortunate that no further data were available to complete this pattern. The local wind record for fig. 4, a period from 1300–1318 on April 29, showed a wind speed at 10 ft of about 6 mi hr^{-1} with variation of $\pm$4 mi hr^{-1}, a speed and fluctuation somewhat in excess of that for fig. 3. The resulting temperature perturbations are now more uniform and have a slightly lower period than those of fig. 3, but due to the short time interval of the sample, no further comparisons can be made.

The temperature field illustrated in fig. 5 is very complex and represents the influence of a third factor in addition to convectional and frictional turbulence. These measurements were made on the afternoon of April 23 from 1250 to 1350 during a period of scattered to broken cumulus-type clouds at about 8000 ft. These clouds had developed to the point where virga could be seen trailing from several of them and for the only time during the month, a very light and intermittent rain was falling. It is presumed that the observed 13C drop in surface temperature from 1257 to 1303 and 7C drop from 1333 to 1336 may be attributed directly to cooling resulting from falling rain. There can be certainly little doubt that a large part of the several extreme temperature drops at levels from 2 to 6 ft must have been due to vertical mixing with air that had been cooled by evaporation from falling rain and to direct cooling by evaporation.[4]

C. Temperatures of aerated thermistors.—An attempt was made to see if the observed microtemperature perturbations could be followed up to 200 ft by using the temperature values obtained from the string of aerated psychrometers using the same two-inch ceramic rods. Although a similar but more extensive switching arrangement using a 5-second alternate dry-bulb and wet-bulb reading made it possible to obtain a sounding in the same period of time (75 seconds), the integrating effect of aeration was sufficient to distort the isotherm pattern so that only the major fluctuations could be identified. Thus while it would be possible to find a crest and trough pattern in the isotherms for aerated temperatures up to our observational limit of 200 ft, the overall pattern of fluctuations was considerably more random than those of the microtemperature measurements and uniform gradients were the exception rather than the rule.

4. Theory fundamentals

A. R_x and R_τ.—Let us now consider the temperature fluctuations $t_{ij} = f(x, \tau)$, where x is distance along the vertical and τ is time. Fluctuations are defined as the deviations of the temperatures T_{ij} from the mean temperature $\bar{T}$, *i.e.*, $t_{ij} = T_{ij} - \bar{T}$. If the temperature fluctuations at a given time for two levels with separation δ: $(t_{x,\tau}t_{x+\delta,\tau})$, or the fluctuations at a given point for a time interval ϵ: $(t_{x,\tau}t_{x,\tau+\epsilon})$, were random, then the mean value of these products averaged over a sufficiently large number of samples to have statistical significance would be zero. However, since the existence of eddies in the atmosphere is dependent upon the existence of a correlation between these fluctua-

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[4] Although it was not possible to determine from the data whether or not any rain had actually hit the exposed thermistors, the uniform and equally rapid temperature rises at all levels following these falls seemed to indicate that the cooling was due essentially to mixing with air which had been cooled by evaporation.

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tions, we may define the following correlation coefficients:

$$R_x = \overline{t_{x,\tau}t_{x+\delta,\tau}}/\sigma_x\sigma_{x+\delta}, \tag{1}$$

$$R_\tau = \overline{t_{x,\tau}t_{x,\tau+\epsilon}}/\sigma_x^2, \tag{2}$$

where

$$\sigma_x = (\overline{t_x^2})^{\frac{1}{2}}$$

and

$$\sigma_{x+\delta} = (\overline{t_{x+\delta}^2})^{\frac{1}{2}}.$$

The curves of R_x versus δ and R_τ versus ϵ then represent the statistical distribution of t_{ij} along the x and τ axes. It should be noted that these correlation coefficients are analogous to those developed by Taylor [7, 8, 9], von Kármán [10, 11], and others [2], which depend on a similar correlation between wind components.

Fig. 6 is a plot of R_x versus δ for several of the previously presented sets of data where the temperatures at various levels for each minute, as interpolated from the isotherms, were correlated with those at the reference or six-foot level over a period of 20–30 minutes. Although the absolute values of the correlation coefficients cannot be used quantitatively due to restricted time samples of the data, their relative values are believed significant. With the exception of the data for thermal stability under near calm conditions, the correlation plots seem to separate thermal stability and instability and indicate only a small variation of the correlation coefficient with wind speed up to 12–15 mi hr^{-1}. A few calculations were made on the aerated thermistor data, but for the most part the correlation coefficients had a very erratic variation with δ. The decrease in correlation due to the use of aerated housings is indicated in fig. 6 where correlations between the temperature at 6 ft and the temperature at 2 ft for aerated-element data are much lower than corresponding correlations from exposed-element data. The aerated data did tend, however, to make the same separation between stable and unstable atmospheres, as indicated in fig. 6 where approximate values of the correlation coefficient for 200 ft of separation using a 6-ft reference were 0.8 and 0.4, respectively.

B. Intensity and scale factors of turbulence.—Following Taylor [8], if R_x falls to zero and remains zero, a length L_x may be defined for temperature fluctuations as suggested by Corrsin [3]:

$$L_x = \int_0^\infty R_x dx, \tag{3}$$

which can be considered as a possible definition of the average size of the eddies. Inasmuch as it was not possible to determine such a length from these data due to the short available height interval, an alternative length L'_x was defined, following a recent study by Priestley and Swinbank [6] and similar to the concept of mixing length,[5] as

$$L'_x = 2\sigma_x(d\overline{T}/dx)^{-1}, \tag{4}$$

where $d\overline{T}/dx$ is the vertical lapse rate of the mean temperature. The standard deviation, σ_x, can be considered as a measure of the intensity of the turbulence present. Computed values of L'_x and σ_x for each

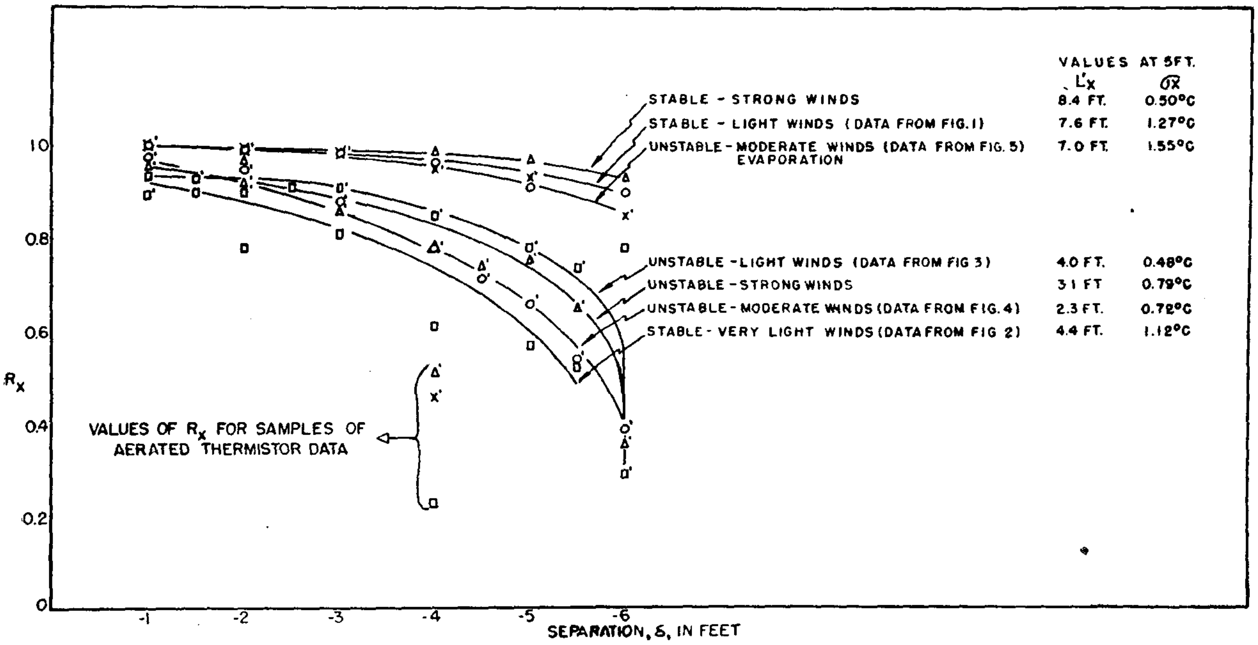

FIG. 6. Correlation coefficient R_x versus separation δ. Based on temperatures from exposed thermistors at the 6-ft level; $\overline{T}$ averaged over 30 minutes.

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[5] Mixing length, l, is defined [1] by the following relationship:

$$\tau_0 = \rho l^2(\partial\bar{u}/\partial z)^2,$$

where τ_0 is the shear stress at the ground and is equal to $\tau_0 = -\rho\overline{u'w'}$. Thus by rewriting the above, and using absolute values of τ_0, we obtain

$$l = (\overline{u'w'})^{\frac{1}{2}}(\partial\bar{u}/\partial z)^{-1},$$

which is equivalent in form to equation (4). The term $(\overline{u'w'})^{\frac{1}{2}}$ can be taken as a measure of the standard deviation of the wind fluctuations.

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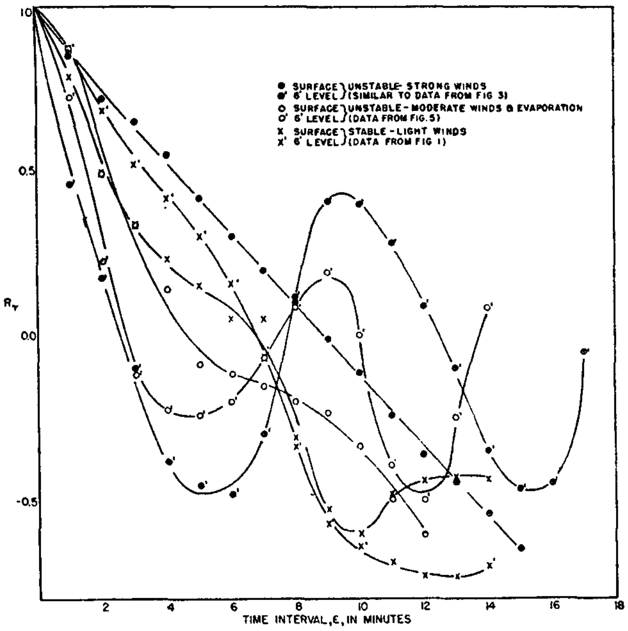

FIG. 7. Correlation coefficient R_T versus time interval ϵ; $\bar{T}$ averaged over 30 minutes.

of the correlation curves at 5 ft are tabulated on fig. 6. Although the values of σ_x are apparently random, the values of L'_x, with the exception of the calm stable case, appear to be a measure of the degree to which the correlation coefficients vary with separation. In other words, if L'_x can be designated as a scale measure of turbulence equivalent to some function of an effective eddy diameter, the value of the correlation coefficient at any point varies directly as the size of the effective eddy. Although there are not at present sufficient data to evaluate the variation of σ_x and L'_x with height, it is apparent that these relationships must be known in order to describe the variation of turbulence and diffusion with height. Moreover, since the value of the correlation coefficient should depend solely on the distance between two points for isotropic turbulence, it should also be possible not only to obtain a measure of the degree of isotropy in any atmosphere at a given level by comparing correlation variations along different axes, but to determine how any atmosphere approaches isotropy with height.

Fig. 7 is a plot of R_T versus ϵ for several of the same sets of data where the temperatures at a given level with varying time separations from 1 to about 20 minutes were correlated with each other. The curves are extremely interesting but they should be studied primarily as a guide for future research since for the most part the data sampling was such that the absolute values of the correlation coefficient are not too significant. Once again the curves clearly delineate between stable and unstable atmospheres and are essentially of a damped sinusoidal nature. The resultant "wave length" is related to the mean isotherm pattern of the original microtemperature fluctuations. The much longer "wave length" for the surface temperature fluctuations as compared to the air fluctuations under unstable conditions is more difficult to understand, but by applying the mean wind-speed transformation it may be that such a "wave length" could be attributed to the much reduced mean wind in the laminar layer.

5. Conclusion

It is planned in future work to extend this concept of a turbulence theory based on temperature fluctuations and to obtain a more comprehensive set of such data. With the installation now being prepared, it will be possible to measure, for selected periods of time, continuous air temperatures simultaneously, using a group of as many as 20 exposed thermistors distributed between the ground and 300 ft. It is hoped that a combination of these measurements along perpendicular axes with supplemental hot-wire recordings of wind speed will enable us to make substantial contributions to the current knowledge of low-level atmospheric turbulence.

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