An Analytical Study on the Feedback between Large- and Small-Scale Eddies

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ABSTRACT

The feedback between large-scale stationary Rossby waves and small-scale high-frequency eddies is computed analytically in the framework of a barotropic and frictionless model atmosphere. The Rossby wave is meant to model blocking situations characterized by high-over-low dipoles and flow splitting, while the eddies represent synoptic disturbances propagating in the deformation field of the blocking pattern. Apart from a westward contribution to the phase speed of the large-scale pattern, the eddy forcing (Reynolds stress), averaged in time, turns out to be a nonlinear function of the large-scale streamfunction amplitude. It is shown that the eddy forcing always determines, in the time average, a steepening of the flow split associated with the block and (possibly) multiple large-scale amplitudes. A comparison between these results and previous studies about eddy forcing of blocking situations is attempted.

1. Introduction

It is well known that models of the atmosphere display significant low-frequency variations when zonally homogeneous, and time-independent boundary conditions are applied. It has been shown that an important source of low-frequency variability is given by the nonlinear interaction between large-scale features and synoptic-scale eddies through a mechanism known as quasigeostrophic turbulence. The work of MacVean (1985) provides a clear illustration of how low-frequency variability arises from initially weak inhomogeneities in the baroclinic wave activity. A feature commonly observed in this context is that high- and low-frequency fluctuations exhibit different types of anisotropy; while high-frequency eddies are elongated in the meridional direction, the low-frequency fluctuations have much longer zonal wavelengths and are often in opposition of phase between high and low latitudes.

These geometric properties, together with quasigeostrophic turbulence, are at the basis of some theoretical and observational studies of blocking persistence and amplification (Shutts 1983; Illari and Marshall 1983; Hoskins et al. 1983; Vautard et al. 1988a, Part I). These studies, conceived of after the idea originally proposed by Green (1977), identify with the eddy forcing, due to synoptic-scale disturbances, the main cause for blocking maintenance. The idea is that synoptic perturbations are, on average, stretched into filaments; conservation of energy and enstrophy thus requires that energy be transferred from the smaller to the larger scales. The bifurcation of the midlatitude jet stream associated with blocking dipoles is precisely a region of vigorous deformation where traveling perturbations are stirred and compressed. This characteristic has been recognized since the first studies about European blocking; Berrgren et al. (1949) report on the progressive reduction of the east–west scale of disturbances entering the region of large-scale flow splitting.

Vautard and Legras (1988b, Part II) show that Reynolds stresses due to small scales can lead to multimodality in the large scale part of the motion. From a very long time integration of a two-layer quasigeostrophic model they choose all the instantaneous system configurations that possess the same values in the large scale. The ensemble average of the Reynolds stress, computed over these "analogues of the large scale flow," turns out in general to be a nonlinear function of the large scales and provides a statistical description of the feedback between small- and large-scale eddies. As a result, Vautard and Legras find three statistically equilibrated large-scale patterns, two of which correspond to a zonal circulation and the other one to a blocked configuration associated with substantial flow splitting.

There is increasing evidence (Hansen and Sutera 1990; Molteni et al. 1990) that atmospheric, planetary-scale wave activity in the Northern Hemisphere shows, as a first approximation, a bimodal probability density distribution. After the work of Charney and Devore (1979), theories have been proposed (Benzi et al. 1986) that tend to explain observed bimodality in terms of multiple equilibria of highly truncated systems. The bending of the orographic resonance, which implies

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bimodality in the wave amplitude and unimodality in the zonal wind parameters, is achieved in Benzi et al. by projecting the streamfunction onto a meridional basis of nonsinusoidal eigenfunctions. If the maximum truncation to a single meridional mode is assumed, the Jacobian nonlinearity is retained in the form of the quadratic term $\delta a a_x$, $a(x, t)$ being the coefficient arising from the streamfunction projection. The physical significance, and mathematical consistency, of this nonlinearity has been questioned by Källén and Reinhold (1988) and discussed by Benzi et al. in a correspondence exchange, but no satisfactory conclusion has been reached. In this paper, it will be shown that such a quadratic nonlinearity has a dynamical origin, arising from the nonlinear feedback between large-scale flow and high-frequency eddies.

Malguzzi and Malanotte-Rizzoli (1985b) also employ the technique of streamfunction projection upon a basis of meridional modes, which are eigenfunctions of a particular Sturm-Liouville problem. They show that the same quadratic nonlinearity discussed above can lead to finite amplitude coherent structures that resemble localized blocking patterns characterized by high-over-low dipoles. One of the aims of the present work is to establish a link between these analytic solutions and eddy forcing due to synoptic eddies. In the following, it will be shown that, in the context of a barotropic model atmosphere, the time-averaged effect of high-frequency transients can be parameterized in terms of linear and nonlinear functions of the large scales, and, in particular, by a quadratic term of the form $\delta a a_x$.

2. A barotropic model

The simplest framework in which scale interaction can be studied is given by the conservation of potential vorticity in a barotropic, frictionless atmosphere. A periodic or infinite β-plane channel is considered for simplicity, where a uniform and time-independent zonal wind, denoted by U, is assumed to blow from west to east. The time evolution of the streamfunction Φ, which determines the deviations of the flow from the basic wind U, is governed by

$$\nabla^2\Phi_t + J(\Phi, \nabla^2\Phi + \beta y) + U\nabla^2\Phi_x = 0 \qquad (1)$$

$$\Phi = 0 \quad \text{at} \quad y = 0, L,$$

where J denotes the Jacobian operator; ∇^2, the Laplace operator; and where partial derivatives are denoted by subscripts. Equation (1) is dimensionless. In the following, a length scale of 10^6 m, time scale of 10^5 s, and velocity scale of 10 m s^{-1} will be assumed. Given these values, a dimensionless difference of one in the streamfunction field will correspond, in physical quantities, to 100 m of geopotential height. The solution of (1) is projected onto two meridional eigenfunctions of the Laplace operator; this is the minimal truncation that retains nonlinear wave–wave interactions. Let

$$\Phi = \phi_a + \phi_s,$$

where ϕ_a denotes the antisymmetric (around the channel center) part of the streamfunction, and ϕ_s is the symmetric one. Hence,

$$\phi_s = a_1(x, t)\sin(\pi y/L) \qquad (2a)$$

$$\phi_a = a_2(x, t)\sin(2\pi y/L). \qquad (2b)$$

The projection of Eq. (1) onto ϕ_a and ϕ_s yields

$$a_{1_{xxt}} - \frac{\pi^2}{L^2}a_{1_t} + (U + 2Ia_2)\left[a_{1_{xxx}} - \frac{\pi^2}{L^2}a_{1_x}\right] + \beta a_{1_x} + Ia_{2_x}a_{1_{xx}} - Ia_1a_{2_{xxx}} + 3I\frac{\pi^2}{L^2}a_1a_{2_x} - 2Ia_{1_x}\left[a_{2_{xx}} - 4\frac{\pi^2}{L^2}a_2\right] = 0, \qquad (3a)$$

$$a_{2_{xxt}} - 4\frac{\pi^2}{L^2}a_{2_t} + \beta a_{2_x} + U\left[a_{2_{xxx}} - 4\frac{\pi^2}{L^2}a_{2_x}\right] + I\left[2(a_{1_x})^2 - \frac{1}{2}(a_1^2)_{xx}\right]_x = 0, \qquad (3b)$$

where I denotes the interaction coefficient:

$$I \equiv \frac{2\pi}{L}\frac{1}{L}\int_0^L \cos(\pi y/L)\sin(2\pi y/L)\sin(\pi y/L)dy = \pi/(2L).$$

Equation (3a), which governs the time evolution of the symmetric part of the streamfunction, is a linear equation with coefficients depending on space and, in general, on time. Equation (3b) is linear as well in the amplitude of the antisymmetric component but is forced by the divergence of vorticity fluxes associated with the symmetric component; consequently, in the following, the term

$$I\left[2(a_{1_x})^2 - \frac{1}{2}(a_1^2)_{xx}\right]_x \qquad (4)$$

will be referred to as eddy forcing. This general split of the Jacobian operator determines basically different roles played by the symmetric and antisymmetric components of the motion: the antisymmetric part modifies the linear propagation of symmetric eddies which, in turn, force the antisymmetric motion. This simple property has been exploited by Shutts (1983) and Vautard et al. (1988) in their studies about eddy forcing on blocking patterns.

3. The blocking pattern

System (3) will be solved in the weak interaction limit; namely, when the eddy forcing (4) is small. This limit may easily be obtained, for instance, by assuming that the symmetric eddies have a small amplitude. Another possibility would be to consider the interaction coefficient I as a small number, which can be accomplished by taking a domain with large meridional extension. In the following, for the sake of simplicity, only the first case is worked out formally, although one can easily convince oneself that the same results hold, with minor modifications, in the second case as well.

At the lowest order, Eq. (3b) admits a class of very simple solutions, namely Rossby waves. A Galilean transformation to the reference frame moving with the phase speed of a particular Rossby wave does not formally change system (3), apart from a redefinition of the uniform wind U. Hence, without loss of generality, it may be assumed that the Rossby wave solution of (3b) is stationary. Accordingly, let

$$\phi_a^{(0)} = A \sin(k_0 x) \sin(2\pi y/L) \equiv a_2(x) \sin(2\pi y/L), \quad (5)$$

where

$$k_0^2 = \beta/U - 4\pi^2/L^2 \quad (6)$$

and the amplitude A is still arbitrary. In this study, solution (5) plays the role of the blocking dipoles of Shutts (1983) and Vautard et al. (1988); in the next sections, the feedback of the symmetric eddies on the blocking pattern (5) will be assessed.

4. Propagation of symmetric eddies

The linear propagation of symmetric disturbances on the deformation field given by (5) is governed by Eq. (3a).

Equation (3a) can be solved in the limit of short waves. In fact, the solution can be written in the following WKB approximation (see Bender and Orszag 1978):

$$a_1(x, t) = \mathrm{Re}\{\exp[k(S_0 + S_1/k + S_2/k^2 + S_3/k^3 + \cdots)]\}, k \geqslant 1. \quad (7)$$

After substitution of (7) into (3a), at the higher order in k, the eikonal equation is obtained:

$$S_{0_t} + (U + 2Ia_2)S_{0_x} = 0, \quad (8)$$

which has a solution:

$$S_0 = i\left[-Ut + \int_0^x \frac{U}{U + 2Ia_2(x')}\,dx'\right] + \text{const.} \quad (9)$$

Here $S_1(x, t)$ satisfies the following equation, obtained by grouping all terms at $O(k^2)$:

$$S_{1_t} + (U + 2Ia_2)S_{1_x} = 3Ia_{2_x}, \quad (10)$$

which admits the particular solution [the homogeneous solution of (10) can be incorporated into (9) without loss of generality]:

$$S_1 = \text{const} + \frac{3}{2}\ln(1 + 2Ia_2/U). \quad (11)$$

Apart from an arbitrary phase, the physical optics approximation of the symmetric eddy structure is, thus, given by

$$a_1 \sim \alpha\left(1 + 2\frac{I}{U}a_2\right)^{3/2} \times \cos\left\{k\left[-Ut + \int_0^x \frac{U}{U + 2Ia_2(x')}\,dx'\right]\right\}, \quad (12)$$

which defines a Rossby wave with the local wavenumber equal to

$$kS_{0_x} = k\frac{1}{1 + 2Ia_2(x)/U}.$$

In expression (12), α represents the arbitrary amplitude of the symmetric eddy and may be considered as the expansion parameter that renders the eddy forcing a small quantity. Expression (12) differs from the exact solution of (3a) by terms of order $1/k^2$.

In Fig. 1, the physical optics approximation is sketched, together with the blocking pattern (5). The physical parameters are set to typical values of the large-scale atmospheric circulation: the mean wind is 10 m s^{-1}, the meridional extension of the domain 6000 km, the gradient of the Coriolis parameter 1.6 $\times 10^{-11}$ m^{-1} s^{-1}, and the amplitude of the blocking wave 100 m. The zonal wavenumber of the blocking wave, given by (6), corresponds to a wavelength of about 9000 km, or to zonal wavenumber 3 at midlatitudes. In the example depicted in Fig. 1, the zonal wavenumber of the symmetric eddy is chosen to be five times that of the blocking wave, in order to guarantee the accuracy of the WKB expansion.

In the diffluent region (the left half of the domain in Fig. 1) the symmetric eddy shortens while its amplitude is reduced; it is in this region that energy is expected to be transferred to the blocking pattern. In fact, the energy density of short eddies, roughly pro-

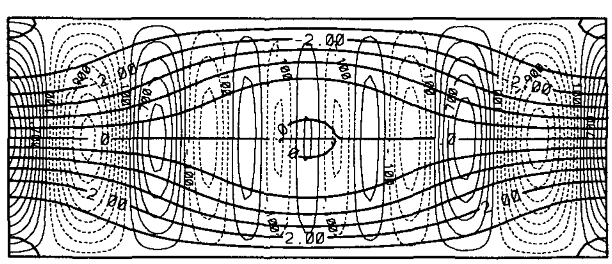

FIG. 1. Contour plot in the x–y plane of the physical optics approximation (12) (thin line positive values, dashed line negative ones). The parameters are: $A = 1$, $U = 1$, $\beta = 1.6$, $L = 6$, $k = 5k_0$, $\alpha^2 = 0.5$. The thick contours represent the blocking pattern (5) (contour interval is 0.5).

portional to the square of the amplitude multiplied by the square of the local wavenumber, decreases like $(1 + 2Ia_2/U)$, while enstrophy is increasing like $1/(1 + 2Ia_2/U)$ due to the shortening of the eddy scale. Exactly the opposite occurs in the confluent region (the right half of the domain): here the short, traveling wave receives, from the blocking pattern, the same amount of energy lost in the diffluent region. The block is thus reinforced upstream and weakened downstream in a symmetric fashion, at least in the frictionless scenario, energy being conserved on the average.

The next order in the WKB expansion is also solved, leading to the following phase and amplitude corrections:

$$S_{2_x} = -4ik_0^2 Ia_2/U + 4i\frac{\beta}{U^2}Ia_2(U + Ia_2)/(U + 2Ia_2), \quad (13)$$

$$S_{2_t} = i\beta, \quad (14)$$

$$S_3 = -3(Ia_2/U)(\beta/U + 8\pi^2/L^2) + (I^2a_2^2/U^2)(10\beta/U - 76\pi^2/L^2) + 3I^2(a_{2_x})^2/U^2 + \text{const.} \quad (15)$$

Equation (13) is the $O(1/k^2)$ correction of the local wavenumber, while Eq. (14) gives the correction $-\beta/k^2$ of the phase speed, which, in the physical optics approximation (12), is given by the constant U. Hence, it is at this order of the WKB expansion that symmetric eddies become dispersive. Finally, expression (15) gives the $O(1/k^2)$ eddy amplitude correction, which is an exponential function of the blocking amplitude. The exponential can be expanded in power series up to orders of $1/k^2$, without loosing mathematical consistency. In summary, the symmetric eddy expression reads:

$$a_1 = \alpha\left(1 + 2\frac{I}{U}a_2\right)^{3/2}(1 + S_3/k^2)[1 + O(1/k^4)] \times \cos\left\{k\left[-(U - \beta/k^2)t + \int_0^x \frac{U}{U + 2Ia_2(x')}dx' + S_2/i/k^2 + O(1/k^4)\right]\right\}. \quad (16)$$

In the next section, the eddy forcing associated with (16) is computed and parameterized in terms of a_2.

5. Eddy forcing

Substituting (16) into (4) and averaging in time over the symmetric eddy period, it turns out that at the lowest order in $1/k$ the following expression for the eddy forcing results:

$$2I^2\alpha^2k^2a_{2_x}/U. \quad (17)$$

At the next order, the correction to the eddy forcing is

$$a^2I\left\{(1 + 2Ia_2/U)\left[3I^2(a_{2_x})^2/U^2 - 2i(1 + 2Ia_2/U)S_{2_x} + 2S_3 + \frac{3}{2}(1 + 2Ia_2/U)\frac{I}{U}k_0^2a_2\right]\right\}_x. \quad (18)$$

Expression (17), which comes from the physical optics approximation alone, is a linear function in the large-scale wave amplitude with a positive coefficient proportional to the energy of small-scale eddies. Figure 2 shows the eddy forcing given by (17) for the synoptic situation and parameters used in Fig. 1. The eddy forcing consists in a quadrupolar pattern one-quarter wavelength out of phase with respect to the blocking wave. A positive (negative) maximum is located upstream of the blocking anticyclone (cyclone), hence implying, after (3b), anticyclonic (cyclonic) vorticity generation upstream of the blocking anticyclone (cyclone)—a feature also observed in many observational and numerical studies (see also Austin 1980; Illari 1984). Because of the east–west symmetry of eddy modification, anticyclonic (cyclonic) vorticity depletion is found one-quarter wavelength downstream of the blocking anticyclone (cyclone). A slow westward propagation of the blocking pattern should be expected as the consequence of upstream block reinforcement and downstream block weakening. In the next section this aspect is analyzed in a more formal way.

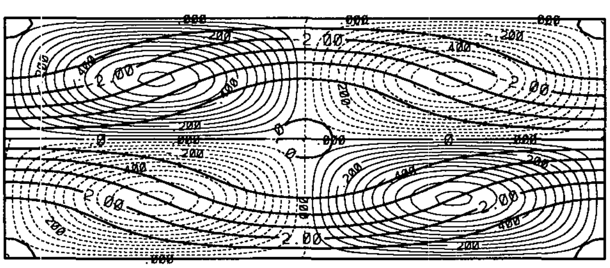

FIG. 2. Eddy forcing derived from the physical optics approximation alone (contour interval is 0.05). Parameters are as in Fig. 1.

Expression (18) is a nonlinear contribution to the time-averaged eddy forcing that, after (13) and (15), reduces to

$$\delta a_2 a_{2_x} + \delta_1 a_2^2 a_{2_x} + \delta_2(a_{2_x})^3 - 2\alpha^2I^2\left(\frac{9}{4}\beta/U + 11\frac{\pi^2}{L^2}\right)a_{2_x}/U, \quad (19)$$

TABLE 1. Absolute value of the ratio δ/δ_1 and δ/δ_2 for different values of the parameters U and L and with $\beta = 1.6$. The numerical value of δ, computed for $\alpha^2 = 0.5$, is also reported.

	$L = 6$, $U = 1$	$L = 6$, $U = 2$	$L = 10$, $U = 1$	$L = 10$, $U = 2$
$\left\|\frac{\delta}{\delta_1}\right\|$	7	6	5	212
$\left\|\frac{\delta}{\delta_2}\right\|$	15	27	11	19
δ	−0.62	−0.14	−0.06	−0.01

where

$$\delta = -6\alpha^2 I^3\left(\frac{\beta}{U} + 36\frac{\pi^2}{L^2}\right)/U^2 \quad (20a)$$

$$\delta_1 = 6\alpha^2 I^4\left(9\frac{\beta}{U} - 76\frac{\pi^2}{L^2}\right)/U^3 \quad (20b)$$

$$\delta_2 = 18\alpha^2 I^4/U^3. \quad (20c)$$

Expressions (17) and (19) give the parameterization of the eddy forcing due to small-scale eddies and constitute the main result of this work. In particular, the first term in (19) is formally identical to the nonlinearity introduced in Benzi et al. (1986) and Malguzzi and Malanotte-Rizzoli (1985b), which was obtained by truncating over a single nonsinusoidal meridional mode. It is also interesting to notice that the sign of δ, always negative in (20a), is consistent with the sign of δ given by expression (19) of Malguzzi and Malanotte-Rizzoli (1985b), hence implying a modification of the sinusoidal blocking wave in the direction of a more localized "modonlike" structure.

In expression (19), besides a linear term similar to (17) (but with opposite sign), other nonlinear terms of higher order in the large-scale wave amplitude appear. However, it turns out, as Table 1 clearly shows, that the quadratic term dominates the cubic ones.

Figure 3a shows the eddy forcing arising from the nonlinear terms alone while Fig. 3b shows the total eddy forcing, that is, the sum of contributions (17) and (19); the comparison with Fig. 2 clearly shows a localization of the eddy-forcing pattern around the center of the flow splitting. In the next section it is shown that this particular eddy-forcing structure can produce a substantial steepening of the splitting itself.

A more general solution for the eddies can be obtained by taking the superposition of solutions like (16) with different values of k. If a time average over a sufficiently long time interval is taken, it is easy to show that the eddy forcing becomes the sum over wavenumber k of expressions (17) and (19). Hence, the linear (nonlinear) contribution to the time-averaged eddy forcing turns out to be proportional to the energy (variance) of the high-frequency part of the motion.

6. Second-order induced flow

It is now possible to compute the correction to the blocking wave due to eddy forcing. The results in section 5 indicate the necessity of introducing a long time scale in the zero-order blocking wave (5), in order to avoid the arising of secularity. Hence, up to the second order in the eddy amplitude, a consistent definition of the antisymmetric part of the streamfunction, averaged in time, is

$$\overline{\phi_a} \approx \{A \sin[k_0(x - c^{(2)}t)] + a_2^{(2)}\} \times \sin(2\pi y/L), \quad (21)$$

where the overbar denotes the time average over the period of the high-frequency eddy. The time average of Eq. (3b), at the second order in α, becomes

$$\frac{1}{2}[\beta a_{2_x}^{(2)} + U(a_{2_{xxx}}^{(2)} - 4\pi^2/L^2 a_{2_x}^{(2)})] = -\mu k_0 A\cos(f) - \frac{\delta}{2}A^2 k_0 \sin(2f) - \frac{1}{4}\delta_1 A^3 k_0(\cos(f) - \cos(3f)) - \frac{1}{4}\delta_2 A^3 k_0^3(\cos(3f) + 3\cos(f)) - \frac{1}{2}\frac{\beta}{U}Ac^{(2)}k_0\cos(f), \quad (22)$$

where $f = k_0(x - c^{(2)}t)$ and μ denotes the sum of the linear coefficients in the eddy-forcing expression. In the derivation of (22) expressions (6), (17), (19), and (21) have been used. In Eq. (22) the terms in the rhs proportional to $\cos(f)$ are an order-zero solution of the lhs; the removal of these secularities determines the following expression for the slow phase speed:

$$c^{(2)} = -\frac{2U}{\beta}\left(\mu - \frac{1}{2}\delta_1 A^2 + \frac{3}{4}\delta_2 A^2 k_0^2\right). \quad (23)$$

After the secular terms have been removed, Eq. (22) finally yields the second-order induced flow:

$$a_2^{(2)} = B\cos(2f) + C\sin(3f),$$

$$B = -\frac{\delta A^2}{6Uk_0^2}, \quad C = -\frac{A^3(\delta_1 - k_0^2\delta_2)}{48Uk_0^2}. \quad (24)$$

Figure 4a shows the second-order induced flow, computed from the above formulas with the parameter setting used in the example of Fig. 1. The second-order streamfunction is antisymmetric in latitude and mainly consists of a zonal wavenumber $2k_0$; this is due to the dominance of the first term in the rhs of (22), both because δA^2 is the leading nonlinearity and because $\cos(2f)$ is closer to resonance. Figure 4b shows the total streamfunction averaged over a period of the symmetric eddy [expression (21)]. The second-order correction deepens the high-over-low blocking pattern and strengthens the jet immediately upstream and downstream of it; the net effect on the total streamfunction is the steepening of the jet split and its confinement in a region of shorter zonal extension. Note that this effect is obtained with all possible settings of the model parameters, the sign of B always being positive [see expressions (20a) and (24)].

The strong eddy-induced modifications observed in Fig. 4b are obtained by setting the amplitude of transient eddies to 70 m ($\alpha^2 = 1/2$). This is not a realistic value for a single-scale eddy of short wavelength that extends over the whole globe. However, as pointed out earlier, α^2 has to be identified with the spatial variance explained by the entire short-scale spectrum, rather than with the amplitude of a single wave. Moreover, on a more local scale, like in storm tracks and their adjacent regions, enhanced short-scale activity can be expected to force the large scales in a more vigorous way.

As a final consideration, it is possible to obtain second-order flow modifications of very large amplitudes by choosing a small value for k_0. In fact, while the eddy forcing goes to zero as k_0 goes to zero, the second-order induced flow diverges as $1/k_0^2$. The mathematical

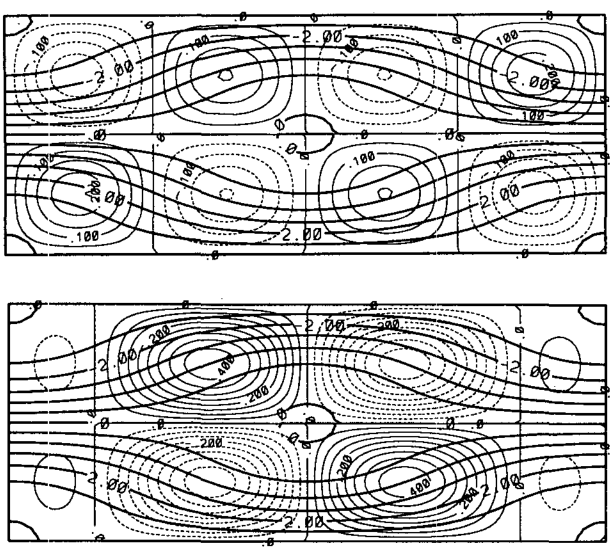

FIG. 3. Nonlinear contributions to the eddy forcing (a) and total eddy forcing (b). Contour interval is 0.05. Parameters are as in Fig. 1.

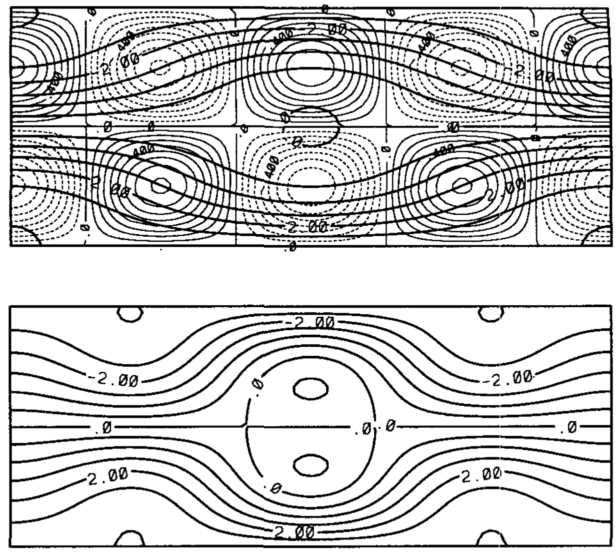

FIG. 4. Second-order induced flow (a) and total streamfunction (b) averaged over a period of the eddy. Contour intervals are 0.1 and 0.5, respectively. Parameters are as in Fig. 1.

reason for this apparently paradoxical behavior is that the nonlinear terms in Eq. (22) become secular as well. The elimination of the singularity would require the introduction of a long space scale in the lowest order streamfunction; more details on this subject are given in the discussion presented in the next section. This result suggests that large-scale flow modifications, caused by Reynolds stresses associated with the high-frequency part of the motion, should be expected, especially at the longest planetary scales.

7. Discussion and conclusions

In this work, the nonlinear feedback between large scales and high-frequency eddies has been analytically computed in the simplest model of the atmospheric circulation. The most important result emerging from this analysis is the retrieval of the nonlinear term which, in previous studies, has lead to multiple equilibration of topographic waves and to coherent structures; here it is shown that this term, which in Benzi et al. (1986) and Malguzzi and Malanotte-Rizzoli (1985b) arises from the streamfunction projection upon nonsinusoidal functions, has a dynamical origin. Another strong analogy with the previous studies is that the nonlinearity appears in the evolution equation of the antisymmetric part of the motion, due to the particular properties of the Jacobian operator.

As far as the (possibly multiple) equilibration of the large-scale blocking wave is concerned, it has to be noted, from the procedure employed in the previous section to solve Eq. (22), that the wave amplitude A

still remains an arbitrary quantity. Nevertheless, there is the possibility that multiple equilibration be achieved at the next order in the eddy-forcing expansion, similar to the case studied by Benzi et al. (1986). This subject is not further pursued here. An interesting exception is, however, given by the case of small k_0. It is easy to show that when $k_0^2 \ll 1$ (or $\beta/U - 4\pi^2/L^2 \ll 1$)[1] Eqs. (16), (17), (19), and (20a,b) become mathematically consistent expressions for any given function $a_2(x)$ of the slow variable x, and that the equilibration problem for a_2 reduces to the following Korteweg–de Vries equation modified by the introduction of a small, cubic nonlinear term associated to δ_1:

$$\frac{U}{2}[a_{2_{xxx}} + k_0^2 a_{2_x}] + \mu a_{2_x} + \delta a_2 a_{2_x} + \delta_1 a_2^2 a_{2_x} = 0. \tag{25}$$

In this case, Eq. (25) completely determines the feedback on the blocking wave, which may lead to multiple solutions (one solution is always given by $a_2 = 0$) of localized or wavy type, depending on the sign of $k_0^2 + 2\mu/U$. From the formal point of view, this analysis presents strong analogies with the case studied in Malguzzi and Malanotte-Rizzoli (1984a), although it must be pointed out that the similarities are more formal than physical. The formal analogy follows from the hypothesis that Rossby dispersion balances nonlinearity, while the physical differences are that in this work the blocking wave is a finite-amplitude "statistical" solution maintained by Reynolds stresses, rather than an exact, weak amplitude solution of the governing equations.

Dissipation and, perhaps more importantly, baroclinic effects have been ignored in this study. In the real atmosphere, synoptic activity is modulated on a long space scale through the generation of storm tracks, where perturbations grow and equilibrate by means of processes not included in this model. However, the present model may describe the essential dynamics of the scales of motion in the inertial range, characterized by zonal scales ranging from the most unstable baroclinic scale to the dissipation scale. Experimental and observational evidence in this sense comes from the study of Mullen (1987). He examines the net forcing of blocking flows by high-pass–filtered synoptic eddies finding, in all the cases analyzed, a quadrature relationship with the blocking pattern, with anticyclonic (cyclonic) eddy forcing located one-quarter wavelength upstream (downstream) of the blocking anticyclone. Robinson (1991) examines the low-frequency variability of a simple GCM and finds that the vorticity flux by synoptic waves reinforces the low-frequency eddies and retards their eastward propagation, with a clear out-of-phase relationship between the low-frequency streamfunction and eddy forcing. Finally, in the numerical experiment performed by Malanotte-Rizzoli and Malguzzi (1987c), a pattern of eddy forcing in clear qualitative agreement with that of Fig. 3b (see their Fig. 8) is found, but in the context of a two-layer quasigeostrophic model where eddies spontaneously originate from the baroclinic instability of the zonal wind.

The idea of nonlinear eddy forcing upon large scales deserves further analysis, both from the theoretical and numerical point of view. One unsatisfactory aspect of the theory proposed here is the meridional truncation employed in section 2. An accurate comparison between the eddy structure shown in Fig. 1 and the same quantity computed by Shutts (1983) reveals some deficiencies linked to the poor meridional resolution. In fact, real synoptic perturbations tend to split somewhat as they enter the blocking region with maxima being carried away along the northern and southern branch of the jet. Moreover, the role played by the storm tracks in the dynamics of planetary waves is an important topic that needs investigation and will be the subject of future work.

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[1] This condition is fulfilled when the largest planetary scale becomes stationary. With a meridional scale of 6000 km and mean wind of, roughly, 14 m s^{-1} it turns out $k_0^2 \approx 0.05$, which corresponds to a zonal wavelength of 28 000 km.

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