One-Parameter Scaling and Exponential-Sum Fitting for Water Vapor and CO_2 Infrared Transmission Functions

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ABSTRACT

A medium-sized band model for water vapor and CO_2 absorption is developed using the one-parameter scaling approximation. The infrared spectrum is divided into 10 bands. The Planck-weighted diffuse transmittance is reduced to a function dependent only upon the scaled absorber amount and fit by an exponential sum. By selecting specific sets of absorption coefficients for exponential-sum fitting, computations of fluxes and cooling rate are made very fast. Compared to a broadband model, the accuracy, speed, and versatility are all enhanced. With absorption due to water vapor line, continuum, CO_2 as well as O_3 included, the parameterization introduces an error of <1.5 W m^{-2} in fluxes and <0.15°C day^{-1} in the tropospheric and lower stratospheric cooling rates.

1. Introduction

In the one-parameter scaling approximation (Goody and Yung 1989), the absorber amount along a nonhomogeneous path is scaled in such a way that the transmission can be computed from the scaled absorber amount and a reference pressure, p_r, and temperature, T_r. This scaling of the absorber amount is equivalent to computing the absorption coefficient by extrapolating from the values at p_r and T_r. Generally, the extrapolation can only be applied to conditions not far from p_r and T_r.

Chou et al. (1991) and Chou and Peng (1983) used the one-parameter scaling approximation to compute water vapor and CO_2 cooling rates in the troposphere and the lower stratosphere. The absorption bands were grouped into two noncontiguous regions: one associated with the band centers and the other associated with the band wings. The reason for grouping the spectrum into such wide spectral regions is to enhance the speed of computation. In this study we will demonstrate that this grouping of spectral regions is not necessarily the most efficient. Flux calculations can be made more efficient by dividing the spectrum into narrower bands for the following reasons. (i) Temperature dependence of absorption can be taken into account more accurately. (ii) The transmittance is reduced to a function only of the scaled absorber amount and can be computed rapidly. (iii) The multiplication approximation for overlapping absorptions can be applied. The increase in speed due to factors (i) and (iii) more than compensates for the reduction in speed due to the use of more spectral bands. (iv) Flux computations can be made more versatile. These advantages of dividing the spectrum into more bands are addressed in this study.

Because of the use of one-parameter scaling, parameterizations developed in this study cannot be applied to accurately computing the cooling rate in the upper stratosphere above the 10-mb level. The transmittance parameterization for the 9.6-μm O_3 band using the one-parameter scaling approximation is found to be less satisfactory than the corresponding parameterizations for water vapor and CO_2 bands. Therefore, the parameterization of Chou and Kouvaris (1991), which uses a two-parameter scaling model and precomputed transmission tables, is used in this study for computing absorption in the O_3 band.

2. Line-by-line calculations

The net upward flux at the pressure level p integrated over a spectral interval, $\Delta\nu$, can be computed from

$$F(p) = \int_{\Delta\nu} d\nu \{ B_\nu(T_s)\tau_\nu(p, p_s) - \int_0^{p_s} B_\nu[T(p')][\partial\tau_\nu(p, p')/\partial p']dp' \}, \quad (1)$$

where ν is the wavenumber, B is the Planck flux, T is the temperature, the subscript s denotes the surface, $\tau_\nu(p, p')$ is the diffuse transmittance given by

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$$\tau_\nu(p, p') = 2\int_0^1 \exp\left\{-\int_{p'}^{p} q(p'')k_\nu[p'', T(p'')]dp''/(g\mu)\right\} \times \mu d\mu, \quad (2)$$

q is the mixing ratio of an absorber, k is the absorption coefficient, g is the gravitational acceleration, and μ is the cosine of the angle between a beam and the vertical. From (1), the contribution of a layer, centered at $p_{i+1/2}$, to the flux at the pressure p is, in a finite-difference form,

$$\int_{\Delta\nu} \{B_\nu[T(p_{i+1/2})][\tau_\nu(p, p_i) - \tau_\nu(p, p_{i+1})]\}d\nu,$$

where p_i and p_{i+1} are the pressures at the boundaries of the layer $i + 1/2$. Therefore, the relevant term in computing fluxes from (1) is

$$\Delta F(p, p') = \int_{\Delta\nu} B_\nu[T(p')]\tau_\nu(p, p')d\nu. \quad (3)$$

Our line-by-line calculations of the absorption coefficient use the Air Force Geophysical Laboratory 1986 edition of the molecular absorption parameters (Rothman et al. 1987). The absorption coefficient is computed at spectral intervals of 0.01 cm^{-1} for water vapor and 0.002 cm^{-1} for CO_2 using the algorithm of Armstrong (1967) for the Voigt line profile. The use of these spectral intervals may not be able to resolve individual lines in the stratosphere but is adequate for computing fluxes and cooling rates of a wide spectral band. Molecular lines located beyond 10 cm^{-1} of a spectral point are excluded in computing the absorption coefficient of that point. This is equivalent to a line cutoff of 10 cm^{-1}. The line half-width is assumed to be proportional to $pT^{-0.5}$. The temperature dependence of the line width varies with broadening gas and the energy level and transition of a line (Delaye et al. 1989). It has been shown by Chou and Kouvaris (1986), however, that the transmission function averaged over a spectral band changes only slightly for a change in the temperature dependence of the line width from $T^{-0.5}$ to $T^{-0.75}$. As in Ridgway et al. (1991), the diffuse transmittance, that is, the third exponential integral, is evaluated from table look-up.

Fluxes and cooling rate are computed using the line-by-line method for five widely different atmospheres taken from McClatchey et al. (1972). The atmosphere is divided into 60 layers with $\Delta p \approx 25$ mb at pressures greater than 100 mb and $\Delta \log_{10} p \approx 0.15$ at pressures smaller than 100 mb. Temperature and the logarithm of humidity are assumed to be linear in the logarithm of pressure. Figure 1 shows the cooling rate profiles due to water vapor "line" absorption for the five atmospheres. The contribution from the far wings of a line (beyond the cutoff of 10 cm^{-1}), which constitutes the "continuum" absorption, is not included. The spectral integration covers the entire thermal infrared

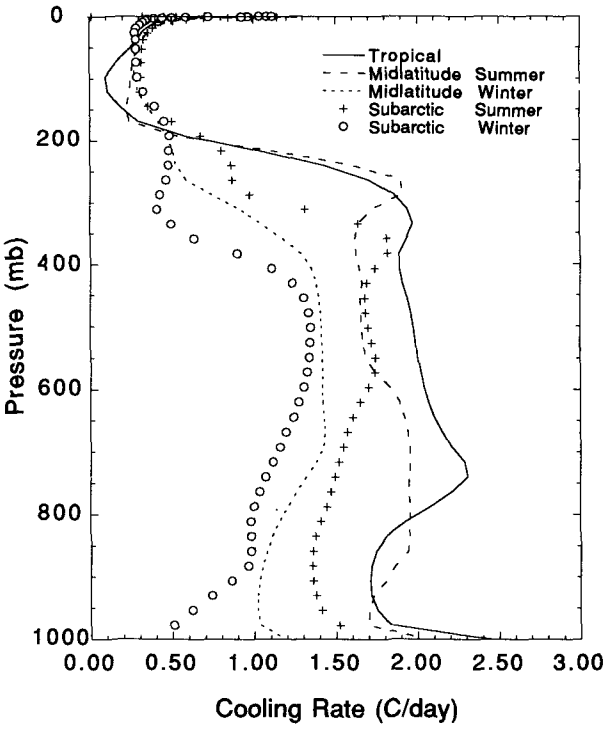

FIG. 1. Cooling rate profiles computed using a line-by-line method for five atmospheres taken from McClatchey et al. (1972). The cooling is due to water vapor line absorption only.

(0–3000 cm^{-1}). It is well acknowledged that the shape at the far wings of a line does not follow the Lorentz function and that the line cutoff should be related to the treatment of continuum absorption (Clough et al. 1992). Because the line cutoff of 10 cm^{-1} is somewhat arbitrary, it is important to know its effect on cooling rate calculations. Figure 2 shows the cooling rate profiles of a midlatitude summer and a midlatitude winter atmosphere using two different choices of line cutoff, 6 cm^{-1} and 10 cm^{-1}. As in Fig. 1, the cooling is due to water vapor line absorption only. The difference between the two sets of cooling rate calculations is generally small, except in the 300–500-mb region where the difference exceeds 0.1°C day^{-1}. The difference in fluxes is ~4 W m^{-2} at the top of the atmosphere (TOA) and ~2 W m^{-2} at the surface. In the rest of this study, all calculations are based on a line cutoff of 10 cm^{-1}.

3. One-parameter scaling approximation

In order to compute efficiently the transmission function of a nonhomogeneous path, it is necessary to scale the path to an equivalent homogeneous path. The two-parameter scaling approximation, such as the Curtis–Godson approximation or the pressure and temperature scaling of Chou et al. (1991), can be applied to a path encompassing both the stratosphere and troposphere. The transmission function is dependent upon three variables: the absorber amount and the two

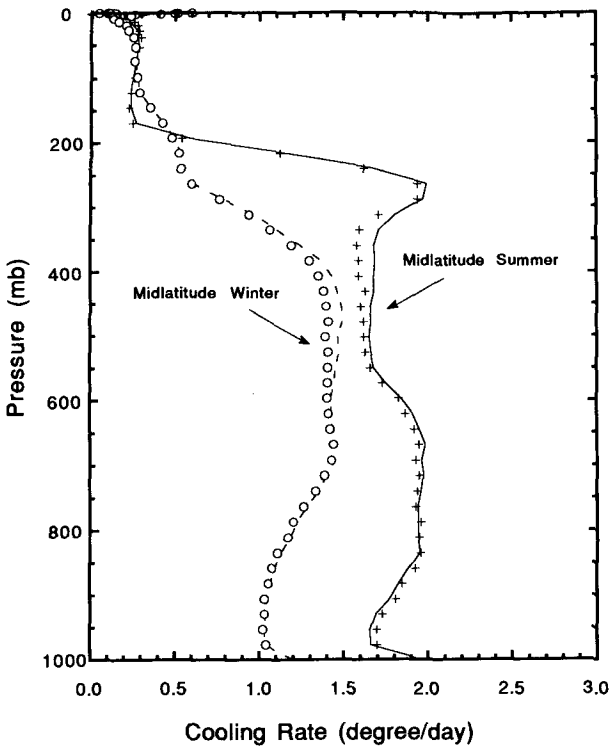

FIG. 2. Cooling rate profiles from line-by-line calculations with a 10 cm^{-1} line cutoff (solid and dashed curves) and a 6 cm^{-1} line cutoff (circles and pulses). The cooling is due to water vapor line absorption only.

scaled parameters. When more than one absorber is involved, the transmission is a function of four or more variables. Accurate calculations of this function become very difficult.

As an alternative to two-parameter scaling, one-parameter scaling reduces a nonhomogeneous path to an equivalent homogeneous path with a reference pressure, p_r, and a reference temperature, T_r. The transmission along a path using the one-parameter scaling approximation is a function only of the scaled absorber amount.

Chou and Arking (1980) and Chou and Peng (1983) demonstrated that the absorption near line center regions saturates quickly, and an accurate treatment of the absorption in these regions is not critical in radiative transfer calculations. Therefore, they used a one-parameter scaling based on the shape of the wings of a line. For this wing-scaling approximation, the absorption coefficient is extrapolated from the absorption coefficient at the reference pressure and temperature according to

$$k_\nu(p, T) = k_\nu(p_r, T_r)(p/p_r)^m f(T; T_r), \quad (4)$$

where m is a positive number generally close to but less than 1, f is the scaling factor for temperature.

The diffuse transmittance of an atmospheric layer with pressure p, temperature T, and the absorber amount u is given by

$$\tau_\nu(u, p, T) = 2\int_0^1 \exp[-k_\nu(p, T)u/\mu]\mu d\mu. \quad (5)$$

With the use of the one-parameter scaling, the diffuse transmittance is reduced to

$$\tau_\nu(u, p, T) \approx \tau_\nu(w) = 2\int_0^1 \exp[-k_\nu(p_r, T_r)w/\mu]\mu d\mu, \quad (6)$$

where w is the scaled absorber amount for water vapor (or CO_2) molecular line absorption given by

$$w = u(p/p_r)^m f(T; T_r). \quad (7)$$

From (4) and (7), it can be seen that scaling of the absorption coefficient is equivalent to scaling of the absorber amount. Because the absorption coefficient is extrapolated from the absorption coefficient at reference pressure and temperature, the one-parameter scaling approximation works better for p and T closer to the reference pressure and temperature. For water vapor, we choose p_r and T_r to be 500 mb and 250 K, which are medium values in the troposphere. The value of m is chosen to be 1 for all water vapor bands. In the 15-μm band, the absorption due to CO_2 overlaps strongly with the absorption due to water vapor. The lower troposphere is quite opaque in this spectral region, and accurate treatments of the absorption are not critical to flux calculations. Therefore, pressures representative of the upper troposphere and the lower stratosphere are chosen for p_r. For a spectral band with stronger absorption, the peak of the cooling profile shifts to a higher altitude, and a smaller reference pressure should be used. Also for a spectral band with stronger absorption, the spectral regions away from line centers become more important because the atmosphere is more opaque near line center regions. Therefore, the value of m should be chosen closer to 1 for a spectral band with stronger absorption. Following Chou and Peng (1983), the reference pressure is chosen to be 30 mb and 300 mb, respectively, for the center region and the wing regions of the 15-μm CO_2 band, and the value of m is chosen to be 0.85 and 0.5, respectively, for the center region and the wing regions of the absorption band. As in the water vapor bands, the reference temperature is chosen to be 250 K. Values of m, p_r, and T_r are given in Table 1 for the 10 bands used in this study.

For the water vapor continuum absorption, the absorption coefficient increases with increasing water vapor partial pressure but with decreasing temperature. The scaled water vapor amount for continuum absorption is computed from (Bignell 1970; Roberts et al. 1976)

$$w = ue\exp[1800(1/T - 1/296)], \quad (8)$$

where e is the water vapor partial pressure in atmospheres. The units of temperature T are in kelvin.

TABLE 1. Spectral bands, reference pressure and temperature, and the coefficients for the pressure and temperature scaling. Units are K^{-1} for a and K^{-2} for b.

Band	Spectral range (cm^{-1})	H_2O p_r (mb)	H_2O T_r (K)	H_2O m	H_2O a	H_2O b	CO_2 p_r (mb)	CO_2 T_r (K)	CO_2 m	CO_2 a	CO_2 b
1	0–340	500	250	1.0	.0021	-1.01×10^{-5}					
2	340–540	500	250	1.0	.0140	5.57×10^{-5}					
3	540–620	500	250	1.0	.0149	6.20×10^{-5}	300	250	0.5	.0179	1.02×10^{-4}
4	620–720	500	250	1.0	.0199	5.57×10^{-5}	30	250	0.85	.0042	2.00×10^{-5}
5	720–800	500	250	1.0	.0231	1.70×10^{-4}	300	250	0.5	.0184	1.12×10^{-4}
6	800–980	500	250	1.0	.0302	2.96×10^{-4}					
7	980–1100	500	250	1.0	.0307	2.86×10^{-4}					
8	1100–1380	500	250	1.0	.0154	7.53×10^{-5}					
9	1380–1900	500	250	1.0	.0008	-3.52×10^{-6}					
10	1900–3000	500	250	1.0	.0096	1.64×10^{-5}					

a. *Pressure scaling*

The validity of pressure scaling is investigated by comparing two sets of calculations. One set of calculations uses the "exact" line-by-line method, which takes into account properly the dependence of the absorption coefficient on pressure and temperature. The other set of the calculations is the same as the "exact" line-by-line calculations, except the absorption coefficient is extrapolated from

$$k_\nu(p, T) = k_\nu(p_r, T)(p/p_r)^m. \quad (9)$$

In both sets of calculations, the effect of temperature on the absorption is correctly taken into account. Therefore, the difference between these two sets of calculations is due to pressure scaling.

The downward flux at the surface, $F\downarrow_{sfc}$, and the upward flux at TOA, $F\uparrow_{top}$, for these two sets of calculations are given in the first two rows of Table 2. The clear midlatitude summer atmosphere given in McClatchey et al. (1972) is used. The calculations cover the entire thermal infrared (0–3000 cm^{-1}) for the water vapor absorption and the 15-μm band (540–800 cm^{-1}) for the CO_2 absorption. Water vapor continuum absorption is not included. The error of the pressure scaling is <1.2 W m^{-2} in the surface flux and <0.5 W m^{-2} in the TOA flux. The differences between the "exact" cooling rate and the cooling rate using the pressure-scaling approximation are shown in Figs. 3 and 4 for the water vapor and CO_2 absorption, respectively. Except in the upper stratosphere, the difference is smaller than 0.05°C day^{-1} for both the water vapor and CO_2 bands.

TABLE 2. The downward flux at the surface, $F\downarrow_{sfc}$, and the upward flux at the top of the atmosphere, $F\uparrow_{top}$, for a clear midlatitude summer atmosphere computed with a line-by-line method and different approximations. Units of the fluxes are watts per square meter. Water vapor continuum absorption is not included.

	Water vapor (0–3000 cm^{-1})		CO_2 (540–800 cm^{-1})	
	$F\downarrow_{sfc}$	$F\uparrow_{top}$	$F\downarrow_{sfc}$	$F\uparrow_{top}$
"Exact" line-by-line	265.5	336.3	74.91	73.07
Pressure scaling	266.7	336.2	75.06	72.61
Temperature scaling	263.1	336.6	74.55	71.93
Equation (18)	266.5	335.0		

b. *Temperature scaling*

The temperature dependence of absorption is primarily due to the effect on line intensity. This effect varies strongly with wavenumber. It is small near the band center region but is very large away from it. Figure 5 shows the spectral variation of the ratio of water vapor

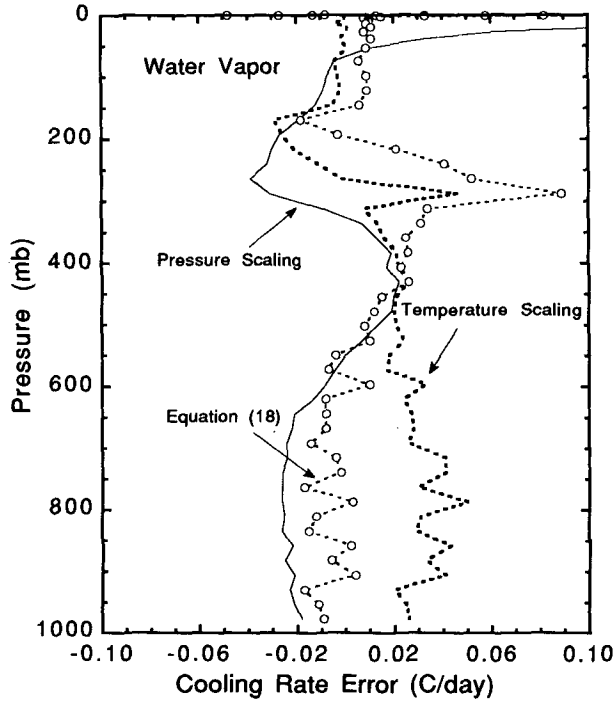

FIG. 3. Error in the water vapor cooling rate calculations induced by the pressure scaling, the temperature scaling, and the approximation for the Planck-weighted diffuse transmittance, $\tau(p, p', T') \approx \tau(p, p'; 250\ K)$ for a clear midlatitude summer atmosphere.

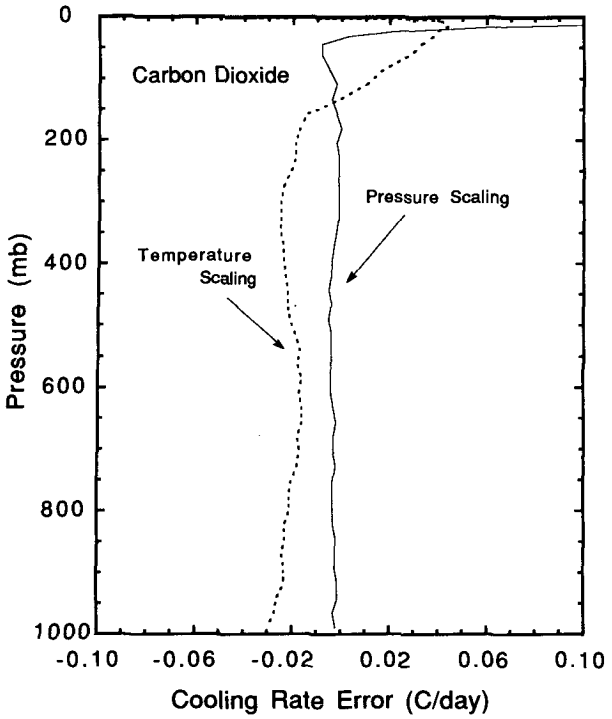

FIG. 4. Error in the CO_2 cooling rate calculations induced by the pressure and temperature scaling for a clear midlatitude summer atmosphere.

line intensities in 20 cm^{-1} intervals at different temperatures. It can be seen that between absorption bands, the line intensity depends strongly on temperature. Near the 10-μm window region, the line intensity increases by a factor of 2–6 for an increase of 50°C in temperature. In the band center regions ($\nu < 300$ cm^{-1} and $\nu \approx 1600$ cm^{-1}), the intensity depends only weakly on temperature. These different temperature dependencies of the line intensity is one of the bases for separating the band-center regions form the band-wing regions in parameterizing the broadband transmittance by Chou and Arking (1980). It can also be seen from Fig. 5 that the line intensity depends nonlinearly on temperature; the dependency is stronger at lower temperature.

Except for a narrow-band model, the spectral variation of the Planck function must be taken into consideration in computing the mean transmittance of a band. The flux term ΔF can be expressed as

$$\Delta F(p, p') = \int_{\Delta\nu} B_\nu(T')\tau_\nu(p, p')d\nu = B(T')\tau(p, p', T'), \quad (10)$$

where $\tau(p, p', T')$ is the Planck-weighted diffuse transmittance of a band defined by

$$\tau(p, p', T') = \int B_\nu(T')\tau_\nu(p, p')d\nu \Big/ \int B_\nu(T')d\nu, \quad (11)$$

$B(T')$ is the Planck flux integrated over the band,

$$B(T') = \int_{\Delta\nu} B_\nu(T')d\nu, \quad (12)$$

and T' is the temperature at p'.

In determining the temperature scaling factor, $f(T; T_r)$, the following procedures are taken. (i) The spectrum is divided into 10 bands shown in Table 1. (ii) The diffuse transmittance, $\tau_\nu(u, p_r, T)$, is computed as a function of u from (5) for three temperatures, T = 200, 250, and 300 K. (iii) The Planck-weighted diffuse transmittance, $\tau(u, p_r, T; T')$, is computed as a function of u for T' = 250 K and three values of T, 200, 250, and 300 K,

$$\tau(u, p_r, T; T') = \int B_\nu(T')\tau_\nu(u, p_r, T)d\nu \Big/ \int B_\nu(T')d\nu. \quad (13)$$

(iv) The transmittance $\tau(u, p_r, 250\text{ K}; T')$ is shifted by a constant, f_1, along the u axis so that the root-mean-square difference between $\tau(u, p_r, 200\text{ K}; T')$ and $\tau(f_1u, p_r, 250\text{ K}; T')$ is minimized. (v) Another constant, f_2, is obtained by shifting $\tau(u, p_r, 250\text{ K}; T')$ so that the difference between $\tau(u, p_r, 300\text{ K}; T')$ and $\tau(f_2u, p_r, 250\text{ K}; T')$ is minimized. (vi) Finally, the temperature scaling factor is fit to f_1 and f_2 using the function

$$f(T; T_r) = 1 + a(T - T_r) + b(T - T_r)^2, \quad (14)$$

where T_r = 250 K, and the constants a and b are determined for each of the 10 bands. Values of a and b for the water vapor and the 15-μm CO_2 bands are given in Table 1. It is noted that a similar temperature scaling was used by Chou et al. (1991) where the water vapor bands are grouped into a band center region and a band wing region. Because the spectral bands of this study are narrower than that of Chou et al. (1991),

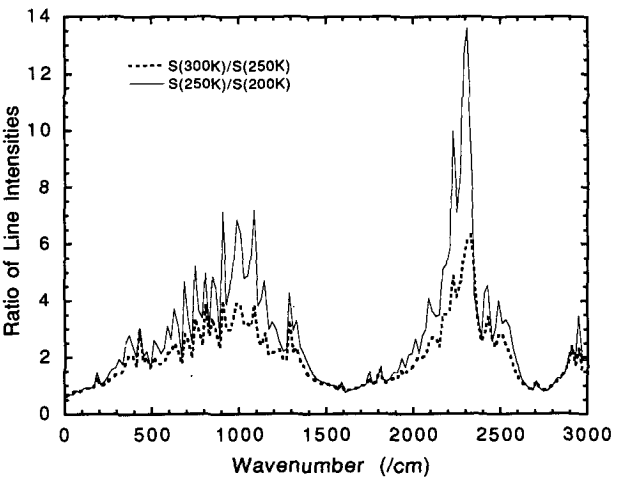

FIG. 5. The ratios of line intensities at different temperatures in 20 cm^{-1} intervals; S is the integrated line intensity.

the variation of the temperature dependence of line intensity within a band is also smaller. Therefore, the temperature scaling of this study can be expected to be more accurate than that of Chou et al. (1991).

Similar to the pressure scaling, the validity of the temperature scaling is investigated by comparing the "exact" line-by-line calculations with the calculations using the following temperature scaling for the absorption coefficient:

$$k_\nu(p, T) = k_\nu(p, T_r) f(T; T_r). \quad (15)$$

The effect of pressure on the absorption coefficient is correctly taken into account in both sets of calculations. Therefore, the difference between these two sets of calculations is due to the error induced by the temperature scaling. Fluxes at the surface and at TOA computed with the temperature scaling are shown in Table 2 for water vapor and CO_2 absorption. The error is -1.0 W m^{-2} in the upward flux at TOA and is -2.4 W m^{-2} in the downward flux at the surface. The cooling rate errors due to the temperature scaling are shown in Figs. 3 and 4, respectively, for the water vapor and CO_2 absorption. They are smaller than 0.05°C day^{-1} throughout the entire atmosphere.

c. *Approximation for Planck-weighted diffuse transmittance*

Equation (11) shows that the mean diffuse transmittance of a spectral band depends not only on the scaled absorber amount but also on the temperature of the layer where radiation originates. This temperature dependence can be neglected only if the band is narrow. For the IR spectrum divided into ten bands as shown in Table 1, this temperature dependency cannot be neglected, but the Planck-weighted diffuse transmittance can be approximated by

$$\tau(p, p', T') \approx \tau(p, p'; T_0) = \int_{\Delta\nu} B_\nu(T_0)\tau_\nu(p, p')d\nu / B(T_0), \quad (16)$$

and the flux term ΔF can be simply computed from

$$\Delta F(p, p') = B(T')\tau(p, p'; T_0), \quad (17)$$

where T_0 is chosen to be 250 K.

Based on (17), the validity of the approximation (16) is investigated by carrying out line-by-line calculations with (3) replaced by

$$\Delta F(p, p') = \left[\int_{\Delta\nu} B_\nu(T_0)\tau_\nu(p, p')d\nu\right] \times [B(T')/B(T_0)]. \quad (18)$$

The fluxes at the surface and at TOA computed for the water vapor absorption using the line-by-line method but with the approximation of (18) are given in Table 2. The difference between these fluxes and the "exact" fluxes is the error arising from the approximation of $\tau(p, p', T')$ by $\tau(p, p'; T_0)$. The error is 1.0 W m^{-2} at the surface and -1.3 W m^{-2} at TOA. The corresponding error in the cooling rate is shown in Fig. 3. Except near the 300-mb region, the cooling rate error is <0.05°C day^{-1}.

4. Exponential-sum fitting (ESFT)

The Planck-weighted diffuse transmittance at the reference pressure and temperature for each of the 10 bands are precomputed using the line-by-line method. Because the transmittance is a function of a single variable using the one-parameter scaling approximation, it can be easily fit by a function that mimics the Goody band model (e.g., Chou et al. 1991), by Pade approximants (Morcrette et al. 1986), or by an ESFT (e.g., Wiscombe and Evans 1977; Asano and Uchiyama 1987; Wang and Shi 1988; Lacis and Oinas 1991).

It has been shown by Chou and Arking (1980) that the diffuse transmittance, given by the third exponential integral, $2E_3(k_\nu u)$, can be approximated by $\exp(-1.66k_\nu u)$, where the constant 1.66 is the diffusivity factor. We have applied this diffuse approximation to the line-by-line calculations of cooling rate and compared the results with the "exact" line-by-line calculations. Line-by-line calculations show that the diffuse approximation induces an error of <0.05°C day^{-1}. Thus, the Planck-weighted diffuse transmittance for a band can be approximated by

$$\tau(w; T_0) = \sum_{i=1}^{m} f(k_i) \exp(-1.66k_i w), \quad (19)$$

where $f(k)$ is the k-distribution function at the reference pressure and temperature derived from line-by-line calculations. The value of k normally ranges by a few decades in a spectral band. Errors arise from using a limited number of terms in (19). Instead of directly using (19), we have made slight adjustments to the k-distribution function so that the Planck-weighted diffuse transmission function is best fit by an exponential sum,

$$\tau(w; T_0) = \sum_{i=1}^{m} c_i \exp(-1.66k_i w), \quad (20)$$

where c_i are the regression coefficients. It is found that a maximum of six values of k (i.e., $m = 6$) is adequate for computing the diffuse transmittance. In order to preserve the physical meaning of the coefficients, that is, the k-distribution function, we impose two conditions in the regression. (a) The sum of c_i is required to be 1. This condition is met by assigning a very large weight for $w = 0$ in the regression. (b) The difference between c_i and $f(k_i)$ is required to be small. This condition is met by fixing alternate terms of c_i (for example, $i = 2, 4, 6$) but deriving the other terms ($i = 1, 3, 5$) from regression. Because the adjustment involves only three coefficients, these coefficients can only differ slightly from $f(k_i)$.

TABLE 3. Coefficients for the transmission function given by Eq. (20) due to water vapor line absorption; k_1 is the first absorption coefficient, and n is the constant that $k_i = nk_{i-1}$. Units of k are square centimeter per gram.

	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6	Band 7	Band 8	Band 9	Band 10
k_1	1.78×10^{1}	2.51×10^{-1}	6.40×10^{-2}	8.00×10^{-3}	1.00×10^{-3}	3.16×10^{-4}	3.16×10^{-4}	1.41×10^{-3}	7.95×10^{-1}	3.16×10^{-4}
n	6	6	8	8	8	6	6	8	6	16
c_1	0.2747	0.1521	0.3153	0.2375	0.2180	0.4654	0.5543	0.1845	0.0740	0.1436
c_2	0.2717	0.3974	0.4604	0.4309	0.4468	0.2991	0.2723	0.2732	0.1636	0.2197
c_3	0.2752	0.1778	0.1326	0.2376	0.2214	0.1343	0.1131	0.2353	0.4174	0.3185
c_4	0.1177	0.1826	0.0798	0.0482	0.0802	0.0646	0.0443	0.1613	0.1783	0.2351
c_5	0.0352	0.0374	0.0119	0.0458	0.0253	0.0226	0.0160	0.1146	0.1101	0.0647
c_6	0.0255	0.0527			0.0083	0.0140		0.0311	0.0566	0.0184

Calculations of the Planck-weighted diffuse transmission function using (20) is very fast for two reasons. (1) For each k, it requires only l exponential operations, where l is the number of atmospheric layers. As a comparison, the commonly used band models require at least $0.5l^2$ exponential operations. (2) Values of k are chosen in such a way that $k_i = nk_{i-1}$, where n is a positive integer. With this choice of k, only a set of l exponential operations for the first value of k is needed. The other exponential terms can be derived from that for the previous value of k, with each derivation involving only ≈ 3 multiplication operations. Because a multiplication operation is ≈ 17 times faster than an exponential operation, flux calculations can be greatly accelerated by using (20) for computing transmittances.

The first absorption coefficient, k_1, the constant n, and values for the coefficient, c_i, are given in Tables 3, 4, and 5 for the water vapor line absorption, the CO_2 absorption, and the water vapor continuum absorption, respectively. Figures 6, 7, and 8 show, respectively, the Planck-weighted diffuse transmittances for the water vapor line absorption, the CO_2 absorption, and the water vapor continuum absorption. The solid curves are computed monochromatically at 0.01 cm^{-1} intervals for water vapor and at 0.002 cm^{-1} for CO_2. In computing the water vapor continuum absorption, we used the function given by Roberts et al. (1976) for the absorption coefficient, which fits laboratory data. The dashed curves are the parameterization using (20). It can be seen in the figures that the Planck-weighted diffuse transmittances for the water vapor line absorption and the CO_2 absorption can be fit very well with an error of <0.01 by using six terms in the exponential-sum fitting transmissivity. For the water vapor continuum absorption, only one or two terms are used in the ESFT. The transmittance error of the ESFT is ≈ 0.02.

TABLE 4. Same as Table 3 except for CO_2 absorption. Units of k are $(\text{cm-atm})_{\text{STP}}^{-1}$.

	Band 3	Band 4	Band 5
k_1	1.60×10^{-5}	1.60×10^{-3}	1.60×10^{-5}
n	8	8	8
c_1	0.2673	0.1970	0.1784
c_2	0.2201	0.3528	0.2432
c_3	0.2106	0.3056	0.3086
c_4	0.2409	0.0861	0.1983
c_5	0.0196	0.0434	0.0424
c_6	0.0415	0.0151	0.0291

5. Results and concluding remarks

Fluxes and cooling rates due to water vapor molecular line absorption, water vapor continuum absorption, and CO_2 absorption are computed from the parameterization that includes: (i) the one-parameter scaling approximations of (7) and (8), (ii) the approximation of the Planck-weighted diffuse transmittance $\tau(p, p', T')$ by $\tau(w, T')$, (iii) the approximation of $\tau(w, T')$ by $\tau(w; T_0)$, and (iv) the approximation of $\tau(w, T_0)$ by the ESFT of (20).

Figure 9 shows the cooling rate difference between the parameterization and the "exact" line-by-line calculations for the five atmospheres taken from McClatchey et al. (1972). Only the absorption due to water vapor molecular lines is included in the calculations. Except for the region above the 20-mb level, where the pressure scaling is not valid, the error of the parameterization is <0.07°C day^{-1}. Table 6 shows the fluxes at the surface and at TOA from line-by-line calculations and from the parameterization for the tropical atmosphere. The error of the parameterization is only ≈ 0.2 W m^{-2} in the total fluxes, although the error in individual bands reaches 1 W m^{-2}.

The results that have been shown are for the absorption due to the water vapor molecular lines only. When the absorption due to CO_2, O_3, and water vapor continuum are included, the total diffuse transmittance for individual bands is approximated by

$$\tau_{\text{total}} = \tau_w \tau_c \tau_0, \tag{21}$$

TABLE 5. Same as Table 3 except for water vapor continuum absorption. Units of k are square centimeter per gram.

	Band 3	Band 4	Band 5	Band 6	Band 7	Band 8
k_1	63.25	31.93	17.47	7.08	5.66	4.67
n				2		
c_1	1.0000	1.0000	1.0000	0.6145	1.0000	1.0000
c_2				0.3855		

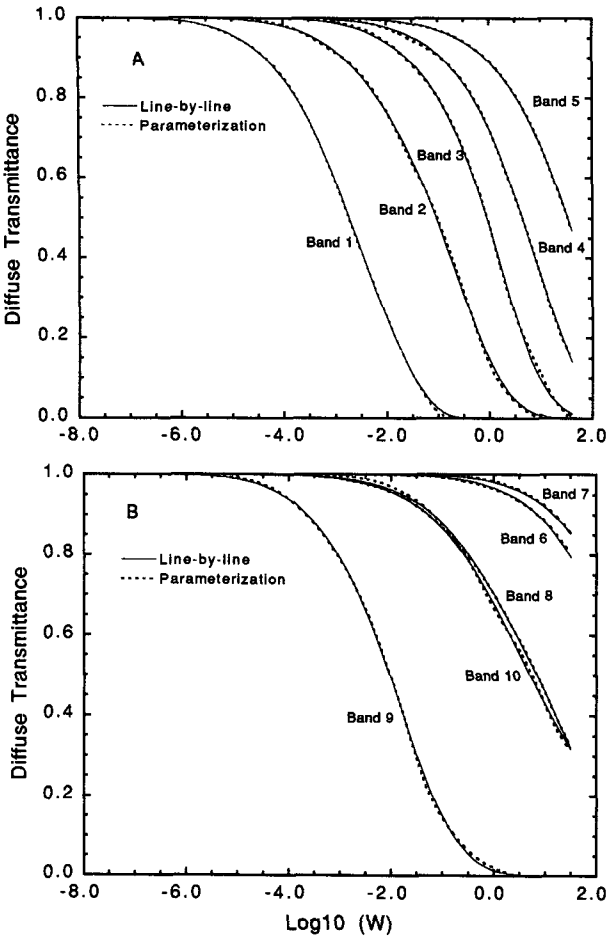

FIG. 6. Planck-weighted diffuse transmittances for the water vapor line absorption computed using a line-by-line method and the exponential-sum fitting of (20). The parameter w is the scaled water vapor amount in grams per square centimeter. The spectral ranges are given in Table 1.

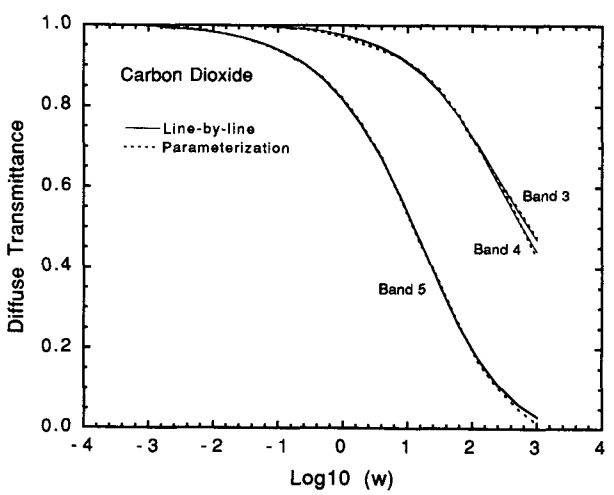

FIG. 7. Same as Fig. 6 except for the CO_2 absorption. The parameter w is the scaled CO_2 amount in $(\text{cm-atm})_{STP}$. The spectral ranges are given in Table 1.

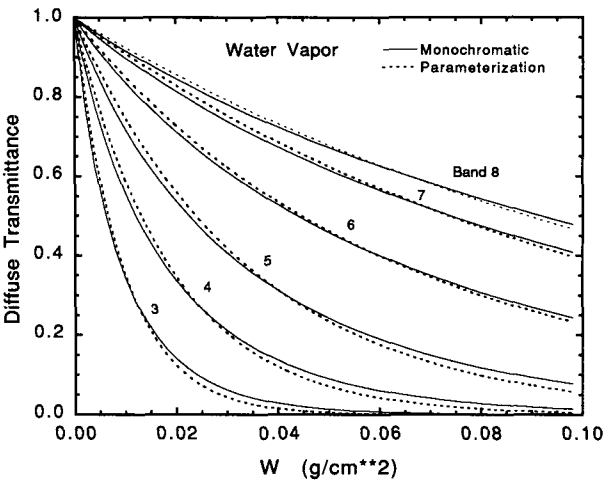

FIG. 8. Same as Fig. 6 except for the water vapor continuum absorption.

where τ_w and τ_c denote, respectively, the Planck-weighted diffuse transmittances for water vapor line and continuum absorption, and τ_0 denotes either the CO_2 transmittance or the O_3 transmittance. The parameterization for the absorption due to O_3 follows that of Chou and Kouvaris (1991). The two-parameter (p and T) scaling is used, and the O_3 transmittance is interpolated from precomputed tables. Figures 10a and b show the cooling rate profiles derived from line-by-line calculations (solid curves) and from the parameterization (dashed curves) for the tropical and the subarctic winter atmospheres. Absorptions due to water

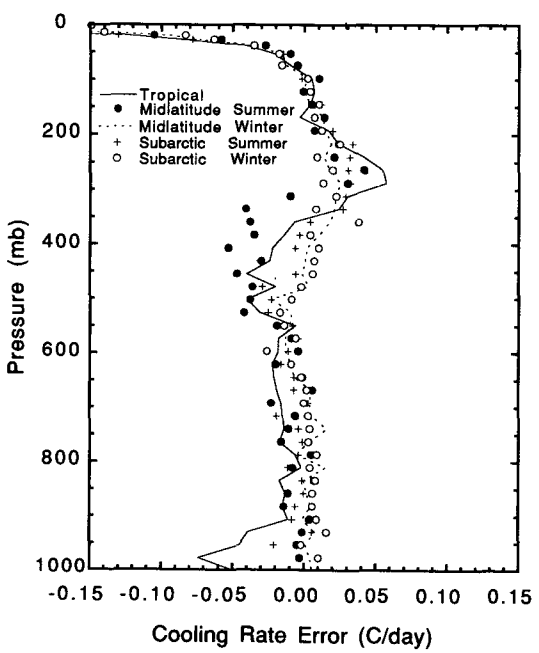

FIG. 9. Cooling rate error of the parameterization for five atmospheres. The cooling is due to water vapor line absorption only.

TABLE 6. Downward fluxes at the surface, $F\downarrow_{sfc}$, and upward fluxes at the top, $F\uparrow_{top}$, for a clear tropical atmosphere computed using a line-by-line method and the parameterization. Units of the fluxes are watts per square meter. Absorption is due to water vapor molecular line only.

Spectral band (cm^{-1})	$F\downarrow_{sfc}$	$F\uparrow_{top}$
0–340		
line-by-line	52.70	34.33
parameterization	52.70	34.02
340–540		
line-by-line	85.26	61.26
parameterization	85.71	60.85
540–620		
line-by-line	34.81	30.35
parameterization	34.23	30.45
620–720		
line-by-line	31.18	40.51
parameterization	31.36	40.52
720–800		
line-by-line	12.57	32.76
parameterization	12.64	32.78
800–980		
line-by-line	9.38	65.57
parameterization	9.79	65.43
980–1100		
line-by-line	3.31	34.03
parameterization	3.50	34.06
1100–1380		
line-by-line	29.15	40.97
parameterization	28.21	42.07
1380–1900		
line-by-line	35.20	7.74
parameterization	35.22	7.82
1900–3000		
line-by-line	3.91	5.96
parameterization	4.19	5.69
Total		
line-by-line	297.47	353.48
parameterization	297.52	353.70

vapor line and continuum, CO_2, and O_3 are all included in the calculations. Figure 10a shows detailed cooling in the troposphere and lower stratosphere, while Figure 10b shows details of upper stratospheric cooling. It can be seen in the figures that the error of the parameterization is $\approx 0.15°C\ day^{-1}$ in the troposphere and lower stratosphere. Above the 5-mb level, errors become large. Fluxes at the surface and at TOA are given in Table 7. The parameterization introduces an error of 1.5 W m^{-2} in the downward surface flux and only <0.3 W m^{-2} in the TOA flux.

The advantage of using narrow spectral intervals is that the Planck-weighted diffuse transmittances can be reduced to a function of a single variable (the scaled absorber amount) using the one-parameter scaling and can be rapidly computed from the ESFT. Overlapping of absorptions can then be approximated by multiplying transmittances of individual absorbers. These simplifications are not possible for broadband parameterizations. Because of the use of the ESFT and the specific selection of absorption coefficients for the ESFT, the computing speed of the present parameterization is much faster than that of the broadband algorithm of Chou et al. (1991). The difference in speed depends on the number of atmospheric layers used. The more the atmospheric layers used, the larger the difference in computing speed. Running on a Cray Y/MP computer, the difference is a factor of 2 for a 20-layer atmosphere and a factor of 4 for a 60-layer atmosphere.

In addition to accuracy and speed, the parameterization for IR radiation developed in this study has a high degree of versatility. For example, the scattering of radiation in the IR window region by cirrus ice crystals is believed to be important, and the graybody assumption for cloud emission may not be valid. Not

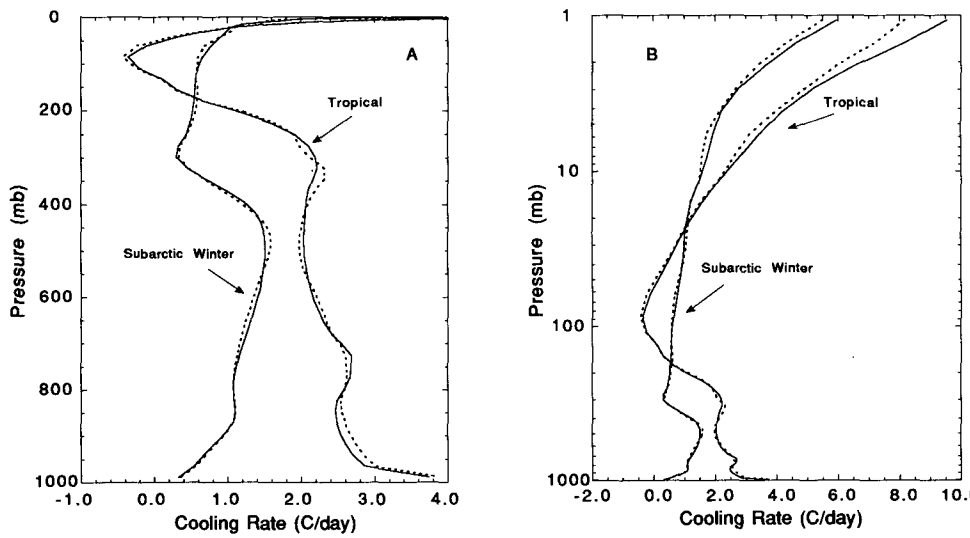

FIG. 10. Cooling rate profiles from line-by-line calculations (solid curves) and from the parameterization (dashed curves), plotted linearly in pressure (A) and logarithmically in pressure (B). The cooling is due to water vapor line, continuum, CO_2, and O_3 absorption.

TABLE 7. Downward fluxes at the surface, $F\downarrow_{sfc}$, and upward fluxes at the top, $F\uparrow_{top}$, for a clear tropical atmosphere computed using a line-by-line method and the parameterization. Units of the fluxes are watts per square meter.

Spectral band (cm^{-1})	$F\downarrow_{sfc}$	$F\uparrow_{top}$	Absorber
0–340			
line-by-line	52.70	34.33	H_2O (line)
parameterization	52.70	34.02	
340–540			
line-by-line	85.26	61.26	H_2O (line)
parameterization	85.71	60.85	
540–620			
line-by-line	38.12	25.60	H_2O (line, continuum)
parameterization	38.15	25.93	CO_2 (300 ppmv)
620–720			
line-by-line	46.64	16.49	H_2O (line, continuum)
parameterization	46.69	15.80	CO_2 (300 ppmv)
720–800			
line-by-line	31.35	25.44	H_2O (line, continuum)
parameterization	31.91	25.26	CO_2 (300 ppmv)
800–980			
line-by-line	42.17	61.25	H_2O (line, continuum)
parameterization	42.25	61.17	
980–1100			
line-by-line	17.16	25.55	H_2O (line, continuum)
parameterization	17.86	24.50	O_3
1100–1380			
line-by-line	36.68	39.64	H_2O (line, continuum)
parameterization	36.26	40.65	
1380–1900			
line-by-line	35.20	7.74	H_2O (line)
parameterization	35.22	7.82	
1900–3000			
line-by-line	3.91	5.96	H_2O (line)
parameterization	4.19	5.69	
Total			
line-by-line	389.19	303.26	
parameterization	390.85	301.75	

only can the ESFT be used to compute radiative transfer in clouds (e.g., Stackhouse and Stephens 1991), but also the cloud optical properties can be varied for each of the various bands.

Because of the use of the one-parameter scaling approximation, the parameterization developed in this study cannot be applied to computing cooling rates in the upper stratosphere. For a climate model extending to the upper stratosphere, either the band models developed by Chou and Kouvaris (1991) and Chou et al. (1991) or the correlated-k distribution method (Fu and Liou 1992; Goody et al. 1989; Lacis and Oinas 1991; Zhu 1992) can be used.

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