The Structure and Nonlinear Evolution of Synoptic-Scale Cyclones. Part II: Wave–Mean Flow Interaction and Asymptotic Equilibration

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ABSTRACT

The life cycle of a baroclinic wave in nearly inviscid, adiabatic flow and its interaction with the zonal mean circulation is here described with primary emphasis focused upon the later stages of evolution. A comparison of simulations performed on the $f$ plane with similarly initialized flows on the $\beta$ plane reveals differences related to the enhanced northward displacement of the cyclone on the $\beta$ plane. The zonal mean flow pattern is displaced northward for simulations on the $\beta$ plane and southern jets are comparatively larger in magnitude than their counterparts in simulations on the $f$ plane. On the $\beta$ plane, wave energy is absorbed at latitudes equatorward of linear disturbance, whereas on the $f$ plane, wave energy is absorbed at this same latitude. Associated with the northward propagation of the cyclone on the $\beta$ plane is the splitting of zones of Eliassen–Palm flux divergence near the surface and the appearance of southward heat fluxes after cyclone cutoff.

Diagnostic analyses of these flows, very late in the life cycle of the wave, reveal that on the $f$ plane an oscillation of the domain-integrated energy budgets occurs that corresponds to a quasi-periodic repetition of the full three-dimensional flow pattern. This dynamically evolving structure bears some resemblance to baroclinic wave vacillations observed in rotating tank experiments. In the $\beta$-plane simulations, late in the life cycle of the wave, the cyclone is observed to split into two vortices and secondary cyclonic waves develop on a shallow front in the southern part of the domain. A multiplicity of vortices is thereby seen to emerge in the $\beta$-plane simulations and these tend to be smaller in horizontal scale.

1. Introduction

Simulations of the life cycles of idealized baroclinic waves have been useful for developing conceptual models of atmospheric cyclogenesis (e.g., Shapiro and Keyser 1990) as well as providing model atmospheres with simplified physics into which complexities can be gradually introduced. A hierarchy of such model simulations exists, ranging from quasigeostrophic to primitive equation dynamics (e.g., Mudrick 1974; Hoskins 1976; Hoskins and West 1979; Heckley and Hoskins 1982; Simmons and Hoskins 1978, 1980; Takayabu 1987; Polavarapu and Peltier 1990; Davies et al. 1991), and has stimulated work providing insight into the various systems of dynamical equations (e.g., Snyder et al. 1991). In our simulations of idealized baroclinic waves (Polavarapu and Peltier 1990, hereafter Part I) we focused upon the influence of beta and the initial state on wave development up to the point of wave maturation. This study is intended to complete the description of the life cycles of nonlinear baroclinic waves presented in Part I by assessing the influence of the $\beta$ effect on the latter stages of the wave life cycle. That is, we focus upon the stage after the peak in disturbance kinetic energy, when vortices form and interact. This stage is too early to warrant a statistical description of vortex evolution or to be free from the influence of the initial state. Thus, it is hoped that this work may bridge the gap between life-cycle studies and those of geostrophic turbulence by clarifying the role of beta on wave development during this stage.

In section 2 we describe the diagnostic tools that will be employed in the subsequent discussion of the results (section 3). The influence of the $\beta$ effect on zonal mean flow evolution and wave–mean flow interaction is presented in subsections 3a and 3b. The impact of beta on vortex development is considered in subsection 3c. Finally, since we have focused attention upon the latter stages of wave evolution when diffusive processes should become important, in subsection 3d we describe a numerical consequence of this fact: the nonconservation of potential vorticity. Our main conclusions are briefly summarized in section 4.

2. Numerical simulations and diagnostic analysis procedures

The model that we have developed for application to these life-cycle analyses is based upon the nonhydrostatic anelastic system of Clark (1977). For an adequately detailed description of this model and for a discussion of the parameter settings employed, the

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reader is referred to Part I. In brief, the initial fields are assumed to consist of a deep baroclinic zone and an alongfront velocity field that are in thermal wind balance. We consider two general classes of such flows, the first consisting of a single upper-level westerly jet and the second containing two oppositely directed jets, the upper-level jet being westerly and the lower-level jet easterly. In all cases, jet maxima occur on the rigid horizontal boundaries. The first class of mean state will be seen to be adequately representative of the “average” midlatitude zonal flow in winter and is denoted by HW (the analytic description of the flow on the f-plane follows that initially employed by Hoskins and West 1979). We consider the second class of flows to be adequately representative of the reverse-shear frontal structures that develop within synoptic-scale cyclones and denote them by HB (after the two-dimensional deformation induced frontogenesis models of Hoskins and Bretherton 1971, 1972). These four mean states, depicted on Fig. 1, are all 8 km deep and 5500 km in the north–south direction. The model is initialized with any of the above mean flows and a perturbation provided by a linear stability analysis of the mean flow (following Moore and Peltier 1987 but with extensions to include anelastic effects and β; see Polavarapu 1989).

Initial wave amplitudes are 1 (0.5) K for the HB, HBB (HWF, HWB) simulations. The boundary conditions employed in all of our simulations have been taken to correspond to zero normal velocity in the vertical and meridional directions and periodicity in the zonal direction. Motions are assumed dry and adiabatic in all analyses and, apart from the introduction of a horizontal spatial filter, the flow is assumed inviscid.

For purposes of diagnostic analysis of these nonlinear simulations, it is useful to consider the kinetic energy to consist of two components: a zonally averaged flow and a deviation from this zonal average (to which we shall refer as the “wave”). If we denote zonal and domain averages by

\[
\left\{ \right\} = \frac{1}{L_x} \int_{0}^{L_x} dx,
\]

and

\[
\langle \rangle = \frac{1}{L_y} \int_{0}^{L_y} \frac{1}{H} \int_{0}^{H} dz dy,
\]

respectively, then the energy balance in the anelastic model may be summarized as follows:

\[
\delta_t \langle ZKE \rangle = -\langle HRS \rangle - \langle VRS \rangle 
\]

(3)

\[
\delta_t \langle EKE \rangle = \langle HRS \rangle + \langle VRS \rangle + \langle VHF \rangle
\]

(4)

\[
\delta_t \langle PE \rangle = -\langle VHF \rangle,
\]

(5)

where \( \bar{u} = \{ u \} \); \( \bar{\theta} = \{ \theta \} \); \( u' = u - \bar{u} \); \( \theta' = \theta - \bar{\theta} \); \( \bar{\theta}^* = \theta'/\bar{\theta} \); and the domain-averaged quantities that appear in (3)–(5) are

\[
EKE = \frac{1}{2} \left\{ \bar{\rho} \frac{x v^2}{\bar{\theta}^*} + \bar{\rho} \frac{v^2 \bar{\theta}^*}{\bar{\theta}^*} + \bar{\rho} \frac{w^2 \bar{\theta}^*}{\bar{\theta}^*} \right\};
\]

\[
ZKE = \frac{1}{2} \bar{\rho} \frac{x \bar{\theta}^*}{\bar{\theta}^*};
\]

\[
PE = -\bar{\rho} \frac{z \bar{\theta}^* \bar{\theta}^*}{\bar{\theta}^*}; \quad HRS = -\left\{ \bar{\rho} \frac{x \bar{\theta}^*}{\bar{\theta}^*} \delta_x \bar{\theta}^* \right\};
\]

\[
VRS = -\left\{ \bar{\rho} \frac{w \bar{\theta}^* \bar{\theta}^*}{\bar{\theta}^*} \right\}; \quad VHF = \left\{ \bar{\rho} \frac{z \bar{\theta}^* \bar{\theta}^*}{\bar{\theta}^*} \right\}.
\]

The difference operators employed in the above are defined as follows:

\[
\delta_i = \frac{\psi_{i+1} - \psi_{i-1}}{2\Delta t}; \quad \bar{\psi}^x = \frac{\psi_{i+1} + \psi_{i-1}}{2}; \quad \bar{\psi} = \frac{\psi_{i+1} + \psi_{i-1}}{2}.
\]

Other diagnostic quantities capable of providing insight into the physical processes occurring in the nonlinear dynamical system include Ertel's potential vorticity (Ertel 1942; Ertel and Rossby 1949),

\[
PV = \frac{1}{\rho} \vec{\xi} \cdot \nabla \theta.
\]

Fig. 1. The four mean states employed. Potential temperature deviation from a background constant value (dash–dot) and alongfront or zonal velocity (solid; negative values dashed) fields are given. Contour intervals are 2 K and 2 m s⁻¹. (a) HBF, (b) HBB, (c) HWF, (d) HWB. North is at the left of each frame.
where \( \vec{\xi} \) is the vector absolute vorticity field, the spatial distribution of the conversion terms in the energy budget, VRS, HRS, and VHF, and the Eliassen–Palm (EP) flux (Eliassen and Palm 1960). For the present application it was necessary to obtain expressions for the EP flux gradients for the anelastic, nonhydrostatic equations. One method of discretizing EP flux components, \( \mathcal{F}_y \) and \( \mathcal{F}_z \), is as follows:

\[
\begin{align*}
\{ v \} &= \{ v^* \} + \frac{1}{\rho} \left( \rho \frac{v_y}{\theta} \right)_z \\
\{ w \} &= \{ w^* \} - \left( \frac{v^*}{\theta} \right)_y 
\end{align*}
\] (7)

\[
\mathcal{F}_y = -\left( \rho u^* v^* - v^* \frac{v^*}{\theta} \right)_y - \frac{\theta}{\rho} \left( \frac{v^*}{\theta} \right)_z 
\] (8)

\[
\mathcal{F}_z = -\left( \rho \frac{v^*}{\theta} - w^* \right)_y + \left( \frac{\theta}{\rho} \right)_z 
\] (9)

where the difference operators have been defined previously. The starred variables refer to the residual circulation (see Andrews and McIntyre 1976) and are defined by (7). The brace brackets refer to the zonal integration operator defined in (1). With this representation, the x-momentum equation can be written as

\[
\delta_\rho \left( \vec{u}^* \right)_x + \delta_y \left( u^* \right)_y + \delta_z \left( u^* \right)_z + \rho f \left( \frac{v^*}{\theta} \right)_y = \delta_x (\mathcal{F}_y) + \delta_z (\mathcal{F}_z). 
\] (10)

Because of the density factor in the first term, we shall be diagnosing changes in the zonal mean momentum rather than in the zonal mean velocity. In what follows we will indicate EP flux components \( \mathcal{F}_y, \mathcal{F}_z \) with arrows and contour the quantity on the right-hand side of (10). The flux component \( \mathcal{F}_y \) defined in (8) consists of two contributions, a horizontal Reynolds stress and the correlation of the horizontal component of absolute vorticity with northward heat flux. The first term is nonzero only when barotropic conversions of energy are occurring. The vertical flux component, \( \mathcal{F}_z \), consists of a vertical Reynolds stress and a correlation of the vertical component of absolute vorticity with the meridional heat flux. We have multiplied this vertical component by \( f_0/N \) so that for flow that is dominated by baroclinic conversions of energy, flux arrows will be vertically oriented.

Here, \( \nabla \cdot \mathbf{F} \) as defined in (7) has units of J m\(^{-4}\) (or Pa m\(^{-1}\)). Since the mass element in Cartesian coordinates is simply \( dm = \rho L_x dy dz \), we note that

\[
\int \rho^{-1} \nabla \cdot \mathbf{F} dm = \int (L_x \nabla \cdot \mathbf{F}) dy dz = \int \Delta dy dz.
\]

As discussed by Edmon et al. (1980), \( \Delta \) is a convenient quantity to contour as it describes the volume between the \( y-z \) plane and the surface a distance \( \Delta \) from it when pressure coordinates are employed. Although the natural units for \( \Delta \) are J m\(^{-4}\) (or Pa m\(^{-1}\)), we are computing EP fluxes for a number of different simulations with a number of different zonal channel lengths, \( L_x \), so it is more convenient to consider \( \nabla \cdot \mathbf{F} \) directly. Thus, the units employed to represent EP flux divergences are J m\(^{-4}\). The EP flux components represent energy transfer through the \( xz \) or \( xy \) surface of a unit volume per unit length in the zonal direction.

3. Results

The following discussion is based on the analysis of four main simulations. Table 1 lists a number of the parameters that distinguish them. The spatial resolution employed in these analyses is approximately 50 km in the horizontal and 0.5 km in the vertical. A detailed qualitative description of the evolution of the individual flow fields was provided in Part I. The more quantitative discussion to follow in the present paper will be introduced through a brief discussion of the energetics of the four main life-cycle analyses.

Figure 2 depicts the time evolution of the various components of the energy budget that appear in (3)–(5) for the four main simulations. In all cases, initial growth is through the conversion of PE to EKE through the vertical heat flux (VHF). Note that the anelastic formulation of the equations of motion (which eliminates the possibility of distinguishing between total potential energy and available potential energy) does not distinguish between zonal and eddy APE so that horizontal heat fluxes (HHF) do not appear explicitly as energy transfer mechanisms in the energy budget (3)–(5). Nevertheless, HHF can be calculated as we shall see in the EP flux diagrams presented below. Inspection of Fig. 2 shows that in all cases, EKE appears to decay at the same rate at which it rises; however, after decreasing to approximately half the peak energy,

<table>
<thead>
<tr>
<th>Run</th>
<th>Grid points (x, y)</th>
<th>Wave no. (x, y, z)</th>
<th>Grid size (km)</th>
<th>Time step (sec)</th>
<th>Initial wave amplitude (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HBF</td>
<td>63 111 17</td>
<td>1.62</td>
<td>50 50 .5</td>
<td>180</td>
<td>1</td>
</tr>
<tr>
<td>HBB</td>
<td>63 111 17</td>
<td>1.80</td>
<td>45 50 .5</td>
<td>180</td>
<td>1</td>
</tr>
<tr>
<td>HWF</td>
<td>63 111 17</td>
<td>1.60</td>
<td>51 50 .5</td>
<td>180</td>
<td>0.5</td>
</tr>
<tr>
<td>HWB</td>
<td>63 111 17</td>
<td>1.60</td>
<td>51 50 .5</td>
<td>180</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1. The model parameters for the various simulations. Wavenumbers have been nondimensionalized by the Rossby radius of deformation, \( NH/\beta = 800 \) km. All domains are 5500 km \( \times \) 8 km in the meridional and vertical directions, respectively. The channel lengths are \( 2\pi NH/(\beta b) \) km for nondimensional wavenumber \( b \).
Fig. 2. The energetics of the four simulations: (a) HBF, (b) HBB, (c) HWF, (d) HWB. Energy quantities minus initial values are given. The dashed lines refer to simulations identical to that represented by the solid lines except for an increase in horizontal resolution. TKE refers to total kinetic energy; i.e., TKE = EKE + ZKE.

EKE increases once more. In Part I we noted that this phase of secondary growth was due to both PE-EKE and ZKE-EKE conversions. Furthermore, on the \( \phi \) plane (Figs. 2a,c), the EKE and ZKE appear to oscillate in a quasi-periodic fashion. On the \( \beta \) plane (Figs. 2b,d), two identical simulations were performed with the exception of the fact that higher horizontal resolution was employed \( [\Delta y = 28 \text{ km}; \Delta x = 30 \text{ (HBB) or } 34 \text{ (HWB) km}] \). Results from these high-resolution simulations are indicated by dashed lines. Note that the energetics of the lower-resolution calculations (and indeed the detailed characteristics of the hydrodynamic fields themselves) are very well reproduced by the high-resolution runs until very late in the simulations (day 14 for HBB shown on Fig. 2b, day 9 for HWB shown on Fig. 2d). The time beyond which the energy budgets begin to diverge clearly indicates a point beyond which the simulations are no longer exactly reproducible. During this time the simulations are influenced by diffusion and are hence a function of the spatial resolution employed. The evolution of the volume integral of potential vorticity is also a useful indicator of reproducibility since it should be exactly conserved in adiabatic inviscid flow. This will be discussed further in section 3d.

An illustration of the cyclogenesis depicted by the energy budgets of Fig. 2 is given in Fig. 3 for the HBF flow. Note that the development of the anticyclone (Figs. 3c-f) lags that of the cyclone. Other vortices may develop but are not seen here. In particular, a second anticyclone appears at upper levels and at northern latitudes for all simulations. In addition, simulations employing \( \beta \) tend to produce more vortices (section 3c). The rough analogy between the flows of Figs. 3c, f is explored in more detail in section 3c. First, however, we consider the influence of the developing vortices on the mean flow (sections 3a and 3b). Through detailed inspection of zonally averaged quantities, we shall be able to further clarify the nature of the influence of the \( \beta \) effect on midlatitude cyclogenesis.

a. The evolution of the zonal mean fields

The role of baroclinic instability in the general circulation is one of maintaining the midlatitude west-
Fig. 3. The evolution of the surface pressure field for HBF at day (a) 4, (b) 6, (c) 8, (d) 10, (e) 12, (f) 14. Contours are shown every 2 mb. Horizontal flow vectors are also indicated. The longest arrow corresponds to the maximum velocity in the field and is indicated above each plot.

erlies (e.g., Holton 1979). The effect of a single disturbance on the zonal mean flow has been illustrated by Simmons and Hoskins (1976) and Hoskins (1983). Specifically, the zonal mean flow is accelerated at latitudes north and south of the disturbance and decelerated at latitudes corresponding to the disturbance. With our life-cycle simulations we can assess the changes in zonal mean flow due to modifications in initial state or due to the $\beta$ effect.

The evolving zonal mean fields for HBF and HBB are compared in Fig. 4, of which panels (a), (b), (c) are for HBF while panels (d), (e), (f) are for HBB. Because of the differing rates of evolution of the waves in these two simulations, the times chosen for com-
Fig. 4. Zonal mean fields for HBF days 3, 9, 12 in (a)–(c) and HBB days 4, 10, 12 in (d)–(f). Potential temperature (dash-dot) and zonal velocity [westerly (solid), easterly (dashed)] contours are given every 2°C and every 2.5 m s⁻¹.

Comparison correspond to similar positions with respect to the evolution of the respective EKE budgets. Similarly, the zonal mean fields for HWF and HWB are compared in Fig. 5. From these figures, it is clear that the generation of a cyclone-anticyclone pair implies a pattern of easterly–westerly–easterly flow at the surface and westerly–easterly–westerly flow at the lid. Comparison of the upper three panels with the lower ones in Figs. 4 and 5 reveals systematic differences due to the presence of β: the whole pattern is shifted northward due to the more northerly position of the warm seclusion in the simulations employing β. A poleward shift of the westerly jet during baroclinic wave development was observed by Randel and Stanford (1985) and suggests that our simulations employing β are slightly more "realistic" since they reproduce this feature. Another difference attributable to the β effect is the enhancement of the southern westerly jet relative to the northern westerly jet at upper levels. Also, near the surface, the southern easterly jet is more intense than its northern counterpart. This enhancement of the southern zonal jets is due to the fact that flow that is essentially geostrophic will develop more intensely to the south where the Coriolis parameter is smaller. Although ageostrophic influence is crucial to the detailed evolution of the flow in all our simulations, at any instant of time and for diagnostic purposes it may be considered essentially geostrophic, as evidenced by the flow vectors in Figs. 7f and 9 of Part I.

In all cases, the zonal mean temperature field reveals a reduction in baroclinicity at central latitudes and enhancement to the north and south of this region (Figs. 4b,e and 5b,e). This change was also seen by Simmons and Hoskins (1980), as reported in Hoskins (1983, Fig. 7.4), and is due to the fact that reduction of temperature gradients in a finite region implies increasing gradients on the edges of the region. Indeed, with further evolution, these two regions of enhanced baroclinicity are reduced, the baroclinicity being further divided among four or more narrow bands (Figs. 4c–f, 5c–f). This division of baroclinicity into finer and finer bands appears here presumably due to the enhanced resolution utilized in our calculations and the minimal smoothing applied to the fields.

Comparison of Figs. 4b and 5b and Figs. 4e and 5e reveals very similar patterns despite the marked difference in initial states. (Obviously, the jets and baroclinicity in HWF and HWB are more intense due to the increased baroclinicity of the initial states relative to those of HBF and HBB.) Evidently, zonal mean fields are more sensitive to the presence or absence of β than to changes in the initial state.
Examination of the spatial distribution of conversion terms of the energy budget (3)–(5) at key points in the cyclone's evolution reveals little difference between simulations. To illustrate this fact we choose to compare the extreme simulations HBF and HWB (which differ in mean state configuration and in rotational dynamics). From Figs. 6a,d we see that the zonally averaged vertical heat flux term is initially similar for all cases. This is because the wave is analogous to the Eady wave of one-dimensional theory in that the vertical velocity perturbation is maximum at midlatitude. As the wave enters the nonlinear regime this pattern shifts and several distinct local maxima in VHF develop: a positive region to the north of a negative region, the whole pattern still focused into the domain center (Fig. 6b,e). Weaker regions of negative and positive VHF conversions flank this central pattern to the north and south. The region of positive baroclinic conversion corresponds to warm air rising at the leading (lower-level) warm front. The negative region corresponds to warm air descending at the anomalous (because of the rigid upper boundary condition) upper-level front and is indicative of the indirect vertical circulations associated with upper tropospheric frontogenesis (e.g., see Shapiro and Keyser 1990). While no tropopause folding can occur in the simulations that we have performed, the sense of the circulation suggests that the upper-level cyclogenesis that does develop here may be analogous to the process of tropopause folding that would occur if this boundary were treated realistically (this is the subject of ongoing study). The southern region of positive VHF is due to rising air above the surface cold front. As tighter temperature gradients appear, the vertical velocity field develops a great deal of "wavelike" fine structure (see Fig. 14 of Part I) and so correspondingly does the VHF pattern shown in Figs. 6c and 6f.

As with the vertical heat flux patterns discussed above, there are common patterns of Reynolds stress evolution observed in all cases. Initially the HRS is focused in two positive and negative regions on both surfaces for the HB mean states (Fig. 7a). The positive regions are more intense, indicating net contribution to EKE or net depletion of ZKE. The opposite is true for HW mean states where HRS is predominantly negative initially (Fig. 7d). At the time of peak EKE (Figs. 7b and 7e), the net result is ZKE support from within the occlusion due to the zonal elongation of the occluded cyclone (see Figs. 6 and 8 of Part I). Changes are always largest at the surface because the jets have largest amplitude there, though presumably the inclusion of surface friction would raise the level of maxi-
mum amplitude to the top of friction layer (e.g., Gall 1976). To the south and north of the center of the domain where baroclinicity is being enhanced (7b and 7c), HRS contributes to EKE. In these regions, therefore, Reynolds stresses act to decelerate the zonal flow and the EWE pattern of the zonal jets seen in Figs. 4 and 5 develops. For the HW mean states the narrowness of the flanking baroclinic regions is evidenced by these narrow positive plumes. In Fig. 7c patterns are not very clear, so we focus attention upon Fig. 7f where a positive region exists throughout the troposphere. This corresponds to a depletion of ZKE by the cutoff cyclone. South of this region is a deep zone of barotropic shearing (negative contours), which increases ZKE. Still farther south, zonal flow is decelerated in shallow baroclinic regions near the surface.

We have seen that the conventional conversion terms have similar spatial patterns for simulations on both the $f$-plane and $\beta$ plane. This is not too surprising, considering the similarities of the general description of cyclogenesis. On the other hand, the evolution of the zonal mean fields is sensitive to the $\beta$ effect. Thus, in order to observe the wave–mean flow interactions that result in this sensitivity, we must consider other diagnostics. As we shall see below, EP flux diagrams point to some rather distinct differences between cyclogenesis on the $f$-plane and that on the $\beta$ plane.

b. Diagnosis of the $\beta$ effect using EP flux diagnostics

With our simulations on the $f$-plane and $\beta$ plane, we are in a position to compare wave–mean flow interactions in these two cases. Edmon et al. (1980) have presented EP flux cross sections for Eady and Charney modes of baroclinic instability, noting that neither adequately represents observed cross sections. Specifically, EP flux diagrams based on observed, averaged seasonal data depict a region of concentrated divergence at the lower boundary and a strong equatorward tilt of flux vectors in the upper troposphere. Simulations of a linear, growing baroclinic wave on a sphere (Hoskins 1983) based on the case of Simmons and Hoskins (1980) do depict the evolution of such a pattern from one resembling a Charney mode at early stages (strong divergence at lower boundary with convergence at wave steering level). The propagation of planetary waves responsible for the equatorward flux becomes important with the occlusion of the original disturbance due to the erosion of the lower-level temperature gradient. Our $f$-plane simulations do not permit planetary wave radiation, yet capture the baroclinic wave life cycle rather well, and, as seen in the previous section, energy conversions differ little from simulations employing $\beta$. How then are wave–mean flow interactions altered by the presence of the $\beta$ effect?
Figures 8 and 9 show time sequences of EP flux and its divergence for the HBF and HBB simulations, respectively. Because the patterns are similar for both flow types (HB and HW), we present EP flux diagnostics only for the HB simulations. Our EP flux diagrams feature the shortening of the arrows with height (Figs. 8a–c) in contrast with the EP flux patterns associated with an Eady wave (Edmon et al. 1980). Of course this problem differs from Eady’s since we consider a two-dimensional mean state and anelastic effects.

In Fig. 8, the regions of nonzero EP flux divergence occur near the upper and lower boundaries. After the peak in EKE, the horizontal component of the EP flux increases (Fig. 8d) due to the increasing barotropy of the flow within the occlusion as it becomes zonally elongated. However, baroclinic processes eventually dominate again (Fig. 8f). This reemergence of deep baroclinic regions results in a second peak in EKE (Fig. 2a,c) and the recreation of similar but somewhat noisier surface fields (Fig. 6c, f and 10c, f of Part I). In Fig. 8f, deep baroclinic zones are reestablished, and the pattern is reminiscent of but noisier than that of Fig. 8c due to the reduction of spatial coherence of the redeveloped cyclonic flow (Fig. 6f of Part I).

On the β plane, nonzero interior divergence of EP flux is more pronounced. The initial modal growth is apparent in Fig. 9a, but the interior region of negative divergence begins to move northward thereafter. The disturbance has penetrated to higher elevation by the time corresponding to Fig. 9c. When the wave activity reaches the lid it turns southward, remaining at upper levels. Downward-pointing arrows due to southward heat flux later appear in the lower half of the troposphere over the occlusion (Figs. 9e, f). Note that the flux patterns have reversed from the situation of baroclinic growth with negative surface divergence and positive midlevel divergence. This reversal in the meridional heat transfer late in the wave cycle was noted by Barnes and Young (1992) for simulations on a sphere and is suggested in winter and summer averages of stationary and transient eddies in the Northern Hemisphere (see Edmon et al. 1980). Edmon et al. suggest the possible influence of diabatic effects or the interaction between stationary and transient waves in producing this feature. Another, simpler explanation is that when barotropic energy conversions dominate, the barotropic shears act to destroy meridional coherence of the cyclone and to force the warm occlusion southward. Indeed, steady southward translation of cyclones was noted during this period (see Part I, Figs. 6, 8, 10, 11) and is consistent with southward heat flux.
Comparison of Figs. 8 and 9 shows that inclusion of the $\beta$ effect permits equatorward flux along with the northward displacement of the lower level region of divergence. As indicated above, analogous diagrams for the HWF, HWB simulations (Polavarapu 1989) reveal the same patterns, again pointing to the overriding influence of beta over the initial state with respect to nonlinear wave evolution. While the $f$-plane wave also decays barotropically, in the absence of Rossby wave dispersion, no mean meridional momentum transport occurs. The wave grows and decays at the same latitude. On the $\beta$ plane, momentum transport reinforces the already developing barotropic shear, accelerating poleward jets and decelerating (easterly acceleration) subtropical jets. The resulting mean flow changes consist of enhanced westerly midlatitude and easterly subtropical jets, as seen in the last section. Since Rossby wave dispersion commences with wave decay, the primary impact of $\beta$ appears to be on the nonlinear evolution, not the initial state. The lack of sensitivity to initial state (flow type) seen here also supports this view. We now proceed to consider two other differences between Figs. 8 and 9.

At the surface, and as the nonlinear wave expands northward, the region of upward EP flux splits into two regions (Figs. 9c–f). Interestingly, this pattern was seen in both simulations on the $\beta$ plane but in neither simulation on the $f$ plane. Similar splitting of the low-level regions of positive EP flux divergence was seen by James and Hoskins (1985) and MacVean and James (1986) but neither offered an explanation. In fact, the splitting of this region is associated with the northward movement of the secluded warm pool and creation of two regions of secondary baroclinic growth. As noted in Part I, we might expect differences early in the life cycle due to the meridional asymmetry implied by the $\beta$ effect for primitive equation dynamics. Surprisingly, however, differences are apparent in these simulations well into the life cycle of the wave.

The reason for the splitting of the region of positive zonally integrated HHF on the $\beta$ plane can be seen through inspection of the spatial distribution of this term. Because quasigeostrophic (hereafter referred to as QG) forms of the EP flux components (not shown) produced similar patterns to the anelastic forms in all cases, it is sufficient to examine the QG form of the vertical component of EP flux in order to comprehend the differences between cyclogenesis on the $f$ and $\beta$ planes. The QG expression analogous to (9) is

$$\mathcal{F}_z^{\text{QG}} = f_0 \rho \frac{\omega}{\theta} \left( \frac{\omega}{\theta} \right)_z$$

Because the denominator of (11) is positive, it is suf-
Fig. 9. Eliassen-Palm flux diagrams for HBB days 5, 7, 9, 11, 12, and 13 in (a)–(f), respectively. Notation is similar to that in Fig. 32.

sufficient to consider the spatial distribution of the meridional heat flux (hereafter MHF).

In Figs. 10 and 11, the evolution of the MHF at 0.5-km elevation for the HBF and HBB simulations, respectively, is illustrated. As one might expect, positive regions are associated with the northward movement of warm air near the surface and the southward movement of cold air. These regions define the cyclonically wrapping tongues of warm and cold fluid in Figs. 10a–c. Cyclonic motions along the west side of the warm tongue imply southward flow and a negative MHF region results. Similarly, a negative region appears along the cold tongue. Prior to the cutoff of the warm pool, the positive region appears as a single elongated structure with its maximum near central latitudes. Note that the region of negative MHF is similarly located and of similar magnitude. The positive region along the neck of the occlusion has two clear maxima only at the time of detachment (Fig. 10d). In contrast, in Fig. 11, we see the meridional elongation of the region of positive MHF with the northward movement of the cyclone. The region of positive MHF has two maxima, one associated with the leading warm front, the second at the base of the neck of the occlusion (Fig. 11b). The negative region also has two maxima (Fig. 11b,c). Upon zonal integration at this level, two regions of baroclinic activity appear, the first at the latitude of the occlusion, the second over the range of latitudes spanned by the protruding mouth of the occlusion. In contrast, the proximity of the two regions of large positive MHF in the meridional direction for the $f$-plane cases creates a single broad maximum upon zonal integration. Thus, the northward propagation of the developing occlusion, on the $f$ plane, causes the apparent splitting of the regions of positive MHF in EP flux diagrams.

A second difference between the EP flux patterns delivered by the $f$ and $\beta$-plane simulations occurs later in the life cycle of the wave; regions of southward heat flux appear in both $\beta$-plane cases but in neither $f$-plane case. Again, this is likely due to the more northerly position of the occluded cyclone on the $\beta$ plane. In Figs. 11e and 11f, we see that once the warm pool cuts off, the region of positive MHF almost instantly collapses while the negative region (southward movement of warm air in the occlusion) retains its previous magnitude. This causes a net negative MHF when zonally integrated. On the $f$ plane, a net positive MHF results at central latitudes even after detachment of the occlusion (Fig. 10e). These patterns were echoed in the plots of MHF at elevation 0.5 km for both the HWF and HWB simulations as well. The disparity in southward
heat flux between $f$-plane and $\beta$-plane simulations may simply be due to the more northerly position of the warm cyclone at the time when horizontal shearing becomes important.

To summarize, on the $f$ plane, in the absence of Rossby wave radiation, wave energy is absorbed at the latitude of the linear disturbance, whereas on the $\beta$ plane, wave energy is absorbed equatorward of this latitude. An important distinction between cyclogenesis on the $\beta$ plane and that on the $f$ plane is the northward propagation of the cyclone in the former case, which results in an apparent splitting of the region of EP flux divergence at the lower boundary as well as the appearance of downward EP flux (southward meridional heat flux) late in the wave life cycle. Reversed meridional heat flux at central latitudes also appears in $f$-plane cases but in areas adjacent to regions of positive baroclinic growth so that in a zonally averaged sense, no net southward heat flux is evident.

c. Flow development after cutoff

As noted in Part I, cyclonic and anticyclonic vortices develop late in the wave life cycle. In this section we consider the influence of $\beta$ on the vortices that develop from the time of the cutoff of the cyclone. This will refer to times onward from day 10 (11, 7, 7) for the HBF (HBB, HWF, HWB) simulation. While this pe-
period is late with respect to the development of a single baroclinic wave, it is too early to permit statistical flow description. The waves have not equilibrated (Fig. 2) and diffusion has become important (section 3d). Our simulations are not forced by a time-continuous applied thermal gradient (as in experiments that explore the nature of geostrophic turbulence) but only by an unstable initial state. Thus, we might expect strong dependence upon initial conditions during this stage of development. As we shall see, however, in determining some characteristics of the flow, the \( \beta \) effect has an influence that overrides that of the initial state.

Figure 2a reveals that for the HBF simulation, this stage is characterized by a quasi-regular oscillation of the eddy and zonal kinetic energy budgets. In order to illustrate the nature of the flow responsible for this oscillation, it is necessary to examine the fully three-dimensional evolution of the wave. The quantity that best illustrates the three-dimensional flow associated with cyclogenesis is Ertel's potential vorticity (PV).
Since PV is conserved on theta surfaces for dry, inviscid, adiabatic motions, a comparison of the motion of PV contours with the height of the isentropic surface qualitatively determines vertical motions. The motion of the $\theta_w$ or $\theta_e$ (wet-bulb or equivalent potential temperature, respectively) surfaces is often used to diagnose large-scale slantwise ascent through a cyclone (e.g., Green et al. 1966; Carlson 1980). The isentropic surface chosen for the analyses to be presented here was one at midlevel in central latitudes. Both upper- and lower-level flow may then be represented on an isentrope that deflects upward and downward and this is most apparent for an isentrope in the center of the initial baroclinic zone.

Figure 12 depicts the evolution of PV on the $\theta = 10^\circ C$ surface for the HBF simulation. Contours of PV below the median value are dashed. The PV is not constant because for the anelastic dynamics, the factor $\rho^{-1}$ in the definition dictates that high PV occurs at low density and low PV occurs in regions of high density. Thus, Fig. 12a reveals an approximately sinusoidal field with low PV in the south (the isentrope has lower elevation there) and higher PV air to the north. The meridional extent coincides with the region of large baroclinicity (see Fig. 1).

We shall now proceed to discuss the flow configuration that precipitated the EKE oscillation in Fig. 1a. The motions to be discussed were diagnosed on the basis of temporal comparisons of PV contours with the height of the isentropic surface. As shown in section 3d, potential vorticity is not conserved late in the wave cycle; however, it takes considerable time for the leakage of spurious PV, which is initiated at the boundaries, to reach the domain interior. In addition, the impact of such leakage is identifiable in the fields by the “noise” that it introduces. We will here confine our diagnosis of motion to scales in excess of those on which such noise appears. In Fig. 12b, the amplitude of the wave has increased and cyclonic rotation is evident in the center of the frame. In Fig. 12c, a spiral is beginning to form where high and low PV air meet in the domain center. The dashed lines north of the spiraling occlusion are associated with anticyclonic flow that ascends from lower levels to the upper surface. Likewise, the southern anticyclone is seen in Fig. 7d as the surface high pressure region that forms to the south of the occlusion. Thus, three vortices exist. The development of the anticyclones precedes the demise of the cyclone. This becomes zonally elongated (Fig. 12e) as fluid is entrained into both anticyclones. However, continued anticyclonic development results in a northwest orientation of the major axis of the cyclone (Fig. 12f). Figures 12g,h,i continue this sequence but at 1-day intervals. Note the rough analogy between Fig. 12g and Fig. 12c. The major axis of the cyclone is now oriented north-northeast. The anticyclonic regions have diminished in horizontal extent. However, over the ensuing two days (Figs. 12h and 12i) these regions develop once more, orienting the major axis of the cyclone zonally once again. Thus, the EKE oscillation is seen to correspond to the tilting of the cyclone axis from roughly NW–SE at times of a peak in EKE to nearly NE–SW at the time of a peak in ZKE. It also corresponds to the alternating dominance of the cyclone and the anticyclones in the flow. The original baroclinic event ending in the cutoff of the cyclone is different from subsequent periods of EKE rise because further cyclonic development occurs through both baroclinic and barotropic conversions of energy.

Oscillations in the energy cycles of finite-amplitude baroclinic waves have also been determined for weakly nonlinear flows (Pedlosky 1970, 1971; Boville 1980) in a two-layer quasigeostrophic model. However, the oscillations reported here are clearly not associated with initially marginally unstable waves. Figure 2a is reminiscent of the damped oscillatory decay of wave kinetic energy in stably stratified Kelvin–Helmholtz flows (Davis and Peltier 1979; Klaassen and Peltier 1985a,b). The oscillation of the energy cycle observed in the $f$-plane simulations is due to two processes: 1) the creation of a cyclone that reduces baroclinicity at central latitudes and intensifies it in regions to the north and south and 2) the development of anticyclones in the baroclinic zones to the north and south, which reduce baroclinicity there and recreate it at central latitudes. In a similar fashion, the finite-amplitude oscillation of Kelvin–Helmholtz waves was seen to correspond to the creation and destruction of regions of high static stability (Klaassen and Peltier 1985a, Fig. 11).

Oscillations have also been observed to characterize the energy budgets of finite-amplitude baroclinic waves in rotating tank experiments (e.g., Hide and Mason 1975). These oscillations can be due to amplitude vacillation or structure vacillation, the former type having been first identified by Pfeffer and Chang (1967). The latter type includes “tilted trough” or “kinetic energy” vacillations and is similar to that described above in a number of respects. Structure vacillations are characterized by changes in wave shape but not amplitude, periodic changes in radial (meridional) structure and energy distributions, and strong vertical coherence (Pfeffer et al. 1980). They may also be characterized by vacillations in the kinetic energy budget due to fluctuations in Reynolds stress associated with the tilting of trough axes from NE to NW. Note that flow vacillation was also seen in the HWF simulation as the cyclone and the upper-level anticyclone periodically produced similar flow patterns while propagating in opposing directions. While the periodic zonal boundary conditions and the channel geometry are crucial to the existence of the oscillation, the large-scale component of atmospheric flow may at times resemble such oscillations (e.g., Quinet 1974).

The flow evolution on the $\beta$ plane is rather different and is depicted by the evolution of the potential vorticity on the $\theta = 10^\circ C$ isentropic surface (Fig. 13). As in HBF and in all other simulations, three main vortices form. Unlike the previous example (Fig. 12), the up-
FIG. 12. The evolution of potential vorticity on the $\theta = 10^\circ$C isentrope for the HBF simulation. Days 4, 6, 8, 10, 12, 14, 15, 16, and 17 are represented in (a)–(i), respectively. Contours are given every 0.02 PV units. Contours below the median value of PV are dashed.
per-level anticyclone is less pronounced, appearing only at day 12 (Fig. 13e), and is smaller in meridional extent. All three vortices appear farther north with respect to their counterparts in Fig. 12. In addition to these main vortices, others later develop. At day 14 the cyclone splits into two vortices that propagate at different latitudes, and a secondary cyclone develops on the sharp shallow front in the south. South of the southern anticyclone, in Fig. 13f, there is a hint of the cyclonic flow that is associated with this secondary cyclone (see Polavarapu and Peltier 1993, hereafter Part II). While the energetics of the other simulation on the β plane (HWB) hint at a possible oscillation (Fig. 2d), quasi-regular repetition of the flow was not seen due to the development of multiple vortices at various latitudes. On day 10 at upper levels and near the northern boundary, a cyclone and an anticyclone formed. This cyclone merged with the midlatitude cyclone on day 11. At lower levels, the midlatitude cyclone split into two cyclones (Fig. 11f of Part I). Here too, secondary disturbances developed on the sharp, shallow front in the south (Fig. 7 of Part II).

To identify systematic differences between the f-plane and β-plane simulations, we compiled a list (Table 2) of features characterizing the three main vortices that form in each simulation (the cutoff cyclone, the upper-level northern anticyclone, and the lower-level southern anticyclone). The values in Table 2 are rough
The nonconservation of PV near the surface allows theta surfaces to lift off the lower boundary, a process documented in two dimensions by Nakamura and Held (1989). We observe the same process through x-y cross sections of theta through the neck of the occlusion. Figure 16 depicts such cross sections for HBB (the case in which the longest occluded front was produced). While wave equilibration was seen by Nakamura and Held as the lifting of an occlusion off the surface, horizontal views show that it is simply the destruction of the neck attaching the secluded warm pool that is occurring. The warm occlusion still exists north of this cross section. The PV changes associated with the lifting of theta surfaces are similar to what one might expect if a thin sheet of infinite PV existed adjacent to boundaries where isentropes intersect the surface. The mathematical concept of infinitesimally thin layers of infinite static stability and potential vorticity was introduced by Bretherton (1966).

From Figs. 14–16, it is clear that even for simulations inviscid apart from a smoothing operator, PV nonconservation is notable in a global sense and severe locally. The quantification of nonconservation here serves to warn against the presumption of PV conservation in the troposphere during meteorologically significant events. Another potential use of domain-integrated PV is in determining the point beyond which a given simulation is no longer reproducible. When PV is no longer conserved, further development depends on the nature and strength of the diffusion.

If diffusion in these simulations is indeed mimicking the role of physical processes in the atmosphere, then a preference for processes that concentrate PV in the regions of strong frontal gradients near the surface as well as near the tropopause might be expected. Presumably a flexible tropopause would fold at this stage of wave development. An interesting question is then whether diffusion in these simulations fulfills the role that tropopause folding does in the atmosphere or whether other processes that concentrate PV might also be implicated. Nearly inviscid simulations with a flexible tropopause might clarify this point if the role of diffusion could be shown to be significantly reduced simply by introducing this more realistic feature into the model.

4. Conclusions

In this work we have attempted to address the relative importance of initial state and rotational dynamics on a number of aspects of nonlinear baroclinic wave development: the evolution of the zonal flow, wave-mean flow interactions, and vortex formation. Because the presence of beta changes not only the dynamics of the problem but also the initial state, a separation of these two effects is not possible even for a given flow type. However, the minor nature of the changes in initial state leads us to propose that results based on varying rotational dynamics for a given initial flow type are due to the impact of beta on the nonlinear evolution of the wave. Differences due to variation of initial flow type for fixed rotational dynamics highlight the influ-
ence of the initial state. The idea that the primary influence of $\beta$ is on nonlinear development rather than the initial state is supported by the work of Nakamura (1993) who found that for QG dynamics the main effect of $\beta$ was the formation of momentum fluxes that reinforce the developing barotropic shear despite the addition of an initial PV gradient.

The similarity of energy budgets (Fig. 2) and zonal mean flows (Figs. 4f, 5f) for radically different initial flows on the $\beta$ plane points to the overriding influence of $\beta$ on the nonlinear evolution of the wave. The northward propagation of the developing wave (see Part I) has a lasting, irreversible impact: northward movement of zonal jets, splitting of regions of EP flux divergence at low levels, the appearance of reversed (southward) heat fluxes during barotropic wave decay, and the propensity to ultimately form many, small vortices. The profound sensitivity of wave evolution to initial state found by Davies et al. (1991) for semi-geostrophic flows is not seen here for primitive equation dynamics. The symmetry inherent in the SG equations (see Snyder et al. 1991) elicits an immediate nonlinear response to asymmetry in the initial state. Finally, while the above results are for simulations that are inviscid apart from horizontal smoothing, Barnes and Young (1992) find that surface friction and thermal damping reduce sensitivity to the initial state.

Because we have utilized nearly inviscid dynamics and have chosen to focus attention upon the later stages of baroclinic wave evolution when dissipative processes should become important, the analogy with atmosphere cyclogenesis is less clear than for earlier stages. A numerical consequence of the increasing importance of dissipative processes at this stage is the nonconservation of PV. With the appearance of sharp fronts near rigid horizontal boundaries, PV was shown to increase. The nonconservation was severe locally and notable globally. Thus, the role of numerical diffusion here is to permit wave equilibration in analogy with the role played by realistic dissipative atmospheric processes.

One issue that has not been directly addressed here is the relevance of the baroclinic adjustment hypothesis of Stone (1966). Gutowski et al. (1989) found static stability and vertical shear (of zonal mean zonal flow) adjustment important, thus implicating the utility of baroclinic adjustment ideas. They also found the generation of barotropic flow insufficient to stabilize the initial flow. However, Barnes and Young (1992) and Nakamura (1993) point to the importance of the "barotropic governor" (James 1987) in mean flow adjustment. Clearly, further study to reconcile these disparate views is needed and is currently in progress.

REFERENCES

---, and B. J. Hoskins, 1985: Some comparisons of atmospheric


