Mean and Transient Spectral Energy and Enstrophy Budgets

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ABSTRACT

The scale-dependent behavior of atmospheric flow on the sphere is investigated in terms of the spectra and spectral budgets of enstrophy and kinetic and available potential energy. The decomposition into both the $\alpha = (n, m)$ and the more usual n-spectral forms is considered. Several novel spectral results are obtained. First, a *direct* test of the extent to which large-scale atmospheric flow satisfies the necessary and sufficient conditions for homogeneous and isotropic two-dimensional turbulent behavior is carried out by calculating "spectral teleconnections" from data. The conditions are satisfied for the higher wavenumber transient component of the flow. Second, the spectral budget equations are extended to include the decomposition into time mean and transient components, a method that, although common and straightforward in real-space calculations, is novel in the spectral domain. The terms in these budgets are evaluated from data. Finally, the flow of energy and enstrophy through α-spectral space is displayed in terms of a novel "spectral potential function," which is related to spectral fluxes.

The results give information about the scales and wavenumbers at which sources and sinks of the mean and transient components of energy and enstrophy are found and how these quantities are transferred between scales and converted from one form to another. The interaction between the high wavenumber homogeneous and isotropic transient components and the low wavenumber inhomogeneous and nonisotropic mean components is seen to be an essential aspect of the spectral budgets.

1. Introduction

There has been a twofold tendency in the development of ideas concerning the behavior of the atmosphere. One tendency is to concentrate on the broad steady features of the flow and to consider the transient perturbations as secondary features. The other tendency is to concentrate on the essential nature of the transient aspects of the flow that are such a feature of the local behavior of the atmosphere, especially in the middle latitudes. These views have had counterparts in the development of theories of the general circulation.

It was perhaps natural that when a growing body of observations permitted it, researchers looked for and stressed the steady aspects of the circulation and attempted to explain these features in physical terms. Lorenz (1967) reviews these early theories, which are also generally characterized by a zonally symmetric viewpoint and which are associated with the names of Hadley, Dove, Maury, Thompson, Ferrel, and others. However, these views began to be contradicted by further observations, balance considerations, and ideas concerning the roles of cyclones and anticyclones. According to Lorenz, Defant was the first to adopt the second point of view and to consider that the motions of the atmosphere were simply turbulence on a very large scale. Jeffreys considered the transient and nonzonal aspects of the atmosphere to be major features of the general circulation.

Another viewpoint emerges when the flow is decomposed not only into mean and transient components but also into zonal and nonzonal components. This point of view seeks to understand the zonal and time-averaged flow in terms of the interactions with the deviations or "eddies" from them. This view is identified with Rossby, Starr, and others and is discussed in detail by Starr (1968). As expressed by Wallace (1978), there is a "gap that separated Rossby, Starr, and others who viewed the time and zonally averaged circulation as a response to forcing by large-scale turbulent eddies, from Palmén, Riehl, Namias, and others who sought to explain the gross features of the time-averaged general circulation as a response to steady forcing by the time-averaged diabatic heating field and large-scale orographic influences."

These viewpoints persist in current approaches to understanding the general circulation. In particular, the decomposition of the flow into time mean and transient parts remains basic, while the fact that the flow has features of large-scale turbulence is used in many studies and especially in considerations of energy exchange with scale and of predictability.

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Here, an attempt is made to interpret the behavior of the mean and transient components of the flow in terms of scale by applying both the turbulence approach and the decomposition of the flow into mean and transient components. A direct test is made of the extent to which the conditions for homogeneous and isotropic two-dimensional turbulence hold; spectral equations and spectral quantities in two dimensions are examined; and a set of equations describing the interaction of mean and transient components of the energy and enstrophy budgets as a function of scale is developed and applied to global data.

2. Spectra

For the sphere the appropriate expansion functions are the spherical harmonics as discussed, for instance, in Boer (1983) and Boer and Shepherd (1983), referred to hereafter as B1 and B–S. Any reasonably smooth scalar field $x(\lambda, \varphi)$ on the sphere may be expressed as

$$x(\lambda, \varphi, p, t) = \sum_{n=0}^{N} \sum_{m=-n}^{n} x_n^m(p, t) e^{im\lambda} P_n^m(\varphi) = \sum_\alpha x_\alpha(p, t) Y_\alpha(\lambda, \varphi), \quad (1)$$

where $\alpha = (n, m)$ represents the wavenumbers, Y_α the spherical harmonics, and x_α the complex expansion coefficients. The mean global variance of x is written as

$$\langle \overline{x^2} \rangle = \sum_\alpha \frac{1}{2} \overline{|x_\alpha|^2} = \sum_\alpha V_\alpha, \quad (2)$$

where the overbar represents a time or ensemble average and the angular brackets the area average over the sphere; V_α is that portion of the total spatial variance associated with wavenumber α.

The spatial spectrum of x may be presented as a function of both wavenumbers $\alpha = (n, m)$ or, more commonly, as function of a single wavenumber as

$$V(m) = \sum_{n=0}^{N} V_\alpha \quad \text{or} \quad V(n) = \sum_{m=-n}^{n} V_\alpha.$$

The first of these is the "m spectrum" associated with the usual Fourier decomposition in longitude [here the truncation wavenumber may be a function of m, i.e., $N(m)$]. This decomposition has a relatively long history of use in analyzing the atmospheric flow, beginning with Saltzman (1957). The n-spectrum representation is somewhat more recent (Baer 1972, 1974; Wiin-Nielsen 1972; Chen and Wiin-Nielsen 1978; Lambert 1981; B1; B–S. On the sphere, the wavenumber n is analogous to the total wavenumber $k = \sqrt{k_x^2 + k_y^2}$ for Fourier series on the plane and, as such, is particularly suitable for analysis of homogeneous and isotropic flow on the sphere.

a. Mean and eddy components

The quantity x is decomposed into its *time mean* $\bar{x}$ and *transient eddy* component x'. The mean component is further decomposed into *mean zonal* $[\bar{x}]$ and *stationary eddy* $\bar{x}^*$ components to give

$$x = \bar{x} + x' = [\bar{x}] + \bar{x}^* + x'.$$

The mean global variance of x is partitioned likewise as

$$\langle \overline{x^2} \rangle = \langle \bar{x}^2 \rangle + \langle \overline{x'^2} \rangle = \langle [\bar{x}]^2 \rangle + \langle \bar{x}^{*2} \rangle + \langle \overline{x'^2} \rangle.$$

The decomposition with spatial scale follows from (1) as

$$\langle \overline{x^2} \rangle = \sum_\alpha \overline{\frac{1}{2} |x_\alpha|^2} = \sum_\alpha V(\alpha)$$

$$\langle \bar{x}^2 \rangle + \langle \overline{x'^2} \rangle = \sum_\alpha \overline{\frac{1}{2} |\bar{x}_\alpha|^2} + \sum_\alpha \frac{1}{2} \overline{|x'_\alpha|^2} = \sum_\alpha V_M(\alpha) + \sum_\alpha V_T(\alpha)$$

and

$$\langle [\bar{x}]^2 \rangle + \langle \bar{x}^{*2} \rangle + \langle \overline{x'^2} \rangle = \sum_{n=0}^{N} \frac{1}{2} |\bar{x}_n^0|^2 + \sum_{\substack{\alpha \\ n \neq 0}} \frac{1}{2} |\bar{x}_\alpha|^2 + \sum_\alpha \frac{1}{2} \overline{|x'_\alpha|^2} = \sum_n V_Z(n) + \sum_\alpha V_S(\alpha) + \sum_\alpha V_T(\alpha).$$

The associated m and n spectra are obtained by summation. The last expression for the two-dimensional spectral decomposition includes the decomposition into the zonal time mean (for which $m = 0$) and the remaining standing and transient components.

b. Homogeneity and isotropy

The appeal of the ideas of homogeneous and isotropic two-dimensional turbulence as applied to the large-scale behavior of the atmosphere resides in the major simplification that they afford and in the body of turbulence theory that is available. As discussed in B1, B–S, and Boer (1984, hereafter B2), the possibility of inertial subranges has important theoretical and practical consequences for atmospheric flow.

Despite the appeal of these ideas, there has not been a direct calculation to test the extent to which atmospheric flow displays the *necessary and sufficient* condition for homogeneity and isotrophy. According to B1, the condition on the rotational part of the flow is that the two-point covariance function of the streamfunction ψ is a function only of separation; that is,

$$C(s_1, s_2) = \overline{\psi(s_1)\psi(s_2)} = C(d),$$

where s_1 and s_2 are two points on the sphere and d is the geodetic distance between them. As shown in B1, the condition on the spherical harmonic expansion coefficients is

$$\overline{\psi_\alpha \psi_\beta^*} = \delta_\alpha^\beta C(n), \tag{3}$$

which states that the covariance is zero unless both wavenumbers are the same, in which case

$$\overline{\psi_\alpha \psi_\beta^*} = \overline{|\psi_\alpha|^2} = C(n) = \overline{|\psi_n^{(m)}|^2} \tag{4}$$

and the spectral densities are a function only of n. Expressed in terms of a complex correlation coefficient, the condition becomes

$$R(\alpha, \beta) = \overline{\psi_\alpha \psi_\beta^*} / (\overline{|\psi_\alpha|^2} + \overline{|\psi_\beta|^2})^{1/2} = \delta_\alpha^\beta.$$

Previous studies have shown only that the *necessary* condition that follows from (4)—that the spectral densities are independent of m and are a function only of n—holds to a reasonable extent for the higher wavenumber region of the atmosphere.

3. Enstrophy and energy equations

The n-spectrum equations are considered in B–S. Here the associated $\alpha = (n, m)$ spectral equations are stressed. They have the same form as the equations in B–S but with α in place of n and with the summation over m omitted. In particular, for enstrophy, kinetic energy, and available potential energy

$$\begin{aligned} \partial G(\alpha)/\partial t &= J(\alpha) + R(\alpha) \\ \partial E(\alpha)/\partial t &= I(\alpha) + S(\alpha) \\ \partial A(\alpha)/\partial t &= M(\alpha) + Q(\alpha), \end{aligned} \tag{5}$$

where the rate of change of enstrophy, kinetic energy, and available potential energy associated with wavenumber α is a consequence of the nonlinear interactions $(J(\alpha), I(\alpha), M(\alpha))$ between wavenumber α and all others plus the source/sink terms $(R(\alpha), S(\alpha), Q(\alpha))$ at that wavenumber. For enstrophy,

$$\begin{aligned} G(\alpha) &= \frac{1}{4} |\zeta_\alpha|^2 \\ J(\alpha) &= -\frac{1}{4} (\zeta_\alpha^* (\mathbf{V} \cdot \nabla \zeta)_\alpha + \zeta_\alpha (\mathbf{V} \cdot \nabla \zeta)_\alpha^*) \\ R(\alpha) &= \frac{1}{4} (\zeta_\alpha^* R_\alpha + \zeta_\alpha R_\alpha^*), \end{aligned} \tag{6}$$

where x_α^* represents the complex conjugate. The corresponding terms in the kinetic energy budget are closely linked to those in the enstrophy budget, as shown in B–S, for instance, by the relationships

$$(E(\alpha), I(\alpha), S(\alpha)) = (G(\alpha), J(\alpha), R(\alpha)) a^2 / n(n+1). \tag{7}$$

The equations for the available potential energy have the same form as (6) with T_α (with the global mean removed) replacing ζ_α and where each quantity is multiplied also by $C_p \gamma$, the specific heat at constant pressure times the usual stability function.

a. Time-averaged equations

As noted in B–S, averaging (5) over one month gives a rate of change term that is small, so that the *interaction* terms equal the negative of the *source/sink* terms. For the kinetic energy equation, for instance, $\bar{I}(\alpha) = -\bar{S}(\alpha)$, which states that at any wavenumber the nonlinear transfer of kinetic energy is balanced by the source/sink at that wavenumber and similarly for enstrophy and available potential energy.

Since the nonlinear interaction term serves only to transfer energy and enstrophy between scales, its net effect must add to zero,

$$\sum_\alpha J(\alpha) = \sum_\alpha I(\alpha) = \sum_\alpha M(\alpha) = 0,$$

which implies also that the interaction term may be expressed as the divergence of a spectral flux. In the α-spectral case considered here this may be written in terms of a wavenumber version of the del operator of the form $\nabla_\alpha = (\partial/\partial m, \partial/\partial n)$ to give

$$\begin{aligned} (J(\alpha), I(\alpha), M(\alpha)) &= \nabla_\alpha \cdot (\mathbf{H}(\alpha), \mathbf{F}(\alpha), \mathbf{P}(\alpha)) \\ &= -\nabla_\alpha^2 (\phi(\alpha), \chi(\alpha), \psi(\alpha)), \end{aligned} \tag{8}$$

where $\mathbf{H}, \mathbf{F}, \mathbf{P}$ are *spectral flux vectors* and ϕ, χ, ψ are the associated *spectral potential functions.*

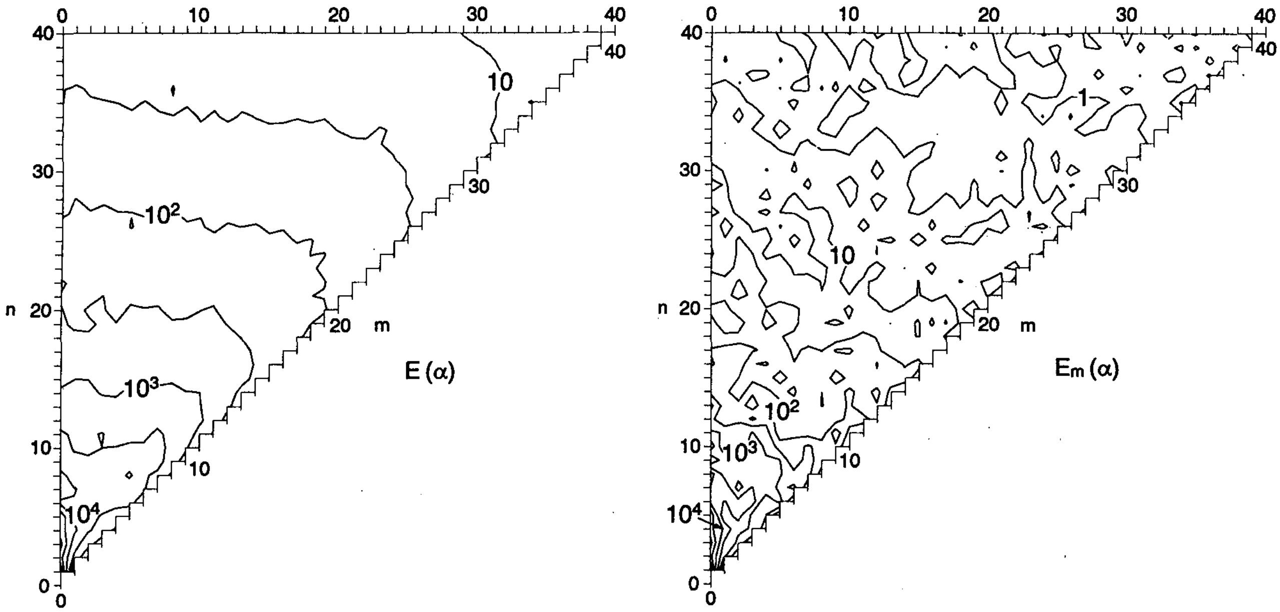
b. Inertial subranges

A particularly simple turbulent situation obtains in an inertial subrange for two-dimensional homogeneous and isotropic turbulence. In this case, the source/sink and nonlinear interaction terms are zero, the spectrum is a function only of n (it is independent of m), and the magnitude of the flux of energy and enstrophy is constant. In terms of the α-spectrum representation this implies, for an *enstrophy cascading* subrange, that

$$\begin{aligned} &E(\alpha) = A\eta^{2/3} n^{-3} / (2n+1) \approx \eta^{2/3} n^{-4}, \quad \mathbf{F}(\alpha) = 0 \\ &\mathbf{H}(\alpha) = -\nabla_\alpha \phi(\alpha) = \frac{-2\eta}{\pi r} (m, n), \quad \phi(\alpha) = \frac{-2\eta}{\pi a} \ln r, \end{aligned} \tag{9}$$

and for an *energy cascading* subrange that

$$\begin{aligned} &E(\alpha) = B\epsilon^{2/3} n^{-5/3} / (2n+1) \approx \epsilon^{2/3} n^{-8/3}, \ \mathbf{H}(\alpha) = 0 \\ &\mathbf{F}(\alpha) = -\nabla_\alpha \chi(\alpha) = \frac{-2\epsilon}{\pi r} (m, n), \quad \chi(\alpha) = \frac{2\epsilon}{\pi a} \ln r, \end{aligned}$$



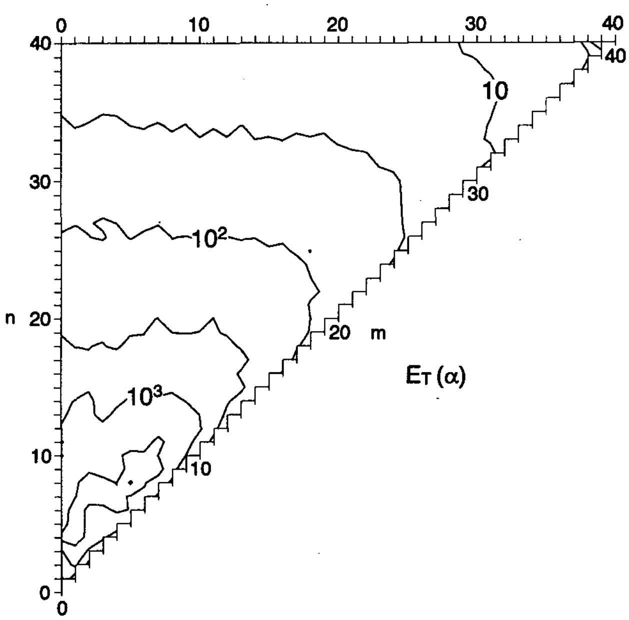


FIG. 1. The vertically integrated rotational kinetic energy spectra as a function of α for (a) the total $E(\alpha)$, (b) the mean component $E_M(\alpha)$, and (c) the transient component $E_T(\alpha)$. Units are joules per square meter.

where $r = |\alpha| = \sqrt{m^2 + n^2}$. The n-spectrum representations given in B–S are obtained by integrating over m. These equations are obtained by treating the wavenumbers as if they are continuous rather than in terms of sum and difference formulas appropriate to quantized values. However, the flux and potential function formulas are essentially definitions, so this simpler continuous form should be sufficient.

c. *Mean and transient spectral budgets*

The decomposition of the atmospheric budget equations into component equations under various forms of averaging is a basic approach, as noted above. Under zonal averaging, the Lorenz (1955) "four box" version of the zonal and eddy equations is standard. Saltzman's (1957) m-spectral budget equations are an extension of this approach where the eddy component of the equations is further decomposed into Fourier components in the east–west.

The further decomposition of the budget equations under time averaging gives a variant of the Lorenz equations where the separation is into mean and transient eddies. Decomposition under both time and zonal averaging provides a "six box" version of the

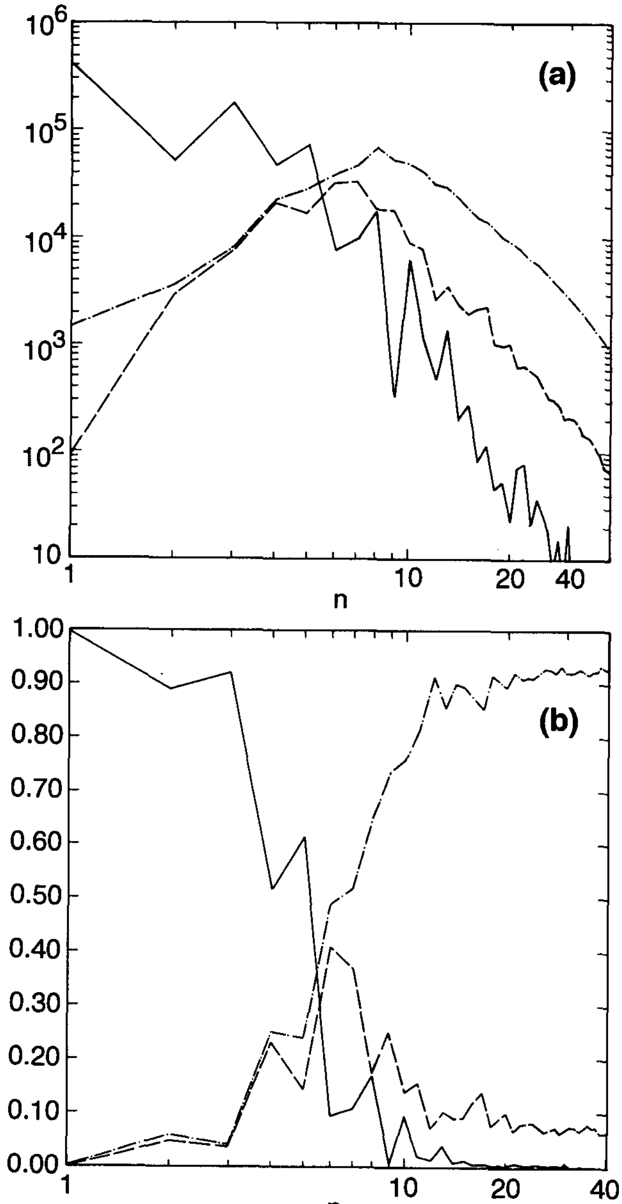


FIG. 2. (a) The vertically integrated rotational kinetic energy spectra as a function of n for the zonal component $E_Z(n)$ (solid line), the stationary component $E_S(n)$ (dashed line), and the transient component $E_T(n)$ (dash–dotted line); and (b) the ratio of these components to the total kinetic energy $E(n)$. Units are joules per square meter.

budgets involving zonal, stationary, and transient eddy components of the kinetic and available potential energy.

A further extension to the budget equations in which the *spectral* energy and enstrophy equations [(5)–(6)] are decomposed into mean and transient components is discussed here. The development of these "component" equations is not straightforward. It is possible to write down any number of component equations that have the standard form. It may be possible to develop several apparently equivalent component equations. The appropriate forms are those that have a useful physical interpretation.

The development of the mean and transient spectral enstrophy and energy equations for the rotational part of the flow proceeds by direct analogy with the budget equations for *systematic* and *random* forecast error as a function of scale given in B2 by replacing an ensemble average over a set of forecasts by the time mean over the data. The appropriate form of the component equations among several possibilities is obtained by requiring that like terms in the enstrophy and energy equations, which are linked by (7), represent the same physical processes; that is, they are interaction, conversion, or source/sink terms in both equations. This leads to the form of the equations in which the redistribution of energy or enstrophy in the mean equation depends only on the mean flow, while in the transient equation the redistributive term displays a suitable symmetry with all possible combinations of two transient and one mean term appearing. Shepherd (1987) has noted the approach developed in B2 but also argues for a different approach in which the conversion and interaction terms are not separated out in this way and where the analogous terms in the enstrophy and energy form of the equations do not play the same redistributive and conversions roles.

The resulting mean and transient equations for enstrophy are

$$\partial G_M(\alpha)/\partial t = J_M(\alpha) + C_G(\alpha) + R_M(\alpha)$$
$$\partial G_T(\alpha)/\partial t = J_T(\alpha) - C_G(\alpha) + R_T(\alpha), \qquad (10)$$

where

$$J_M(\alpha) = -\sum_\alpha \frac{1}{2}(\bar\zeta^*_\alpha(\bar{\mathbf{V}}\cdot\nabla\bar\zeta)_\alpha + \bar\zeta^*_\alpha(\bar{\mathbf{V}}\cdot\nabla\bar\zeta)^*_\alpha)$$
$$J_T(\alpha) = J(\alpha) - J_M(\alpha) \qquad (11)$$

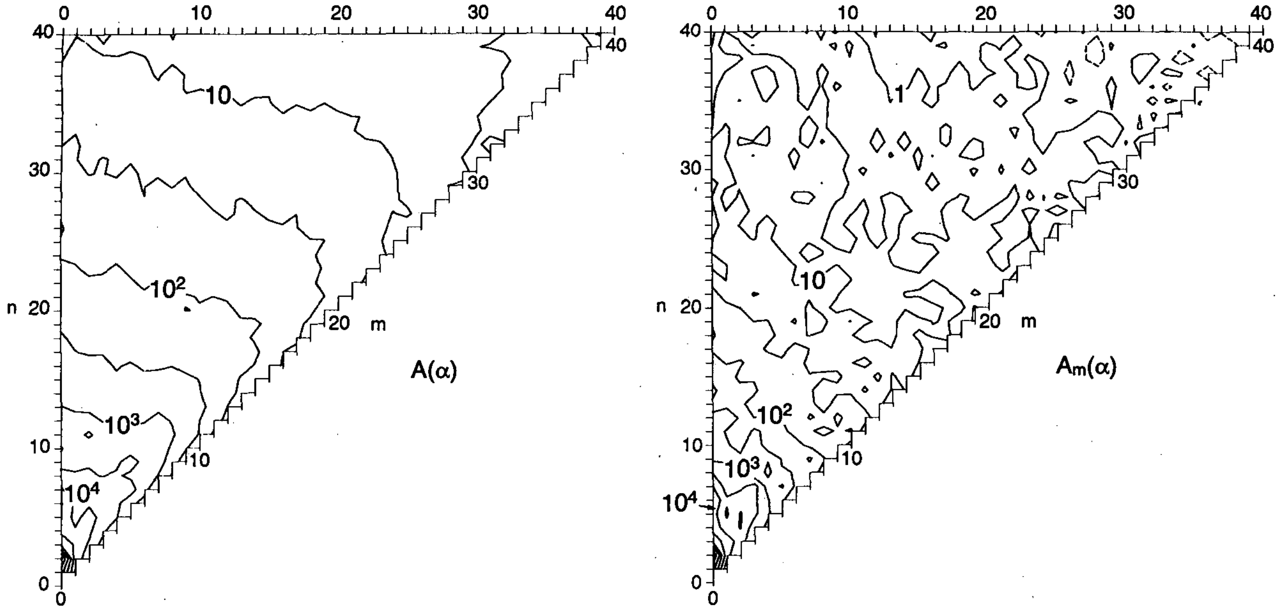
for $J(\alpha)$ given in (6), and where the conversion term has the form

$$C_G(\alpha) = \sum_\alpha \frac{1}{2}(\bar\zeta_\alpha(\overline{\mathbf{V}'\cdot\nabla\zeta'})_\alpha + \bar\zeta^*_\alpha(\overline{\mathbf{V}'\cdot\nabla\zeta'})^*_\alpha). \qquad (12)$$

The source/sink term is decomposed straightforwardly as

$$R(\alpha) = R_M(\alpha) + R_T(\alpha)$$
$$= \frac{1}{2}\sum_\alpha \bar\zeta_\alpha\bar R^*_\alpha + \bar\zeta^*_\alpha\bar R_\alpha) + \frac{1}{2}\sum_\alpha \overline{\zeta'_\alpha R'^*_\alpha} + \overline{\zeta'^*_\alpha R'_\alpha}).$$

As before, the corresponding kinetic energy terms are obtained using (7), while the analogous available po-



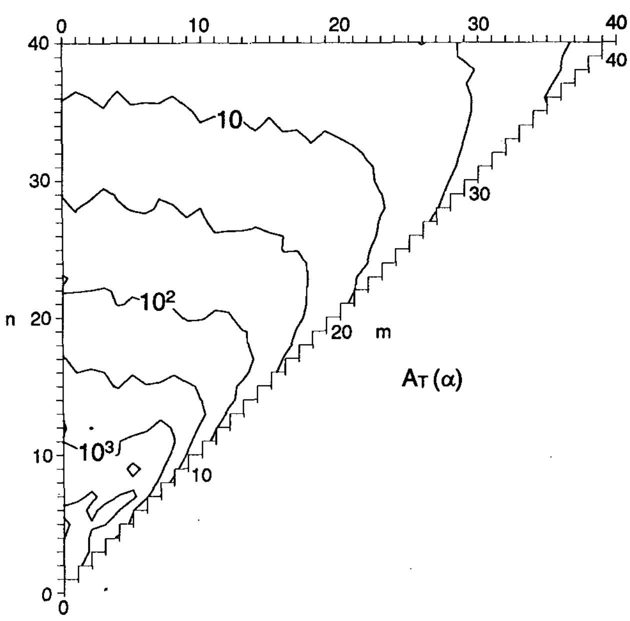


FIG. 3. The vertically integrated available potential energy spectra as a function of α for (a) the total $A(\alpha)$, (b) the mean component $A_M(\alpha)$, and (c) the transient component $A_T(\alpha)$. Units are joules per square meter.

tential energy equations are obtained by replacing ζ with T and multiplying by $C_p\gamma$ as before.

4. Data

Global data from the European Center for Medium-Range Weather Forecasts (ECMWF) First Global GARP Experiment (FGGE) III-b dataset are used to evaluate the various spectra and the terms in the spectral enstrophy and energy budgets. These data consist of the two velocity components for 00 and 12 GMT for each day of January 1979.

The data are available on a 1 1/8 degree latitude–longitude grid, but for the calculations reported here are interpolated to the transform grid consistent with triangular T40 truncation [$N = 40$ in (1)] before transforming to spectral coefficients. The data are available on 15 pressure levels from 10 to 1000 mb.

5. Results

The vertically integrated spectrum $E(\alpha)$ has been presented in B–S for the related National Meteorological Center (NMC) FGGE III-a dataset (note that in

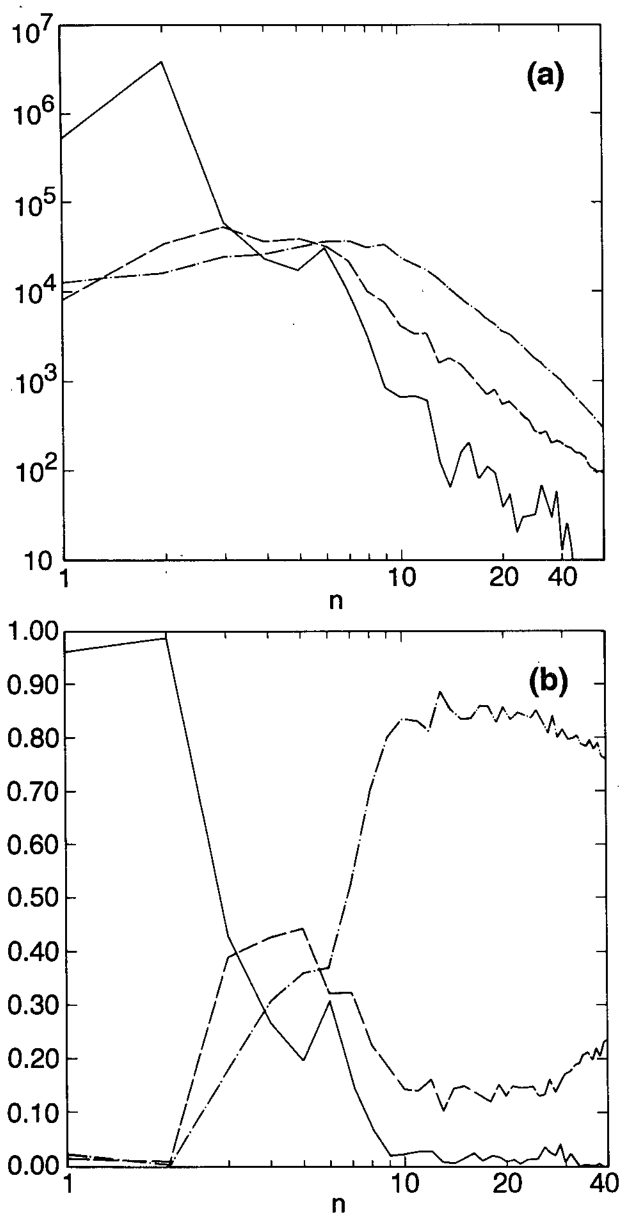


FIG. 4. (a) The vertically integrated available potential energy spectra as a function of n for the zonal component $A_Z(n)$ (solid line), the stationary component $A_S(n)$ (dashed line), and the transient component $A_T(n)$ (dotted line) and (b) the ratio of these components to the total kinetic energy $A(n)$. Units are joules per square meter.

B–S the units differ since the terms were divided by 10^5 representing the mass of the column). The total spectrum $E(\alpha)$ together with the mean $E_M(\alpha)$ and transient $E_T(\alpha)$ components are presented in Fig. 1. In all such figures, the wavenumber "triangle" is plotted with (m, n) in the (x, y) directions. As in B–S, the dominance of the mean structures for small wavenumbers and of the transient component for wavenumbers beyond about 10 is clear. The lines of constant spectral density in this high wavenumber region are more or less horizontal, implying their independence of m. The most notable exception to this is in the small region near the hypotenuse of the triangle where spectral densities decrease.

Figure 2 shows the corresponding n spectra for which the decomposition in this case is into the zonal $E_Z(n)$, stationary $E_s(n)$, and transient $E_T(n)$ components, while Fig. 2b shows the ratio of these components to the total energy $E(n)$. This diagram emphasizes a feature that was also notable in Fig. 1: namely, E_Z, the zonal mean $(m = 0)$ structure of the flow, is dominant at low wavenumbers. The stationary eddies are an appreciable fraction of the total only in the region $5 < n < 10$, while once again, the transients are seen to dominate the high wavenumbers.

Figures 3 and 4 show the comparable diagrams for the available potential energy. The overall features of the spectra are similar although the stationary eddy structure $A_S(\alpha)$ is seen to be comparatively more important at smaller scales for the available than for the kinetic energy according to Fig. 4b.

a. A test for homogeneity and isotropy on the sphere

Figures 1 and 3 suggest that the spectral densities for kinetic energy [and therefore for enstrophy through (7)] as well as available potential energy are more or less independent of m, at least for larger values of n. This is a *necessary* condition for homogeneous and isotropic turbulence on the sphere. However, the *necessary and sufficient* condition (3) demands that the spec-

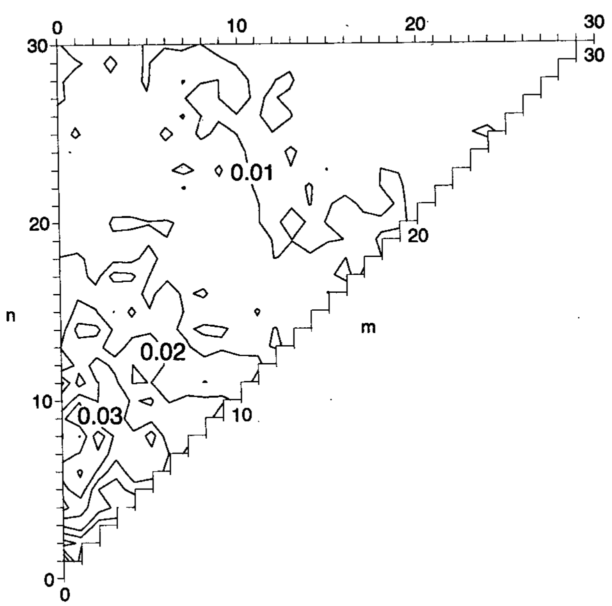
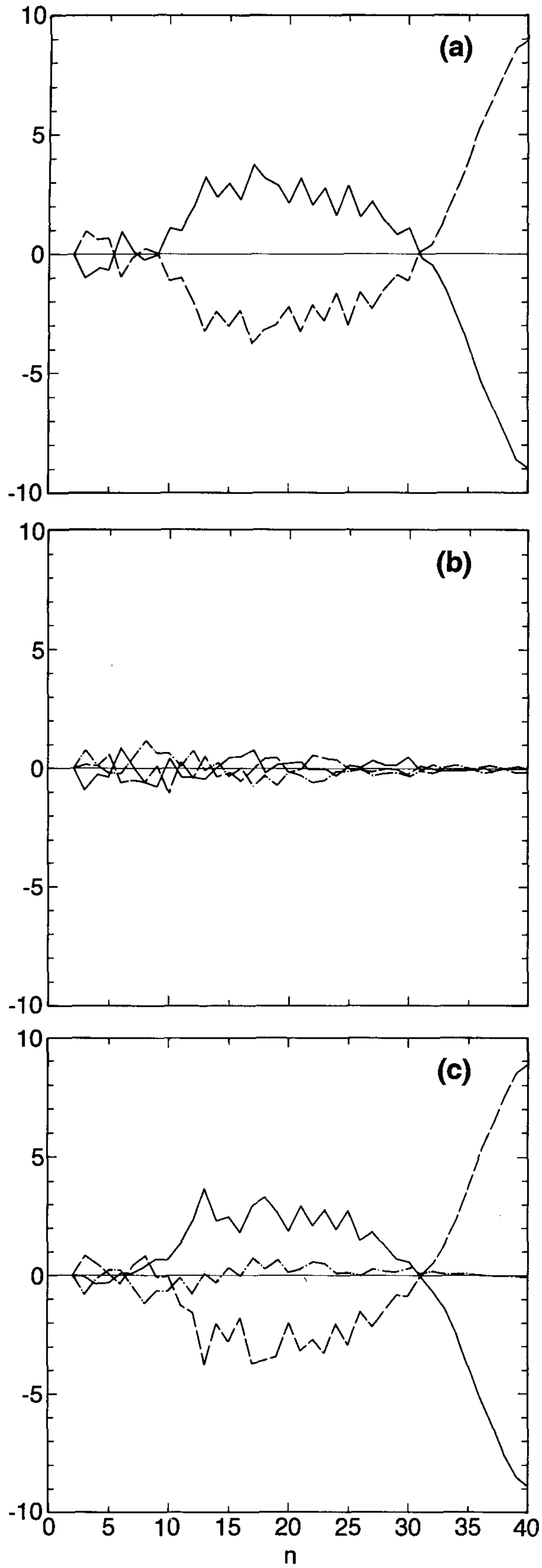


FIG. 5. Spectral correlation function $R^2(\alpha)$ indicating the extent to which the streamfunction ψ'_α for wavenumber α is correlated with all other wavenumbers on average.



tral expansion coefficients of the streamfunction are uncorrelated. This is tested by direct calculation.

Values of the streamfunction expansion coefficients ψ_α are calculated for each data time. The test is applied to the transient component of the flow $\psi'_\alpha = \psi_\alpha - \overline{\psi}_\alpha$. Mean components $\overline{\psi}_\alpha$ represent a nonisotropic and inhomogeneous aspect of the flow. The covariance of each expansion coefficient with every other is calculated as

$$C(\alpha, \beta) = \int_0^{p_0} \overline{\psi'_\alpha \psi'^*_\beta}\, dp/g,$$

where the associated variances are special cases $\sigma^2(\alpha) = C(\alpha, \alpha)$ and the complex correlation coefficient is

$$R(\alpha, \beta) = C(\alpha, \beta)/\sigma(\alpha)\sigma(\beta).$$

The necessary and sufficient condition (3) demands that $R(\alpha, \beta) = \delta_\alpha^\beta$. The calculation is simplified somewhat by considering the squared amplitude of R, which nevertheless requires that $R^2(\alpha, \beta) = \delta_\alpha^\beta$.

The set of $R^2(\alpha, \beta)$ is large. For each value of α_0, $R^2(\alpha_0, \beta)$ represents a *spectral teleconnection* map, which could be graphed on a triangular plot similar to those of Fig. 1. If condition (3) is satisfied, this map would have zero entries everywhere except at $\alpha = \alpha_0$ where $R^2(\alpha_0, \alpha_0) = 1$. It is not feasible to inspect all of the spectral teleconnection maps, so a spectral average of R^2 is calculated as

$$R^2(\alpha) = \frac{1}{M} \sum_{\beta \neq \alpha} R^2(\alpha, \beta),$$

where the sum is over all entries in the teleconnection map for wavenumber α. If (3) holds, then $R^2(\alpha)$ will be zero for all α. The map of $R^2(\alpha)$ is displayed in Fig. 5.

Although a formal statistical test for the hypothesis that $R^2 = 0$ is not straightforward, it is clear that R^2 is generally very small with maximum values of the order of 0.04. In the region with $n > 10$, for which the necessary condition roughly applies, generally $R^2 < 0.02$, and this small value supports the contention that the transient component of atmospheric flow satisfies both the necessary and sufficient condition for homogeneous and isotropic turbulence for these wavenumbers.

b. *Spectral budgets as a function of n*

The terms in the n-spectral enstrophy and kinetic and available potential budgets are discussed in B–S as

FIG. 6. Terms in the vertically integrated n-spectral (a) net, (b) mean, and (c) transient enstrophy budget equations. The nonlinear interaction term $J(n)$ (dashed line), the source/sink term $R(n)$ (solid line), and the conversion term $C_G(n)$ between mean and transient components (dash–dotted line) are shown for the various budgets. (Units: s^{-3} kg m^{-2}.)

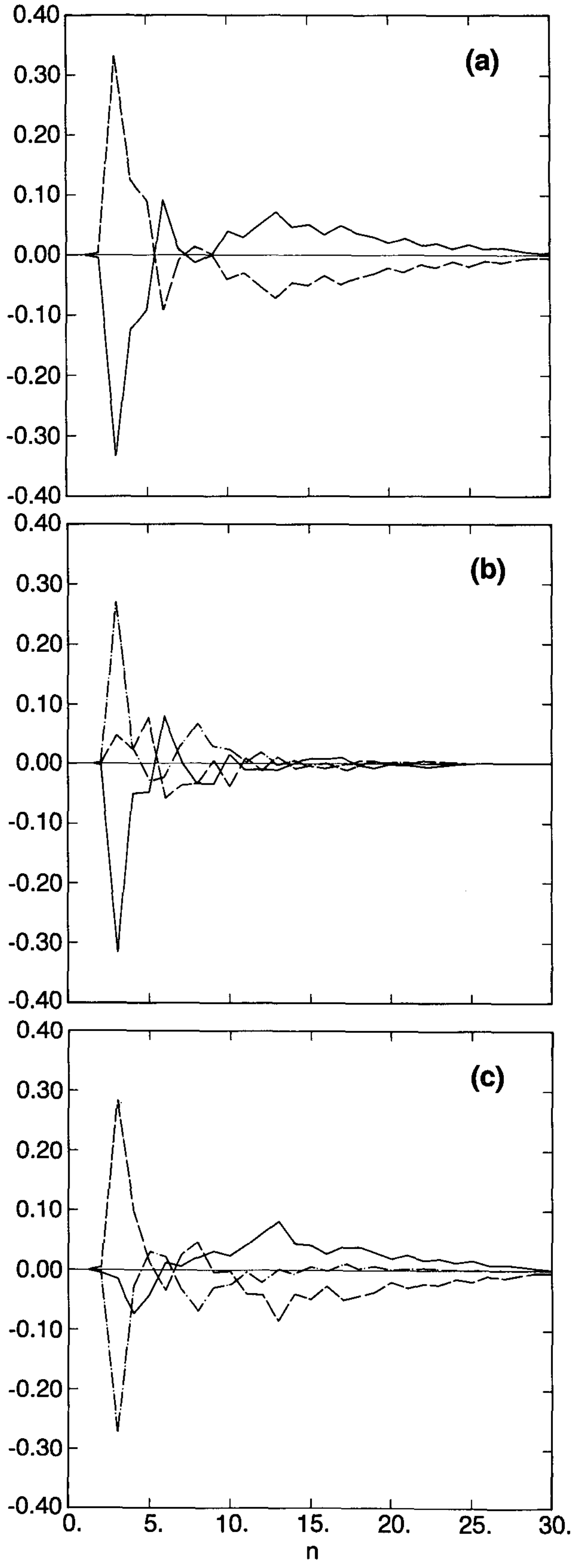


FIG. 7. Terms in the vertically integrated n-spectral (a) net, (b) mean, and (c) transient kinetic energy budget equations. The nonlinear interaction term $I(n)$ (dashed line), the source/sink term $S(n)$ (solid line), and the conversion term $C_E(n)$ between mean and transient components (dash–dotted line) are shown for the various budgets. Units are watts per square meter.

evaluated from FGGE IIIa data. Terms in the spectral budgets that have been further decomposed into mean and transient spectral components following (10)–(12) are shown in Figs. 6–8.

The net energy and enstrophy budgets, top panels of Figs. 6–8, show 1) a distributed source region over middle wavenumbers for kinetic energy and enstrophy, 2) a strong sink at low wavenumbers for kinetic energy and a more distributed sink at high wavenumbers for enstrophy, and 3) a strong localized source at $n = 2$ for available potential energy and a broad sink region in middle wavenumbers. The associated net fluxes of enstrophy and energy are the solid lines of Fig. 9.

The mean and transient budgets of enstrophy, kinetic, and available potential energy are shown in the remaining panels of Figs. 6–8. The transient component dominates the spectrum and the budget equations for enstrophy, as shown in Fig. 6c, as the terms in the equation for the mean component, as shown in Fig. 6b, are all small. This is consistent with the result that the high wavenumber transients in the atmosphere are roughly characterizable as homogeneous and isotropic turbulence.

Both mean and transient components are important in the kinetic energy budget of Fig. 7, and there is a strong interaction between them. The transient kinetic energy budget of Fig. 7c is characterized by a broad source region at middle wavenumbers, upscale transfer, and conversion to mean kinetic energy at low wavenumbers. The mean kinetic energy budget of Fig. 7b is dominated by the low wavenumbers where the conversion from the transients is balanced by the sink term at these wavenumbers. The mean budget shows a very modest source region and associated upscale transfer.

The mean and transient available potential energy budgets of Fig. 8 are in many respects the inverse of the kinetic energy budgets. There is a strong source of mean APE at low wavenumbers, that is converted to transient APE there and transferred downscale to a distributed sink at middle wavenumbers by the transient interaction term.

The fluxes of energy and enstrophy from source to sink regions are displayed in Fig. 9 where the net flux together with contributions arising from the interactions between *mean* components, between *mean and transient* components, and between *transient* components are plotted separately. These diagrams show: 1) The fluxes associated with the interactions among mean components of the flow (dashed lines) are generally

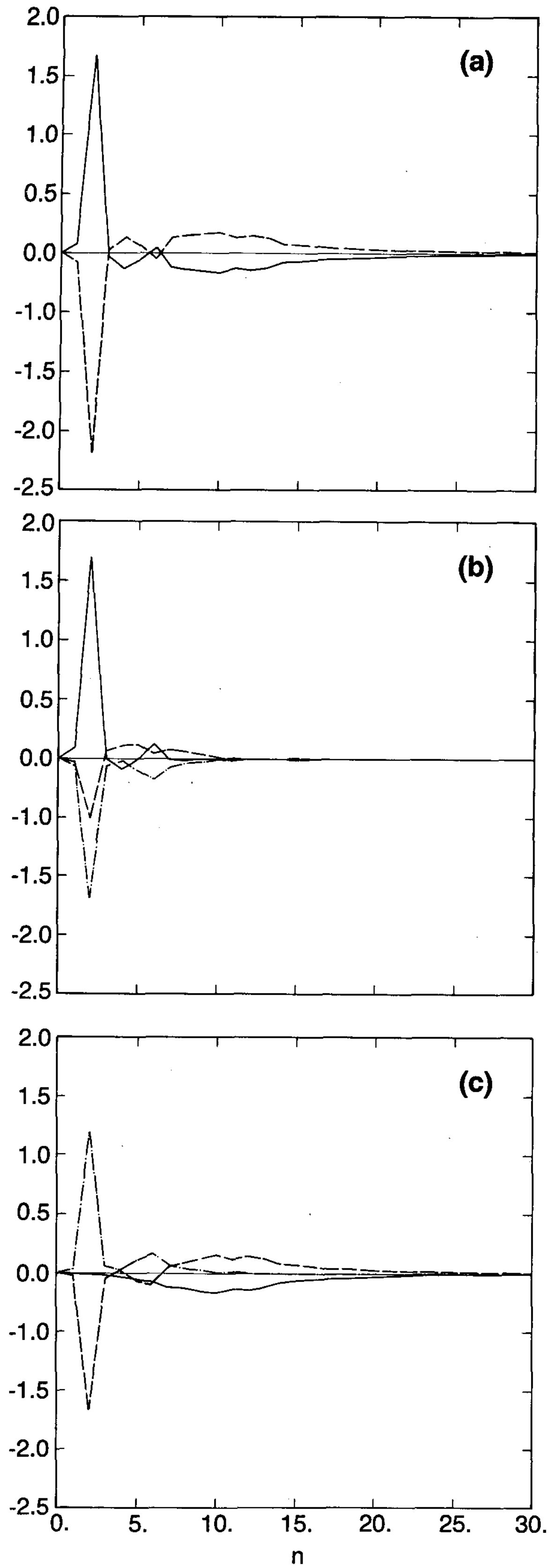


FIG. 8. Terms in the vertically integrated n-spectral (a) net, (b) mean, and (c) transient available potential energy budget equations. The nonlinear interaction term $M(n)$ (dashed line), the source/sink term $Q(n)$ (solid line), and the conversion term $C_A(n)$ between mean and transient components (dash–dotted line) are shown for the various budgets. Units are watts per square meter.

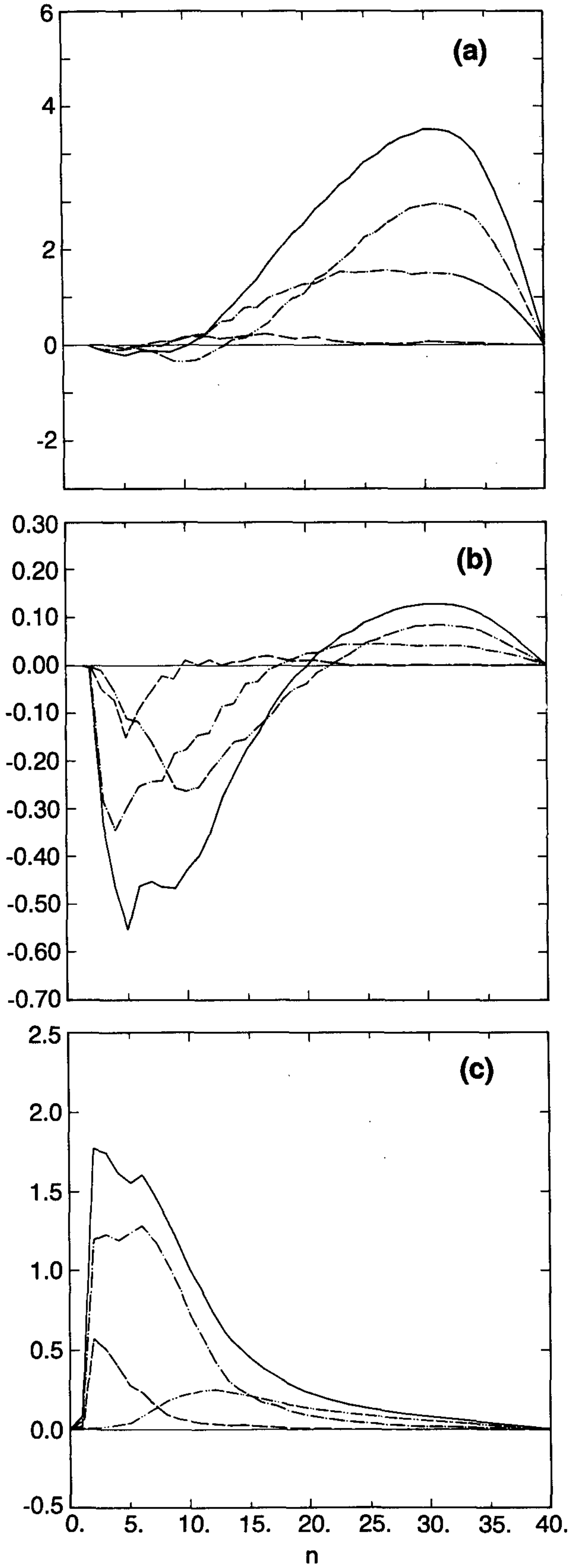
small; 2) in all cases the fluxes associated with the interaction between mean and transient components (dash–dot lines) are important, especially the flux of available potential energy; 3) the fluxes associated with the interaction of transient components (dash–dot–dot lines) are strongest at high wavenumbers for enstrophy and at middle wavenumbers for kinetic energy. Pure transient flux is a surprisingly small part of the total for available potential energy.

These n-spectrum mean and transient energy budgets extend the traditional view of the energy cycle. Low wavenumber mean available potential energy is converted to transient available potential energy at these wavenumbers and this APE is spectrally transferred to smaller scales where some of it provides the source for transient kinetic energy. The transient KE is, in turn, spectrally transported upscale, where it is converted into low wavenumber mean kinetic energy and dissipated at these scales. These budgets provide additional scale information beyond the usual Lorenz representation and provide also a mean and transient decomposition of a scale-dependent representation of the energy budget, which could be applied to the Saltzman m-spectrum energy budget.

In interpreting these budgets, it must be noted that the interaction and flux terms for the transient budgets are composed of both a pure transient and mixed mean–transient terms. The mixed term is particularly important and indicates that the atmosphere cannot be thought of as a freely evolving field of transient turbulence but that many wavenumbers are involved in important interactions with the low wavenumber anisotropic flow in which the mean, especially the mean zonal, structures are dominant.

c. Spectral budgets as a function of α

For the time-averaged budgets, the interaction terms are the negative of the source/sink terms, for instance, $I(\alpha) = -S(\alpha)$ for kinetic energy and similarly for enstrophy and APE, as is noted above. The interaction terms together with the associated potential functions (8) of the fluxes connecting sources with sinks are shown in Fig. 10. (Note that these calculations necessarily include only the scales explicitly resolved in the data.) The fluxes of energy and enstrophy through the spectrum are implicit in the potential function in the usual way; that is, the fluxes are directed perpendicular to the lines of constant potential from high values (in-



dicated by shading) to low values with magnitudes inversely proportional to the spacing between them. Figure 10 shows the following: 1) A broad enstrophy source region $R(\alpha) = -J(\alpha)$ indicated by negative values of $J(\alpha)$ at intermediate wavenumbers. The interaction term transfers this enstrophy to smaller scales as graphically indicated by the enstrophy potential function $\phi(\alpha)$, which shows that large downscale fluxes dominate the picture with much smaller upscale fluxes converging into the zonal component. 2) Kinetic energy sources $S(\alpha) = -I(\alpha)$ are indicated by negative regions of $I(\alpha)$ at middle to low wavenumbers. The interaction term transfers this energy strongly upscale where it converges into low zonal wavenumbers, as indicated by the potential function $\chi(\alpha)$. A much weaker flux to high wavenumbers is also seen. 3) The available potential energy displays a very strong low wavenumber zonal source $Q(\alpha) = -M(\alpha)$, and this energy is transferred down the spectrum to higher wavenumbers.

The implied energy cycle has a strong APE source at low zonal wavenumbers with this energy transferred downscale to a distributed sink region at low/middle wavenumbers. Part of the APE at these scales serves as the source of KE at these scales while the remainder represent the (negative) generation term. The KE source at low/middle wavenumbers provides the energy for a strong upscale energy transfer to a sink of KE at low zonal wavenumbers.

A more detailed look at the energy cycle is provided by Figs. 11 and 12. There is a considerable amount of detail in these figures and no attempt is made to discuss all features. The KE diagram, for instance, displays the mean $S_M(\alpha)$ and transient $S_T(\alpha)$ source/sink terms, the conversion term $C_E(\alpha)$, and the three components of the interaction term involving mean terms $I_M(\alpha)$, mixed mean and transient terms $I_{MT}(\alpha)$, and pure transient terms $I_{TT}(\alpha)$. In the budget equations (9)–(10) the transient interaction term is the sum of the latter two terms $I_T(\alpha) = I_{MT}(\alpha) + I_{TT}(\alpha)$.

The source of APE in Fig. 12 is associated with the mean component $Q_M(\alpha)$ and is found predominantly at low zonal wavenumbers. Some of this APE is transferred to an adjacent mean sink region at low nonzonal wavenumbers by the mean interaction term $M_M(\alpha)$, while much of the remaining energy is converted to transient APE by the conversion term $C_A(\alpha)$ whence it is transferred downscale to larger wavenumbers by the

FIG. 9. Fluxes through wavenumber space of (a) enstrophy, (b) kinetic energy, and (c) available potential energy. The net fluxes (solid line) are shown together with the contributions arising from interactions between mean components (dashed), between mean and transient components (dash–dotted), and between pure transient components (dash–dot–dot). (Units are 10^{-3} s^{-3} kg m^{-2} for enstrophy and W m^{-2} for energy fluxes.)

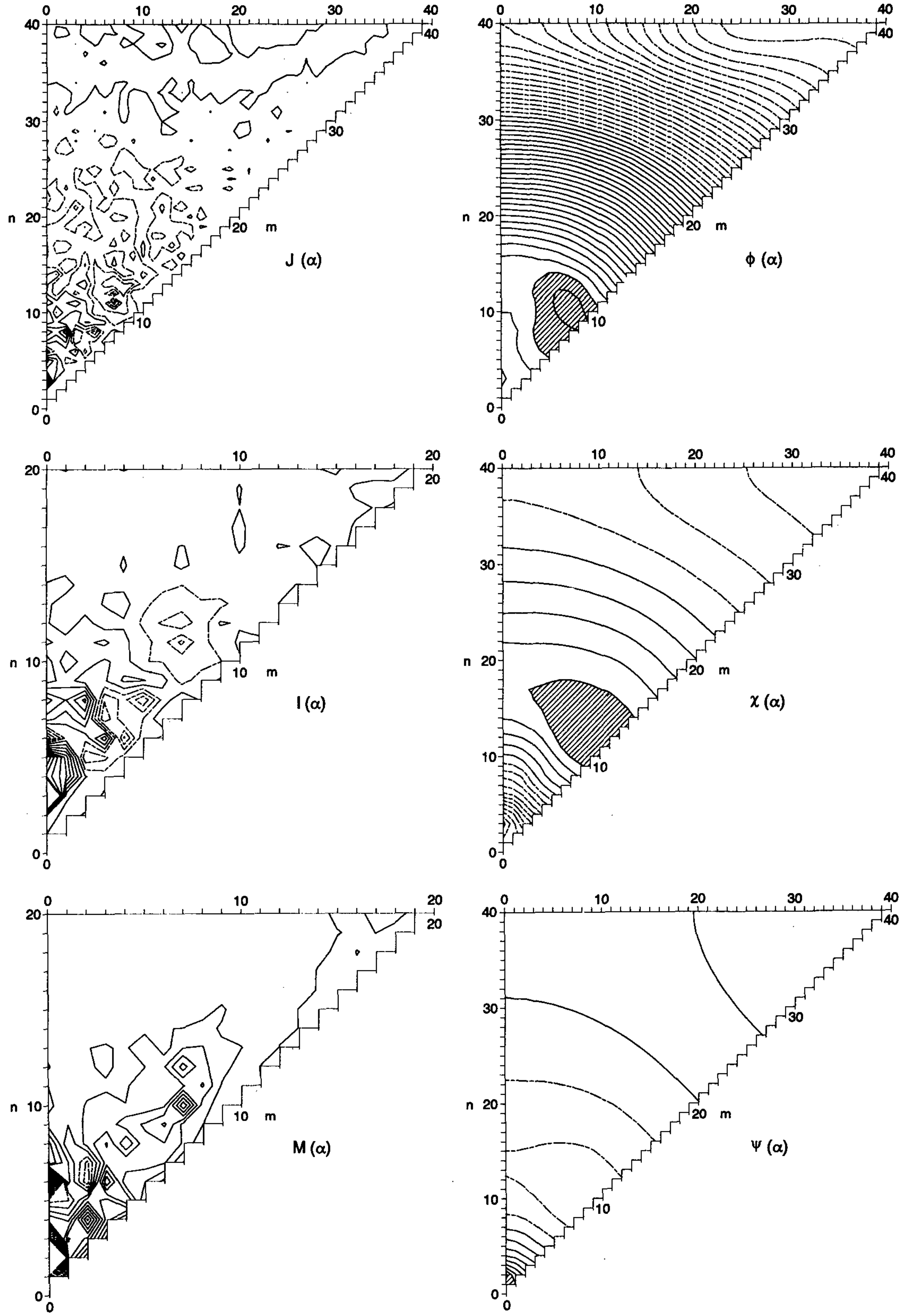


FIG. 10. Nonlinear interaction terms for enstrophy $J(\alpha)$, kinetic energy $I(\alpha)$, and potential energy $M(\alpha)$. For the time-averaged budget equations considered here, the associated source/sink $R(\alpha)$, $S(\alpha)$, and $Q(\alpha)$ are simply the negative of these quantities. The associated spectral potential functions indicate the flux of energy and enstrophy through spectral space. Negative contours are dashed. Units and contour intervals are $J(\alpha)$ (10^{-15} s^{-3} kg m^{-2}, 20); $\phi(\alpha)$ (10^{-16} kg s^{-3}, 5); $I(\alpha)$ and $M(\alpha)$ (10^{-3} W m^{-2}, 10); $\chi(\alpha)$ (10^{-4} W, 2) and $\psi(\alpha)$ (10^{-4} W, 10). Not all contours are drawn in regions of very large values for $m = 0$.

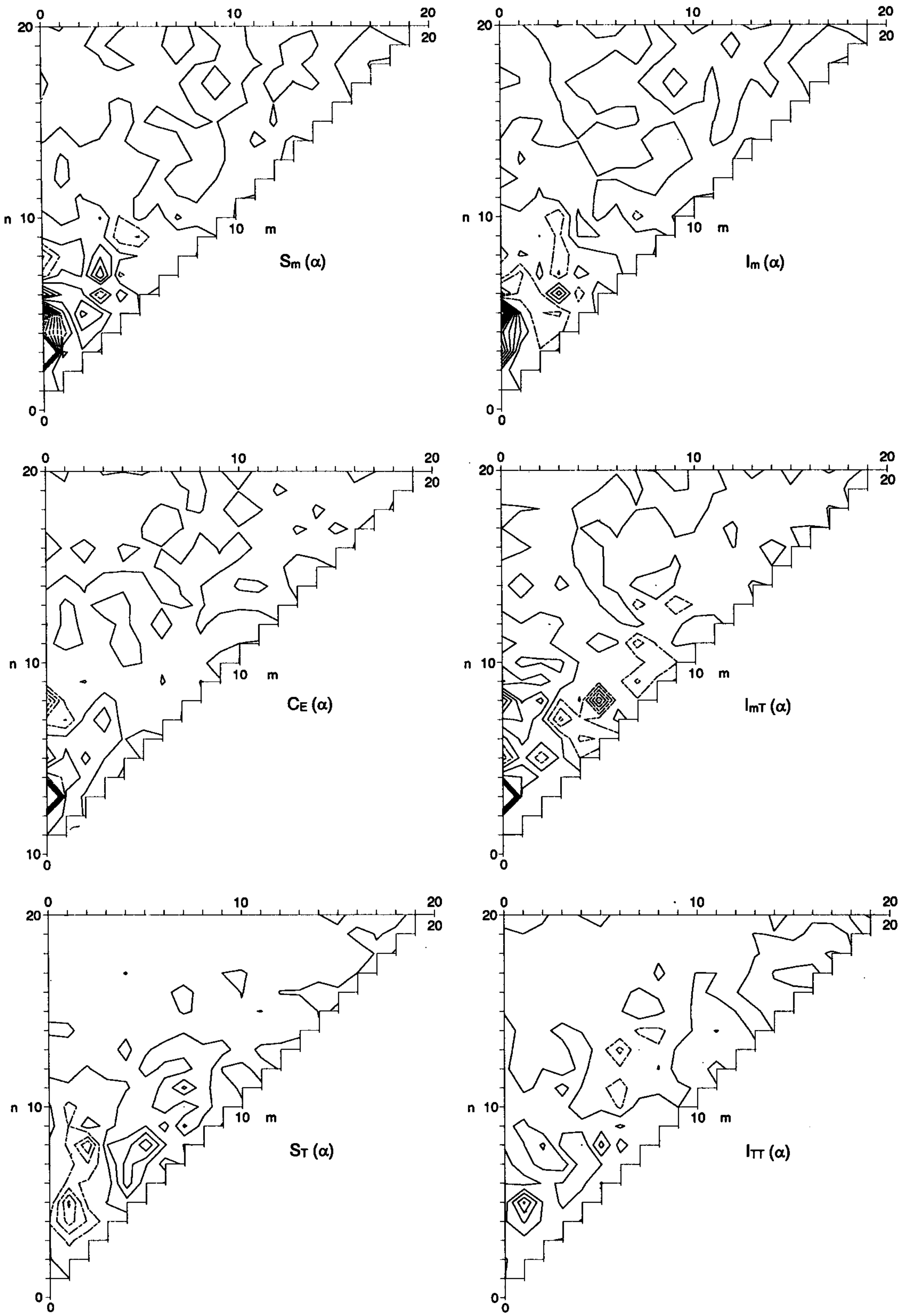


FIG. 11. Terms in the mean and transient kinetic energy budget. Negative contours are dashed. Units and contour intervals as in Fig. 10.

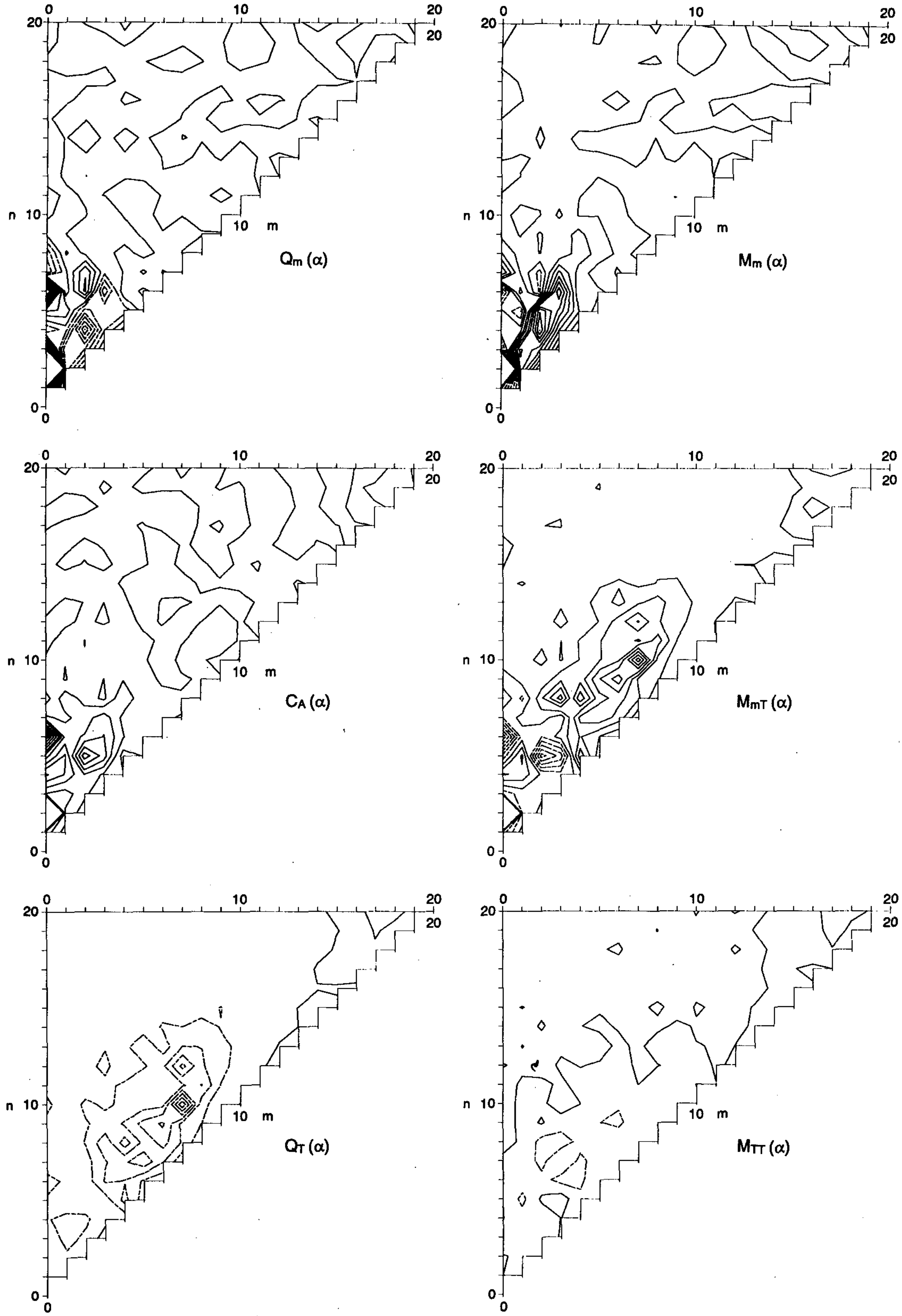


FIG. 12. Terms in the mean and transient available potential energy budget. Negative contours are dashed. Units and contour intervals as in Fig. 10.

interactions between the mean and transient components $M_{MT}(\alpha)$ (the pure transient transfer $M_{TT}(\alpha)$ is surprisingly small) to the transient sink $Q_T(\alpha)$. This sink term represents both the conversion to transient KE and any generation of transient APE.

The transient KE source $S_T(\alpha)$ represents conversion from APE, which outweighs the dissipation of KE. The transient KE is transported largely upscale by the various interaction terms and passed to the low zonal wavenumbers where a strong conversion to mean KE provides the energy for the concentrated sink region of $S_M(\alpha)$. Once again, the flavor is that of the original Lorenz budget but with the complexity of the interactions and sources and sinks at different wavenumbers made explicit.

6. Concluding remarks

The degree to which the large-scale atmospheric flow satisfies the conditions for homogeneous and isotropic two-dimensional turbulence is directly tested from data by calculating "spectral teleconnection maps" for the streamfunction. The results indicate that the high wavenumber transient component of the flow satisfies these conditions to a reasonable degree.

The mean and transient components not only of the spectra of enstrophy, kinetic, and available potential energy but also of the spectral budgets of these quantities are calculated from the ECMWF FGGE III-b dataset for January 1979. These quantities are displayed both in terms of the wavenumber $\alpha = (n, m)$ and separately, after summing over the Fourier wavenumber m, to give the n-spectral version, which is appropriate when the conditions for two-dimensional homogeneous and isotropic turbulence hold.

Both α- and n-spectral budgets provide a more detailed scale-selective look at components of the atmospheric energy budget than does the usual Lorenz approach. A strong source of APE in low, especially low zonal, wavenumbers is transferred downscale largely by interactions involving mean and transient components where some of it is converted into transient KE, which is transferred upscale, converted to mean KE at low, especially low zonal, wavenumbers, and dissipated.

The α-spectral mean and transient budgets give information as to the particular scales and wavenumbers at which the sources and sinks of energy and enstrophy are found and how they are transferred between scales and converted from one form to the other. The potential functions associated with the spectral fluxes through wavenumber space provide a novel view of this transfer. The importance of the mean components and of the interactions between mean and transient components, especially at the lower wavenumbers, as well as the direct test of the necessary and sufficient condition for homogeneity and isotropy shows that the interpretation of atmospheric behavior as large-scale turbulence is suitable only for high wavenumber transients and that the lower wavenumber inhomogeneous and nonisotropic mean structures interact in an important way with the transient turbulence-like structures.

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