Strongly Curved Flow Profiles in Quasigeostrophic Stability Theory

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ABSTRACT

Piecewise linear flow profiles are commonly used in quasigeostrophic (QG) stability analyses as idealizations of the smooth profiles found in nature. In such studies the QG potential vorticity (PV) equation is generally solved in layers of uniform PV with solutions matched at interfaces. While such a procedure is formally valid, it will be shown that nearby smooth flow profiles, which the piecewise linear profiles are supposed to represent, violate QG scaling assumptions. A scaling analysis suggests that the maximum vertical curvature in the isentropes consistent with quasigeostrophic theory is $O(U/fL^2)$, where U is the tropospheric scale of the zonal wind, f is the Coriolis parameter, and L is the horizontal length scale. Comparisons are made between the stability characteristics of a piecewise linear flow profile and nearby smooth flows that better satisfy this curvature constraint, and significant qualitative differences are found. The authors conclude that extreme caution must be exercised when using piecewise linear flow profiles in QG stability theory.

1. Introduction

Piecewise linear flow profiles have been used in studies of hydrodynamic stability for over a century. Such profiles can considerably simplify the mathematical analysis, and much insight into the dynamics of continuous flows has been gained by their employ. The approach is to solve the appropriate governing equation (e.g., Rayleigh, Taylor–Goldstein, etc.) separately in layers and to match solutions at interfaces by imposing continuity of pressure (a dynamic constraint) and normal velocity (a kinematic constraint). Generally, the solutions are simple analytic functions, and a straightforward dispersion relation is obtained.

The method dates back to Kelvin's (1871) classic analysis of a stratified shear layer, which he approximated as a vortex sheet between layers of uniform but different density (see Drazin and Reid 1981). Rayleigh (1894) investigated unstratified shear layers of finite width by considering a layer of uniform shear sandwiched between layers of uniform velocity. Taylor (1931) and Goldstein (1931) extended this analysis to the stratified case, again with piecewise linear velocity and density profiles. Since the publication of these seminal papers, many other authors have adopted this approach with great success.

Nevertheless, there are well-known problems with Kelvin's method, in particular, discrepancies in the stability characteristics of piecewise linear flow profiles and the continuous profiles they are supposed to represent. These generally manifest themselves as the generation of spurious modes (Leibovich 1969) or the failure to produce all of the unstable modes (Howard and Drazin 1964; Blumen et al. 1975, and references therein; see also Maslowe 1985).

Given the long history and relative success of piecewise linear flow profiles in hydrodynamics, it is not surprising that some authors have tried to adapt their use to the problem of atmospheric baroclinic instability (e.g., Tang 1975; James and Hoskins 1985; Weng and Barcilon 1987; Bell and White 1988; Müller 1991). This is typically accomplished by suitably modifying the Eady problem, that is, by solving the quasigeostrophic potential vorticity equation in layers and matching solutions at interfaces. While such an approach is formally valid, nearby smooth flows that the piecewise linear profiles are supposed to represent violate quasigeostrophic scaling assumptions used to derive the QG potential vorticity equation in the first place. Thus, a quasigeostrophic stability analysis of a piecewise linear flow may not be relevant, since a nearby smooth flow profile cannot be quasigeostrophic, and nongeostrophic effects may be important. This issue, while not particularly new, does not appear to have been discussed in the literature.

The purpose of this article is to clarify which basic states are acceptable in QG stability theory, in general, and to discuss some discrepancies in results for the smooth and piecewise linear semibounded shear layer as an illustrative example.

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2. Allowable basic states in QG theory

We begin by deriving the linear quasigeostrophic potential vorticity equation from the linearized primitive equations, paying particular attention to the background wind and static stability profiles. Using Hoskins and Bretherton's (1972) pseudoheight coordinate, the linearized primitive equations are

$$\left[\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right]u'+(\bar{U}_y-f)v'+\bar{U}_z w'=-\frac{\partial\phi'}{\partial x} \quad (1a)$$

$$\left[\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right]v'+fu'=-\frac{\partial\phi'}{\partial y} \quad (1b)$$

$$\frac{\partial\phi'}{\partial z}=\frac{g}{\theta_{00}}\theta' \quad (1c)$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \quad (1d)$$

$$\left[\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right]\theta'-\frac{f\theta_{00}}{g}\bar{U}_z v'+\bar{\Theta}_z w'=0, \quad (1e)$$

where overbars denote zonally averaged quantities and primed variables are deviations from the zonal mean. Note that in writing (1e) we have used the thermal wind balance of the basic state:

$$f\bar{U}_z=-\frac{g}{\theta_{00}}\bar{\Theta}_y. \quad (2)$$

We render (1) dimensionless by scaling each of the variables (e.g., Pedlosky 1987) in the usual way:

$$x,y\mapsto L(\tilde{x},\tilde{y}) \quad \theta'\mapsto\frac{UfL\theta_{00}}{Hg}\tilde{\theta}'$$

$$z\mapsto H\tilde{z} \quad \phi'\mapsto UfL\tilde{\phi}'$$

$$\frac{df}{dy}=\beta\mapsto\frac{U}{L^2}\tilde{\beta} \quad u',v'\mapsto U(\tilde{u}',\tilde{v}')$$

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\mapsto\frac{U}{L}\left(\frac{\partial}{\partial\tilde{t}}+\tilde{\mathcal{U}}\frac{\partial}{\partial\tilde{x}}\right) \quad w'\mapsto\frac{UH}{L}\tilde{w}'$$

$$\mapsto\frac{U}{L}\tilde{D}.$$

In addition, we scale the basic-state wind and static stability by

$$\bar{U}\mapsto U\tilde{\mathcal{U}}(\tilde{y},\tilde{z})$$

$$\bar{\Theta}_z\mapsto\left(\frac{\theta_{00}N_0^2}{g}\right)\tilde{\eta}(\tilde{y},\tilde{z}). \quad (3)$$

Generally speaking, dimensionless quantities such as $\tilde{\mathcal{U}}$ are $O(1)$, but derivatives such as $\tilde{\mathcal{U}}_{\tilde{z}\tilde{z}}$ can be much larger than $O(1)$. Simply scaling the zonal wind $\bar{U}$ by U does not preclude the possibility of strong curvature.

Such a scaling as (3) naturally produces two dimensionless groups that characterize the flow. These are the Rossby number $\mathrm{Ro}=U/fL$ and the Burger number

$$B=\left(\frac{N_0H}{fL}\right)^2=\left(\frac{L_d}{L}\right)^2,$$

where L_d is the internal Rossby deformation radius $L_d=N_0H/f$. On the synoptic scale one generally considers $\mathrm{Ro}\ll 1$, $B\sim 1$, and $\tilde{\eta}=\tilde{\eta}(\tilde{z})$, but we will not make this restriction just yet.

Differentiating the thermal wind relation (2) and using (3), we find

$$\tilde{\eta}_{\tilde{y}}=-\frac{\mathrm{Ro}}{B}\tilde{\mathcal{U}}_{\tilde{z}\tilde{z}}, \quad (4)$$

so that in regions of strong curvature or small B one cannot ignore the y-dependence in static stability.

Equation (1) can be reduced to the dimensionless system

$$\mathrm{Ro}^2\tilde{D}\tilde{\zeta}'-\mathrm{Ro}\tilde{w}'_{\tilde{z}}+\mathrm{Ro}^2(\tilde{\beta}-\tilde{\mathcal{U}}_{\tilde{y}\tilde{y}})\tilde{v}'=\mathrm{Ro}^2(\tilde{\mathcal{U}}_{\tilde{y}\tilde{z}}\tilde{w}'_{\tilde{z}}-\tilde{\mathcal{U}}_{\tilde{y}}\tilde{w}'_{\tilde{z}}+\tilde{\mathcal{U}}_{\tilde{z}}\tilde{w}'_{\tilde{y}})$$

$$\mathrm{Ro}^2\tilde{D}\tilde{\theta}'-\mathrm{Ro}^2\tilde{\mathcal{U}}_{\tilde{z}}\tilde{v}'+\mathrm{Ro}B\tilde{\eta}\tilde{w}'=0, \quad (5)$$

where $\tilde{\zeta}'$ is the perturbation vorticity $\tilde{\zeta}'=\tilde{v}'_{\tilde{x}}-\tilde{u}'_{\tilde{y}}$. To this point, no approximations have been made.

To derive the (linearized) QG potential vorticity equation, we must now assert the smallness of the Rossby number and expand perturbation variables in power series of this parameter; that is,

$$\tilde{u}'=\tilde{u}_0+\mathrm{Ro}\tilde{u}_1+O(\mathrm{Ro}^2)$$

$$\tilde{v}'=\tilde{v}_0+\mathrm{Ro}\tilde{v}_1+O(\mathrm{Ro}^2)$$

etc.

One can easily show (e.g., Pedlosky 1987) that

$$\tilde{u}_0=-\tilde{\phi}_{0y}$$

$$\tilde{v}_0=\tilde{\phi}_{0x}$$

$$\tilde{\theta}_0=\tilde{\phi}_{0z}$$

$$\tilde{w}_0=0.$$

Dropping the tildes now, and neglecting terms of $O(\mathrm{Ro}^3)$ in (5), we find that

$$D\zeta_0-w_{1z}+(\beta-\mathcal{U}_{yy})v_0=0$$

$$D\theta_0-\mathcal{U}_z v_0+B\eta w_1=0. \quad (6)$$

So far the approach has been conventional except that we have allowed for an explicit y dependence (however weak) in the static stability. In writing (6) we have neglected the term proportional to $(\mathcal{U}_y w'_z-\mathcal{U}_{yz}w'_z-\mathcal{U}_z w'_y)$ on the basis that it is $O(\mathrm{Ro})$ smaller than the other terms. This is reasonable provided $\mathcal{U}$ is continuous in y and z and that w'_y and w'_z are not too large. (Pathological flow profiles such as the cycloid of revolution are also disallowed.)

Differentiating (6b), we find that

$$w_{1z} = -D\left(\frac{1}{B\eta}\theta_0\right)_z + \left(\frac{\mathcal{U}_z}{B\eta}\right)_z v_0 + \mathrm{Ro}\left(\frac{\mathcal{U}_z}{B\eta}\right)_z v_1 + O\left[\mathrm{Ro}^2\left(\frac{\mathcal{U}_z}{B\eta}\right)_z\right], \quad (7)$$

where we have explicitly included higher-order terms in v since we recognize that $(\mathcal{U}_z/B\eta)_z$ may be larger than $O(1/\mathrm{Ro})$ in some regions. Substituting (7) into (6a) yields

$$D\left[\zeta_0 + \left(\frac{1}{B\eta}\theta_0\right)_z\right] + \left[\beta - \mathcal{U}_{yy} - \left(\frac{\mathcal{U}_z}{B\eta}\right)_z\right]v_0 = \mathrm{Ro}\left(\frac{\mathcal{U}_z}{B\eta}\right)_z v_1 + O\left[\mathrm{Ro}^2\left(\frac{\mathcal{U}_z}{B\eta}\right)_z\right]. \quad (8)$$

This is just the QG potential vorticity equation provided that the right-hand side is $O(\mathrm{Ro})$, which occurs when

$$\left(\frac{\mathcal{U}_z}{\eta}\right)_z \sim O(B). \quad (9)$$

Alternatively, the first term on the right-hand side of (8) becomes important and can no longer be neglected when

$$\left(\frac{\mathcal{U}_z}{\eta}\right)_z \geqslant \frac{B}{\mathrm{Ro}}. \quad (10)$$

In dimensional form the condition (9) becomes

$$\left(\frac{\bar{U}^*_{z^*}}{N^2}\right)_{z^*} \sim O\left(\frac{U}{f^2L^2}\right). \quad (11)$$

This is a condition on the isentropic slope S^* of the basic state if the basic state is in thermal wind balance, since

$$S^* = -\frac{\Theta^*_{y^*}}{\Theta^*_{z^*}} = f\frac{\bar{U}^*_{z^*}}{N^2},$$

and (11) becomes

$$S^*_{z^*} \sim O\left(\frac{U}{fL^2}\right). \quad (12)$$

We see that the curvature of the basic-state isentropes is constrained by (12) in order for that basic state to be consistently quasigeostrophic.

A number of comments are now in order. For cases in which static stability depends only weakly on height (e.g., no tropopause), condition (11) becomes

$$\bar{U}^*_{z^*z^*} \sim O\left(\frac{N^2U}{f^2L^2}\right).$$

Likewise for atmospheres with uniform shear, (11) yields a constraint on the acceptable curvature of the static stability

$$\left(\frac{1}{N^2}\right)_{z^*} \sim O\left(\frac{H}{f^2L^2}\right).$$

Piecewise uniform stability profiles are particularly commonplace in baroclinic stability analyses (e.g., Blumen 1979).

Equation (8) also implies a horizontal curvature constraint. In order that the basic-state PV gradient be $O(1)$, we require that $U_{yy} \sim O(1)$ or

$$\bar{U}^*_{y^*y^*} \sim O\left(\frac{U}{L^2}\right).$$

Now for $\mathrm{Ro} = U/fL \ll 1$, we see that

$$\bar{U}^*_{y^*} \sim O\left(\frac{U}{L}\right) \ll f \quad (13)$$

Equivalently, if $\bar{U}^*$ varies in y^* on a length scale $\mathcal{L}$ different from L, then (5) becomes

$$\mathrm{Ro}^2\tilde{D}\tilde{\zeta}' - \mathrm{Ro}\tilde{w}'_{\tilde{z}} + \mathrm{Ro}^2(\tilde{\beta} - \tilde{\mathcal{U}}_{\tilde{y}\tilde{y}})\tilde{v}' = \frac{L}{\mathcal{L}}\mathrm{Ro}^2(\tilde{\mathcal{U}}_{\tilde{y}\tilde{z}}\tilde{w}'_{\tilde{z}} - \tilde{\mathcal{U}}_{\tilde{y}}\tilde{w}'_{\tilde{z}}) + \mathrm{Ro}^2(\tilde{\mathcal{U}}_{\tilde{z}}\tilde{w}'_{\tilde{y}}),$$

and the first term on the right-hand side can only be neglected when $L/\mathcal{L} \ll 1/\mathrm{Ro}$ or

$$\tilde{U}^*_{y^*} \sim O\left(\frac{U}{\mathcal{L}}\right) \ll \frac{U}{L}\frac{1}{\mathrm{Ro}},$$

from which (13) obtained.

The form of (11) reveals the important role of rotation in the QG curvature constraint. Weakly rotating flows are only weakly constrained. The studies mentioned above employing piecewise linear flow profiles in the context of the Taylor–Goldstein equation, of course, had no rotation. Equation (9) demonstrates the role of scale in the QG curvature constraint. Since we have already restricted ourselves to small Rossby number, we require $L \geqslant L_d$; that is, $B \leqslant 1$. For length scales on the order of L_d, (9) becomes $(\mathcal{U}_z/\eta)_z \sim O(1)$, but

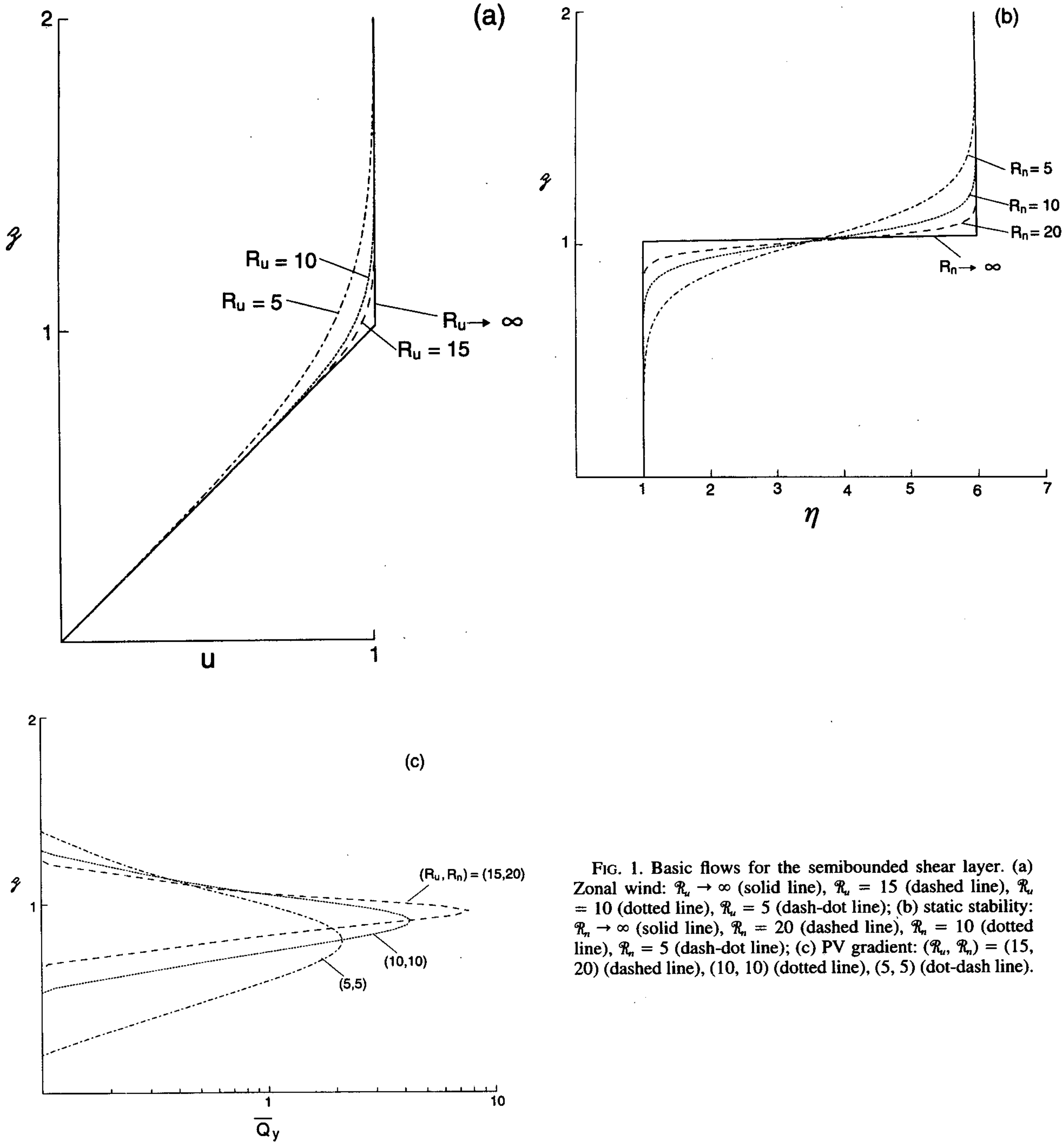

FIG. 1. Basic flows for the semibounded shear layer. (a) Zonal wind: $\mathcal{R}_u \to \infty$ (solid line), $\mathcal{R}_u = 15$ (dashed line), $\mathcal{R}_u = 10$ (dotted line), $\mathcal{R}_u = 5$ (dash-dot line); (b) static stability: $\mathcal{R}_n \to \infty$ (solid line), $\mathcal{R}_n = 20$ (dashed line), $\mathcal{R}_n = 10$ (dotted line), $\mathcal{R}_n = 5$ (dash-dot line); (c) PV gradient: $(\mathcal{R}_u, \mathcal{R}_n) = (15, 20)$ (dashed line), (10, 10) (dotted line), (5, 5) (dot-dash line).

for longer length scales the constraint is even stronger. This is because a mode that has large meridional extent will sense large differences in N^2 across its length, even for small flow curvatures, through the thermal wind relation (4). We also note from (9) that QG theory can accommodate larger flow curvature in regions of large static stability. Finally, it should be mentioned that even though we derived our scaling law (9) from the linearized primitive equations there is nothing fundamentally linear about our arguments, and fully nonlinear QG theory is subject to the same curvature constraint.

3. Example

We have found that strongly curved flow profiles are inconsistent with the QG approximation. If the sharp transition between layers is smoothed enough so that (9) is satisfied, however, one may legitimately solve

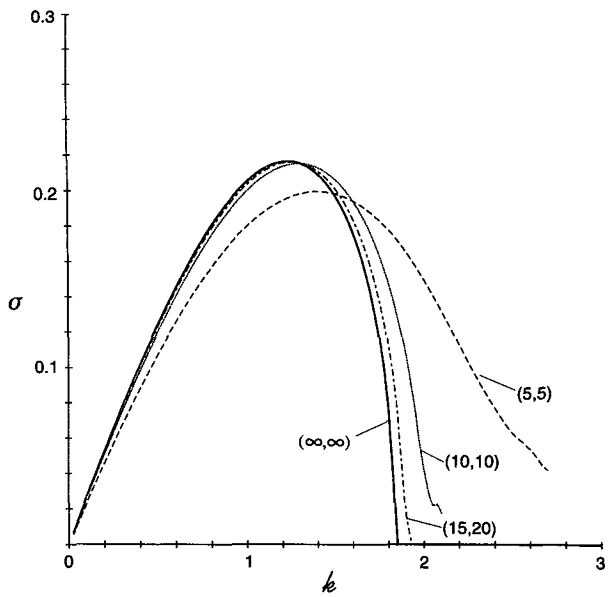

FIG. 2. Dimensionless growth rates from linear stability analyses of the QG potential vorticity equation for the basic flows in Fig. 1: $(\mathcal{R}_u, \mathcal{R}_n) \to (\infty, \infty)$, solid line; (15, 20), dash–dot line; (10, 10), dotted line; (5, 5), dash line.

the QG potential vorticity equation—though numerical techniques will likely be required.

Consider the basic-state velocity:

$$\bar{U}(z) = U\left\{A - \frac{1}{\mathcal{R}_u}\log\left[\frac{1}{2}\left(e^{-\mathcal{R}_u/H(z-H)} + e^{-(\mathcal{R}_u/H)(\lambda_2/\lambda_1)(z-H)}\right)\right]\right\}, \quad (14)$$

with

$$\lambda_1 = \frac{U}{H} > 0$$

$$\lambda_2 \leq 0; \quad |\lambda_2| \leq \lambda_1.$$

In this flow profile $\mathcal{R}_u$ is a curvature parameter, and in the limit $\mathcal{R}_u \to \infty$ we recover the piecewise linear profile

$$\bar{U}(z) = \begin{cases} UA + \lambda_1(z - H) & z \leq H \\ UA + \lambda_2(z - H) & z > H. \end{cases}$$

Here, A is an arbitrary constant, which we conveniently define as

$$A = \frac{1}{\mathcal{R}_u}\log\left[\frac{1}{2}\left(e^{\mathcal{R}_u} + e^{\lambda_2/\lambda_1 \mathcal{R}_u}\right)\right],$$

so that $\bar{U}(0) = 0$. Note that in the $\lim_{\mathcal{R}_u \to \infty}$, $A = 1$. For finite values of $\mathcal{R}_u$, (14) represents a smooth flow profile.

Differentiating (14), we find that

$$\bar{U}_z(z) = \frac{\lambda_1 e^{-\mathcal{R}_u/H(z-H)} + \lambda_2 e^{(-\lambda_2/\lambda_1)(\mathcal{R}_u/H)(z-H)}}{e^{-\mathcal{R}_u/H(z-H)} + e^{(-\lambda_2/\lambda_1)(\mathcal{R}_u/H)(z-H)}},$$

where

$$\lim_{\mathcal{R}_u \to \infty} \bar{U}_z(z) = \begin{cases} \lambda_1 & z < H \\ \lambda_2 & z > H. \end{cases}$$

Consider also the background static stability profile given by

$$N^2(z) = N_0^2\left\{\left(\frac{n1 + n2}{2}\right) + \left(\frac{n2 - n1}{2}\right)\tanh\left[\frac{\mathcal{R}_n}{H}(z - H)\right]\right\}. \quad (15)$$

This is the continuous analog of Blumen's two-layer model used by Nakamura (1988), where n_1 and n_2 are the (dimensionless) asymptotic static stabilities in the troposphere and stratosphere, respectively, and $\mathcal{R}_n$ is a curvature parameter.

We solve the QG potential vorticity equation on the f plane for the basic state defined by (14) and (15) with the following fixed parameters:

$$\mathrm{Ro} = 0.3$$

$$B = 1$$

$$n1, n2 = 1, 6.$$

In this problem the condition (10) becomes a condition on the dimensionless background PV gradient:

$$\bar{Q}_y \geq 3.333\cdots,$$

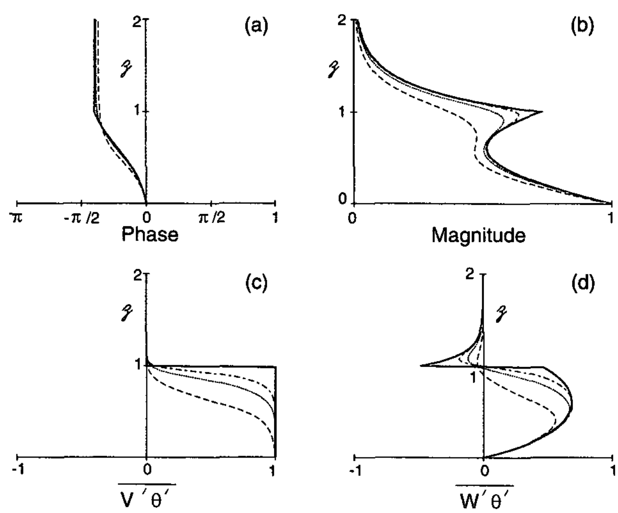

FIG. 3. Vertical structure of the fastest-growing $l = 1$ mode: (a) phase, (b) magnitude, (c) horizontal heat flux, and (d) vertical heat flux.

so that in order for the QG potential vorticity equation to be relevant in this case we require

$$\bar{Q}_y < 3.333\cdots. \quad (16)$$

The semibounded shear layer

We investigate a special cases of (14), where $\lambda_1 = U/H$: $\lambda_2 = 0$. This profile represents a shear layer that is bounded from below but free above. The asymptotic form $\mathcal{R}_u \to \infty$ has been used in several studies [e.g., the semi-internal jet of Bell and White (1988); Tang (1975)].

We are primarily interested in differences in the stability characteristics of the flow represented by (14) and (15) when $\mathcal{R}_u$ and $\mathcal{R}_n$ are reduced from very large values to small values, representing the transition from a piecewise linear flow profile to a smooth profile satisfying (16). For the semibounded shear layer these profiles are shown in Fig. 1 for $\lambda_1 = 3 \times 10^{-3}\ \mathrm{s}^{-1}$; $\mathcal{R}_u = 5$, 10, 15, and $\mathcal{R}_u \to \infty$; and $\mathcal{R}_n = 5$, 10, 20, and $\mathcal{R}_n \to \infty$. For finite $\mathcal{R}_u$ and $\mathcal{R}_n$, the QG potential vorticity equation was solved using finite differences on an 80-point grid. The tropopause is fixed at $H = 8$ km, and there is a rigid lid at $z = 20$ km. In the limit $\mathcal{R}_u \to \infty$ and $\mathcal{R}_n \to \infty$, a generalization of Blumen's (1979) two-layer analytic solution was used. Notice that only the flow profile represented by $(\mathcal{R}_u, \mathcal{R}_n) = (5, 5)$ is strictly QG according to (16).

Growth rates for each of the flows in Fig. 1 are shown in Fig. 2 for meridional wavenumber $l = 1$. Qualitative differences exist between the discontinuous case $(\mathcal{R}_u, \mathcal{R}_n) \to (\infty, \infty)$ and the QG case $(\mathcal{R}_u, \mathcal{R}_n) \to (5, 5)$. In particular, the latter exhibits strong shortwave destabilization as well as smaller growth rates and larger wavenumbers for the fastest-growing modes.

Figure 3 shows the structure of the eigenfunctions and heat fluxes for the fastest-growing waves of Fig. 2, and again qualitative differences are apparent.

4. Discussion

Piecewise linear flow profiles have a long history in hydrodynamic stability theory. Over the last 20 years a number of authors have attempted to apply them to the problem of atmospheric baroclinic instability in the context of quasigeostrophic theory. This is despite the well-known, though (evidently) little-discussed, fact that strong curvature is formally disallowed in QG theory, as it violates a scaling assumption essential to the derivation of the QG potential vorticity equation. In this note we have suggested that the curvature of the isentropes can be no greater than $O(U/fL^2)$. We then compared the stability characteristics of the piecewise linear semibounded shear layer with nearby smooth profiles that better satisfy this curvature constraint and found significant qualitative differences. Given this, and the fact that piecewise linear flow profiles can be problematic even when there are no curvature constraints (e.g., the Taylor–Goldstein equation), we conclude that great care must be exercised when applying them to the QG baroclinic stability problem.

In fact, the difficulty extends beyond the realm of quasigeostrophic theory. We have discussed the fact that *any* vertical curvature in the zonal wind must necessarily lead to meridional variations in static stability. Thus, even one-dimensional *primitive equation* eigenvalue problems cannot have curvature in the zonal wind because variations in static stability will preclude any hope of simple (e.g., sinusoidal) meridional structure in the normal mode, rendering the problem nonseparable.

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