

## Parameterizations for Water Vapor IR Radiative Transfer in Both the Middle and Lower Atmospheres

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### ABSTRACT

Water vapor contributes a maximum of 1°C/day to the middle atmospheric thermal infrared (IR) cooling. This magnitude is small but not negligible. Because of the small amount of mass involved and the extremely narrow molecular absorption lines at pressures less than 1 mb, only a few existing parameterizations can compute accurately the water vapor cooling in this region. The accuracy and efficiency of two IR parameterizations are examined in this study. One is the correlated- $k$  distribution method, and the other is the table look-up using precomputed transmission functions. Both methods can accurately compute the cooling rate from the earth's surface to 0.01 mb with an error of only a few percent. The contribution to the cooling rate at pressures < 1 mb comes from a very small fraction (<0.005) of the spectrum near the centers of the absorption bands, where the absorption coefficient varies by four orders of magnitude. It requires at least 100 terms of the  $k$ -distribution function to accurately compute the cooling profile. The method of table look-up is, therefore, much faster than the correlated- $k$  distribution method for computing the water vapor cooling profile involving both the middle and lower atmospheres.

### 1. Introduction

There are many efficient radiative transfer parameterizations for computing the atmospheric thermal infrared (IR) fluxes and cooling rate in climate models. They include band models (e.g., Briegleb 1992; Chou and Peng 1983; Kratz et al. 1991; Morcrette et al. 1986; Rosenfield 1991; Schwarzkopf and Fels 1991), the  $k$ -distribution method (e.g., Chou et al. 1993; Fu and Liou 1992; Grossman and Grant 1992; Hollweg 1993; Lacis and Oinas 1991; Goody et al. 1989; Wang and Shi 1988; Zhu 1992), and table look-up using precomputed transmission functions (e.g., Chou and Kouvaris 1991; Fels and Schwarzkopf 1981). Most of these parameterizations apply only to the troposphere and lower stratosphere, and only a few can compute accurately the IR cooling rate in the middle atmosphere (0.01–30 mb). There are a number of reasons that accurate calculations of middle atmospheric cooling rate are difficult: 1) Atmospheric pressure ranges by four orders of magnitude. It is difficult to properly take into account the effect of pressure on absorption in a path where the

variation in pressure is large. 2) The molecular line half-width is dominated by the pressure broadening in the troposphere and lower stratosphere, but by the Doppler broadening in the middle atmosphere. Most band models are developed based on the pressure broadening of line widths. To take into account the effect of the Doppler broadening, empirical adjustments are made. As a result, errors in the middle atmospheric cooling rate calculations are often large. 3) Molecular absorption lines in the middle atmosphere are very narrow compared to line spacing, and the absorption changes rapidly with wavenumber near the center region of a line. Radiative transfer calculations using the  $k$ -distribution method require a very high spectral resolution in the absorption coefficient (or in the  $k$ -distribution function). 4) Because of the small amount of mass involved in the middle atmosphere, calculations of fractional transmittance need to be accurate to the sixth digit.

With the sophistication of treating the atmospheric dynamical and physical processes enhanced, many atmospheric general circulation models are extended to include the middle atmosphere, and there is a need to improve the radiative transfer calculations in this region. The IR cooling in the middle atmosphere is primarily due to CO<sub>2</sub> and secondarily due to O<sub>3</sub>, with a maximum cooling of 10°C/day. The water vapor concentration in the middle atmosphere is small, and only the spectral regions near the centers of absorption bands (the pure rotational region and the rotational–vibrational band) have a meaningful contribution to the

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cooling. Maximum cooling due to water vapor is  $1^\circ\text{C}/\text{day}$ , which is relatively small but is not negligible. In this study, two methods for IR radiative transfer calculations are examined by applying them to three spectral bands with strong water vapor absorption. The two methods are the correlated  $k$ -distribution method and the table look-up using precomputed transmission functions. The accuracy of these methods is validated with line-by-line calculations, and their applications are discussed. Although these methods are applied in this study to the three strong water vapor absorption bands, the same conclusions can be extended to the other spectral bands involving other absorbers.

## 2. IR radiative transfer and line-by-line calculations

In the absence of scattering, the upward flux  $F\uparrow(p)$  and downward flux  $F\downarrow(p)$  at the pressure level  $p$  can be written as

$$F\uparrow(p) = \int d\nu \left\{ B_\nu(T_s) \tau_\nu(p, p_s) - \int_p^{p_s} B_\nu(T') [\partial \tau_\nu(p, p') / \partial p'] dp' \right\} \quad (1)$$

$$F\downarrow(p) = \int d\nu \left\{ \int_0^p B_\nu(T') [\partial \tau_\nu(p, p') / \partial p'] dp' \right\}, \quad (2)$$

and the cooling rate is

$$-\partial T(p) / \partial t = -\frac{1}{c_p g} \partial [F\uparrow(p) - F\downarrow(p)] / \partial p, \quad (3)$$

where  $T$  is the temperature,  $B$  is the Planck flux,  $\tau$  is the flux transmission function,  $\nu$  is the wavenumber,  $g$  is the gravitational acceleration,  $c_p$  is the heat capacity of air at constant pressure, and the subscript  $s$  denotes the surface. The flux transmission function is defined as

$$\tau_\nu(p, p') = 2 \int_0^1 e^{-u_\nu(p, p') / \mu} \mu d\mu, \quad (4)$$

where the optical thickness is given by

$$u_\nu(p, p') = \int_p^{p'} k_\nu(p'', T'') dw(p'') \quad (5)$$

in which  $w$  is the water vapor amount, and  $k$  is the absorption coefficient.

Three spectral regions with strong absorption due to water vapor are studied. They are bands  $0\text{--}340\text{ cm}^{-1}$ ,  $340\text{--}540\text{ cm}^{-1}$ , and  $1380\text{--}1900\text{ cm}^{-1}$ . The first band is the core region of the water vapor rotational region, and the last band is the core region of the  $6.7\text{-}\mu\text{m}$  rotational-vibrational band. To serve as the basis for the parameterizations and as the benchmark for validating results, the transmittance and fluxes are computed from

(1), (2), and (4) using a line-by-line method. Our line-by-line program uses the 1992 version of the molecular line parameters compiled by Rothman et al. (1987). The line profile is assumed to follow the Voigt function, and the absorption coefficient  $k_\nu$  is computed at spectral points  $0.0005\text{ cm}^{-1}$  apart. The Voigt function has a Doppler core and Lorentz wings. It is well acknowledged that the molecular line profile at wavenumbers far from the line center does not follow the Lorentz function. The absorption coefficient at wavenumbers  $> 10\text{ cm}^{-1}$  from the line center is taken to be zero, which is equivalent to a line cutoff of  $10\text{ cm}^{-1}$ . The choice of the line cutoff is somewhat arbitrary. It will have certain effect on the middle and upper tropospheric cooling rate calculations. The appropriate treatment of the absorption in the far wings of a line is a subject of much on-going research. It is not a subject of this study.

Fluxes and cooling rate profiles are computed for a midlatitude summer atmosphere and a subarctic winter atmosphere. These two atmospheres are taken from McClatchey et al. (1972), except for the specific humidity in the stratosphere. Following the ICRCCM (Intercomparison of Radiation Codes in Climate Models) specification (Ellingson and Fouquart 1991), the specific humidity is fixed at  $4 \times 10^{-6}\text{ g g}^{-1}$  above the tropopause. Line by line calculations of fluxes at the top of the atmosphere and at the surface are shown in Table 1, and the cooling rate profiles are shown in Figs. 1–3 (solid curves). The total cooling rate attains a local maximum at the stratopause with a magnitude of  $\sim 1^\circ\text{C day}^{-1}$ . The large cooling rate at  $0.01\text{ mb}$  for the  $0\text{--}340\text{ cm}^{-1}$  band is a result of strong cool-to-space. It is interesting to note that in the midlatitude summer case, IR radiative transfer causes a noticeable heating in the band  $1380\text{--}1900\text{ cm}^{-1}$  at  $p < 0.1\text{ mb}$  (upper panel of Fig. 3). In this spectral band, the Planck function decreases rapidly with decreasing temperature. A layer in the region with  $p < 0.1\text{ mb}$  emits very little IR radiation due to low temperature but absorbs relatively large radiation emitting from below where the temperature is much higher. The net effect is to cause a heating in this region. Apparently, the cool-to-space approximation does not apply in this case.

## 3. Parameterizations

Within each band, the shape of the Planck flux  $B_\nu$  does not change significantly with temperature, and we have the following approximation:

$$\int_{\Delta\nu} B_\nu(T) \tau_\nu(p, p') d\nu / \int_{\Delta\nu} B_\nu(T) d\nu \approx \int_{\Delta\nu} B_\nu(T_0) \tau_\nu(p, p') d\nu / \int_{\Delta\nu} B_\nu(T_0) d\nu, \quad (6)$$

where  $T_0$  is chosen to be  $250\text{ K}$ . Equations (1) and (2)

TABLE 1. The upward flux at the top of the atmosphere  $F\uparrow(\text{top})$  and the downward flux at the surface  $F\downarrow(\text{sfc})$  in the three strong water vapor absorption regions. The units are Watts per squared meter.

	$F\uparrow(\text{top})$	$F\downarrow(\text{sfc})$
0–340 $\text{cm}^{-1}$		
Midlatitude Summer		
Line-by-line	34.2	51.2
Table look-up	34.2	51.0
Correlated $k$ -distribution	34.0	51.0
Subarctic W-scalinginter		
Line-by-line	32.1	40.4
Table look-up	31.9	40.4
Correlated $k$ -distribution	31.9	40.4
340–540 $\text{cm}^{-1}$		
Midlatitude Summer		
Line-by-line	60.5	80.9
Table look-up	59.9	81.3
Correlated $k$ -distribution	59.8	81.1
Subarctic Winter		
Line-by-line	52.0	47.4
Table look-up	51.8	47.4
Correlated $k$ -distribution	51.7	48.1
1380–1900 $\text{cm}^{-1}$		
Midlatitude Summer		
Line-by-line	7.3	30.6
Table look-up	7.5	30.3
Correlated $k$ -distribution	7.4	30.3
Subarctic Winter		
Line-by-line	4.9	10.1
Table look-up	4.9	10.1
Correlated $k$ -distribution	4.9	10.1

then reduce to the following representations for fluxes in a spectral band  $\Delta\nu$ ,

$$F\uparrow(p) = B(T_s)\tau(p, p_s) - \int_p^{p_s} B(T')[\partial\tau(p, p')/\partial p'] dp' \quad (7)$$

$$F\downarrow(p) = \int_0^p B(T')[\partial\tau(p, p')/\partial p'] dp', \quad (8)$$

where  $B(T)$  is the spectrally integrated Planck flux given by

$$B(T) = \int_{\Delta\nu} B_\nu(T) d\nu, \quad (9)$$

and  $\tau(p, p')$ , referred as the Planck-weighted flux transmittance by Fels and Schwarzkopf (1981), is given by

$$\tau(p, p') = \frac{\int_{\Delta\nu} B_\nu(T')\tau_\nu(p, p') d\nu}{\int_{\Delta\nu} B_\nu(T') d\nu} \approx \frac{\int_{\Delta\nu} B_\nu(T_0)\tau_\nu(p, p') d\nu}{\int_{\Delta\nu} B_\nu(T_0) d\nu}. \quad (10)$$

Parameterizations for the Planck-weighted flux transmittance are the central problem in the IR radiative transfer calculations.

#### a. Table look-up

It is well acknowledged that the atmospheric cooling is primarily contributed from nearby layers (e.g., Chou and Kouvaris 1991; Wu 1980). Because variations in temperature and pressure are usually small among nearby layers, the effective temperature and pressure of a path encompassing a number of layers can be approximated by

$$p_{\text{eff}} = \int pdw / \int dw \quad (11)$$

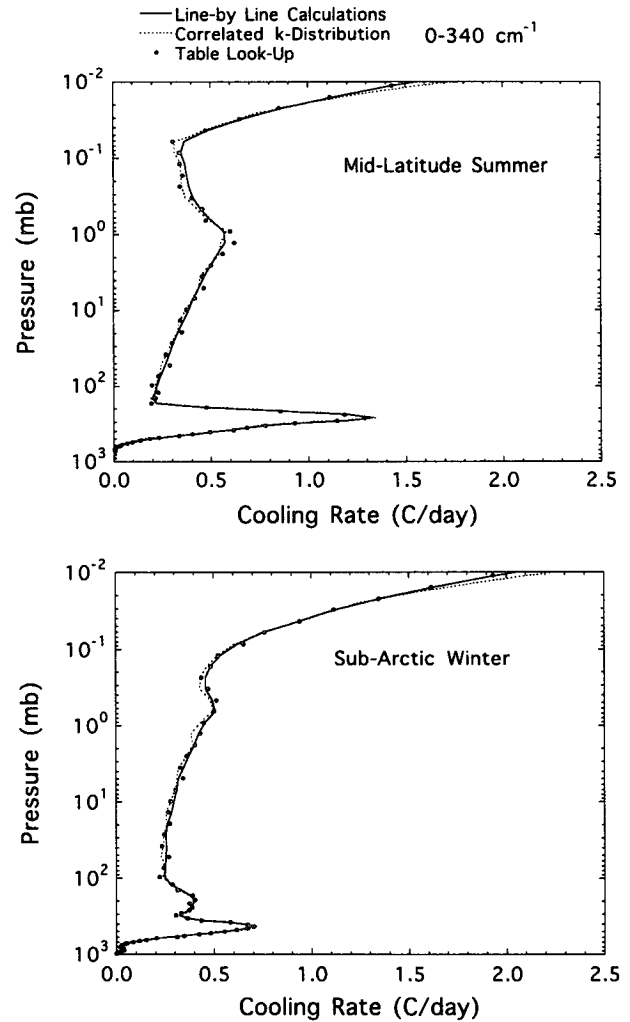


FIG. 1. The cooling rate profiles in the 0–340  $\text{cm}^{-1}$  band for the midlatitude summer atmosphere (upper panel) and the subarctic winter atmosphere (lower panel). Solid curves are line by line calculations, solid circles are calculated using the method of table look-up, and the dashed curves are calculated using the correlated- $k$  distribution method with 595  $k$ -distribution intervals.

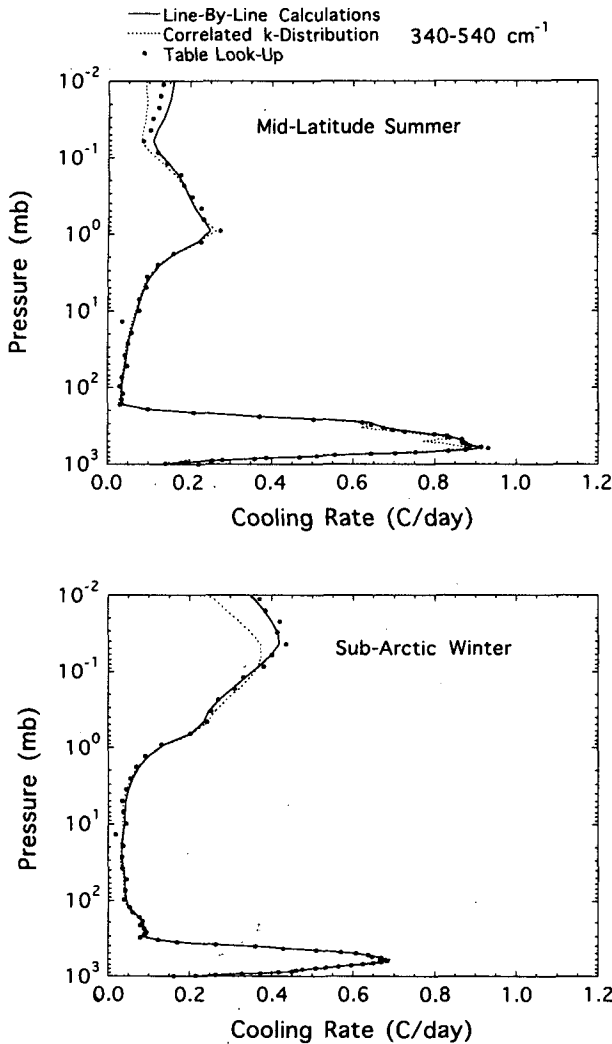


FIG. 2. Same as Fig. 1 except for 340–540 cm<sup>-1</sup>.

$$T_{\text{eff}} = \int T dw / \int dw, \quad (12)$$

where the integration is over the length of a path. With this scaling approximation, the Planck-weighted flux transmittance  $\tau(p, p')$  becomes a function of  $w, p_{\text{eff}}, T_{\text{eff}}$  and can be precomputed from (4) and (10) using the line-by-line method. The flux transmittance is a smooth function of temperature, and the three-dimensional tables of the Planck-weighted flux transmittance can be reduced to three two-dimensional tables using a quadratic fit,

$$\tau(w, p_{\text{eff}}, T_{\text{eff}}) = a(w, p_{\text{eff}}) + b(w, p_{\text{eff}})(T_{\text{eff}} - 250) + c(w, p_{\text{eff}})(T_{\text{eff}} - 250)^2, \quad (13)$$

where  $a, b,$  and  $c$  are the coefficients that best fit the transmittance given in (13) to the line-by-line calcu-

lated transmittance, and the unit of  $T_{\text{eff}}$  is kelvin. Figure 4 shows the percentage error (relative) in the Planck-weighted flux absorption  $(1 - \tau)$  due to the quadratic fit for the spectral band 340–540 cm<sup>-1</sup> and for  $p = 0.1$  mb (upper panel) and 500 mb (lower panel). Points in the figures are for temperature ranging from 170 K to 330 K and for the water vapor amount ranging from 10<sup>-8</sup> to 10 g cm<sup>-2</sup>. It can be seen from the figure that the error in absorption is generally <4%. The transmittances for the other two bands (0–340 cm<sup>-1</sup> and 1380–1900 cm<sup>-1</sup>) are less sensitive to temperature than the band 340–540 cm<sup>-1</sup>, and the error caused by the quadratic fit is smaller. The coefficients  $a, b,$  and  $c$  were precomputed for  $w$  ranging from 10<sup>-8</sup> to 10 g cm<sup>-2</sup>, with an interval of  $\Delta \log_{10} w = 0.2$ , and for  $p$  ranging from 0.01 mb to 1000 mb, with an interval of  $\Delta \log_{10} p = 0.2$ .

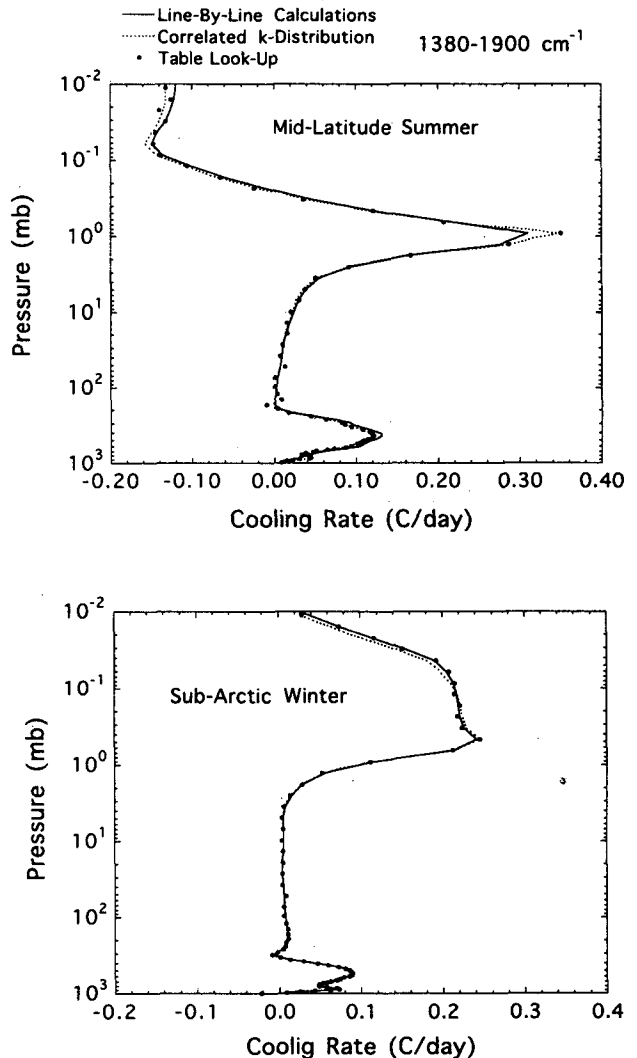


FIG. 3. Same as Fig. 1 except for 1380–1900 cm<sup>-1</sup>.

With the coefficients  $a$ ,  $b$ , and  $c$  precomputed, the Planck-weighted flux transmittance can be simply derived using table look-up, and fluxes can be computed from (7) and (8). The cooling rate profiles computed using the table look-up for the flux transmittance are shown in Figs. 1–3 (solid circles), respectively, for the three bands. The upper panels are for the midlatitude summer atmosphere and the lower panels are for the subarctic winter atmosphere. Compared to line-by-line calculations, the cooling rate error is generally less than  $0.05^\circ\text{C}/\text{day}$ . The small-scale fluctuations around the line-by-line cooling profiles are caused by the interpolation of the transmission function using the precomputed tables. The upward flux at the top of the atmosphere and the downward flux at the surface are given in Table 1. The difference between the method of table look-up and the line-by-line calculations is negligible.

*b. The  $k$ -distribution method*

In a homogeneous layer where pressure and temperature are constant, wavenumbers with the same absorption coefficient are radiatively identical and can be treated as one identity. The integration over a narrow spectral band for deriving the mean optical properties can be replaced by the integration over the  $k$ -distribution function (cf. Arking and Grossman 1972),

$$\int_{\Delta\nu} ( ) d\nu/\Delta\nu = \int_0^1 ( ) dg, \quad (14)$$

where  $g$  is the cumulative  $k$ -distribution function for which the fraction in  $\Delta\nu$  having an absorption coefficient  $< k_g$  is  $g$ . When the  $k$ -distribution method is applied to a nonhomogeneous path between  $p$  and  $p'$ , the beam transmittance of the path is approximated by

$$\int_{\Delta\nu} [e^{-\int_p^{p'} k_\nu(p'', T'') dw(p'')} ] d\nu/\Delta\nu \approx \int_0^1 [e^{-\int_p^{p'} k_g(p'', T'') dw(p'')} ] dg. \quad (15)$$

Because pressure and temperature vary along a nonhomogeneous path, the absorption coefficients at any two spectral points will not be equal in all conditions, and no wavenumbers are radiatively identical. When (15) is applied to a nonhomogeneous path, it implicitly assumes that different wavenumbers with the absorption coefficient in a given range of  $g$  at an atmospheric level will remain in the same range of  $g$  at all other levels. One approach to applying the  $k$ -distribution method is to apply an assumption that separates the dependence of the absorption coefficient on wavenumber from the dependence on pressure and temperature (Chou et al. 1993; Hollweg 1993). The absorption coefficient as a function of  $g$  is computed for a reference temperature and pressure. The absorption coefficient at other pressure and temperature is then extrapolated

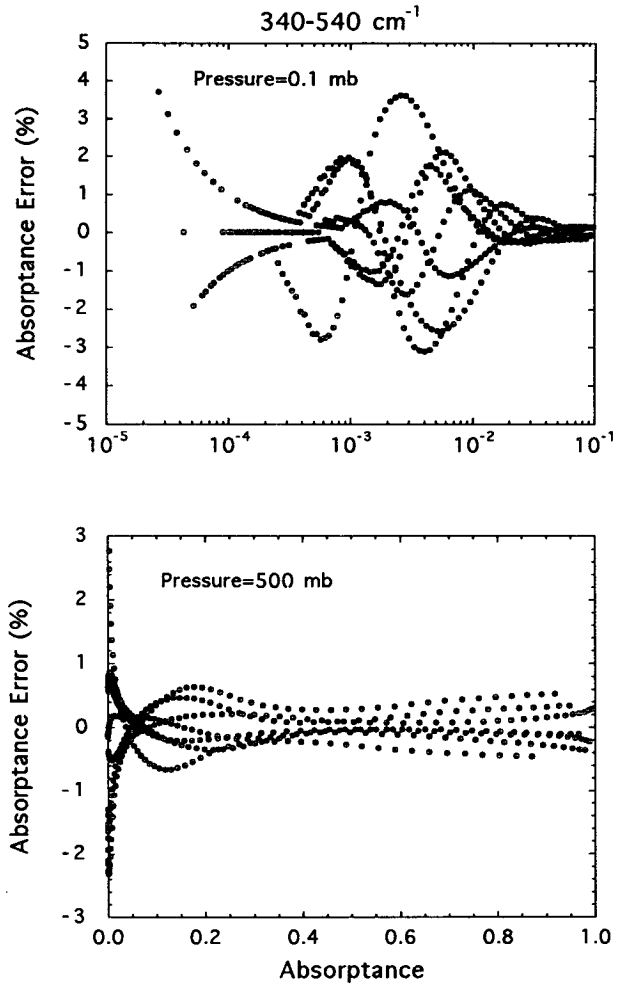


FIG. 4. Errors in the absorbance due to the regression of (13). Points are computed for water vapor amount ranging from  $10^{-8}$  to  $10 \text{ g cm}^{-2}$  and for temperature ranging from 170 K to 330 K.

from the reference values. The far-wing pressure scaling of Chou et al. (1993) is computationally very fast and can accurately compute fluxes and cooling rate in the troposphere and the lower stratosphere, but the cooling rate is highly underestimated in the middle atmosphere because of the extrapolation to very low pressures. Figure 5 shows the absorption coefficient as a function of the cumulative- $k$  distribution for the band  $0\text{--}340 \text{ cm}^{-1}$ . Neighboring curves are for pressures one decade apart. Except for a very small or very large absorption coefficient, the spacing of neighboring curves are also one decade apart. It explains why the linear pressure scaling of the absorption coefficient used by Chou et al. (1993) can be applied to accurately compute the tropospheric and lower-stratospheric cooling rate. At the high end of  $g$ , the absorption coefficient varies rapidly with  $g$  and is no longer a linear function of pressure. This is the reason that the linear pressure

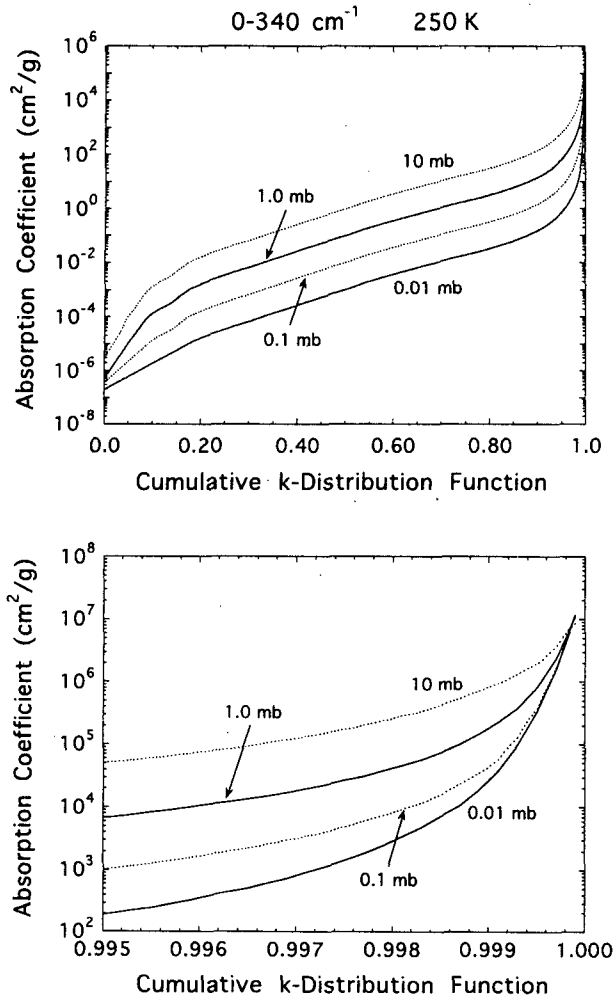


FIG. 5. The absorption coefficient as a function of the cumulative- $k$  distribution at 250 K and various pressures for the 0–340  $\text{cm}^{-1}$  band. The absorption coefficient is computed using the line-by-line method. The upper panel covers the entire range of the cumulative- $k$  distribution, and the lower panel shows the details of the absorption coefficient in the 0.995–1.0 range of the cumulative- $k$  distribution.

scaling cannot be used for computing middle atmospheric cooling rate.

Another approach is the correlated- $k$  distribution method (e.g., Fu and Liou 1992; Goody et al. 1989; Lacis and Oinas 1991; Wang and Shi 1988; Zhu 1992), which computes  $k_g$  at individual atmospheric levels and assumes that wavenumbers in a given range  $\delta g$  at one level remain in the same  $\delta g$  at all other levels so that (15) can be applied to computing mean transmittance. In this study, we further investigate the applicability of this method to the calculation of the middle atmospheric cooling rate. Consider a homogeneous layer with pressure  $p$ , temperature  $T$ , and the water vapor amount  $w$ . If we divide a spectral band  $\Delta\nu$  into a number of small intervals with widths  $\Delta\nu_i$  and apply the  $k$ -

distribution method of (14), then the Planck-weighted flux transmittance becomes

$$\tau(w) = \frac{\sum_i \left\{ [B_i(T_0)\Delta\nu_i] \int_0^1 \tau_i(g, w) dg \right\}}{\sum_i [B_i(T_0)\Delta\nu_i]}, \quad (16)$$

$$\sum_i \Delta\nu_i = \Delta\nu, \quad (17)$$

where  $B_i(T_0)$  is the mean Planck flux in the spectral interval  $i$  at temperature  $T_0$ , and  $\tau_i$  is the flux transmittance defined by (4). The division into a number of small intervals is necessary if the range of  $B_\nu(T_0)$  in  $\Delta\nu$  is large.

In applying the  $k$ -distribution method, the number and size of the cumulative- $k$  intervals are fixed for all heights. It can be seen from the lower panel of Fig. 5 that the absorption coefficient increases rapidly and nonlinearly as the cumulative  $k$ -distribution function approaches 1. This nonlinearity of the  $k$  values should be properly taken into account in deriving the effective values of  $k$  in individual  $g$  intervals. By dividing the cumulative  $k$ -distribution function into a number of intervals,  $\Delta g_j$ , Eq. (16) becomes

$$\tau(w) = \frac{\sum_i \left\{ [B_i(T_0)\Delta\nu_i] \sum_j \int_{\Delta g_j} \tau_i(g, w) dg \right\}}{\sum_i [B_i(T_0)\Delta\nu_i]} = \sum_j \tau_j(w) \Delta g_j \quad (18)$$

where  $\tau_j(w)$  is the Planck-weighted flux transmittance in the  $j$ th  $g$  interval given by

$$\tau_j(w) = \frac{\sum_i \left\{ [B_i(T_0)\Delta\nu_i] \int_{\Delta g_j} \tau_i(g, w) dg / \Delta g_j \right\}}{\sum_i [B_i(T_0)\Delta\nu_i]} \quad (19)$$

and

$$\sum_j \Delta g_j = 1. \quad (20)$$

It is noted that in the above equations, we have omitted the expression for the dependence of the transmittance on pressure and temperature. Because the Planck-weighted flux transmittance  $\tau(w, p, T)$  is for a homogeneous layer, the  $k$ -distribution method is exact. Values of  $\tau_j(w, p, T)$  are precomputed from (18) using the line-by-line method for  $w$  ranging from  $10^{-8}$  to  $10^3 \text{ g cm}^{-2}$  with  $\Delta \log_{10} w = 0.2$ ,  $p$  ranging from  $10^{-2}$  to  $10^3 \text{ mb}$  with  $\Delta \log_{10} p = 0.2$ , and  $T$  ranging from 170 to 330 K. The transmittance for each  $j$  is then approximated by

$$\tau_j(w, p, T) \approx e^{-k_j(p, T)w} \quad (21)$$

such that

$$\sum_w [e^{-k_j(p, T)w} - \tau_j(w, p, T)]^2 = \min \quad \text{for} \\ 10^{-8} \text{ g cm}^{-2} \leq w \leq 10 \text{ g cm}^{-2}. \quad (22)$$

Corresponding to each  $\Delta g_j$ , we have  $k_j(p, T)$ . It is noted that  $\tau_j$  is the flux transmittance given by (4). Thus, the coefficient  $k_j$  is the effective absorption coefficient of the range  $\Delta g_j$  multiplied by a diffusivity factor, which is commonly taken to be 1.66.

The variation of the absorption coefficient with temperature in the atmosphere is relatively small as compared to the variation with pressure. Following Chou and Kouvaris (1986) and Fu and Liou (1992), the logarithm of  $k_j$  is fit to a quadratic function in  $T$ ,

$$\ln k_j(p, T) = \alpha_j(p) + \beta_j(p)(T - 250) \\ + \gamma_j(p)(T - 250)^2. \quad (23)$$

The coefficients  $\alpha_j$ ,  $\beta_j$ , and  $\gamma_j$  are derived for pressure ranging from  $10^{-2}$  to  $10^3$  mb with  $\Delta \log_{10} p = 0.2$ . It is found that the percentage error of the effective absorption coefficient for flux transmittance is generally less than 5% for  $w$  ranging from  $10^{-8}$  to  $10 \text{ g cm}^{-2}$ ,  $p$  ranging from  $10^{-2}$  to  $10^3$  mb, and  $T$  ranging from 170 to 330 K.

Similar to the approximation (15) for the beam transmittance, the Planck-weighted flux transmittance between  $p$  and  $p'$  for a nonhomogeneous path is approximated by

$$\tau(p, p') = \sum_j \left[ \prod_l e^{-k_j(p_l, T_l)w_l} \right] \Delta g_j, \quad (24)$$

where the subscript  $l$  denotes the atmospheric layers between  $p$  and  $p'$ . Because of the nonlinear dependence of the absorption coefficient on wavenumber, pressure, and temperature, the wavenumbers in a given  $\Delta g_j$  at an atmospheric level are generally different from the wavenumbers in the same  $\Delta g_j$  at all other level. Equation (24) implicitly assumes that the wavenumbers in a given  $\Delta g_j$  are the same at all levels and is called the correlated- $k$  distribution method.

It is clear from Fig. 5 that to be computationally efficient, the size of the  $g$  interval should vary with  $g$  itself. For  $g > 0.995$ , the magnitude of  $k$  varies by an order of 5. To achieve the maximum accuracy, we have computed the cooling profiles using  $\Delta g = 0.01$  for  $g < 0.95$  and  $\Delta g = 10^{-4}$  for  $g > 0.95$ . The results are shown in Figs. 1, 2, and 3 (dashed curves) for the bands  $0\text{--}340 \text{ cm}^{-1}$ ,  $340\text{--}540 \text{ cm}^{-1}$ , and  $1380\text{--}1900 \text{ cm}^{-1}$ , respectively. It can be seen from the figures that the correlated- $k$  method can compute very accurately cooling rate throughout the middle and lower atmosphere. Moderately large errors (0.05–0.1°C/day) are found near 0.01 mb for the  $0\text{--}340 \text{ cm}^{-1}$  band and 0.01–0.1 mb for the band  $340\text{--}540 \text{ cm}^{-1}$ . In the middle tropo-

sphere (500 mb), an error of 0.1°C/day is also found in the  $340\text{--}540 \text{ cm}^{-1}$  band for the midlatitude summer atmosphere (upper panel of Fig. 2). The cause of this relatively large error is not clear.

The upward fluxes at the top of the atmosphere and the downward fluxes at the surface are given in Table 1. As in the cases of table look-up, the flux errors caused by using the correlated- $k$  method are very small with a maximum of  $\approx 1.5\%$ .

The sizes of  $\Delta g$  used for computing cooling profiles shown in Figs. 1–3 are much too small to be computationally efficient. In order to study the effect of  $\Delta g$  on the cooling rate calculations,  $\Delta g$  is degraded to  $10^{-3}$  for  $g > 0.95$ . The result for band  $0\text{--}340 \text{ cm}^{-1}$  in the midlatitude case is shown in Fig. 6 (dot-dashed curve). Also shown in Fig. 6 are the cooling profile from line-by-line calculations (solid curve) and the correlated- $k$  method with  $\Delta g = 10^{-4}$  (dashed curve). Although the size  $\Delta g = 10^{-3}$  is very small, the cooling rate error above the 3-mb level is very large. To further investigate the causes of this large error, the contributions to the total cooling from the small fractions at the high end of the cumulative  $k$ -distribution function are computed. The results are shown in Fig. 7. The upper panel is for the band  $0\text{--}340 \text{ cm}^{-1}$  and the midlatitude summer atmosphere, and the lower panel is for the band  $1380\text{--}1900 \text{ cm}^{-1}$  and the subarctic winter atmosphere. In both cases, the cooling above the 1-mb level is contributed nearly totally from the 0.5% of the spectrum with the largest absorption coefficient. For the band  $0\text{--}340 \text{ cm}^{-1}$ , the cooling above the 0.1-mb level is contributed from the 0.01% of the spectrum with the largest absorption coefficient. The high sensitivity of the

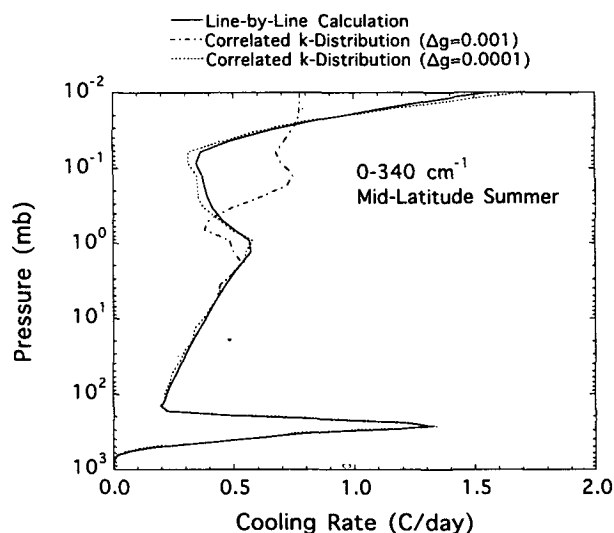


FIG. 6. The cooling rate profiles in the  $0\text{--}340\text{-cm}^{-1}$  band for the midlatitude summer atmosphere computed from line-by-line calculations (solid curve) and the correlated- $k$  distribution method with  $\Delta g = 10^{-4}$  (dashed curve) and  $\Delta g = 10^{-3}$  (dot-dashed curve).

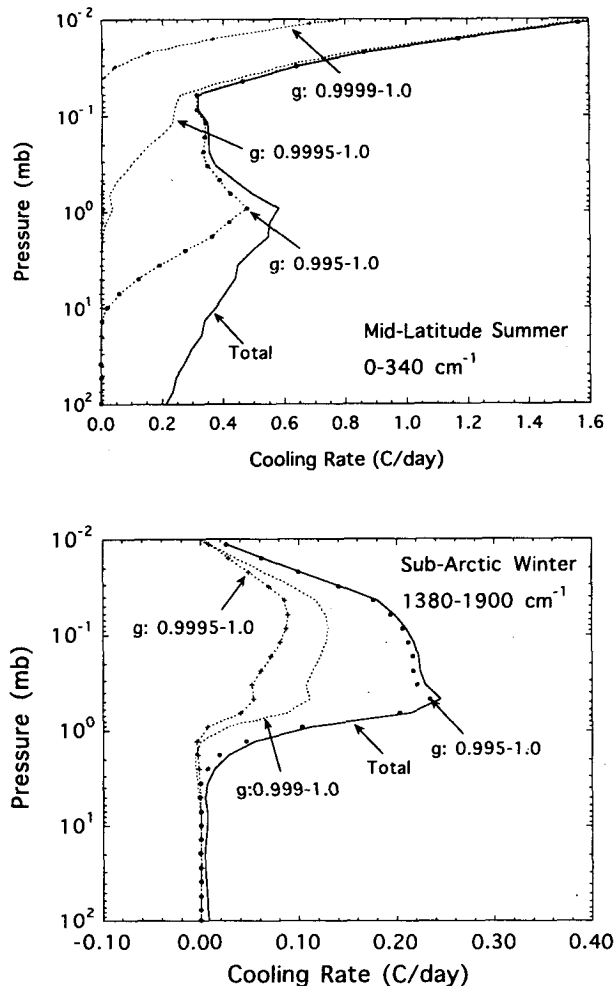


FIG. 7. The cooling rate profiles contributed from various sections of the cumulative- $k$  distribution function  $g$ . The upper panel is for the 0–340- $\text{cm}^{-1}$  band and the midlatitude summer atmosphere, and the lower panel is for the 1380–1900- $\text{cm}^{-1}$  band and the subarctic winter atmosphere.

cooling rate calculation to the size of  $\Delta g$  was also found by Grossman and Grant (1992).

#### 4. Concluding remarks

Chou et al. (1993) have shown that the  $k$ -distribution method with linear pressure scaling can compute accurately the water vapor and  $\text{CO}_2$  cooling rate in the troposphere and the lower stratosphere. In this method, the absorption coefficients at a reference pressure (500 mb) and temperature (250 K) are derived for a few (6) of the  $k$ -distribution intervals. The absorption coefficient is linearly extrapolated to other pressures. Because of the linear extrapolation, this method cannot be applied to the middle atmosphere ( $p < 20$  mb). In this study, we investigate two parameterizations for the computation of the water vapor cooling rate in

three strong absorption regions (0–340  $\text{cm}^{-1}$ , 340–540  $\text{cm}^{-1}$ , and 1380–1900  $\text{cm}^{-1}$ ). These methods are the correlated- $k$  distribution method and the method of table look-up. Both methods are equally accurate in the troposphere and the middle atmosphere. The cooling rate error is  $< 0.1^\circ\text{C}/\text{day}$ , and the flux error is negligible.

It is found that the cooling in the lower pressure region ( $p < 1$  mb) is contributed from a very small fraction ( $< 0.005$ ) of the spectrum where the absorption coefficient is the largest and where the absorption coefficient varies by four orders of magnitude. It requires at least 100 terms of the  $k$ -distribution in each band when the correlated- $k$  method is used. The correlated- $k$  method is, therefore, significantly slower than the method of table look-up.

With the considerations of accuracy and speed, we recommend that the  $k$ -distribution method with linear pressure scaling be used if a climate model does not extend to the height far above the 20-mb level. For a climate model involving both the troposphere and the middle atmosphere, the method of table look-up can be used for flux and cooling rate calculations in the three strong water vapor absorption bands, as well as the  $\text{CO}_2$  and  $\text{O}_3$  bands (Chou and Kouvaris 1991). The  $k$ -distribution method with linear pressure scaling can still be used for flux and cooling rate calculations in the water vapor spectral regions other than the three strong absorption bands.

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