NOTES AND CORRESPONDENCE

At What Optical Thickness Does a Cloud Completely Obscure the Sun?

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It appears to be well known (among those who know it well) that the sun is completely obscured by a cloud with optical thickness of about 10 as a consequence of contrast reduction. The origins of this belief are obscure, although they appear to stem from a paper by Twomey et al. (1967), in which one can find the following assertion: "With $\tau = 10$, the sun's image would be reduced to the brightness of a gas or kerosene flame and effectively obliterated." No evidence, theoretical or experimental, is adduced in support of this assertion, which is almost a parenthetical remark embedded in a paper devoted to reflection and transmission by clouds. Deirmendjian (1969, pp. 113–114) estimates that when the sun can be seen only faintly the optical thickness of the intervening cloud is 16. Without proof, van de Hulst (1980, p. 660) asserts that the "sun's disk is not visible at all . . . roughly at cloud decks with optical depth [along the observer's line of sight to the sun] ≥ 14."

The problem of rigorously computing the optical thickness at which the sun completely disappears is difficult. First, there is to our knowledge no exact theory of transmission of light from a finite source by a cloud. Then there is the complicating factor that the contrast threshold depends on the angular width of an object viewed through a turbid medium. That is, the object can be decomposed by Fourier analysis into a superposition of periodic patterns with different spatial frequencies; the contrast threshold is different for each spatial frequency. Even if the computations could be done, would they be preferable to an experiment? We think not.

Although we never doubted the value $\tau = 10$ as an approximate limit beyond which the sun cannot be seen through a cloud (in part because it squares with our observations of the sun through fog of approximately known optical thickness), we do think it appropriate that experimental evidence be provided to support what everyone knows, but which no one seems to have verified.

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What is to be determined is the optical thickness at which one can no longer see the sun through clouds. The word "see" signals that human subjects are required. These were the three authors of this paper, and the results presented are a composite of their observations since no two humans see exactly alike.

Uniform atmospheric clouds of known optical thickness and particle size distribution cannot be produced or maintained at will, so we made our observations through artificial clouds composed of particles of known shape and (narrow) size distribution. These clouds were composed of polystyrene spheres suspended in distilled water in a glass tank with dimensions 26 × 26 × 50 cm. The spheres were supplied by the Duke Scientific Corporation (1992). We used spheres of three different diameters: 0.652 μm (std dev = 0.0048 μm), 5.3 μm (std dev = 1.2 μm), and 15.9 μm (std dev = 2.9 μm). At visible wavelengths, polystyrene spheres are negligibly absorbing, as evidenced by the fact that a dense suspension of them is white. The refractive index of polystyrene at visible wavelengths is about 1.59.

The artificial sun was an ordinary incandescent light bulb (60 W) positioned so that it looked like a disk when observed through the tank. The distance of the light bulb from the tank was such that it was more or less uniformly illuminated; the observers were positioned at a distance from the bulb so that it subtended the same angle as the sun (~0.5°).

The polystyrene spheres were packaged as aqueous suspensions (10% solids). We gradually added drops of these suspensions to the water in the tank to vary the optical thickness τ:

$$\tau = NC_{sca}h, \qquad (1)$$

where N is the number density of polystyrene spheres in the water, C_{sca} is the scattering cross section of polystyrene spheres averaged over the size distribution, and h is the physical thickness of the tank measured along the transmission path. For a fixed h and fixed volume of water (much larger than the volume of drops) to which drops are added, the relation between optical thickness and number of drops N_d uniformly mixed in the water is therefore

$$\tau = CN_d, \tag{2}$$

where C is a constant.

To determine C we measured spectral radiances of the light bulb with a PhotoResearch SpectraScan PR-704 spectral radiometer. The field of view of the radiometer was such that it received light from the central half of the light bulb. The measured radiance is an average over the field of view (1°) of the detector.

If multiple scattering is negligible, the radiance L transmitted by a cloud of particles of optical thickness τ is

$$L = L_0 e^{-\tau}, \tag{3}$$

where L_0 is the radiance of the light bulb with water in the tank but without particles suspended in the water. Underlying Eq. (3) is the assumption that the detector receives no near-forward scattered light, a condition that is never strictly satisfied.

Consider a monodirectional beam and ignore multiple scattering (absorption also negligible). The *effective* scattering cross section measured in a transmission experiment is

$$C_{\text{sca}}\left(1 - \int_{\Omega_d} p d\Omega\right), \tag{4}$$

where C_{sca} is the scattering cross section measured by a perfect instrument (one with zero acceptance angle), p is the phase function (normalized to 1), and Ω_d is the acceptance solid angle of the detector at the particle. For isotropic scattering, the correction factor multiplying C_{sca} in Eq. (4) is $1 - \Omega_d/4\pi$, which makes sense: the effective scattering cross section is reduced by the fraction of light isotropically scattered into the detector. From Eqs. (1) and (4) it follows that the correction to be applied to the optical thickness because of scattered light collected by the detector is

$$1 - \int_{\Omega_d} p d\Omega. \tag{5}$$

To determine this correction factor we first transform to the variable $\mu = \cos\theta$, where θ is the scattering angle. With this transformation the correction factor is

$$1 - \int_{\mu_d}^{1} p(\mu) d\mu, \tag{6}$$

where $\mu_d = 1 - \Omega_d/4\pi$ and the integral of $p(\mu)$ between -1 and 1 is unity. To estimate Eq. (6) we use the Henyey–Greenstein phase function

$$p(\mu) = \frac{1}{2} \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}, \tag{7}$$

where g is the asymmetry parameter (mean cosine of the scattering angle). Subject to the restriction that $g\Omega_d/\pi(1 - g)^2 \ll 1$, Eq. (7) substituted in Eq. (6) yields the approximate correction factor

$$1 - \frac{\Omega_d}{4\pi} \frac{1 + g}{(1 - g)^2}. \tag{8}$$

Note that for isotropic scattering ($g = 0$), the correction factor Eq. (8) is $1 - \Omega_d/4\pi$, as required.

The acceptance solid angle of our detector is about 2.4×10^{-4} sr. For $g = 0.9$ (the largest value for the particles of interest), the correction factor is 0.9964. Thus, the error associated with collecting near-forward scattered light by our detector and for our particles is less than 1%.

For sufficiently small optical thicknesses, Eq. (3) applies to our suspensions. Hence, the constant C was determined by the slope of the logarithm of the measured ratio of radiances versus N_d in the limit of zero drops (zero optical thickness):

$$C = \lim_{N_d \to 0} \left[-\frac{d}{dN_d} \ln\left(\frac{L}{L_0}\right) \right]. \tag{9}$$

All measured radiances are for a wavelength of 700 nm, chosen to be large enough to eliminate selective scattering by the smallest polystyrene spheres but small enough to be below absorption bands of liquid water.

Each time that drops containing polystyrene spheres were added to the tank water, it was stirred thoroughly. Before measuring the radiance and making our obser-

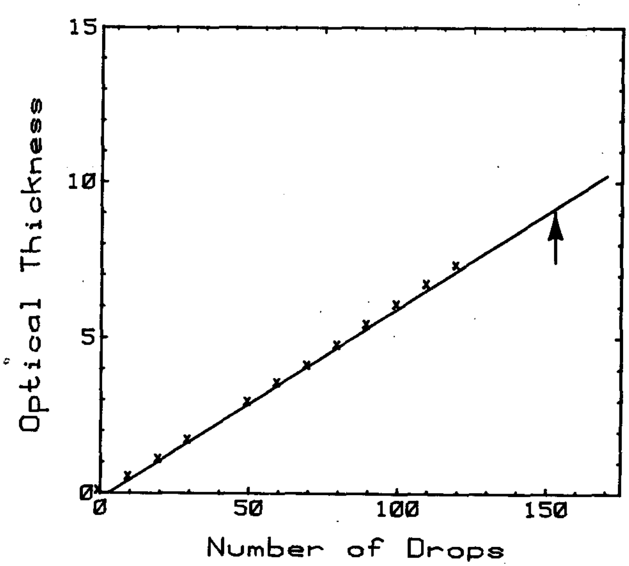

FIG. 1. Optical thickness of distilled water in a glass tank to which drops of an aqueous suspension of polystyrene spheres of mean diameter 0.652 μm were added. The crosses indicate values of $-\ln(L/L_0)$, where L is the measured transmitted radiance (at a wavelength of 700 nm) from an incandescent light bulb and L_0 is the measured radiance with no spheres in the distilled water. The straight line is a least-squares fit to the data over the range for which the relation between optical thickness and number of drops is approximately linear. An arrow marks the optical thickness at which the light bulb could not be seen by three observers.

vations we waited long enough to eliminate turbulence but not so long that the particles settled appreciably.

The results of our observations are shown in Figs. 1–3, plots of $-\ln(L/L_0)$ versus N_d. The number of drops, hence optical thickness, at which all three observers agreed that the light bulb could no longer be seen is shown on these figures. The range of optical thicknesses for which the observers did not agree on whether the bulb could be seen was small. Note that the critical optical thickness is around 10, although its precise value depends on mean particle diameter d. The larger the diameter, the greater the critical optical thickness: $\tau = 9.4$ for $d = 0.652$ μm; $\tau = 11.6$ for $d = 5.3$ μm; and $\tau = 12.7$ for $d = 15.9$ μm. This is not surprising: Scattering becomes increasingly peaked in the forward direction the larger the particle. Using data shown by van de Hulst (1980, Fig. 10.3) we estimated the asymmetry parameter g for our particles (relative refractive index of 1.2) to be about 0.8, 0.85, and 0.9 for mean diameters of 0.652 μm, 5.3 μm, and 15.9 μm, respectively. As g increases so does the optical thickness at which the sun is completely obscured. Hypothetical particles that scatter *only* in the forward direction ($g = 1$) would not obscure the sun regardless of cloud optical thickness. Our conclusion about the relation between g and the optical thickness necessary to obscure the sun is consistent with the similarity principle discussed by van de Hulst (1980, p. 479), according to which two clouds are optically similar if their optical thicknesses and asymmetry parameters satisfy

$$\tau(1 - g) = \tau'(1 - g'). \qquad (10)$$

As an aside we note that the smaller the particle, the greater the range over which the relation between $-\ln(L/L_0)$ and N_d is linear. This is expected because the larger the particle (the greater the value of g), the more that multiply scattered light remains within the direct beam, whereas strict exponential attenuation requires that all scattered light never return to the beam.

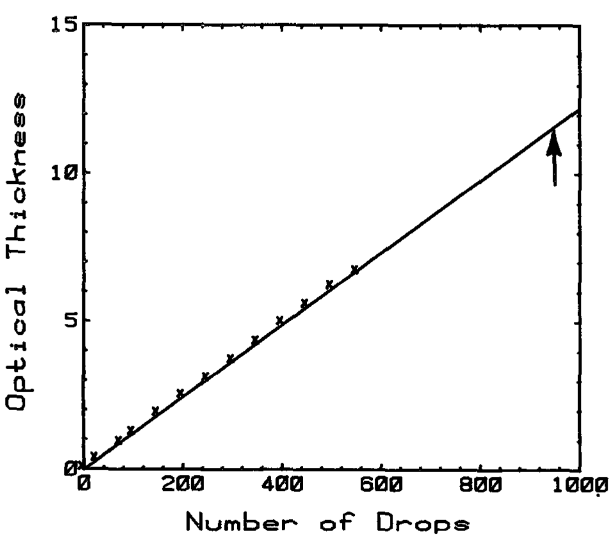

FIG. 2. Same as for Fig. 1 except that the mean diameter of the polystyrene spheres is 5.2 μm.

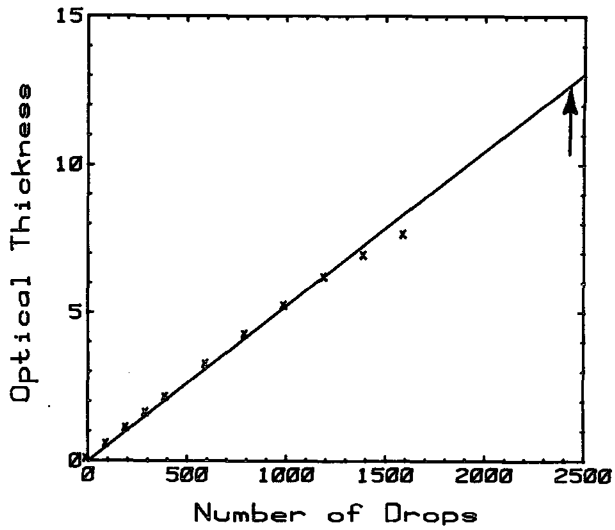

FIG. 3. Same as for Fig. 1 except that the mean diameter of the polystyrene spheres is 15.9 μm.

We have verified by controlled experiments using human observers of a laboratory sun seen through laboratory clouds of polystyrene spheres that a cloud optical thickness of 10 is a good rule of thumb for the upper limit beyond which the real sun cannot be seen through atmospheric clouds. In addition, we have shown that the precise optical thickness for complete obscuration increases with particle size, although not so much as to invalidate the rule of thumb.

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