NOTES AND CORRESPONDENCE

The Dependence of the Hadley Circulation on the Thermal Relaxation Time

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ABSTRACT

Analytic and numerical solutions are found for the nonlinear Hadley circulation problem with respect to the dependence of the strength and the extent of the Hadley circulation on the thermal relaxation time. The dependence on the thermal relaxation time is a crucial parameter to investigate since the simplifications used in previous studies assumed a large thermal relaxation time, to justify the geostrophic assumption, but in the presence of moist convection, thermal relaxation may be fast in the convection regions. In this study, a primitive equation model is used to investigate the effect of different latitudinal distribution of thermal relaxation time on the extent of the circulation cells, the zonal wind, the temperature distribution, and the strength of the meridional circulations. It is found that the extent of the Hadley circulation is insensitive to the value of the thermal relaxation time τ, while the strength of the circulation is very sensitive to τ (but in a way that is predictable based on the $1/\tau$ scaling).

1. Introduction

In the study of the nonlinear axially symmetric circulations in the ''nearly inviscid'' limit, Held and Hou (1980) developed a simple approximate theory based on the conservation of absolute angular momentum and potential temperature. The theory predicts, as a function of Rossby number Ro, the extent of the Hadley circulation, the latitude of the upper-level jet of the zonal wind, and the region of surface easterlies and westerlies. This theory also predicts the total poleward heat flux, the mass flux in the surface boundary layer, and the surface zonal wind distribution, which are found to be inversely proportional to the thermal relaxation time. It is noted in Held and Hou (1980) that this simple theory is self-consistent only in a restricted parameter domain of Rossby number (Ro), Ekman number (E), and the thermal relaxation time (τ). One of the requirements is that the relaxation time (made dimensionless by multiplying by 2Ω, $\Omega = 2\pi/$day) be large, that is, $\tau \gg 1$. In their studies, τ is considered to be a constant throughout the model domain.

In Fang and Tung (1996), we found that to model the Hadley circulation with moist convection it is desirable to allow the thermal relaxation time to be a function of space. It is suggested that results from moist convective models can be incorporated and reinterpreted in the Newtonian cooling formulation by assuming a small τ in the convecting region, which leads to a fast convective adjustment to a moist adiabat. A simple case was considered where it was assumed that τ is so small in the ITCZ (intertropical convergence zone) that the local temperature there quickly relaxes to its thermal equilibrium value. In light of this reinterpretation, the thermal relaxation time τ can vary widely with latitude and can range between 0 and ∞.

To understand the axisymmetric circulation in a more realistic moist-convecting atmosphere, it is useful to have a systematic study of the effect of the thermal relaxation time on the structure and the magnitude of the circulation. In this study, using both scaling arguments and numerical results, we derive the extent of the Hadley circulation (in the equatorially symmetric case) for any constant τ and for a latitudinal distribution of τ. Since geostrophy fails at small values of τ, the primitive equations are used here. The dependence of the strength of the circulation and the temperature deviation from its equilibrium value on τ are also discussed.

2. The model

A set of axially symmetric primitive equations is used here for a Boussinesq fluid on a sphere of radius a rotating with rate Ω, confined between a solid bottom surface and a stress-free lid at height H. The set of nondimensional equations is given in Fang and Tung (1996) and is listed below:

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$$\frac{\partial u}{\partial t} + \mathrm{Ro}\left(v\frac{\partial u}{\partial \phi} + w\frac{\partial u}{\partial z} - uv\tan\phi\right) - \sin\phi v = E\frac{\partial^2 u}{\partial z^2} \quad (1)$$

$$\frac{\partial v}{\partial t} + \mathrm{Ro}\left(v\frac{\partial v}{\partial \phi} + w\frac{\partial v}{\partial z} + u^2\tan\phi\right) + \sin\phi u = E\frac{\partial^2 v}{\partial z^2} - \frac{\partial \Phi}{\partial \phi} \quad (2)$$

$$\frac{1}{\cos\phi}\frac{\partial(v\cos\phi)}{\partial \phi} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

$$\frac{\partial \Phi}{\partial z} = \Theta \quad (4)$$

$$\frac{\partial \Theta}{\partial t} + \mathrm{Ro}\left(v\frac{\partial \Theta}{\partial \phi} + w\frac{\partial \Theta}{\partial z}\right) = \frac{E}{\mathrm{Pr}}\frac{\partial^2 \Theta}{\partial z^2} + \frac{Q}{C_p}, \quad (5)$$

where Θ is the potential temperature and (u, v, w) the velocity of the fluid in the longitudinal, latitudinal (ϕ), and the vertical (z) direction respectively; $\mathrm{Ro} \equiv U/(2\Omega a)$ is Rossby number where U is a typical zonal velocity, $E \equiv \nu/(2\Omega H^2)$ is Ekman number, and $\mathrm{Pr} \equiv \nu/\kappa$ is Prandtl number. We suppose that the diabatic heating can be approximated by the Newtonian cooling law with a variable relaxation time:

$$\frac{Q}{C_p} = \frac{\Theta_E - \Theta}{\tau(\mu)}, \quad (6)$$

where $\mu \equiv \sin\phi$. The equilibrium potential temperature Θ_E is given as a function of latitude and height. Held and Hou (1980) used the following as their (dry) equilibrium potential temperature

$$\frac{\Theta_E}{\Theta_{00}} = \Theta_0 - \Delta_H\mu^2 + \Delta_V z. \quad (7)$$

It needs to be modified to include regions of moist convection, where Θ_E should be interpreted as the moist convective radiative equilibrium value. Thermal relaxation time τ is generally a function of space. Here we confine our attention to a latitude-dependent profile $\tau(\mu)$ only.

The boundary conditions for the velocities are no-slip conditions at the bottom and stress-free lid at the top. The boundary is considered to be insulated so that no heat flux crosses it. There is no poleward velocity, that is, $v\cos\phi = 0$, at the poles. These are the same as in Fang and Tung (1996).

To test the dependence of the Hadley circulation on thermal relaxation time τ, we have also performed a series of numerical calculations for various relaxation time profiles. The value of the parameters used in those calculations are $\Delta_H = 1/6$, $\Gamma = 6$ K/km, $\Gamma_d = 9.8$ K/km, $H = 15$ km, $T_0 = 300$ K, $\nu = 3.5$ m s^{-2}. These lead to $\Delta_v \equiv \Gamma_d - \Gamma = 3.8$ K/km and $E = 1 \times 10^{-4}$. The typical velocity U is taken from the thermal equilibrium solution as $gH\Delta_H/(2\Omega a)$ and it leads to a thermal Rossby number $\mathrm{Ro} = R = gH\Delta_H/(2\Omega a)^2 = 0.0283$. The solution sought in this study is the steady solution.

3. Scaling

In the "nearly inviscid limit" $(E \to 0^+)$, from Eq. (5) we can see that the steady state of (v, w) is proportional to Q and therefore is scaled by $1/\tau_0$ except in the viscous boundary layers. Here τ_0 is a typical value of τ. We can rescale them by introducing $v = \hat{v}/\tau_0$, $w = \hat{w}/\tau_0$. The steady version of the angular momentum equation (1) can be rewritten as

$$\mathrm{Ro}\left(\hat{v}\frac{\partial u}{\partial \phi} + \hat{w}\frac{\partial u}{\partial z} - u\hat{v}\tan\phi\right) - \sin\phi\hat{v} = E\tau_0\frac{\partial^2 u}{\partial z^2}. \quad (8)$$

It can be seen that if $E\tau_0 \ll 1$, the absolute angular momentum is conserved along streamlines away from viscous boundary layers; that is,

$$\hat{v}\frac{\partial L}{\partial \phi} + \hat{w}\frac{\partial L}{\partial z} = 0 \quad \text{for} \quad E\tau_0 \ll 1, \quad (9)$$

where $L \equiv \cos^2\phi + 2\mathrm{Ro}u\cos\phi$ is the absolute angular momentum.

The meridional momentum equation (2) can be rewritten as

$$\frac{\mathrm{Ro}}{\tau_0^2}\left(\hat{v}\frac{\partial \hat{v}}{\partial \phi} + \hat{w}\frac{\partial \hat{v}}{\partial z}\right) + \mathrm{Ro}u^2\tan\phi + \sin\phi u = \frac{E}{\tau_0}\frac{\partial^2 \hat{v}}{\partial z^2} - \frac{\partial \Phi}{\partial \phi}. \quad (10)$$

For $\tau_0 \gg 1$, (10) results in a statement about cyclostrophic balance:

$$\mathrm{Ro}u^2\tan\phi + \sin\phi u = -\frac{\partial \Phi}{\partial \phi}. \quad (11)$$

The thermodynamic equation (5) can be rewritten as, for the case of uniform $\tau = \tau_0$,

$$\mathrm{Ro}\left(\hat{v}\frac{\partial\Theta}{\partial\phi} + \hat{w}\frac{\partial\Theta}{\partial z}\right) = \frac{E\tau_0}{\mathrm{Pr}}\nabla^2\Theta + (\Theta_E - \Theta). \quad (12)$$

If Pr = $O(1)$ or larger, the thickness of the thermal diffusive layer has the same or higher order as that for the zonal wind. Under the assumption of $E\tau_0 \ll 1$, we have

$$\mathrm{Ro}\left(\hat{v}\frac{\partial\Theta}{\partial\phi} + \hat{w}\frac{\partial\Theta}{\partial z}\right) = (\Theta_E - \Theta), \quad (13)$$

which actually is presumed in the beginning of the rescaling of v and w.

Held and Hou (1980) discovered a simple formula for the extent of the Hadley circulation. The derivation of that formula is based on a few assumptions:

1) The absolute angular momentum is conserved in the upper branch of the circulation, from (9).
2) The cyclostrophic balance is satisfied, from (11).
3) The potential temperature is conserved in the bulk of the Hadley circulation; that is, the integral of the right-hand side of (13) over the whole circulation region is zero. [This can also be derived more generally from the integration of Eq. (5) since the integral of the left-hand side is zero and the integral of the diffusion term is also zero because of the insulated boundary condition.]

What we have briefly shown above is that these ''assumptions'' are derivable from asymptotic scaling. What we have further shown is that for the case of *uniform* $\tau = \tau_0$ and *under the assumption of* $\tau_0 \gg 1$ and $E\tau_0 \ll 1$, *the extent of the Hadley circulation, the zonal wind, and the potential temperature are independent of* τ, *while the strength of the meridional circulation is inversely proportional to* τ. This is because in the scaled equations (9), (11), (13), (3), and (4), τ no longer appears.

To confirm this conclusion, which was based on scaling arguments, we carried out a numerical solution of the full primitive equations (1)–(7) for various uniform values of τ. The solution is obtained by time stepping until a steady state is reached. The result is shown in Fig. 1. The strength of the Hadley circulation is found to be proportional to $1/\tau$ for all values of τ. The extent of the circulation is found to be independent of τ.[1] The same is also true for the deviation of the temperature from its equilibrium value. The calculated zonal winds show that the location of the jet does not change with τ, but the horizontal gradient at the poleward side of the jet becomes steeper, and the absolute angular momentum conserving behavior is more obvious for smaller τ.

[1] Note that the numerical solution is not ''nearly inviscid.'' A value of E was used for practical reasons, which is not as small as one would have preferred. Consequently, the extent of the circulation in the numerical solution is affected by the viscous boundary layer when $\tau \geq 250$.

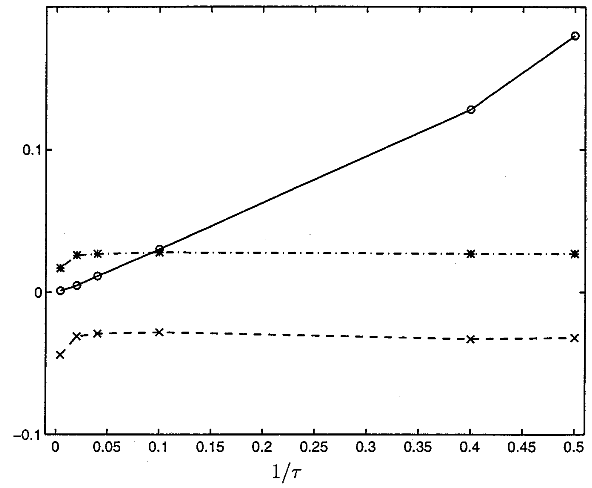

FIG. 1. The strength of the circulation and the deviation of the temperature from its equilibrium value, as a function of the thermal relaxation time: solid line with ''○'' is for the strength of the circulation; dashed line with ''×'' is for the temperature deviation at $\mu = 0$, $z = 0.5$; dashed–dotted line with ''*'' is for the deviation at $\mu = 0.3$ and $z = 0.5$.

4. The more general case

a. *The effect of* τ *on the extent*

The discussion in the last section shows that the extent of the Hadley circulation is independent of τ for a uniform τ, in the limit $\tau \gg 1$. We shall now discuss the case of a latitudinal-dependent $\tau(\mu)$. The full primitive equations are used here because $\tau(\mu)$ cannot be considered as uniformly large.

The ''equal area'' rule for potential temperature conservation (Held and Hou 1980; Lindzen and Hou 1988), which is valid for uniform τ, is

$$\int_0^{\mu_H} (\overline{\Theta}_E - \overline{\Theta})\, d\mu = 0, \quad (14)$$

where μ_H is the edge of the Hadley circulation and $\overline{(\cdot)} \equiv \int_0^1 (\cdot)\, dz$, is here modified for the case of variable $\tau(\mu)$ to

$$\int_0^{\mu_H} \frac{\overline{\Theta}_E(\mu) - \overline{\Theta}(\mu)}{\tau(\mu)}\, d\mu = 0. \quad (15)$$

Equation (15) is obtained by integrating the steady, nearly inviscid version of (5) vertically and meridionally. We still have (9), which is a statement of the conservation of angular momentum $L \equiv \cos^2\phi + 2\mathrm{Ro}u\cos\phi$ in the upper branch of the Hadley circulation in the nearly inviscid limit. Thus, the zonal wind at the ''top'' in the Hadley region is given by

$$u_t = \frac{\sin^2\phi - \sin^2\phi_1}{2\mathrm{Ro}\cos\phi}, \quad (16)$$

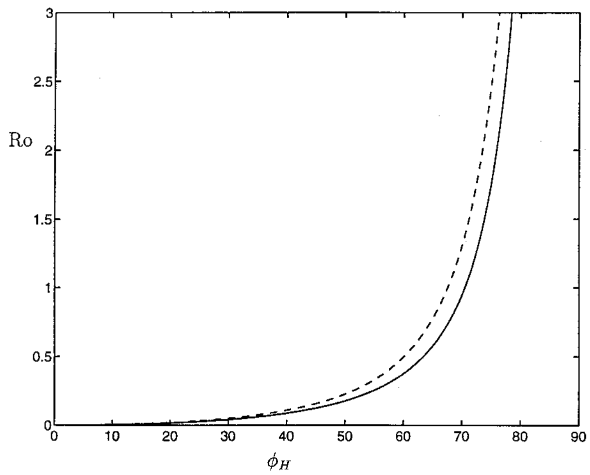

FIG. 2. The extent of the Hadley circulation for an equatorial symmetric potential temperature: solid line is for $\tau = 0$ and dashed line is for $\tau \gg 1$.

where $\phi = \phi_1$ is where $u = 0$ (and $L = \cos^2\phi_1$). The subscript t denotes the value at the top branch.

From the primitive equation (2) for the meridional velocity we have, for the upper (horizontal) branch of the Hadley circulation and in the inviscid limit,

$$\mathrm{Ro}\left(v_t\frac{\partial v_t}{\partial\phi} + u_t^2\tan\phi\right) + \sin\phi u_t = -\frac{\partial\Phi_t}{\partial\phi}. \quad (17)$$

The same equation, (2), when applied to the lower, no-slip boundary yields

$$\left.\frac{\partial\Phi}{\partial\phi}\right|_{z=0} = E\left.\frac{\partial^2 v}{\partial z^2}\right|_{z=0}.$$

This quantity has been shown in Fang and Tung (1996) to be zero in the limit $E \to 0^+$ for an equatorially symmetric Θ_E. We can thus set $\Phi(z = 0)$ to zero. The hydrostatic equation (4) can be integrated from $z = 0$ to the top to yield

$$\Phi_t = \overline{\Theta}. \quad (18)$$

Equation (18) can be substituted in (17) and integrated meridionally from $\mu = 0$ to μ, yielding

$$\frac{\mathrm{Ro}}{2}v_t^2 = \overline{\Theta}(0) - \overline{\Theta}(\mu) + \frac{\mathrm{Ro}}{2}(v_t^2(0) + u_t^2(0) - u_t^2(\mu)).$$

For the symmetric case, $u = 0$ and $v = 0$ at the equator (see Fang and Tung 1996). The use of (16) then gives

$$\frac{\mathrm{Ro}}{2}v_t^2 = \overline{\Theta}(0) - \overline{\Theta}(\mu) - \frac{\mu^4}{8\mathrm{Ro}(1 - \mu^2)}. \quad (19)$$

The extent of the Hadley circulation is given by the region where $v_t^2 > 0$, with the edge μ_H determined by $v_t^2(\mu_H) = 0$. Therefore, to find the location of the edge of the Hadley circulation, we use

$$\frac{\mu_H^4}{8\mathrm{Ro}(1 - \mu_H^2)} = \overline{\Theta}(0) - \overline{\Theta}(\mu_H). \quad (20)$$

Held and Hou (1980) introduced the constraint that at the edge of the Hadley circulation, the vertically integrated temperature is continuous and hence is given by its equilibrium value outside the circulation region; that is,

$$\overline{\Theta}(\mu_H) = \overline{\Theta}_E(\mu_H). \quad (21)$$

Consequently, (20) becomes

$$F(\mu_H) = c, \quad (22)$$

where

$$F(\mu) \equiv -\frac{\mu^4}{8\mathrm{Ro}(1 - \mu^2)} - \overline{\Theta}_E(\mu) + \overline{\Theta}_E(0)$$

is a known function of μ, and

$$c \equiv \overline{\Theta}_E(0) - \overline{\Theta}(0)$$

is always positive because the equator is in the upwelling region where $\Theta - \Theta_E < 0$. In the extreme (but unrealistic) case of $\tau \to 0$, $\Theta \to \Theta_E$ and so $c \to 0$. The extent of the circulation in that case is determined from

$$F(\mu_H) = 0 \text{ as } \tau \to 0. \quad (23)$$

For the symmetric $\overline{\Theta}_E(\mu)$ given by (7) and used by previous authors, $\Theta_E(\mu)$ has the dimensionless form

$$\overline{\Theta}_E(\mu) = \overline{\Theta}_E(0) - \mu^2. \quad (24)$$

Equation (23) can be solved explicitly to yield

$$\mu_H = \sqrt{\frac{8\mathrm{Ro}}{1 + 8\mathrm{Ro}}}. \quad (25)$$

For nonzero τ, the extent μ_H can be determined graphically from (22) for a given $c > 0$. However, c cannot be arbitrarily given; it has to satisfy a constraint similar to the the equal-area rule (15), which in the present case can be written as

$$\int_0^{\mu_H}\frac{F(\mu) - c}{\tau(\mu)}\,d\mu = \int_0^{\mu_H}\frac{\mathrm{Ro}}{2\tau(\mu)}v_t^2\,d\mu > 0. \quad (26)$$

For the case of large, uniform τ, considered previously by Held and Hou (1980), (26) becomes

$$c = c(\infty) \equiv \frac{1}{\mu_H}\int_0^{\mu_H}F(\mu)\,d\mu \quad (27)$$

since v_t^2 is negligible because of its $O(1/\tau_0^2)$ scaling. Substituting (27) into (22) then yields an equation for μ_H:

$$\frac{1}{3}(16\mathrm{Ro} - 1)\mu_H^3 - \frac{\mu_H^5}{1 - \mu_H^2} - \mu_H + \frac{1}{2}\ln\frac{1 + \mu_H}{1 - \mu_H} = 0, \quad (28)$$

which was previously obtained by Held and Hou (1980).

It is seen that, in general, the extent of the circulation depends on τ. We denote this dependence by writing $\mu_H = \mu_H(\tau)$. We have now found two extreme values, $\mu_H(0)$ from (25) and $\mu_H(\infty)$ from (28). These are plotted in Fig. 2 as functions of Ro, $\mu_H(0)$ as a solid line and $\mu_H(\infty)$ as a dashed line. We see that these two have a

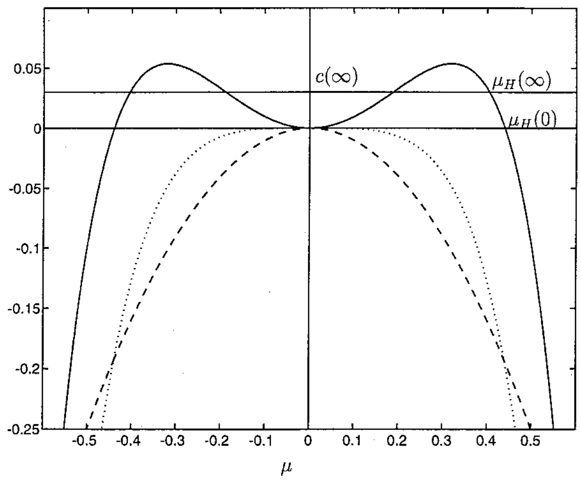

FIG. 3. The graphical solution for μ_H. The solid curve is $F(\mu)$. The top horizontal line is $c(\infty)$, while the lower horizontal line is $c(0)$. The two parts that make up $F(\mu)$ are also shown: $1/2\mathrm{Ro}u_t^2$ by the dotted line and $\Theta_E(\mu) - \Theta_E(0)$ by the dashed line.

difference of less than two degrees of latitude for all values of Ro.

For other values of τ between these two limits, we can easily show that

$$\mu_H(\infty) \leq \mu_H(\tau) \leq \mu_H(0) \quad (29)$$

for the case of constant τ. For this case, $c = c(\tau)$ is given by

$$c(\tau) = \frac{1}{\mu_H}\int_0^{\mu_H} F(\mu)\, d\mu - \frac{1}{\mu_H}\int_0^{\mu_H} \frac{\mathrm{Ro}}{2} v_t^2\, d\mu. \quad (30)$$

In Fig. 3, $F(\mu)$ is plotted in the solid curve. In the absence of the last term in (30), c would be determined by the equal-area rule to be the line $c(\infty)$ that intersects $F(\mu)$ in such a way that there is an equal area bounded by the curve $F(\mu)$ above and below the line $c(\infty)$. The outermost intersection of the $F(\mu)$ curve and the c line then determines μ_H. Equation (30) states that for τ not necessarily large, c should be smaller than the value determined by the equal area rule since the last term in (30) always subtracts from $c(\infty)$; that is,

$$0 \leq c(\tau) \leq c(\infty). \quad (31)$$

From Fig. 3, the graphical solution for (30), we see that a c between $c(\infty)$ and 0 should yield a $\mu_H(\tau)$ between its two limiting values, thus the inequality in (29).

For a variable $\tau(\mu)$, we still have (22), which implies, since $c \geq 0$,

$$F(\mu_H) \geq 0. \quad (32)$$

Thus, only the positive portion of $F(\mu)$ needs to be considered in our graphical solution for μ_H. We can immediately conclude that

$$\mu_H(\tau) < \mu_H(0) \text{ for } \tau(\mu) > 0. \quad (33)$$

For the (realistic) case where $\tau(\mu)$ varies in such a way that the larger values of $\tau(\mu)$ are found away from the equator, toward μ_H, we can show that

$$c(\tau(\mu)) < c(\infty). \quad (34)$$

If (34) were not true, the left-hand side of Eq. (26) would have been negative, a contradiction. For $c < c(\infty)$, (22) can be satisfied only if

$$\mu_H(\tau) > \mu_H(\infty) \quad (35)$$

(see the graphical solution in Fig. 3). Thus we have shown that the extent of the Hadley circulation is bounded by

$$\mu_H(\infty) < \mu_H(\tau) < \mu_H(0). \quad (36)$$

This is true for a uniform τ as well as for a variable $\tau(\mu)$, provided that $\tau(\mu)$ does not decrease with latitude in the subtropics. Furthermore, since the upper and lower limits have been found to be within two degrees of each other, we conclude that the extent of the Hadley circulation is insensitive to $\tau(\mu)$. The simple, explicit, formula (25) for $\mu_H(0)$ can be used as an approximate formula for μ_H in general.

It should be pointed out that the statement that the upper and lower bounds on μ_H differ from each other by less than two degrees of latitude depends on the particular profile of equilibrium temperature we have chosen. While that smooth profile may be appropriate for a dry radiative equilibrium temperature, in the presence of moisture Θ_E should be considered as the convective–radiative equilibrium, and so Θ_E can be very different in regions of moist convection as compared with the dry regions. Even for this more general case, we still have the conclusion (36) except that the bounds do not necessarily differ from each other by two degrees of latitude. The quantitative values need to be calculated case by case, but this is a simple task since these two limits, $\mu_H(0)$ and $\mu_H(\infty)$, can be calculated easily using the procedure outlined above.

b. *The effect of τ on the strength*

In contrast to the insensitivity of the extent of the Hadley circulation to the distribution of the relaxation time, it is found that the strength of the circulation is strongly dependent on τ. It is found that the strength of the circulation is inversely proportional to τ in the case of uniform τ, in which the deviation of the temperature from its equilibrium value was found not to change with τ. This turns out not to be generally true for latitudinally varying $\tau(\mu)$. The change of τ in a very narrow region could have a strong impact on the temperature and hence on the strength of the circulation.

Based on our work in Fang and Tung (1996) and Fang (1995), where explicit solutions were obtained for various piecewise constant τ profiles, the following conclusions can be inferred. As long as $\tau(\mu)$ in the whole circulation region is longer than the timescale of transport of the large-scale circulation (i.e., $\tau \gg 1$), the temperature in the Hadley circulation will be determined hydrostatically from the zonal angular momentum transport by the Hadley circulation. In this case the temperature distribution will then be independent of τ. Hence the strength of the Hadley circulation will be inversely proportional to τ_0, same as discussed previously for the uniform τ case.

If somewhere in the circulation region τ attains a value much smaller than the dynamical transport time (i.e., $\tau(\mu_1) \ll 1$ for some μ_1) as in the case of local moist convection, the temperature locally will be determined by this faster thermal relaxation to the local moist radiative–convective equilibrium value. If outside this region τ values are still large, the Hadley circulation will still attempt to homogenize the temperature horizontally, as discussed in Fang and Tung (1996) for the general case. The strength of the circulation is still given approximately by the negative of the temperature deviation from the radiative equilibrium temperature divided by τ. However, the temperature achieved is influenced strongly by the equilibrium distribution in the region of small τ (i.e., at μ_1). Aside from this complication, the strength of the circulation still scales approximately as $1/\tau$ in the dry regions.

The reader is referred to Fang (1995) for the quantitative results for various piece-wise constant τ cases treated.

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