An Assessment of the Balance Approximation in Hurricanes

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ABSTRACT

The validity of the traditional balance approximation for the asymmetric flow above the boundary layer generally in hurricanes is examined here. Scaling considerations of the divergence equation show that the validity of the balance approximation hinges on the smallness of the nondimensional product $(\delta_n'/\zeta_n') \times (n\overline{v}/\overline{\eta}r)$. The first term represents the ratio of asymmetric horizontal divergence to asymmetric vertical vorticity for azimuthal wavenumber n, while the second term represents a Rossby number based upon the azimuthal mean tangential wind and absolute vertical vorticity of the hurricane vortex. Wind observations of Hurricane Gloria (1985) indicate that this product is not at all small in the near-vortex region (several hundred kilometers beyond the radius of maximum tangential winds) where asymmetric convergence forced by surface friction and cumulus convection is typically large. Although the Gloria observations represent only a single case, there are dynamical reasons to expect this product to be $O(1)$ just above the hurricane boundary layer in steadily translating hurricanes. The meteorological relevance of these results to the problem of balance dynamics in hurricanes is briefly discussed.

1. Introduction

Although primitive equation (PE) models will always be the most accurate, approximate balance models still serve a useful purpose in geophysical fluid dynamic (GFD) studies. Whether they are used to diagnose PE models (Möller and Jones 1998; Davis et al. 1996) or perform idealized simulations (Montgomery and Kallenbach 1997), balance models often provide fundamental insight into the dynamics of geophysical vortex flows. Previous work (Shapiro and Montgomery 1993, hereafter SM) proposed an asymmetric balance (AB) theory for rapidly rotating geophysical vortices, such as hurricanes. Unlike other balance models, the AB formulation is the only one that is formally valid when the Rossby number is large and the divergence is not small. Observations, however, have generally been inadequate to demonstrate the meteorological need for such a balance formulation and only recently has a dataset blending the vortex core and environmental flow of an intense hurricane become available for detailed analysis (Franklin et al. 1993, hereafter FLFM).

The physical basis and range of validity of the AB formulation in hurricane vortices was discussed in SM, and its dynamical consistency for linear dynamics was demonstrated by Kallenbach and Montgomery (1995) for asymmetric disturbances on a stable hurricane-like vortex in shallow water. The weakly nonlinear dynamics of the shallow water AB formulation has recently been investigated for azimuthal wavenumbers zero through four and the formulation has been shown to yield quantitatively similar results to the PE (Möller and Montgomery 1998, manuscript submitted to *J. Atmos. Sci.*). The more commonly used balance models, such as the nonlinear balance equations (BE: McWilliams 1985; Charney 1973) or the semibalance equations (SB; Raymond 1992), have received comparatively little scrutiny at the smaller mesoscales found within the near-vortex region of hurricanes. Upcoming work will compare the AB, BE, and SB models against the primitive equations in hurricane-like flows. But before embarking on this journey, it is first necessary to have a clear understanding of the regime of formal validity of these latter two balance models in hurricanes. Although the BE and SB models are popular intermediate models and are also believed useful for rapidly rotating vortices, a clear justification of the balance approximation underlying both formulations in hurricane vortices has been lacking. Beginning with a linearized PE model, section 2 presents a scale analysis that clarifies the conditions under which the balance approximation is and is not formally valid

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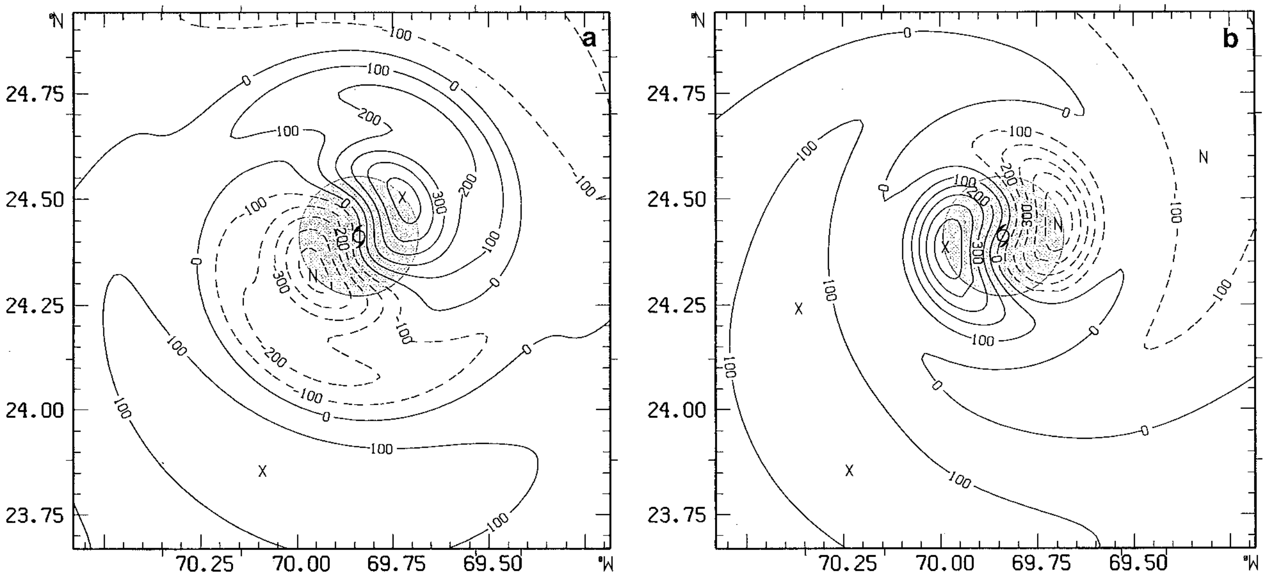

FIG. 1. Wavenumber-one component of (a) relative vorticity ζ_1' and (b) horizontal divergence δ_1' for the Doppler region (meshes 1–3 of FLFM) of Hurricane Gloria at a height of 700 mb. Maximum value of (a) and (b) is approximately 500×10^{-6} s^{-1}. Stippling indicates data-sparse region inside the eyewall within 13 km of the center. The contour interval is 100×10^{-6} s^{-1}.

above the boundary layer in hurricanes. Section 3 augments the scale analysis with new kinematic analyses of Hurricane Gloria based on the FLFM dataset. Section 4 concludes with a discussion of the relevance of the key results to hurricane dynamics.

2. A priori scaling

Above the boundary layer and within 500 km from its center, a hurricane may be approximated as a symmetrically stable, circular vortex in hydrostatic and gradient balance, plus small-amplitude asymmetries (SM, Fig. 1). As a zeroth-order model of the asymmetric flow above the boundary layer we therefore consider the inviscid linearized primitive equations on a circular baroclinic vortex in gradient balance in cylindrical coordinates (radius r, azimuth λ) using pseudoheight (z) as the vertical coordinate. Whether the ideas developed below are generalizable to a nonlinear regime comprising episodes of extensive vorticity mixing (Schubert et al. 1998, manuscript submitted to *J. Atmos. Sci.*) is an open question.

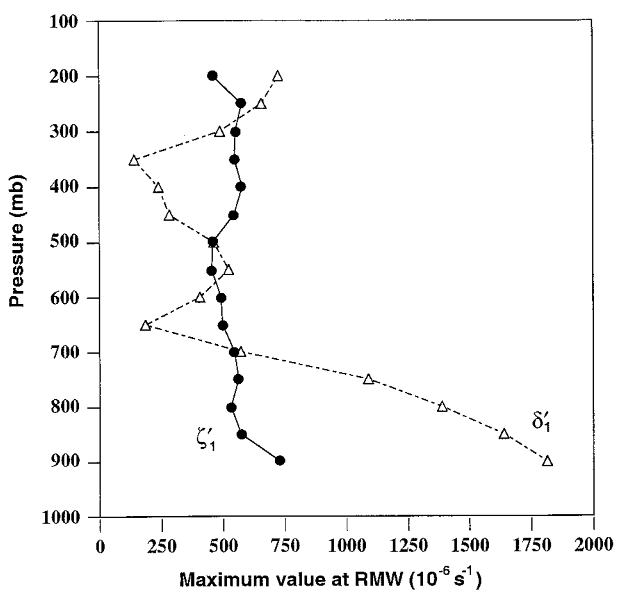

FIG. 2. Wavenumber-one amplitude of relative vorticity ζ_1' and horizontal divergence δ_1' at the RMW of Hurricane Gloria.

Denoting the substantial derivative following the mean circular vortex as $D_V/Dt = \partial/\partial t + (v/r)(\partial/\partial\lambda)$ the linearized f-plane momentum, continuity, and thermodynamic equations are, respectively,

$$\frac{D_V}{Dt}u' - \overline{\xi}v' = -\frac{\partial\phi'}{\partial r}, \qquad (2.1)$$

$$\frac{D_V}{Dt}v' + w'\frac{\partial\overline{v}}{\partial z} + \overline{\eta}u' = -\frac{\partial\phi'}{r\partial\lambda}, \qquad (2.2)$$

$$\frac{g}{\theta_0}\theta' = \frac{\partial\phi'}{\partial z}, \qquad (2.3)$$

$$\frac{\partial}{r\partial r}(ru') + \frac{\partial v'}{r\partial\lambda} + \frac{\partial}{\rho\partial z}(\rho w') = 0, \qquad (2.4)$$

$$\frac{D_V}{Dt}\theta' + \frac{g}{\theta_0}\overline{\xi}\frac{\partial\overline{v}}{\partial z}u' + \frac{\theta_0}{g}N^2 w' = Q'. \qquad (2.5)$$

In (2.1)–(2.5), ϕ' denotes the perturbation geopotential incorporating the motion of the vortex (SM); (u', v', w') denote the perturbation radial, tangential, and ver-

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tical velocity in storm-centered coordinates, respectively; θ' the perturbation potential temperature; $\overline{v} = \overline{v}(r, z)$ the basic-state tangential wind; $N^2 = (g/\theta_0)(\partial\overline{\theta}/\partial z)$ the basic-state static stability; Q' the perturbation heating rate; ρ the pseudodensity; θ_0 a reference potential temperature (300 K); g the gravitational acceleration; $\overline{\xi} = f + 2\overline{v}/r$ the modified Coriolis parameter; and $\overline{\eta} = f + \partial(r\overline{v})/r\partial r$ the basic-state absolute vertical vorticity. In this idealized model, boundary layer processes that force an interior response are incorporated into the boundary condition at $z = 0$. Explicit formulas for the lower boundary condition have been provided elsewhere, invoking the gradient balance approximation (e.g., Ooyama 1969) or retaining full radial accelerations in the boundary layer (Shapiro 1983).

Because the observations of section 3 are most comprehensive in the mid- to lower troposphere, we limit our subsequent discussion to the lower half of the troposphere where $\partial\overline{v}/\partial z$ in (2.2), (2.5) can be neglected in the first approximation. Forming $\partial[r(2.1)]/r\partial r + \partial[2.2]/r\partial\lambda$ then gives the barotropic divergence equation:

$$\frac{D_V}{Dt}\delta' - \frac{1}{r}\frac{\partial}{\partial r}(r\overline{\xi}v') + \frac{\overline{\eta}}{r}\frac{\partial u'}{\partial\lambda} + \frac{d\overline{\Omega}}{dr}\frac{\partial u'}{\partial\lambda} = -\nabla^2\phi'. \tag{2.6}$$

Here $\delta' = \partial(ru')/r\partial r + \partial v'/r\partial\lambda$ is the perturbation divergence, ∇^2 is the horizontal Laplacian, and $\overline{\Omega} = \overline{v}/r$ is the mean angular velocity.

The underlying assumption of BE and SB is that the horizontal divergence is small compared to the vertical vorticity. If one assumes the existence of a single horizontal length scale L and a single vertical length scale H, which characterize the flow within the near-vortex region, then the BE are justified when both the aspect ratio squared H^2/L^2 and the Froude number squared $F^2 = V^2/N^2H^2$ are small compared to unity (McWilliams 1985). Here (V, N^2) denotes a characteristic horizontal velocity and a characteristic static stability, respectively. In the near-vortex region of a hurricane, $H \approx$ scale height, $L \approx$ radius of maximum tangential winds (RMW), and $V \approx \overline{v}_{\rm max}$. Although the first requirement, $H^2/L^2 \ll 1$, is generally met in hurricanes, we will see that the second requirement, $F^2 \ll 1$, is generally insufficient to justify the balance approximation in hurricanes.

In the context of (2.6), the *balance approximation* neglects $D_V\delta'/Dt$ and approximates (u', v') by the rotational wind. The approximate divergence equation is then a two-dimensional Poisson equation for ϕ' given the rotational winds. This limiting equation is called the *balance equation*. For the approximation to be valid $D_V\delta'/Dt$ must be small compared to the remaining terms in (2.6). For exactly nondivergent flow, $\delta' = 0$, and thus $D_V\delta'/Dt$ is trivially zero. For weakly nondivergent flow, we expect δ' to be small. Specifically, we expect

$$\delta' \ll \zeta', \tag{2.7}$$

where $\zeta' = \partial(rv')/r\partial r - \partial u'/r\partial\lambda$ is the perturbation vertical vorticity. In such circumstances $D_V\delta'/Dt$ will remain small compared to the other terms in (2.6). Explicitly, for advective dynamics $D_V\delta'/Dt \sim \delta'_n n\overline{v}/r$, where n is the azimuthal wavenumber and δ'_n is the wavenumber-n component of δ'. Taking $\overline{\eta}\partial u'/r\partial\lambda \sim \overline{\eta}\zeta'$ as a typical term on the left-hand side of (2.6), the ratio of $D_V\delta'/Dt$ to $\overline{\eta}\zeta'$ then scales as

$$\frac{\delta'_n}{\zeta'_n}\frac{n\overline{v}}{\overline{\eta}r}, \tag{2.8}$$

where ζ'_n denotes the wavenumber-n component of ζ'. Therefore, when (2.7) is satisfied and when $n\overline{v}/\overline{\eta}r$ is not large, the neglect of $D_V\delta'/Dt$ is justified.[1] In these circumstances the balance approximation serves as an accurate zeroth-order approximation relating the asymmetric height and wind fields in the near-vortex region of hurricanes.

On the other hand, when $\delta' \sim \zeta'$ on advective timescales, the neglect of $D_V\delta'/Dt$ is no longer justified a priori. Section 3 presents observations that suggest that $\delta' \sim \zeta'$ throughout the near-vortex region of hurricanes. Anticipating that $\delta'_n \sim \zeta'_n$ on advective timecales, the ratio of $D_V\delta'/Dt$ to $\overline{\eta}\zeta'$ then scales as

$$\frac{n\overline{v}}{\overline{\eta}r}, \tag{2.9}$$

a Rossby number based upon the azimuthal-mean tangential wind and absolute vertical vorticity of the vortex. In intense hurricanes the anticyclonic shear vorticity nearly cancels the curvature vorticity outside the RMW. Consequently, in such regions $n\overline{v}/\overline{\eta}r$ is $O(1)$. An illustrative example for which these considerations are relevant corresponds to the *stationary* asymmetric flow forced by surface friction in a steadily translating hurricane (Shapiro 1983). This asymmetric flow can be interpreted as a stationary wave response to surface friction in a steadily translating vortex.

Shapiro's analysis is relevant in two respects. First, unlike the naive GFD scaling, which assumes $\delta' \sim F^2\zeta'$ throughout the entire fluid, his calculation shows that δ' near the top of the boundary layer in a translating hurricane scales with the asymmetric frictional stress. Second, from his Fig. 5 one can infer that $\delta' \sim \zeta'$ outside the RMW. Although asymmetric vorticity was not among the fields reported in Shapiro's analysis, he has provided the numerical output of a representative boundary layer calculation by way of personal communication. For the case provided, the vortex's maximum azimuthal-mean tangential velocity is 61.8 m s^{-1},

[1] In the special event that ζ' is initially zero (e.g., a frictionless symmetric vortex suddenly subject to vertical shear) the scaling (2.8) breaks down. In this case $\overline{\eta}\partial u'/r\partial\lambda$ is more accurately represented as $\overline{\eta}nu'_n/r$ and (2.8) is replaced by $(\delta'_n r/nu'_n)\times(n\overline{v}/\overline{\eta}r)$, which is finite. The neglect of $D_V\delta'/Dt$ is then justified when $n\overline{v}/\overline{\eta}r$ is not large provided $\delta'_n \ll nu'_n/r$.

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which occurs at a RMW of 35 km. The vortex's translation speed is 10 m s^{-1}. The asymmetric convergence and relative vorticity are found to be typically dominated by the azimuthal wavenumber-one component, but the wavenumber-two component is not insignificant outside the RMW, where it contributes almost equally to the asymmetric convergence. The maximum amplitude of the wavenumber-one convergence is found to occur just inside the RMW with a value of 31.7 $\times$ 10^{-4} s^{-1}. In contrast, the maximum relative vorticity for wavenumber one occurs at the RMW with a value of 24 $\times$ 10^{-4} s^{-1}. The amplitude of asymmetric convergence is furthermore found to be of the same order or greater than the amplitude of asymmetric vorticity as far out as the vortex environment ($\sim$500 km).

While Shapiro's calculations only predict asymmetric convergence and vorticity *at the top* of the hurricane boundary layer, they nevertheless provide useful insight into the strength and structure of asymmetric forcing underneath a translating hurricane, which is ultimately coupled to the interior flow via cumulus convection. To the extent that convective processes in a rapidly rotating vortex project onto the slow manifold (Schubert et al. 1980; Ooyama 1982; Montgomery and Kallenbach 1997), we should not expect the naive scaling $\delta' \sim F^2\zeta'$ to be universally valid. Thus in the event that $\delta' \sim \zeta'$ in a horizontally and vertically widespread region of the vortex, an altogether different balance approximation permitting order one asymmetric divergence in its zeroth-order approximation is desirable. Asymmetric balance theory was developed with such applications in mind (SM, section 6).

3. Observations

FLFM described the kinematic structure of Hurricane Gloria as determined from nested analyses of Omega dropwindsonde (ODW) and airborne Doppler radar data. The data were obtained during a ‘‘synoptic-flow’’ experiment conducted by the Hurricane Research Division (HRD) of the Atlantic Oceanographic and Meteorological Laboratory/National Oceanic and Atmospheric Administration (AOML/NOAA) using two NOAA WP-3D research aircraft. The nested multiscale analyses simultaneously describe Gloria's eyewall and synoptic-scale features. The combination of environmental and vortex core observations is the most comprehensive kinematic dataset obtained in a hurricane to date. Sections 2 and 3 of FLFM thoroughly discuss the data, analysis algorithm, and methodology for the Gloria analyses. Readers unfamiliar with the analyses may wish to refer to the discussion in FLFM or the appendix of Shapiro and Franklin (1995), which gives a brief overview.

Figure 1 shows the azimuthal wavenumber-one contribution to the perturbation relative vorticity (ζ_1') and perturbation divergence (δ_1') of Hurricane Gloria at 700 mb. The wavenumber-one vorticity is nearly an order

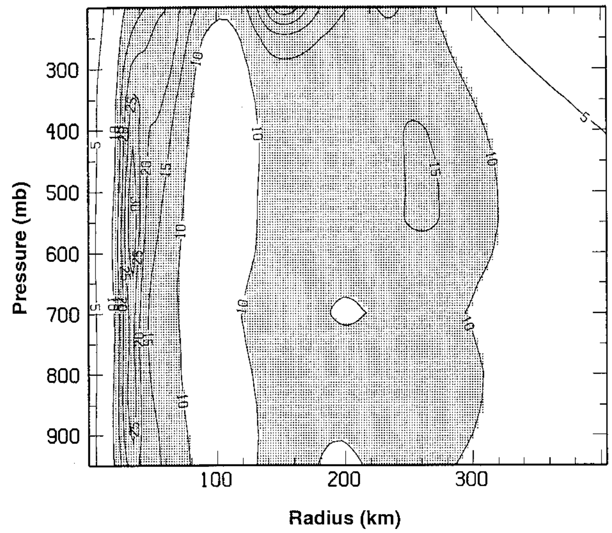

FIG. 3. Radius–height plot of $n\overline{v}/\overline{\eta}r \times 10$ for $n = 1$ in the core and mean environment of Hurricane Gloria corresponding to the time of Figs. 1 and 2. Regions where $n\overline{v}/\overline{\eta}r > 1$ are indicated by stippling.

of magnitude smaller than the total absolute vorticity (not shown). In contrast, the total divergence is dominated by the wavenumber-one contribution. The contribution from higher azimuthal wavenumbers has also been examined and the results suggest that the wavenumber-one component is dominant. The features depicted in Fig. 1 were observed by airborne radar to be long lived (FLFM, 2448), ruling out the possibility that they are merely convective transients. The most noteworthy feature of Fig. 1 is that the wavenumber-one divergence and vorticity possess similar magnitudes throughout the near-vortex region, that is, $\delta_1' \sim \zeta_1'$. The reason for their similarity is thought to lie with the asymmetric divergence structure associated with a convective band near Gloria's center (FLFM, Fig. 12). As further confirmation of the robustness of this result, Fig. 2 plots the maximum magnitude of the wavenumber-one vorticity and divergence at the RMW ($\sim$18 km) as a function of height (pressure). Figure 2 indicates that δ_1' is greater than or equal to ζ_1' above the boundary layer, in the midtroposphere, and in the upper troposphere. As a means of verifying that such behavior is not peculiar to the RMW, the δ_1' and ζ_1' fields at the 850-mb level have also been examined (not shown), confirming that $\delta_1' \sim \zeta_1'$ throughout the near-vortex region (meshes 1–3 of FLFM).

Section 2 showed that the neglect of the divergence tendency following the mean vortex in the divergence equation generally hinged upon the smallness of the nondimensional product $(\delta_n'/\zeta_n') \times (n\overline{v}/\overline{\eta}r)$. The above results show that the first term in this product, δ_n'/ζ_n', is of order unity within the entire near-vortex region. Figure 3 displays the second term in this product, $n\overline{v}/\overline{\eta}r$ for $n = 1$, in Hurricane Gloria. The $\overline{v}$ and $\overline{\eta}$ fields used

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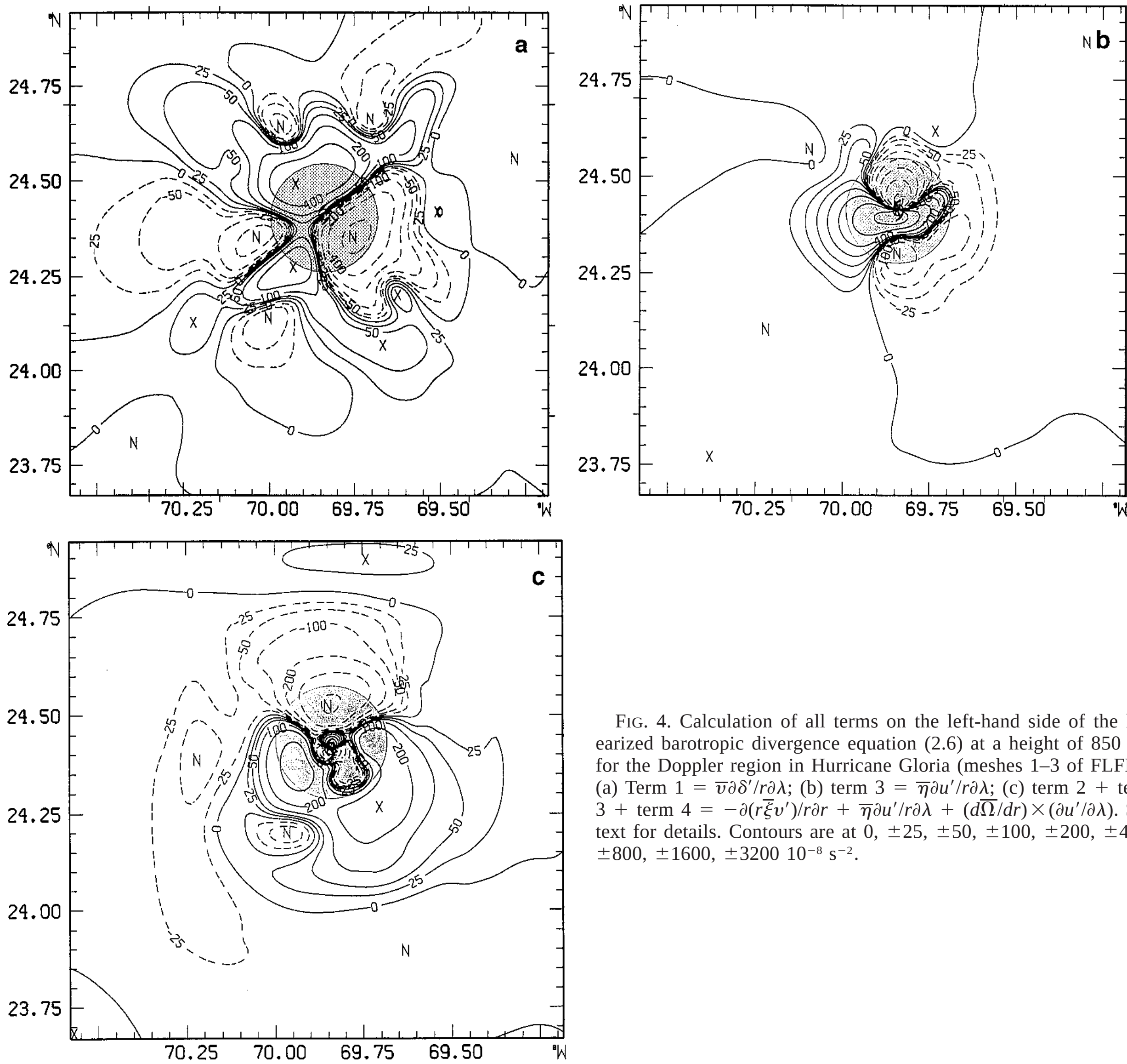

FIG. 4. Calculation of all terms on the left-hand side of the linearized barotropic divergence equation (2.6) at a height of 850 mb for the Doppler region in Hurricane Gloria (meshes 1–3 of FLFM). (a) Term 1 = $\overline{v}\partial\delta'/r\partial\lambda$; (b) term 3 = $\overline{\eta}\partial u'/r\partial\lambda$; (c) term 2 + term 3 + term 4 = $-\partial(r\overline{\xi}v')/r\partial r + \overline{\eta}\partial u'/r\partial\lambda + (d\overline{\Omega}/dr)\times(\partial u'/\partial\lambda)$. See text for details. Contours are at 0, ±25, ±50, ±100, ±200, ±400, ±800, ±1600, ±3200 10^{-8} s^{-2}.

in this plot correspond to the observations shown in Figs. 1 and 2. It is evident that this Rossby number is near unity or greater for several hundred kilometers outside the RMW. This behavior is quite unlike the distribution of the *local* Rossby number squared for azimuthal wavenumber one, R_1^2, of AB theory (SM, Fig. 3), which is less than unity throughout the hurricane vortex except in a very narrow region just outside the RMW. Since the radial scale of the region in which $R_1^2 > 1$ is smaller than the local Rossby deformation radius for disturbances forced by convection, its effect tends to be mitigated by the elliptic operator governing the balanced motion. No such argument can be made for $n\overline{v}/\overline{\eta}r$, however, since it is near unity or greater over a much more extensive region (radius ~300 km) of the hurricane.

For our final calculation, we compute each term of the left-hand side of the divergence equation (2.6) at the 850-mb and 700-mb levels. Although azimuthal wavenumber one tends to dominate the asymmetric structure, each term is evaluated with the total asymmetric fields. We evaluate $\overline{v}\partial\delta'/r\partial\lambda$ as a proxy for $D_v\delta'/Dt$ and define this as term 1. If the asymmetric flow were stationary in the moving system this would be exact. Although a reliable estimate for $\partial\delta'/\partial t$ is not available, we have no a priori reason to expect it to be significantly compensated by azimuthal advection. Term 2 is defined as $-\partial(r\overline{\xi}v')/r\partial r$, term 3 is defined as $\overline{\eta}\partial u'/r\partial\lambda$, and term 4 is defined as $(d\overline{\Omega}/dr)\times(\partial u'/\partial\lambda)$. Figures 4 and 5 summarize the results of these calculations at the 850-mb and 700-mb levels, respectively. From Fig. 4 we find that within the near-core region of the vortex

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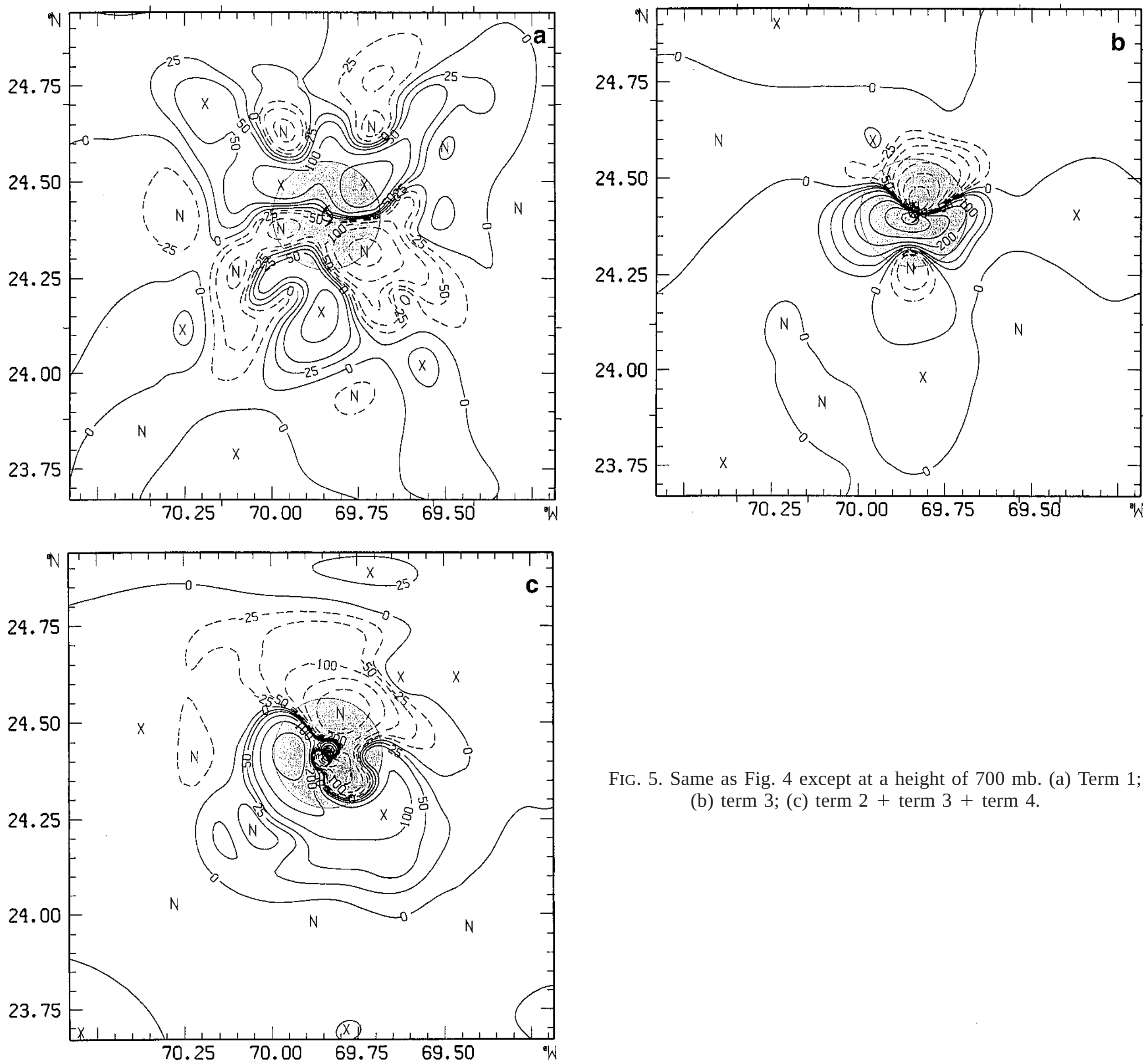

FIG. 5. Same as Fig. 4 except at a height of 700 mb. (a) Term 1; (b) term 3; (c) term 2 + term 3 + term 4.

(several RMW units) the divergence tendency is generally not small compared to either term 3 or the sum of terms 2, 3, and 4, but is of the same magnitude. Figure 5 shows similar results.

These are noteworthy results because, although the near-vortex asymmetries have previously been observed to have small magnitudes in comparison to the mean vortex (SM, Fig. 1), these observations are the first to suggest that the asymmetries are not rotationally dominated in the sense that $\delta' \ll \zeta'$.

4. Discussion and conclusions

Although the balance systems of McWilliams (1985) and Raymond (1992) are popular intermediate balance models for GFD studies of stably stratified rotating flow, a formal justification of the balance equation underlying both systems when applied to the asymmetric flow in hurricanes has been lacking. This work fills this void by developing a scale analysis for linearized dynamics indicating when the balance equation is and is not valid in hurricane-like vortices. In the event that the asymmetric flow is rotationally dominated, the balance equation is shown to serve as a useful first approximation connecting the asymmetric rotational winds to the asymmetric height field. On the other hand, if the asymmetric flow evolving on the advective timescale is not rotationally dominated and the Rossby number based on the mean tangential wind and absolute vertical vorticity of the vortex is of order unity, the balance equation is shown to be formally invalid. In this case the balanced asym-

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metric height field cannot be obtained accurately by solving just the balance equation given the rotational winds. As with other physical approximations encountered in mathematical physics and GFD, it may turn out in practice that the balance equation gives qualitatively correct results in circumstances where it ought not be based on criteria for formal validity (e.g., Spall and McWilliams 1992). But this has yet to be demonstrated in hurricane vortices.

Boundary layer considerations of a translating hurricane forced by surface friction at the air–sea interface, as well as observations of Hurricane Gloria (1985), indicate that hurricane asymmetries are not rotationally dominated and the balance equation for the asymmetric flow is invalid in the near-vortex region. In such circumstances the alternative AB formulation appears naturally suited, at least for small but finite amplitude asymmetric disturbances on a circular vortex. These theoretical considerations are motivated in part by the desire to diagnose the influence of three-dimensional vortex Rossby waves on the hurricane vortex (Montgomery and Kallenbach 1997). For in order to assess their attendant eddy-heat and eddy-momentum flux divergences in the dynamics of the hurricane it is necessary to devise an accurate method for inferring the balanced asymmetric temperature field that contains a minimal gravity–inertia wave component. To the extent that balanced dynamics captures the essence of the hurricane this is inextricably linked to the determination of the balanced asymmetric height field.

As a practical matter these ideas raise the following question: What is the ‘‘best’’ manner to extract the balanced asymmetric height field in hurricanes from the wind data obtained from Doppler radar? One method proceeds by first solving the balance equation to deduce a zeroth-order guess for the height (e.g., Shapiro and Franklin 1995). Improvements would then be obtained in an iterative fashion upon inverting the BE omega and continuity equations and inserting the results back into the divergence equation to yield improvements to the balanced height. This procedure would be carried out to convergence. As long as the divergence tendency is small on advective timescales, this method should converge. In hurricanes, however, we have argued that the asymmetric divergence and asymmetric divergency tendency cannot generally be regarded as small on advective timescales, so an alternative procedure may be necessary. In such circumstances one would begin as before by solving the balance equation to obtain a zeroth-order guess for the asymmetric height. The geopotential evolution equation of the AB formulation [Eq. (3.10) of SM] would then be solved for the asymmetric height tendency as in Möller and Jones (1998). The height tendency would next be inserted into the AB divergence equation

$$\frac{D_V}{Dt}\delta' - \frac{1}{r}\frac{\partial}{\partial r}(r\overline{\xi}v') + \overline{\eta}\frac{\partial u'}{r\partial\lambda} + \frac{\partial\overline{\Omega}}{\partial r}\frac{\partial u'_\eta}{\partial\lambda} = -\nabla^2\phi'$$

to obtain an improvement to the asymmetric height; this new height would then be inserted back into the AB tendency equation and the iteration process would be carried out to convergence. The AB divergence equation is analogous to the linearized divergence equation of the PE [cf. (2.6)]. Here $\delta' = \partial(ru'_\eta)/r\partial r + \partial v'_\xi/r\partial\lambda$ denotes a pseudodivergence based on the pseudomomenta $u'_\eta = (-1/\overline{\eta}r)\times(\partial\phi'/\partial\lambda)$ and $v'_\xi = (1/\overline{\xi})\times(\partial\phi'/\partial r)$ of AB theory, $\partial\overline{v}/\partial z$ has been neglected for simplicity, and u' and v' denote the asymmetric radial and tangential winds in storm-centered coordinates inferred from Doppler radar. Unlike the former scheme whose convergence requires a small divergence tendency, the AB formulation permits an $O(1)$ divergence tendency but requires that the square of the local Rossby number (R_n^2) for wavenumber n be small compared to unity. In hurricanes, R_n^2 can only be considered small for azimuthal wavenumber one. In fact, in full-fledged hurricanes $R_1^2 \sim 0.5$ throughout the bulk of the near-vortex region (SM, Fig. 3). An apparent advantage of extracting the balanced height field using the AB theory as opposed to the balance equation, however, is that R_1^2 typically only exceeds unity in a localized region just outside the RMW, while the relevant small parameter for the balance equation, $(\delta'_n/\zeta'_n)\times(n\overline{v}/\overline{\eta}r)$, has been shown to generally exceed unity in a much more extensive region of the vortex. It stands to reason that the height field extracted with the balance equation may suffer more widespread distortions than the AB formulation. It remains to be determined, however, which formulation is superior in practical situations. Further examination of this problem will be reported in due course.

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