# 3D Radiative Transfer in Weakly Inhomogeneous Medium. Part I: Diffusive Approximation

V. L. GALINSKY AND V. RAMANATHAN

*Center for Atmospheric Sciences, Center for Clouds, Chemistry and Climate, Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California*

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ABSTRACT

The solution of the three-dimensional radiative transfer equation in weakly horizontally inhomogeneous medium has been obtained in the diffusion approximation using the expansion of the three-dimensional delta-Eddington approximation. The solution approach, referred as the gradient correction (GC) method, expands the horizontal fluxes and the source function in terms of the horizontal gradient of the extinction coefficient and/or the cloud-top boundary. In the transfer equation, only the zeroth- and first-order gradient terms are retained and hence the following limitations apply. First, the length of the horizontal variations of optical properties of the medium should be large in comparison to the mean radiative transport length. Second, the ratio of the vertical to horizontal scales should be small enough so that fluxes from boundaries may be neglected.

Since there are no restrictions to the amplitude of the optical properties variations, this method may even be applicable to a medium with strong horizontal variations of optical properties, as long as scales of the variations are large enough in comparison to the radiative transport length. The analytical solutions are in excellent agreement with the more accurate numerical solutions. The solution also shows the solar zenith angle dependence of the albedo, similar to that observed in analyses of satellite imagery.

The GC approach may be useful as a fast and computationally inexpensive method both for the correction of the independent pixel approximation used for extraction of cloud fields from satellite imagery and possibly for the calculation of the radiation fluxes in climate models.

## 1. Introduction

The transfer of solar radiation through the earth's atmosphere plays a major role in determining climate and climate change. Many field measurements and studies based on global circulation models (e.g., Ramanathan 1987; Liou 1992) suggest that large uncertainties exist in our understanding of the radiation transfer through cloudy atmosphere. On the other hand, the retrieval of cloud properties from satellite and aircraft images requires a correct description of the interaction of solar and infrared radiation with an inhomogeneous, both horizontally and vertically, cloud field.

The ''bulletproof'' approach to this problem is well known and requires the solution of linear integro-differential equation with variable coefficients, namely, the radiative transfer equation (RTE). Many methods exist for solving the RTE in two and three dimensions, including the most commonly used Monte Carlo simulations, the interaction principle in three dimensions (Stephens 1988), the Fourier–Riccati method (Gabriel et al. 1993), the spherical harmonic spatial grid method (Evans 1993), and the spherical harmonics discrete ordinate method (Evans 1998). The necessity to specify detailed cloud geometry in three dimensions as well as intracloud variability on various scales makes these approaches computationally extensive and practically unusable both for GCMs and for the cloud retrieval.

Mainly due to the above-mentioned complexity of the RTE solution techniques for inhomogeneous medium, rather than relying on scientifically proven justifications, the two widespread methods used in GCMs and in the satellite imagery are based on the plane-parallel treatment of the radiative transfer in clouds.

The first method, used mainly in GCMs, is the plane-parallel approximation method (PPA) (Fig. 1), where the inhomogeneous cloud field is approximated by two parameters, the mean optical thickness and the fraction of cloud cover. In the PPA calculations the mean optical depth yields the values of the vertical plane-parallel radiative fluxes in the atmosphere and the domain averaging is subsequently performed by simply weighting the fluxes with the cloud fraction. In this procedure the divergences of the horizontal radiative fluxes are neglected. Furthermore, the domain averaging with probability distribution function (PDF) of the cloud field is

*Corresponding author address:* Dr. Vitaly Galinsky, Center for Atmospheric Sciences, Center for Clouds, Chemistry and Climate, Scripps Institution of Oceanography, University of California, San Diego, 9500 Gilman Drive, MC # 0221, La Jolla, CA 92093-0221.
E-mail: vitaly@fiji.ucsd.edu

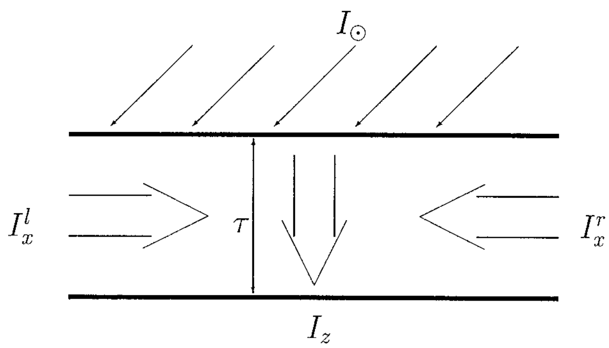


FIG. 1. Plane-parallel radiative transfer ($I_z = I_z(z)$, $I_x = I_x^l + I_x^r = 0$).

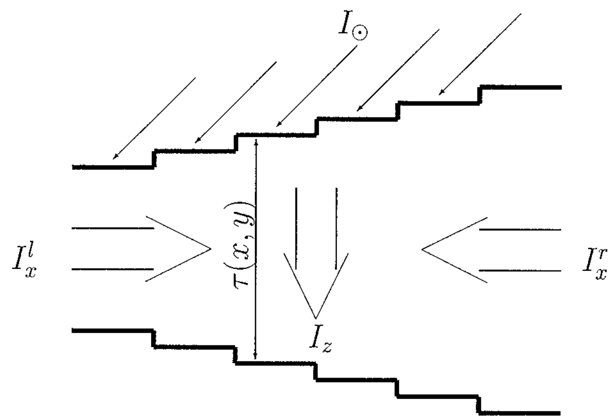


FIG. 2. Independent pixel radiative transfer ($I_z = I_z(x, z)$, $I_x = I_x^l + I_x^r = 0$).

replaced with the multiplication by the zeroth moment of the PDF, which is the fractional cloud cover.

The second method, used mainly for retrieval of optical properties from the satellite imagery, is the independent pixel approximation (IPA) method (Fig. 2). The IPA applies the plane-parallel assumptions to each individual pixel in the image and solves the radiative transfer equation for each pixel independently. This method yields more accurate estimates of the domain-averaged radiative properties on scales larger than the scales of individual pixels (Cahalan et al. 1994a; Cahalan et al. 1994b), because it accounts for the basic horizontal variability of the cloud field. However, this method neglects the horizontal radiative fluxes on pixel and image scales in addition to neglecting the subpixel variability of optical properties.

Several attempts were made to incorporate the horizontal variability into plane-parallel computation in order to obtain a solution of the radiation transfer for three-dimensional geometry (Fig. 3) without diving into computationally expensive direct solution of the RTE. These approximate methods are based either on a perturbation expansion (Romanova 1975; Li et al. 1994; Romanova 1996; Li et al. 1995), which assumes the amplitude of the horizontal inhomogeneity to be small compared to the mean properties of the medium, or on source closure technique (Gabriel and Evans 1996), in which the plane-parallel (or independent pixel) calculations are modified by a more precise representation of the direct beam source term. Although the latter technique may be used for practical calculations in GCMs or in satellite imagery, the restriction of the small-amplitude perturbations of the horizontal variations and the necessity to solve three-dimensional Laplace equation limits the applicability of the former method.

The importance of horizontal flux divergences in the process of radiative transfer in marine stratocumulus clouds (which are, without any doubt, the best candidates for the application of the plane-parallel model) has been pointed out recently by several authors (Marshak et al. 1995; Davis et al. 1997a; Davis et al. 1997b; Loeb and Coakley 1997). These conclusions are based on the Monte Carlo simulations of the fractal model of the cloud field (Cahalan et al. 1994a), as well as on the analyses of the observational data. However, the Monte Carlo computations are too time consuming for satellite applications. We need a simplified analytical approach to the problem of 3D radiative transfer.

Toward this goal, we develop a technique that takes into account the horizontal variability of the optical and/or geometrical properties of the medium by means of a correction to the plane-parallel or independent pixel approximation methods. The correction depends only on the local gradients of the extinction coefficient or the height of the layer and, therefore, can be calculated almost as easily as the unmodified solution of the PPA/IPA method. The basic requirements for the validity of the gradient correction method are the following.

- The characteristic scale of horizontal variations should be large in comparison to the mean radiative transport length. The horizontal variations are treated as local disturbances, which modify the entire spatial and angular distribution of the radiative field.
- The horizontal extent of the medium should be larger than the vertical size, that is, the medium may rep-

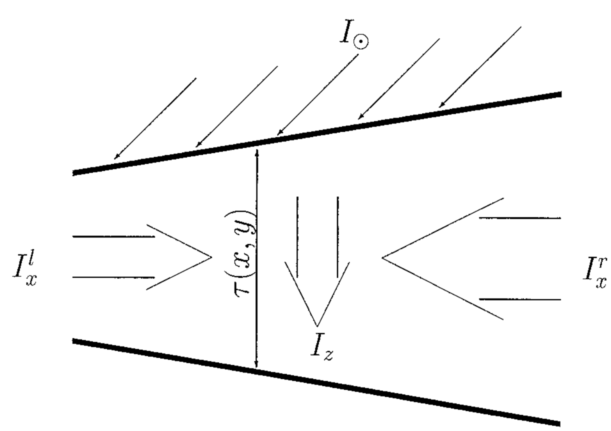


FIG. 3. 3D radiative transfer ($I_z = I_z(x, z)$, $I_x = I_x^l + I_x^r \neq 0$).

resent a horizontal cloudy layer with variations in internal optical properties and/or in geometrical height/position.

The paper is organized as follows. In section 2 a general formulation of the problem of radiation transfer in three-dimensional inhomogeneous medium subject to the diffusion approximation is presented. The formulation results in an equivalent one-dimensional equation, similar to the delta-Eddington approximation equation for the mean intensity of the radiative field but with additional terms due to horizontal fluxes. The gradient correction solutions for the layer with geometrical variations of the upper boundary and for the layer with internal variations of extinction coefficient are derived in sections 3 and 4, respectively, for the conservatively scattering medium. In section 5 the comparison of the above solutions with the results of numerical simulations of the three-dimensional RTE (spherical harmonics discrete ordinate method; Evans 1998) is made and the applications of the technique to the optical depth retrieval from satellite imagery are discussed.

## 2. Initial equations

We will use the diffusion approximation in our analysis and therefore will start from the delta-Eddington approximation in three dimensions [the details of derivation of this approximation from the three dimensional radiative transfer equation may be found elsewhere (Lenoble 1993; Li et al. 1994)]:

$$\nabla\cdot\mathbf{I} = -3k'(1 - \omega')I_0 + F_0^s \tag{1}$$

$$\nabla I_0 = -k'(1 - \omega'g')\mathbf{I} + \mathbf{F}^s, \tag{2}$$

where $\mathbf{I} = \{I_x, I_y, I_z\}$ and $I_0$ are the flux and averaged intensity, respectively; $F_0^s$ and $\mathbf{F}^s = \{F_x^s, F_y^s, F_z^s\}$ are the source terms, which may be expressed as

$$F_0^s = \frac{3k'\omega'}{4}F, \tag{3}$$

$$\mathbf{F}^s = \frac{3k'\omega'g'}{4}F\mathbf{\Omega}_0, \tag{4}$$

$F = F(x, y, z)$ and $\mathbf{\Omega}_0$ being the incident solar beam intensity and unit vector, respectively.

In scalar form these equations represent the standard notation for the three-dimensional delta-Eddington approximation

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = -3k'(1 - \omega')I_0 + \frac{3k'\omega'}{4}F, \tag{5}$$

$$\frac{\partial I_0}{\partial x} = -k'(1 - \omega'g')I_x + \frac{3k'\omega'g'}{4}\nu_0 \cos\phi_0 F, \tag{6}$$

$$\frac{\partial I_0}{\partial y} = -k'(1 - \omega'g')I_y + \frac{3k'\omega'g'}{4}\nu_0 \sin\phi_0 F, \tag{7}$$

$$\frac{\partial I_0}{\partial z} = -k'(1 - \omega'g')I_z + \frac{3k'\omega'g'}{4}\mu_0 F, \tag{8}$$

where $\mu_0$ and $\nu_0$ are the cosine and sine of the solar zenith angle, respectively, and $\phi_0$ is the solar azimuth angle ($\mathbf{\Omega}_0 = (n_x\nu_0, n_y\nu_0, \mu_0)$ and $\mathbf{n} = (n_x, n_y) = (\cos\phi_0, \sin\phi_0)$).

Variables with prime $k'$, $\omega'$, $g'$ represent the extinction coefficient, the single scattering albedo, and the first moment of the phase function with the correction due to the forward peek of the phase function (so-called delta approximation) included. The relationship between the delta corrected and the original quantities is given by the following expressions:

$$\begin{aligned} k' &= k(1 - \omega g^2) \\ \omega' &= \omega(1 - g^2)/(1 - \omega g^2) \\ g' &= g/(1 + g). \end{aligned} \tag{9}$$

Let us consider a weakly horizontally inhomogeneous medium. We will assume for simplicity that either the extinction coefficient $k'$ or position of the upper boundary of the layer $H_0$ is the slowly varying function of the horizontal coordinates $x$ and $y$ and that the single scattering albedo is a constant throughout the layer. We will also introduce a coordinate transform, both direct,

$$u = \alpha x, \qquad \upsilon = \alpha y, \qquad \tau = \mathcal{T}(x, y, z),$$

and inverse,

$$x = u/\alpha, \qquad y = \upsilon/\alpha, \qquad z = Z(u, \upsilon, \tau).$$

In order to compensate for the dependence of the horizontal fluxes on the asymmetry parameter $g'$, we scaled the $x$ and $y$ variables by a parameter, $\alpha$. We let $\alpha = 1 - g'$, because the horizontal flux divergences should depend on the fraction of the total radiation scattered out of the forward direction. In the diffusion approximation this fraction equals $1 - g'$. It also implies that the effects of the horizontal fluxes shall decrease as the forward-scattering peak of the phase function increases.

The basic rule of coordinate transformation can be written in this case as

$$\frac{\partial G}{\partial\{u, \upsilon, \tau\}} = \frac{\partial G}{\partial x}\frac{\partial x}{\partial\{u, \upsilon, \tau\}} + \frac{\partial G}{\partial y}\frac{\partial y}{\partial\{u, \upsilon, \tau\}} + \frac{\partial G}{\partial z}\frac{\partial z}{\partial\{u, \upsilon, \tau\}}. \tag{10}$$

Based on these expressions for the coordinate transformation we can rewrite the delta-Eddington approximation equations (5)–(8) in the following form:

$$\alpha\frac{\partial I_x}{\partial u}+\alpha\frac{\partial I_y}{\partial v}-\alpha\frac{\partial I_x}{\partial\tau}\frac{\partial z}{\partial u}\left(\frac{\partial z}{\partial\tau}\right)^{-1}-\alpha\frac{\partial I_y}{\partial\tau}\frac{\partial z}{\partial v}\left(\frac{\partial z}{\partial\tau}\right)^{-1}+\frac{\partial I_z}{\partial\tau}\left(\frac{\partial z}{\partial\tau}\right)^{-1}=-3k'(1-\omega')I_0+F_0^s, \qquad (11)$$

$$\alpha\frac{\partial I_0}{\partial u}-\alpha\frac{\partial I_0}{\partial\tau}\frac{\partial z}{\partial u}\left(\frac{\partial z}{\partial\tau}\right)^{-1}=-k'(1-\omega'g')I_x+F_x^s, \qquad (12)$$

$$\alpha\frac{\partial I_0}{\partial v}-\alpha\frac{\partial I_0}{\partial\tau}\frac{\partial z}{\partial v}\left(\frac{\partial z}{\partial\tau}\right)^{-1}=-k'(1-\omega'g')I_y+F_y^s, \qquad (13)$$

$$\frac{\partial I_0}{\partial\tau}\left(\frac{\partial z}{\partial\tau}\right)^{-1}=-k'(1-\omega'g')I_z+F_z^s. \qquad (14)$$

These equations are by no means simpler than the original ones. They are also three-dimensional and include the derivatives of coordinate transformation functions. In order to simplify them we invoke additional assumptions. First of all, suppose that the scales of horizontal variations of optical and geometrical properties of the medium are large, such that the medium is weakly inhomogeneous. The implication of this assumption is that deviations from the independent pixel approximation due to horizontal variations are not very big. In this case the coordinate transformation to the optical depth variable will include the horizontal variations as a parameter.

If, in addition, the overall horizontal scales of the medium are large, the horizontal boundary effects may be neglected. After eliminating $\partial I_x/\partial u$ and $\partial I_y/\partial v$ (or including them as next-order corrections), we are left with a one-dimensional problem instead of a three-dimensional one. We can then drop the derivative of the fluxes and intensity with respect to $u$ and $v$. The solution will differ from the independent pixel approximation solution, because it will include the horizontal flux corrections resulting from geometrical or optical inhomogeneity. Those corrections arise not only through the parametric dependence of $k'$ and/or $H_0$ on $x$ and $y$, but also through their derivatives. Formally this one-dimensional equation can be obtained if we assume that $I_0$ and $\mathbf{I}$ depend on $\tau$ only. Ideally, choosing the $\mathcal{T}$ to be a surface of constant total flux (both direct and diffuse) will guarantee this one-dimensionality, but finding the constant flux surface is equivalent to the solution of the original problem. Therefore, we can only hope that the new optical depth variables will give us a better approximation than the plane-parallel or independent pixel one.

After the above-mentioned elimination of $I_x$, $I_y$, and $I_z$ from the equations (11)–(14), the resulting one-dimensional equation may be written as

$$\begin{aligned}
&\frac{1}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial}{\partial\tau}\left[\frac{1}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial I_0}{\partial\tau}\right]-3(1-\omega')(1-\omega'g')I_0+\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\left(\frac{\partial z}{\partial u}\frac{\partial}{\partial\tau}\left[\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial z}{\partial u}\frac{\partial I_0}{\partial\tau}\right]+\frac{\partial z}{\partial v}\frac{\partial}{\partial\tau}\left[\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial z}{\partial v}\frac{\partial I_0}{\partial\tau}\right]\right)\\
&\quad-\frac{\alpha}{k'}\left(\frac{\partial}{\partial u}\left[\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial z}{\partial u}\right]+\frac{\partial}{\partial v}\left[\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial z}{\partial v}\right]\right)\frac{\partial I_0}{\partial\tau}\\
&\quad=-(1-\omega'g')\frac{F_0^s}{k'}+\frac{1}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\frac{\partial}{\partial\tau}\frac{F_z^s}{k'}-\frac{\alpha}{k'}\left(\frac{\partial z}{\partial\tau}\right)^{-1}\left(\frac{\partial z}{\partial u}\frac{\partial}{\partial\tau}\frac{F_x^s}{k'}+\frac{\partial z}{\partial v}\frac{\partial}{\partial\tau}\frac{F_y^s}{k'}\right).
\end{aligned} \qquad (15)$$

In order to use this equation the inverse transform function $Z$ should be specified. That is not a trivial task and in most cases it may be done only numerically. Let us consider, for example, the choice of $\mathcal{T}$ used in the plane-parallel or in the independent pixel computations,

$$\tau=\mathcal{T}(x,y,z)=\int_{H_0(x,y)}^{z}k'(x,y,z')\,dz'. \qquad (16)$$

This equation may only be inverted numerically in the general case of arbitrary functions $k'(x, y, z)$ and the shape of upper boundary denoted by $H_0(x, y)$.

The important point in the analysis of Eqs. (11)–(14) with the coordinate transform (16) is that one can check the validity of the independent pixel approximation quite easily. Indeed, the independent pixel approximation can be obtained by dropping all terms containing derivatives with respect to $u$ and $v$, and it can be justified only if the $(\partial z/\partial u)(\partial z/\partial\tau)^{-1}$ and $(\partial z/\partial v)(\partial z/\partial\tau)^{-1}$ are much less than 1; that may not be the case in many situations of optical depth retrieval from satellite imagery. It should be noted also that we are not restricted to use the above expression (16) for the vertical optical depth as a new variable; for example, in order to simplify the direct source terms in the delta-Eddington approximation the function $\mathcal{T}$ may be chosen the following way:

$$\tau_d=\mathcal{T}_d(x,y,z)$$

$$=\int_{H_0(x',y')}^{z}k'\left\{x'+[z'-H_0(x',y')]\frac{\Omega_{0x}}{\Omega_{0z}},\ y'\right.$$

$$+ [z' - H_0(x', y')]\frac{\Omega_{0y}}{\Omega_{0z}}, z' \Bigg\} dz', \quad (17)$$

where $\mathbf{\Omega}$ is the unit vector in the direction of the incident source of radiation and $x'$ and $y'$ can be determined from geometry of direct beam propagation,

$$\frac{x - x'}{z - H_0(x', y')} = \frac{\Omega_{0x}}{\Omega_{0z}}, \qquad \frac{y - y'}{z - H_0(x', y')} = \frac{\Omega_{0y}}{\Omega_{0z}}.$$

The $\tau$ in the expression (16) represents the vertical optical depth and the $\tau_d$ in (17) is the projection of the optical depth along the path of the direct beam to the vertical direction. The difference between the two vanishes for a plane-parallel homogeneous medium.

The expression (17) was used by Gabriel and Evans (1996) for calculating the source terms in (5) and (8), although they treated the rest of the problem using the independent pixel approximation.

For simplicity we assume that there are no downward-directed diffuse fluxes at the top of the layer and we take the surface albedo to be zero. As a result, there is no upward-directed diffuse irradiance at the bottom. The boundary conditions in this case can be written as

$$F^{\downarrow}(u, v, 0) = \pi\left[I_0(u, v, 0) + \frac{2}{3}I_z(u, v, 0)\right] = 0, \quad (18)$$

$$F^{\uparrow}(u, v, \tau_*) = \pi\left[I_0(u, v, \tau_*) - \frac{2}{3}I_z(u, v, \tau_*)\right] = 0, \quad (19)$$

where $\tau_* = \mathcal{T}(x, y, 0)$.

The effective domain-averaged albedo is the ratio of the upward-directed diffuse irradiance to the incident downward-directed irradiance, thus

$$A = \frac{1}{\pi\mu_0 F_0^0}\frac{1}{S}\int_{\mathcal{D}(S)} F^{\uparrow}(u, v, 0)\, ds$$
$$= \frac{1}{\mu_0 F_0^0}\frac{1}{S}\int_{\mathcal{D}(S)} \left[I_0(u, v, 0) - \frac{2}{3}I_z(u, v, 0)\right] ds. \quad (20)$$

## 3. Geometrical variations in a conservatively scattering medium

Let us first consider the case of an extinction coefficient that is independent of $u$ and $v$. The deviations from the independent pixel approximation in this case will arise due to $u$ and $v$ dependence of the geometrical characteristics of the medium, namely, $H_0(u, v)$. From Eq. (16) we obtain

$$\frac{\partial z}{\partial u} = \frac{\partial H_0}{\partial u}, \qquad \frac{\partial z}{\partial v} = \frac{\partial H_0}{\partial v}, \qquad \frac{\partial z}{\partial \tau} = \frac{1}{k'}. \quad (21)$$

First, assume that the variations of $H_0$ are slow in comparison to the mean transport length, that is,

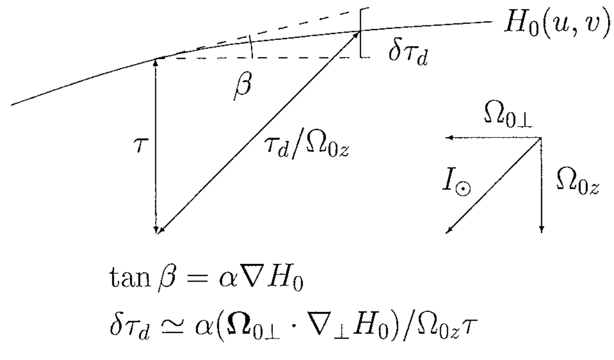


FIG. 4. Linearization of the direct beam optical depth $\tau_d$. The horizontal displacement of the direct beam entry point ($\mathbf{\Omega}_{0\perp}/\Omega_{0z}\tau$) is used to find the linear correction $\delta\tau_d$ to the vertical optical depth $\tau$.

$$\frac{\alpha}{k'H_0}|\mathbf{\nabla}_\perp H_0| = \epsilon \ll 1, \quad (22)$$

where $\mathbf{\nabla}_\perp = (\partial/\partial u, \partial/\partial v)$. Restricting the solutions to solar zenith angles that are not very oblique ($\Omega_{0\perp}/\Omega_{0z} \leq 1/\epsilon$), we may write from (17)

$$\tau_d \simeq \tau(1 + \alpha(\mathbf{\Omega}_{0\perp}\cdot\mathbf{\nabla}_\perp H_0)/\Omega_{0z}), \quad (23)$$

and the source term becomes

$$F = F(u, v, \tau) = F_0^0 e^{-\tau_d/\mu_0}$$
$$\simeq F_0^0 e^{-\tau/\mu_0}\left(1 - \alpha(\mathbf{n}\cdot\mathbf{\nabla}_\perp H_0)\tau\frac{\nu_0}{\mu_0^2}\right), \quad (24)$$

where $\mathbf{n} = (n_x, n_y) = (\cos\phi_0, \sin\phi_0)$. Figure 4 illustrates the expansion (23) of the direct beam optical depth $\tau_d$. Although it is valid only for small enough values of $\tau$, the expansion of the source function itself in (24) is accurate even for large values of $\tau$, because the direct beam intensity decays exponentially (Li et al. 1994).

After substituting (21) into (15), we get

$$\frac{\partial^2 I_0}{\partial\tau^2} - \frac{\alpha^2\nabla_\perp^2 H_0}{k'(1 + \alpha^2|\mathbf{\nabla}_\perp H_0|^2)}\frac{\partial I_0}{\partial\tau} - \frac{3(1-\omega')(1-g'\omega')}{1 + \alpha^2|\mathbf{\nabla}_\perp H_0|^2}I_0$$
$$= \frac{1}{1 + \alpha^2|\mathbf{\nabla}_\perp H_0|^2}$$
$$\times\left[-(1-\omega' g')\frac{F_0^s}{k'} + \frac{\partial}{\partial\tau}\frac{F_z^s}{k'} - \alpha\mathbf{\nabla}_\perp H_0\cdot\frac{\partial}{\partial\tau}\frac{\mathbf{F}_\perp^s}{k'}\right], \quad (25)$$

where all quantities with $\perp$ symbol are in the $u$–$v$ plane.

Using expressions (3), (4), and (24) for the source terms, keeping only the first-order terms (i.e., ignoring $\nabla_\perp^2 H_0$ and $|\mathbf{\nabla}_\perp H_0|^2$ when compared with terms containing $\mathbf{\nabla}_\perp H_0$), and taking $\omega' = 1$, Eq. (25) becomes

$$\frac{\partial^2 I_0}{\partial\tau^2} = -\frac{3}{4}F_0^0 e^{-\tau/\mu_0}\left(1 - \alpha(\mathbf{n}\cdot\mathbf{\nabla}_\perp H_0)\frac{\nu_0}{\mu_0^2}\tau\right), \quad (26)$$

General expressions for a solution of (26) and for a local albedo are obtained in appendix A. We present here as an illustration an asymptotic ($\tau_* \to \infty$) expression for the local albedo $R$ for a spherically symmetric ($g = 0$) phase function:

$$R = 1 - \frac{\nu_0}{\mu_0}(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0) - \frac{1}{4 + 3\tau_*} \times \left[3\mu_0 + 2 - 2\left(3\nu_0 + \frac{\nu_0}{\mu_0}\right)(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)\right]. \quad (27)$$

The inequality (22) is not very restrictive. For sufficiently optically thick clouds it will be satisfied if the aspect ratio of the variations is less than 1. Indeed, for typical stratocumulus clouds with optical depth $\tau \sim 5$ and asymmetry factor $g \sim 0.85$ the inequality (22) reduces to $|\boldsymbol{\nabla}_\perp H_0| < \tau(1 - g^2) \sim 1.4$. For an optically thin medium, on the other hand, the applicability condition becomes more restrictive. For $\tau \sim 1$ it gives $|\boldsymbol{\nabla}_\perp H_0| < \tau(1 - g^2) \sim 0.28$.

## 4. Internal inhomogeneity in a conservatively scattering medium

Next, we consider the case of a layer with constant geometrical height, such that the inhomogeneity is created by variations of the internal optical properties of the medium. The extinction coefficient $k'$ will be a function of $u$ and $v$ only. In this case we can write from (16)

$$\frac{\partial z}{\partial u} = -\frac{\tau}{k'^2}\frac{\partial k'}{\partial u}, \qquad \frac{\partial z}{\partial v} = -\frac{\tau}{k'^2}\frac{\partial k'}{\partial v}, \qquad \frac{\partial z}{\partial \tau} = \frac{1}{k'}. \quad (28)$$

After expanding the optical depth along the path of the direct beam, once again taking into account the weakness of the inhomogeneity, we obtain from (17)

$$\tau_d \simeq \tau(1 - \alpha\tau(\boldsymbol{\Omega}_{0\perp}\cdot\boldsymbol{\nabla}_\perp k')/(2\Omega_{0z}k'^2)), \quad (29)$$

and for the source term we let

$$F = F(u, v, \tau) = F_0^0 e^{-\tau_d/\mu_0} \simeq F_0^0 e^{-\tau/\mu_0}\left(1 + \alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{2k'^2}\frac{\nu_0}{\mu_0^2}\tau^2\right). \quad (30)$$

We have linearized the expression for the source term. This is not a good approximation for realistic conditions. The expansion in (30) is even less accurate than that in (24). We really do not have to make this approximation. The source term can be calculated exactly (as in Gabriel and Evans 1996) if we know the spatial dependence of the extinction coefficient. However, our motivation is the inverse problem, where this distribution is not known a priori, but rather requires determination.

The equation for the mean intensity of the radiation in the internally inhomogeneous medium may be written as

$$\frac{\partial^2 I_0}{\partial \tau^2} = -\frac{3}{4}F_0^0 e^{-\tau/\mu_0}\left(1 + \alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{2k'^2}\frac{\nu_0}{\mu_0^2}\tau^2\right). \quad (31)$$

Once again we will direct the reader to appendix B for the general solution and will present here only an asymptotic ($\tau_* \to \infty$) expression for the local albedo $R$ for a spherically symmetric ($g = 0$) phase function,

$$R = 1 + \nu_0\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2} - \frac{1}{4 + 3\tau_*} \times \left[3\mu_0 + 2 + \nu_0(9\mu_0 + 2)\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2}\right]. \quad (32)$$

## 5. Results and discussion

We have derived the solutions for the diffuse radiative fluxes and the mean intensity in a weakly inhomogeneous medium, where the inhomogeneity is created either by internal variations of the extinction properties of the medium [Eqs. (37), (38)] or by variations of the geometrical boundary of the scattering layer [Eqs. (33), (34)]. In order to verify the validity and accuracy of the solutions we compared them to the results of numerical simulations of the three-dimensional radiative transfer equation using the spherical harmonics discrete ordinate code written by Evans (Evans 1998).

We will first show the comparison for the case of geometrical variations of the upper boundary of the layer. In Fig. 5a the local reflectances calculated by the plane-parallel approximation (PPA), the independent pixel approximation (IPA), and the three-dimensional spherical harmonics discrete ordinate method (SHDOM) are compared with the gradient correction solution (GC), Eq. (36). The spatial grid for the SHDOM calculations in $x$–$z$ plane was $128 \times 128$, and for angular resolution there were 8 or 16 streams with 16 or 32 azimuthal modes. The reflectances are plotted as functions of position for a conservatively scattering layer with flat bottom surface and sinusoidal variations of upper surface, $H_0(x) = \overline{Z}(1 - \delta\sin(2\pi x/L_x))$. The optical depth profile is shown at the bottom of the figure. The cosine of solar zenith angle $\mu_0$ is 0.7, the asymmetry parameter $g$ is 0.75, the domain-averaged optical depth $\overline{\tau}$ is 16, the domain-averaged geometrical depth $\overline{Z}$ is 0.8 km, and $L_x = 14.08$ km. The amplitude of relative geometrical variations $\delta$ is equal to 0.25. Figure 5b shows the plot of the difference between SHDOM and IPA reflectances, where the IPA reflectance was computed numerically by SHDOM, and the difference between GC and IPA reflectances, where the IPA reflectance was calculated by the delta-Eddington approximation.

Figures 6a and 6b show the local reflectances and the differences between SHDOM and IPA reflectances, respectively, as in the previous case, but for the optically thin medium. Optical depth $\tau$ varies from 0.5 to 3.5

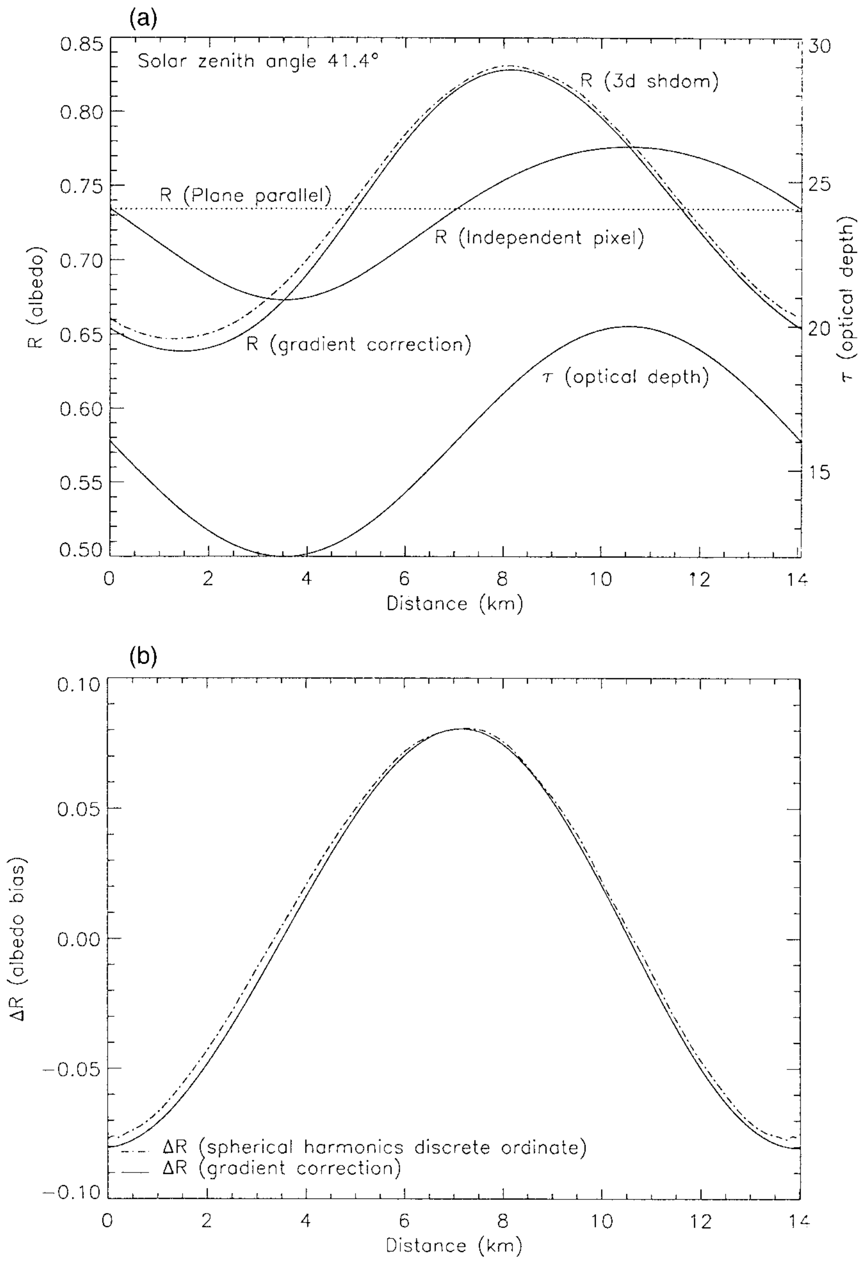


FIG. 5. (a) Plot of reflectances for plane-parallel, independent pixel, and gradient correction approximations and 3D spherical harmonics discrete ordinate method for optical depth profile shown by bottom curve, where $\overline{\tau} = 16$, $\tau(x) = kH_0(x) = k\overline{Z}(1 - \delta \sin(2\pi x/L_x))$, $k = 20$ km$^{-1}$, $\delta = 0.25$, $\overline{Z} = 0.8$ km, $g = 0.75$, and $L_x = 14.08$ km. (b) Independent pixel approximation reflectance error: ΔR(spherical harmonics discrete ordinate) = R(shdom) − R(ipa) and ΔR(gradient correction) = R(gc) − R(ipa).

with the mean value of 2. The cosine of solar zenith angle is 0.3.

Figures 7a,b are similar to Figs. 5a,b except for the linear variations of the upper surface of the layer. Because in this case the gradient of $H_0$ has singularities, a simple Gaussian filter was used to eliminate them and provide smooth behavior of the gradient.

From all these figures one can see that the errors of the GC are quite small and almost always lie within the errors of the delta-Eddington approximation itself, although the deviations of both the GC and the SHDOM reflectances from the IPA solution are quite substantial.

In the case of internal variations of the extinction coefficient in the geometrically plane-parallel layer (Figs. 8a,b) the agreement between the GC and the SHDOM methods is again excellent, although the de-

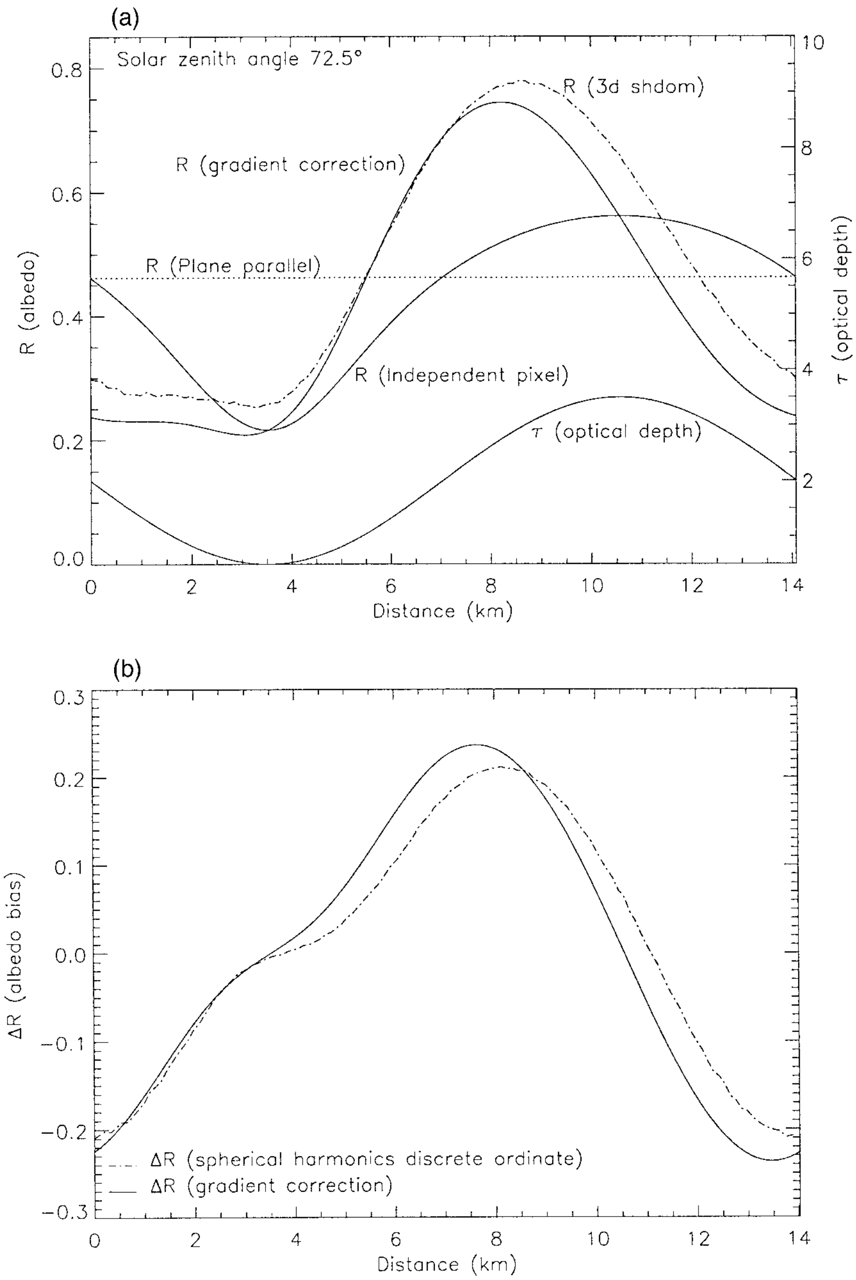


FIG. 6. (a) Plot of reflectances for plane-parallel, independent pixel, and gradient correction approximations and 3D spherical harmonics discrete ordinate method for optical depth profile shown by bottom curve, where $\overline{\tau} = 2$, $\tau(x) = kH_0(x) = k\overline{Z}(1 - \delta\sin(2\pi x/L_x))$, $k = 4.666\ \mathrm{km}^{-1}$, $\delta = 0.75$, $\overline{Z} = 0.429$ km, $g = 0.75$, and $L_x = 14.08$ km. (b) Independent pixel approximation reflectance error: ΔR(spherical harmonics discrete ordinate) = R(shdom) − R(ipa) and ΔR(gradient correction) = R(gc) − R(ipa).

viations of reflectances from the IPA solution are much smaller than in the former case of geometrical variations.

The fact that geometrical variations of cloud surface (cloud-top bumps) are more important for the reflectance calculations than the internal variations of the optical properties of the clouds, was previously reported from the results of Monte Carlo simulations (Loeb and Davies 1996). From our solution one can see that the parameter responsible for the corrections to the IPA reflectance in the geometrical variations case $(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)$ is bigger than the corresponding parameter in the case of the internal inhomogeneity $(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')/k'^2$ for optically thick medium. Indeed,

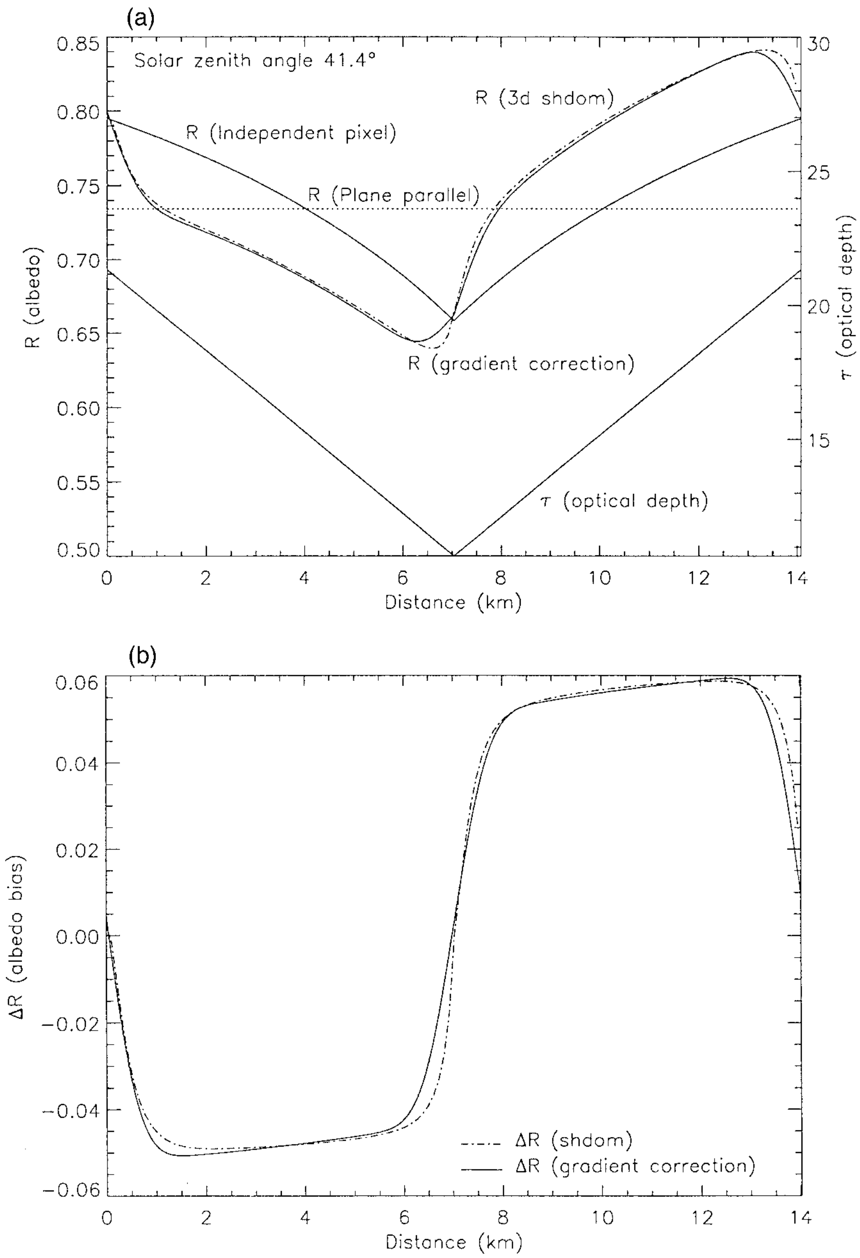


FIG. 7. (a) Plot of reflectances for plane-parallel, independent pixel, and gradient correction approximations and 3D spherical harmonics discrete ordinate method for optical depth profile shown by bottom curve, where $\overline{\tau} = 16$, $\tau(x) = kH_0(x) = k\overline{Z}(1 + \delta(|2x - L_x|/L_x - 0.5))$, $k = 21.333\ \mathrm{km}^{-1}$, $\delta = 0.5$, $\overline{Z} = 0.75$ km, $g = 0.75$, and $L_x = 14.08$ km. (b) Independent pixel approximation reflectance error: ΔR(spherical harmonics discrete ordinate) = R(shdom) − R(ipa) and ΔR(gradient correction) = R(gc) − R(ipa).

$$(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0) \sim (\mathbf{n}\cdot\boldsymbol{\nabla}_\perp \tau_*)/k',$$

$$(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')/k'^2 \sim (\mathbf{n}\cdot\boldsymbol{\nabla}_\perp \tau_*)/k'/\tau_*,$$

and therefore, their ratio is of order of optical depth $\tau_*$.

The GC solution is in agreement with that observed in an inhomogeneous medium shift of the spatial distribution of the radiative field with respect to the geometrical characteristics of the medium (and hence to the IPA computed distribution of radiation). In addition to the shift, the spatial variations of the reflectance are higher for the GC computed local albedo than for the pixel by pixel solution. It would imply that a solution of the inverse problem of optical depth retrieval by the IPA method without taking into account the variability of the surface will generally have

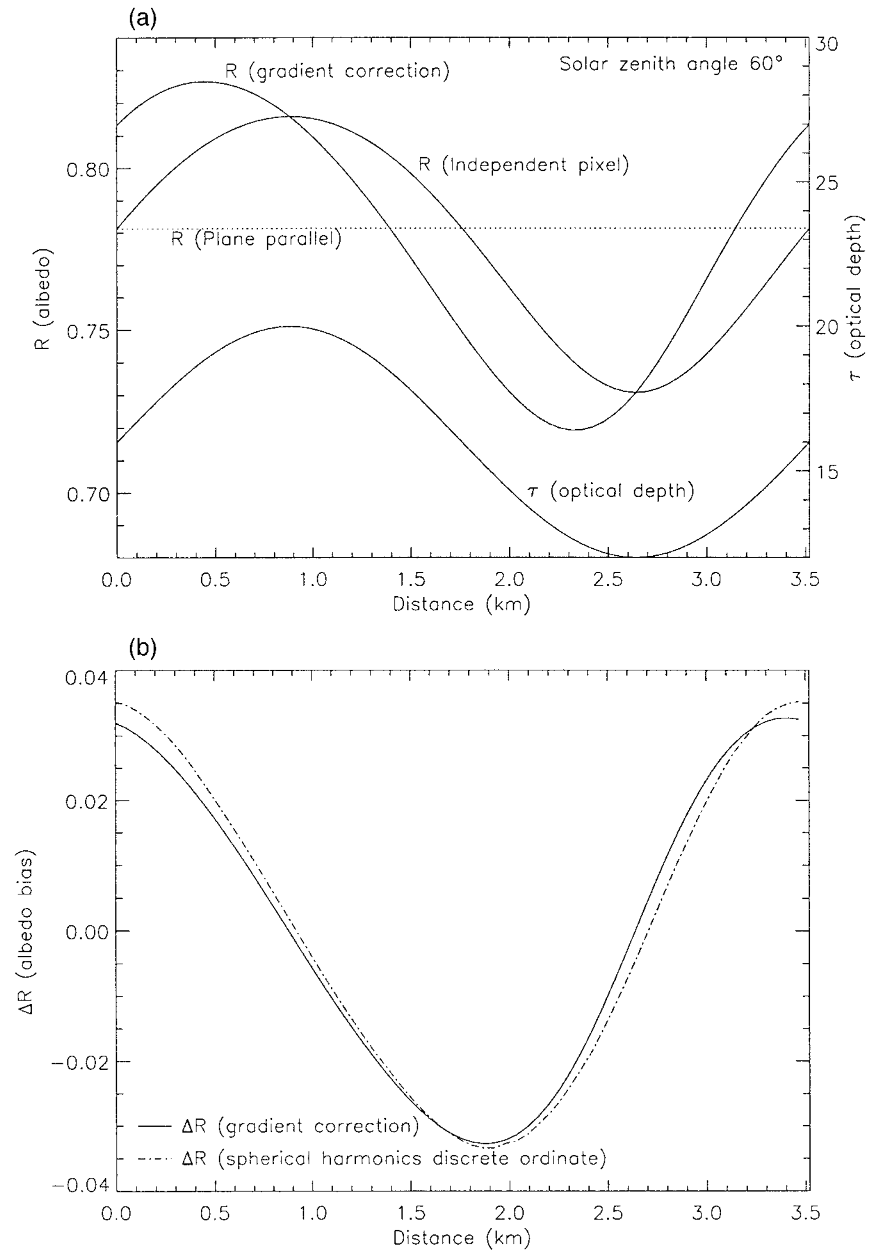


FIG. 8. (a) Plot of reflectances for plane-parallel, independent pixel, and gradient correction approximations for optical depth profile shown by bottom curve, where $\overline{\tau} = 16$, $\tau(x) = k(x)H_0 = \overline{k}\, H_0(1 + \delta \sin(2\tau x/L_x))$, $\delta = 0.25$, $\overline{k} = 16$ km$^{-1}$, $H_0 = 1$ km, $g = 0.75$, and $L_x = 3.52$ km. (b) Independent pixel approximation reflectance error: ΔR(spherical harmonics discrete ordinate) = R(shdom) − R(ipa) and ΔR(gradient correction) = R(gc) − R(ipa).

higher variations and hence bigger errors in the computed optical depth.

Our solution also gives analytical insights into the reason for an unphysical solar zenith angle dependence of retrieved satellite optical depths. As reported by Loeb et al. (1997) and Loeb and Davies (1996), the retrieved optical depths spuriously increase with increasing solar zenith angle. In the limit of large optical depth this dependence may be approximated by the second term in the right-hand side of Eq. (36) and, hence, is proportional to the $\nu_0/\mu_0$. In order to illustrate this dependence we plotted local albedo as a function of $\mu_0$ for several values of $\tau$ in Fig. 9a. We used $g = 0.86$ and $-0.01$ as a value of the parameter $(\mathbf{n}\cdot\mathbf{\nabla}_{\perp}H_0)$. In satellite imagery the analysis of an albedo dependence on solar zenith angle is often based on a subset of brightest pixels (and it is that case which shows the strongest deviation from the plane-parallel approximation), which justifies

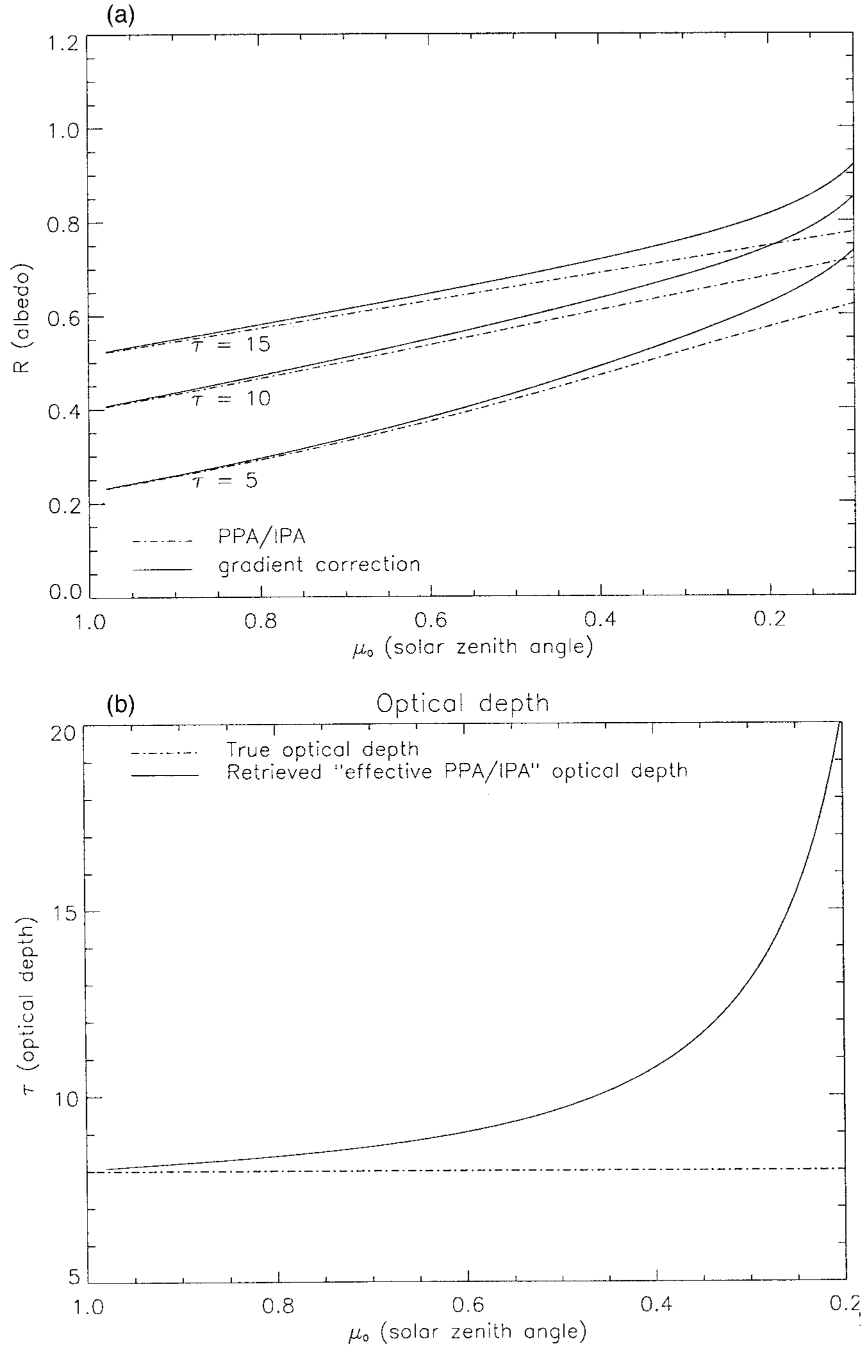


FIG. 9. (a) Plot of dependence of local albedo on solar zenith angle. (b) Plot of dependence of IPA retrieved optical depth on solar zenith angle.

the use of a single value of the parameter $(\mathbf{n}\cdot\boldsymbol{\nabla}_{\perp}H_0)$ for the illustration. As shown in Fig. 9a, the PPA can result in significant errors at values of $\mu_0 < 0.5$. We should also mention that in an inhomogeneous medium a local albedo is not restricted to be less then 1.

We also plot solar zenith angle dependence of the optical depths retrieved by the IPA from a local albedo calculated for various values of the parameter $(\mathbf{n}\cdot\boldsymbol{\nabla}_{\perp}H_0)$ and solar zenith angles, but for one value of optical depth. This dependence is shown in Fig. 9b, where the optical depth has been retrieved by the PPA method. The initial albedo has been calculated by the GC method for the medium with the optical depth $\tau$ equal to 8, and the parameter $(\mathbf{n}\cdot\boldsymbol{\nabla}_{\perp}H_0)$ equal to $-0.02$. Figures 11a,b from Loeb and Davies (1997) show very similar dependence in datasets obtained from Earth Radiation

Budget Satellite (ERBS) measurements and from *NOAA-11* and *NOAA-14*, although we should take into account that in their analysis this dependence is averaged over various values of the parameter $(\mathbf{n}\cdot\nabla_\perp H_0)$.

The insights into the source of the spurious zenith angle dependence is very important and may be used in optical depth retrieval either as an additional method of validation of optical depth field being retrieved (if the position of the upper boundary is known from other sources) or as an additional parameter during retrieval.

## 6. Summary and conclusions

The simple analytical solution for the problem of radiative transfer in an inhomogeneous medium that takes into account the horizontal radiative fluxes has been derived using the delta-Eddington approximation. The horizontal fluxes may result either from variations of the geometrical boundaries of the layer, that is, bumps on top surface, or from variations of the internal optical properties of the medium, that is, dependence of the extinction coefficient on the horizontal coordinate. In both cases the following requirements should be satisfied in order for this solution to be valid.

- The characteristic scale of horizontal variations should be large in comparison to the mean radiative transport length, thus the medium is weakly inhomogeneous.
- The horizontal extent of the medium should be larger than the vertical size, thus the horizontal boundary effects may be neglected.

Note that the second requirement is not specific for the present method, but it is the requirement for the validity of the plane-parallel and independent pixel approximations as well.

The solution has been compared to the results of numerical simulations of the three-dimensional radiative transfer equation using the spherical harmonics discrete ordinate method (Evans 1998), and a good agreement has been found.

The solution shows dependence of the reflectance on solar zenith angle and shift of spatial distribution of the radiative field with respect to the properties of the medium. Both effects are neglected in optical depth retrieval from satellite imagery by the IPA method, but they may be very important, especially for large solar zenith angles ($\mu_0 < 0.5$).

Although the solution accurately describes the spatial distribution of the radiative field in an inhomogeneous medium, the reported deviations of the angular distribution (dependence of bidirectional reflectance on view angle) from the plane-parallel results (Loeb and Coakley 1998) are not addressed in this work and require further investigation. We should also point out that the solutions (36) and (40) are the first-order corrections to the PPA–IPA method. We restrict ourselves to the first-order calculations, despite the fact that Eq. (15) formally contains the second-order terms. The main reason for this restriction lies in a strong coupling between angular and spatial distributions of radiation in an inhomogeneous medium. Both angular and spatial deviations of the radiative field from the plane-parallel solution depend on the scales of the inhomogeneity. Because the delta-Eddington approximation is equivalent to the expansion of the angular dependence of the radiance up to first order only, it is questionable that it can be used for spatial corrections of the orders higher than one.

The computational complexity of the solution is comparable to the independent pixel approximation method. As in the IPA method, the reflectance is calculated on a pixel by pixel basis and the corrections due to horizontal fluxes are included through the local gradients of either the geometrical boundary of the layer or the extinction coefficient. Thus this solution may be used as an effective method in remote sensing of clouds from satellite imagery, although the implementation of this technique, particularly the modification of the method for using radiances instead of hemispheric fluxes, is a subject of future research.

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APPENDIX A

### Local Albedo in a Medium with Geometrical Variations

After integration of Eq. (26) the following solution can be obtained:

$$I_0 = C_1 + C_2\tau + \frac{3}{4}[\nu_0\alpha(\mathbf{n}\cdot\nabla_\perp H_0)(\tau + 2\mu_0) - \mu_0^2]F_0^0 e^{-\tau/\mu_0}, \tag{A1}$$

$$I_z = -\frac{1}{1-g'}C_2 + \frac{3}{4}\left[\nu_0\alpha(\mathbf{n}\cdot\nabla_\perp H_0)\left(\frac{\tau}{\mu_0} + \frac{1}{1-g'}\right) - \mu_0\right]F_0^0 e^{-\tau/\mu_0}, \tag{A2}$$

where from boundary conditions (18), (19),

$$C_2 = F_0^0 \frac{3(1-g')}{4+3(1-g')\tau_*}\left\{-\nu_0\alpha(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)\left(\frac{3}{2}\mu_0 + \frac{1}{2(1-g')}\right) + \frac{3}{4}\mu_0^2 + \frac{1}{2}\mu_0 - e^{-\tau_*/\mu_0}\right.$$
$$\left.\times\left[-\nu_0\alpha(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)\left(\frac{3}{4}\tau_* - \frac{\tau_*}{2\mu_0} + \frac{3}{2}\mu_0 - \frac{1}{2(1-g')}\right) + \frac{3}{4}\mu_0^2 - \frac{1}{2}\mu_0\right]\right\}. \quad \text{(A3)}$$

The local albedo may be written in this case as

$$R = \frac{1}{\mu_0 F_0^0}\left(I_0(0) - \frac{2}{3}I_z(0)\right) = -\frac{4}{3}\frac{I_z(0)}{\mu_0 F_0^0}$$
$$= 1 - \frac{\nu_0}{\mu_0}\frac{\alpha(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)}{1-g'} - \frac{1}{4+3(1-g')\tau_*}\left[-\frac{\alpha(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)}{1-g'}\nu_0\left(6(1-g') + \frac{2}{\mu_0}\right) + 3\mu_0 + 2 - e^{-\tau_*/\mu_0}\right.$$
$$\left.\times\left[-\frac{\alpha(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp H_0)}{1-g'}\nu_0\left(\frac{3\mu_0-2}{\mu_0^2}(1-g')\tau_* + 6(1-g') - \frac{2}{\mu_0}\right) + 3\mu_0 - 2\right]\right].$$

## APPENDIX B

### Local Albedo in a Medium with Internal Inhomogeneity

The solution of Eq. (31) is

$$I_0 = C_1 + C_2\tau - \frac{3}{8}\left[\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2}(\tau^2 + 4\mu_0\tau + 6\mu_0^2) + 2\mu_0^2\right]F_0^0 e^{-\tau/\mu_0}, \quad \text{(B1)}$$

$$I_z = -\frac{1}{1-g'}C_2 - \frac{3}{8}\left[\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2}\left(\frac{\tau^2}{\mu_0} + \frac{2(\tau+\mu_0)}{1-g'}\right) + 2\mu_0\right]F_0^0 e^{-\tau/\mu_0}, \quad \text{(B2)}$$

where from boundary conditions (18), (19),

$$C_2 = -F_0^0\frac{3(1-g')}{4(4+3(1-g')\tau_*)}\left\{\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2}\left(9\mu_0^2 + \frac{2\mu_0}{(1-g')}\right) + 3\mu_0^2 + 2\mu_0 - e^{-\tau_*/\mu_0}\right.$$
$$\times\left[\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2}\left(\frac{3\mu_0-2}{2\mu_0}\tau_*^2 + \frac{6\mu_0(1-g')-4}{1-g'}\tau_* + \frac{(9\mu_0(1-g')-4)\mu_0}{1-g'}\right)\right.$$
$$\left.\left. + 3\mu_0^2 - 2\mu_0\right]\right\}, \quad \text{(B3)}$$

and the expression for the local albedo is

$$R = \frac{1}{\mu_0 F_0^0}\left[I_0(0) - \frac{2}{3}I_z(0)\right] = -\frac{4}{3}\frac{I_z(0)}{\mu_0 F_0^0}$$
$$= 1 + \nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2(1-g')} - \frac{1}{4+3(1-g')\tau_*}\left\{\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2(1-g')}(9\mu_0(1-g') + 2) + 3\mu_0 + 2 - e^{-\tau_*/\mu_0}\right.$$
$$\times\left[\nu_0\alpha\frac{(\mathbf{n}\cdot\boldsymbol{\nabla}_\perp k')}{k'^2(1-g')}\left(\frac{(3\mu_0-2)(1-g')}{2\mu_0^2}\tau_*^2 + \frac{6\mu_0(1-g')-4}{\mu_0}\tau_*\right.\right.$$
$$\left.\left.\left. + 9\mu_0(1-g') - 4\right) + 3\mu_0 - 2\right]\right\}. \quad \text{(B4)}$$

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