A New Approach to Detect and Characterize Intermittent Atmospheric Oscillations: Application to the Intraseasonal Oscillation

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(Manuscript received 17 April 1998, in final form 8 September 1999)

ABSTRACT

This paper presents a method, the local mode analysis (LMA), that makes it possible to extract the most persistent oscillations present in the time evolution of an atmospheric field. This method is particularly suitable to analyze intermittent tropospheric oscillations related to dynamic or thermodynamic instabilities such as the intraseasonal oscillation (ISO). These intermittent oscillations generally exhibit various spatial structures that succeed one another in time and that are difficult to isolate in a simple and comprehensive manner using conventional approaches such as empirical orthogonal functions or composite analyses. The main objective of the LMA approach is to identify the different structures of a given oscillation in order to better understand its physical origin and to test the applicability of different theoretical hypotheses. The LMA also makes it possible to test the representativity of a mean structure in regard to actual modes that succeed one another in time.

The LMA is applied to the National Oceanic and Atmosphere Administration—Advanced Very High Resolution Radiometer outgoing longwave radiation time series in order to study the variability of the convective perturbation at the intraseasonal timescale (30–60 days). The LMA depicts the most intense and persistent modes of the ISO very well and shows the strong variability of the spatial organization of the convective perturbation at this timescale. Results exhibit interannual and seasonal variations of the mean period and amplitude of the ISO with a tendency to have less persistent modes and smaller periods of the oscillation during El Niño years and during summer. The maximum perturbation of the convection by the ISO is not located on the equator but rather around 10°–15° in the summer hemisphere. Several persistent modes exhibit neither the phase opposition between the Indian and Pacific Oceans nor the eastward equatorial propagation that characterize the average mode of Northern Hemisphere winter. Inspecting the ensemble of ISO modes, this eastward propagation of the convective perturbation is well defined only over the Indian Ocean. The convective perturbation over the Maritime Continent is basically stationary, and the eastward propagation over the Pacific Ocean appears only for the strongest convective perturbations.

1. Introduction

Part of the tropical atmospheric variability is organized in quasiperiodic oscillations such as the El Niño–Southern Oscillation (ENSO), the intraseasonal oscillation (ISO), or equatorial easterly waves, giving a possibility to understand, simulate, and predict some elements of tropical climate at the corresponding timescales. These tropical oscillations are, however, difficult to detect and characterize in a simple and comprehensive way because they are generally intermittent and mixed with the atmospheric variability on other timescales. The intermittence of these oscillations leads to a variety of spatial structures (e.g., amplitude distribution and lag between regions) and temporal characteristics (e.g., duration and dominant timescale) from one event to another. These properties characterize a particular mode that exists for a particular date. A given type of oscillation, such as the ISO, may be seen as a succession of such modes with time. In most cases, this variability must be considered typical of the oscillation and not a random noise hiding a hypothetical ideal structure. For this reason, even if a mean spatial structure may give useful condensed information, the analysis of this variability is also required to better understand the physical origin of the phenomenon.

Conventional methods, such as composite or cross-spectral analyses, are generally designed to extract only the mean structure of a given oscillation. For intermittent oscillations, this mean structure may be difficult to interpret because it is constrained by the strongest events that are not necessarily representative of the ensemble of events. If spatial distributions of amplitude and lag are very variable from one oscillation to another, this mean structure also may be influenced strongly by the reference region used to define the dates or the different phases of the oscillations for composite analyses or to
characterize phase lags for cross-spectral analyses. Time–longitude or Hovmöller diagrams [see, e.g., Nakazawa (1988) or Rui and Wang (1990) for the intraseasonal oscillation] give useful information on the variability of a given oscillation. These diagrams give condensed information on the time evolution of the longitudinal structure (amplitude, propagation) of the perturbations. However, for these time–longitude diagrams the latitude band over which the signal [e.g., wind divergence, outgoing longwave radiation (OLR)] is averaged also must be specified. This may sometimes confuse the interpretation if the latitudinal structure of the perturbation is variable from one oscillation to another or if the main structure is not purely longitudinal.

Different structures or modes may be extracted by computing empirical orthogonal functions (EOFs). For example, interannual variability over the Pacific (Barnett 1985, 1991), teleconnection patterns in midlatitudes (Wallace and Gutzler 1981), and the ISO (Lorenc 1984; Lau and Chan 1985, 1986; Gutzler and Madden 1989) were analyzed using EOF analysis. The EOF analysis separates the time–space structure of a geophysical field according to a basis of orthogonal eigenvectors. Each eigenvector gives a spatial distribution of correlation or amplitude that represents the spatial structure of the mode. For complex EOFs (CEOFs), the eigenvector also gives the spatial distribution of relative phases (or lags), informing on the propagative or stationary character of the oscillation (Wallace and Dickinson 1972; Barnett 1983). However, for intermittent oscillations, different structures that are not necessarily orthogonal may be present in the time series. An analysis of long time series, compared with the timescale of the studied oscillation, may thus provide the first EOF (corresponding to the largest eigenvalue) with either an average spatial structure not representative of any of these modes or a spatial structure representative only of some of the strongest modes of the series. In many cases, because of the orthogonality constraint, the remaining EOFs (e.g., second, third, etc.) are not easily interpretable. This shortcoming of the EOF analysis exists for different types of EOFs, including the CEOF and the extended EOF (Weare and Nasstrom 1982). To avoid this problem, some attempts have been made to rotate the eigenvectors (rotated EOF) following different criteria such as the Varimax technique (Horel 1984). However, this approach is not always efficient, and the choice of a rotation solution is “only one of an infinity of alternative solutions” (Horel 1984) that give the impression that any spatial structure can be obtained. It is thus of interest to develop an approach that makes it possible to test the meaning of such a mean spatial structure.

This paper proposes an approach, the local mode analysis (LMA), that extracts and characterizes the different modes of a particular type of oscillation. The LMA breaks the condition of orthogonality between modes that succeed one another in time by applying a series of successive CEOF analyses on relatively short time windows of the whole time series. The choice of short time windows, analogous to the expected duration of a mode, avoids the mixture of different modes and thus gives a far more straightforward interpretation of the CEOF results. We select the CEOF analysis because it provides the distribution of amplitude and of relative phase on a single eigenvector. The CEOF is thus very efficient to describe any phase relationship, including propagative or stationary features, by a single component (a real instead of a complex EOF analysis requires two eigenvectors to describe a propagative signal). The proposed approach makes it possible to detect and characterize the most persistent modes of a time series. Note that the LMA, like the other analysis methods mentioned above, is based on the processing of a given time series representing the fluctuations of a given parameter in a given region and for a given timescale. In some cases, what we call different modes of an oscillation may actually characterize different types of oscillations with different physical origins that coexist in the same frequency range.

Section 2 of this paper contains a description and a justification of the LMA procedure. In section 3, the basic properties of this method are illustrated using an idealized signal. In section 4, the ISO of deep convection is studied using the LMA approach. Advantages and shortcomings of the LMA technique and its field of potential application for the study of various geophysical phenomenons are discussed in section 5.

2. Analysis procedure

The analyzed time series \( s(x, t) \) represents the time evolution of a given parameter for each region \( x \) at each time step \( t \) with \( 1 \leq x \leq N \) and \( 1 \leq t \leq T \). The aim of the analysis is to extract characteristics of the oscillations of this field at a given timescale. In the following, we will show an example in which the signal \( s(x, t) \) is the OLR with a spatial resolution of 5° and a time step of one day. In this case, the analysis will be done to extract spatial structures at an intraseasonal timescale (20–80 days) in order to characterize the ISO over the Indian and the Pacific Oceans.

The original signal \( s(x, t) \) must generally be filtered in time to retain only the oscillations at the selected timescale. The use of a filtered and centered time series, noted \( \tilde{s}(x, t) \), is necessary to avoid a mixing of timescales that may perturb the detection of the modes. Also, the fluctuations must be quasiperiodic for the relative phase field to be interpretable in terms of propagation speed or, more generally, in terms of time lag between the perturbations over different regions.

The CEOF analysis in the frequency domain, introduced by Wallace and Dickinson (1972), is the basis of the LMA. The CEOF analysis is based on the computation of \( N \) real eigenvalues \( \lambda_n \) and complex eigenvectors \( z_n(x) \) from the \( N \times N \) cross spectral matrix \( \mathbf{\xi} \) defined as
\[ \xi(x, x') = \sum_{k=1}^{N} \tilde{F}(x, k) \tilde{F}^*(x', k), \]  

where * represents the complex conjugate, \( k \) is the harmonic number (for the whole spectrum \( k_1 = 1 \) and \( k_2 = T/2 \)), and \( \tilde{F}(x, k) \) represents the Fourier coefficients defined as

\[ \tilde{F}(x, k) = \frac{\sqrt{2}}{T} \sum_{t=1}^{T} S(x, t) e^{-2\pi ikt/T}. \]  

For each region \( R \), the summation over the harmonics \( [k_1, k_2] \) of the squared module of \( \tilde{F}(x, k) \) gives the variance of the filtered time series \( S(x, t) \). Since the spectral variance of this filtered signal is negligible outside a given temporal harmonic interval, \( k_1 \) and \( k_2 \) may be chosen to cover only the frequency range of the filtered signal. The consideration of this restricted interval only avoids useless computation in spectral domains with negligible variance but is not a correct filtering method. (If a filtering is required, it is more appropriate to first filter the entire time series and then to apply the LMA to this filtered time series.)

Each CEOF corresponds to a percentage of variance \( v_n = \lambda_n/\text{Tr} \), where \( \text{Tr} \) is a real number representing the trace of the hermitian matrix \( \xi \) or the total variance of the time series \( S(x, t) \) for the \( N \) regions. Since the eigenvectors are sorted by decreasing \( v_n \), the first eigenvector represents the largest percentage of variance. The complex eigenvectors \( Z_n(x) \) may be normalized to obtain

\[ \sum_{x=1}^{N} Z_n(x) Z_{m}^*(x) = \delta_m^\alpha \lambda_n \delta_{mn} \left\{ \begin{array}{ll} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n. \end{array} \right. \]  

The complex spatial eigenvector \( Z_n(x) \) is used to compute the spatial distributions of amplitude \( A_n(x) \) and phase \( \phi_n(x) \) of the mode using

\[ A_n(x) = |Z_n(x)| = \sqrt{Z_n(x) Z_{n}^*(x)} \] and
\[ \phi_n(x) = \arg[Z_n(x)]. \]  

The normalized spectrum (the principal component), giving the time–frequency characteristics of the mode, is defined as

\[ P_n(k) = \frac{1}{\lambda_n} \sum_{x=1}^{N} Z_n(x) \tilde{F}(x, k), \]  

with

\[ \sum_{k=1}^{k_2} P_n(k) \tilde{P}_{n}^*(k) = \delta_n^\alpha. \]  

To interpret the relative phase field, the spectrum \( P_n(k) \) must be characteristic of a quasi-periodic feature. If a characteristic timescale \( \tau \) can be extracted from this spectrum, a difference of \( \pi \) in the relative phase of two regions represents a lag of \( \tau/2 \) between the fluctuations of the two regions. Appendix A gives an alternative computation method of \( P_n \) and \( Z_n \) that saves computation time when the number of harmonics considered \( (k_2 = k_2 + 1) \) is smaller than the number of regions \( (N) \).

The temporal evolution of the signal of the mode is given by

\[ C_n(t) = \sqrt{2} \sum_{k=1}^{k_2} P_n(k) e^{2\pi ikt/T}. \]  

With this definition, the \( C_n \) are orthogonal:

\[ \frac{1}{2T} \sum_{i=1}^{T} C_n(t) \overline{C_m(t)} = \delta_{nm}. \]  

The input filtered and centered signal \( \tilde{S}(x, t) \) is recovered by computing the real part of complex series:

\[ \tilde{S}(x, t) = \sum_{n=1}^{N} Z_n(x) C_n(t). \]  

The imaginary part of \( \tilde{S}(x, t) \) is the Hilbert transform of \( S(x, t) \) (Thomas 1969; Horel 1984). From this Hilbert transform, one can obtain a spatial distribution of amplitude \( A_n(x) \) and phase \( \phi_n(x) \) at each time step \( t \) using

\[ A_n(x) = |\tilde{S}(x, t)| \] and
\[ \phi_n(x) = \text{arg}[\tilde{S}(x, t)]. \]  

These distributions are what we call instantaneous spatial structures (ISSs) that have a meaning only for a quasiperiodic signal. It can be shown (appendix B) that for a CEOF analysis, the first eigenvector (i.e., largest percentage of variance) has a spatial structure that minimizes the distance relative to the ISS ensemble of the considered time series. For example, if the input signal has the form \( S(x, t) = a(x)b(t) \cos[\omega t + \varphi(x)] \), the whole series is a single mode, explaining 100% of the variance, which can be described at any time step by a single amplitude field \( a(x) \) [apart from a spatially uniform factor \( b(t) \)] and a single relative phase field \( \varphi(x) \) (apart from a spatially uniform phase shift \( \omega t \)). Invariant ISSs thus have the same relative phase distribution and the same amplitude distributions apart from a constant factor. If the ISSs are almost invariant over the period of time considered, the first eigenvector will explain a large percentage of variance. If the ISSs are very variable with time, the percentage of variance of the first eigenvector will be small. In this case, the spatial structure will be either an average structure not representative of any of the modes present along the time series or a structure representative only of some of the most energetic modes.

A particular mode representing a given oscillation is present in a time series only if consecutive ISSs are similar over a sufficient time window (at least one oscillation). For a long time series, the spatial structure extracted from a mean analysis (e.g., composite, cross-spectral, EOF) is mainly affected by the strongest and/or the more reproducible ISSs. In order to avoid a mixing between nonorthogonal persistent modes that suc-
ceed one another in time, it is possible to compute a series of CEOFs on time windows sufficiently small enough to contain relatively uniform ISSs. This is the principle of the LMA.

For an original time series with $T$ time steps, a series of $M = (T - L)/\Delta L + 1$ CEOF analyses is performed where $L$ is the number of time steps of the time window considered for each analysis and $\Delta L$ is the lag between two analyses. The value chosen for $L$ depends on the expected duration of a mode. The sensitivity of the results with regard to the choice of $L$ and $\Delta L$ will be discussed in the following sections. By definition, $L$ will be relatively small, and it becomes necessary to weigh the time series using a Welch window to minimize end effects and to conserve as much as possible the spectral characteristics of the time interval under study. For the analysis number, $m$, the input time series will be

$$S_m(x, t) = S[x, t + (m - 1) \Delta L] W(t),$$

with $1 \leq t \leq L$, $1 \leq m \leq M$, and

$$W(t) = 1 - \frac{(t - 0.5) - 0.5L}{0.5L}^2.$$  (14)

An important effect of the Welch window is also to overweight the ISSs on the central part of the time window relative to the extremities. The Welch window thus reduces the influence of possible strong oscillations at the beginning or end of the time window. These strong modes are detected in previous or later local analyses.

Only the first CEOF will be retained for each local analysis $m$; hereafter the mode represented by this first eigenvector $Z(x)$ is called a local mode (LM). The variance percentage $\gamma_m(m)$ of this first eigenvector will depend on the stability of the ISSs along the time window $L$ (see appendix B). For a large variance percentage, the ISSs are almost invariant and the LM is thus persistent. The time evolution of the variance percentage may thus be used to reduce the number of modes. Each local maximum in the variance percentage series represents an interval for which the ISSs are more uniform than for nearby intervals. The selection of local maxima of the variance percentage makes it possible to reduce the $M$ LM into a subgroup of principal modes (PMs).

3. LMA of an idealized signal

This section aims to highlight the basic properties of the LMA. To this end, the LMA is applied to an idealized signal representing five periodic modes that succeed one another in time over 120 regions (Fig. 1a). The signal $S(x, t)$ is defined as

![Figure 1](image_url)
and after time step 300 (Fig. 1).

series is extended by 40 time steps before time step 1 maxima for the different time window lengths, the time
59, and 53 modes. To detect the variance percentage 60, 90, and 120 and a lag performed with different lengths of the time window
between the modes. For this example, the LMA is
where

\[ W \]

The variable \( W_{\alpha}(t) \) represents the Welch window with
a length of 60 points applied only to avoid abrupt changes between the modes. For this example, the LMA is
performed with different lengths of the time window \( L = 60, 90, \) and 120 and a lag \( \Delta L = 5 \), giving \( M = 65, 59, \) and 53 modes. To detect the variance percentage maxima for the different time window lengths, the time series is extended by 40 time steps before time step 1 and after time step 300 (Fig. 1).

The time evolution of the total variance \( \text{Tr}(\zeta) \) for each analysis has strong fluctuations for short time windows but is nearly constant for a time window of 120 time steps (Fig. 1b). Nevertheless, the variance percentage of each LM always has strong and coherent fluctuations with maximum values for time windows centered at time steps \( t = 30, 90, 150, 210, \) and 270 (Fig. 1c). There are thus five PMs extracted for time windows centered on time steps corresponding to the maximum amplitude of
the five input modes. For \( L = 90 \), these time windows, composed by more uniform ISSs, correspond to analysis number \( m = 6, 18, 30, 42, \) and 54 [Eq. (13)]. Note that maximum variance percentages are smaller for larger time windows because of the larger mixing with the other modes. For this idealized signal, the variance percentage of the five PMs is, however, larger than 0.9 for all considered time windows. For analyses corresponding to a transition between two modes (time window centered at \( t = 60, 120, 180, 240 \)), the variance percentage is minimal because of the strongest mixing between two different successive modes at the beginning and end of the time window.

In order to inspect the stability of the LMs from one analysis to another, we use the normalized distance \( d^2(m) \) between the first eigenvector of the analysis \( m \) and the first eigenvector of the analysis \( m + 1 \). From Eq. (B5) this normalized distance is defined as

\[
d^2(m) = 1 - \frac{\sum_{i=1}^{N} |Z^m_i(x)| |Z^{m+1}_i(x)| \cos[\phi^m_i(x) - \phi^{m+1}_i(x)] - \phi_{\text{min}}}{\sum_{i=1}^{N} |Z^m_i(x)|^2 + |Z^{m+1}_i(x)|^2},
\]

where

\[
\phi_{\text{min}} = \arctg \left( \frac{\sum_{i=1}^{N} |Z^m_i(x)| |Z^{m+1}_i(x)| \sin[\phi^m_i(x) - \phi^{m+1}_i(x)]}{\sum_{i=1}^{N} |Z^m_i(x)| |Z^{m+1}_i(x)| \cos[\phi^m_i(x) - \phi^{m+1}_i(x)]} \right)
\]

is the phase that minimizes the distance between the vectors. Such a minimization is necessary, since two eigenvectors that are identical except for a constant phase difference represent actually the same mode. The normalization gives a distance of 1 for orthogonal vectors and 0 for identical vectors apart from a constant phase difference. For consecutive LMs, this distance is large only for time windows at a transition between two
modes (Fig. 1d). For time windows around a maximum variance percentage, the LMs are very stable for this idealized signal. This means that the time step $\Delta L$ between two analyses could have been augmented without changing the structure of the five PMs detected.

The spectra $P_l(k)$ clearly illustrate the periodicity of the input signal for each of the five PMs (Fig. 1e). For a physical signal, this will make it possible to retrieve the spectral characteristics of the studied phenomenon. From the time evolution $C_1(t)$ [Eq. 8], it is possible to define a mean characteristic timescale $\tau$ as

$$\tau = \frac{2\pi}{R\eta}.$$  \hspace{1cm} (18)

where

$$\eta = \frac{\sum_{i=2}^{L} \Delta \phi(t) A(t)}{\sum_{i=2}^{L} A(t)}$$

$$\Delta \phi(t) = \text{arg}[C_1(t)] - \text{arg}[C_1(t - 1)]$$

$$A(t) = |C_1(t)|^2 + |C_1(t - 1)|^2.$$  \hspace{1cm} (19)

The variable $\eta$ is the sum of the phase differences between two time steps weighted by the average amplitude of these two time steps and $R$ is the number of time steps per time unit (i.e., the number of time steps per day for a characteristic timescale, or mean period, in days). Note that before this computation, it is necessary to modify the time series of the phase $\text{arg}[C(t)]$ by adding or subtracting $2\pi$ for phase jumps larger than $\pi$. From this definition of the mean period, one can also define a deviation as

$$\Delta \tau = \frac{\pi}{R(\eta - \Delta \eta)^2} - \frac{\pi}{R(\eta + \Delta \eta)^2}$$

$$\Delta \eta = \frac{\sum_{i=2}^{L} (\Delta \phi(t) - \eta)^2 A(t)}{\sum_{i=2}^{L} A(t)}.$$  \hspace{1cm} (20)

This deviation is zero for a harmonic signal and increases as the spectrum dispersion increases.

The PMs $Z_m(t)$ obtained with $L = 90$ for analyses $m = 6, 18, 30, 42, 54$ [Eq. (13)] are presented in Fig. 2a. The five PMs obtained with $L = 60$ or 120 are identical to these modes. In Fig. 2a, the regional amplitude of the PMs is proportional to the length of the segments, and the relative phase is represented by a clockwise rotation for increasing time. Note that the
phase is relative for a given mode but is independent from one mode to another. The modes of Fig. 2 are identical to the five input modes (Fig. 1a), confirming that a local maximum of the variance percentage can depict the different modes present in a time series. For example, the two PMs obtained for \( m = 40 \) and for \( m = 52 \) exhibit a wavelength of 40 and 60 regions, respectively, and an “eastward” propagation for the input signal.

A CEOF analysis over the whole time series (time steps 1–300) has been performed to compare the results to the LMA approach. The five modes extracted from the five first eigenvectors do not represent any of the input modes (Fig. 2b). Instead, the first two eigenvectors exhibit improper in-phase or out-of-phase standing oscillations between areas centered on regions 20 and 100. The three remaining eigenvectors are not interpretable. This result is due to both the constraints of maximization of the variance on the first CEOF and of orthogonality between the CEOF. This is not surprising, since the four modes that succeed one another in time in the input time series are not orthogonal.

4. Application to the ISO

The applicability of the LMA to the study of real atmospheric fluctuations is tested on the relatively well-known ISO (see Madden and Julian 1994 for a review of observational studies of the ISO). The analysis is performed on time series of the OLR coming from the National Oceanic and Atmosphere Administration, Advanced Very High Resolution Radiometer (NOAA AVHRR) measurements (Liebmann and Smith 1996). We consider daily mean values (day and night average) of the OLR for \( 5^\circ \times 5^\circ \) regions over a large tropical area (32.5°N–32.5°S, 32.5°–237.5°E) for the period of June 1974–July 1993 (year 1978 is missing). The signal is first filtered with a recursive filter (Murakami 1979), having half-power for periods 20 and 80 days.

a. Influence of the length of the time window (\( L \))

The length \( L \) of the time window is a critical parameter, since it may affect the results of the LMA, in particular the number of PMs detected. The minimum length of the time window is determined by the timescale under study. For the ISO, this timescale is roughly in the range of 30–60 days, and \( L \) must not be smaller than 60 days. The maximum value of \( L \) is determined by the expected maximum duration of a mode. For the ISO, which depends strongly on season (Lau and Chan 1988; Salby and Hendon 1994), we may not expect mode duration larger than 6 months. The length of the time window \( L \) for the ISO must thus be in the 60–180-day range. To test the influence of \( L \) on the detection of the modes, the LMA is applied with \( L = 60, 90, 120, 150, \) and 180 days. All analyses are performed with \( k_1 = 1 \) and \( k_2 \) corresponding to the 10-day harmonic [Eq. (1)]. For all analyses, the lag \( \Delta L \) between two analyses is 10 days [Eq. (3)]. The number of PMs decreases strongly from 135 for \( L = 60 \) to 55 for \( L = 180 \) (Table 1). Since the length \( L \) of the time window is chosen subjectively in a given range, the results of the analysis must not be too dependent on this length. This means that all the PMs extracted for a large time window must also appear for smaller time windows. This also means that the PMs added by the use of a small time window must be less representative (smaller variance and/or variance percentage) than the PMs that already appear for larger time windows. The time evolution of the variance percentage for different time window length is shown in Fig. 3a, where A PM is obtained for each maximum of the variance percentage.

For a large time window, two or more energetic modes may merge into a single average mode that does not correspond to any real mode (i.e., to a series of uniform ISSs) present in the time window, thus giving the same shortcomings as those for an average CEOF. On the other hand, if a PM obtained with a long time window is similar to a PM obtained with a short time window, it effectively represents a real mode. To examine this point, we compute the normalized distance [Eq. (16)] between the eigenvectors of the PMs obtained with different \( L \). The eigenvectors are first normalized by dividing each component by the square root of their eigenvalue [cf. Eq. (3)]. If this distance is zero, the spatial distribution of amplitude and phase are strictly equal apart from the normalization factor for the amplitude and from a constant phase difference for all regions. The lowest agreement between the different \( L \) is obtained for \( L = 60 \). In this case, there are, however, always more than 96% of the PMs that have a distance smaller than 0.15, and the worst pair has a distance of only 0.27 (Table 1). Note that for each comparison, only one pair has a distance larger than 0.2 and corresponds to one of the less persistent modes. In addition, distances lower than 0.3 correspond to modes that are still similar. Results from Table 1 thus show that all the modes detected for a given \( L \) also appear for a smaller \( L \), at least for the range of \( L \) considered here. This is in large part related to the Welch window that maximizes the signal in the center of the time window. For this analysis, there

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<th>( L ) (days)</th>
<th>60</th>
<th>90</th>
<th>120</th>
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<td>PMs</td>
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<td>60</td>
<td>—</td>
<td>0.18 (99)</td>
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<td>0.24 (96)</td>
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<td>90</td>
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<td>150</td>
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is thus no merging of persistent modes into a single PM that does not correspond to any real mode, even for the largest time window of 180 days.

As the time window length increases, a decreasing number of PMs are detected. One may thus question the significance of the modes lost by the use of a longer time window. Note that when comparing the results for $L = 60$ and $L = 180$, only six PMs are lost because of the loss of time steps near the boundaries of the time series. The decreasing number of PMs as the length of the time window increases is primarily due to the preponderance of the most energetic mode inside a time window. As the time window increases, some PMs (local maxima of the variance percentage) are smoothed out, since the first eigenvector will always represent the most energetic mode even if this mode is located near the edges of the “running” time window (despite the weighting by the Welch window). This means that a persistent (variance percentage) mode may not be detected as a PM if it occurs together with a more energetic (variance) mode in the same time window. Two equivalent modes (variance and variance percentage) may thus either vanish or persist for longer time windows, depending on their vicinity. This is, for example, the case for the somewhat persistent mode of December 1987 (Fig. 3a), which is smoothed out for $L = 180$ because of the very large variance in March 1988 (Fig. 3b). At the beginning of 1989, the two PMs also merge into a single one for $L = 180$ only because of the higher persistence of the January mode (variances are equivalent). Note that the total variance represented in Fig. 3b is not very dependent on the time window length and is equivalent to a temporal running mean of the OLR variance of the filtered signal. The Welch window weighting reduced the variance by roughly a factor of 2 for a signal with uniform variance over the time window.

In order to quantify the loss of PM as $L$ increases, it is convenient to consider the most persistent modes (MPMs) obtained using each time window length. These MPMs are the PMs with the largest variance percentage. They represent the most significant modes and should thus be almost independent of the length $L$ of the time window subjectively chosen in a “reasonable” range (60–180 days for the ISO). For the ISO, the 20 MPMs are almost equivalent for all $L$ (Table 2) and appear around the same date (Fig. 3a). As expected, the weakest agreement is obtained comparing $L = 60$ and $L = 180$ but with only four different MPMs. An inconsistency between the MPMs obtained with different $L$ is due either to somewhat persistent modes that are smoothed out as $L$ increases or to different arrangements of the
variance percentage of the PMs. When considering a number of MPMs equal to the total number of PMs obtained with a given time window, the number of inconsistent pairs relative to a smaller time window simply represents the number of smoothed out MPMs. Results of Table 3 show that most of the modes smoothed out when the time window length increases are modes representing a small variance percentage. For example, among the 80 PMs added when reducing \( L \) from 180 (55 PMs) to 60 days (135 PMs), only 18 are in the 55 MPMs obtained with \( L = 60 \). When the reduction of the time window is less important (e.g., from 150 to 120 or from 120 to 90), the proportion of inconsistencies becomes much smaller (5/68 and 7/86 respectively).

This section shows that in many aspects, the result of the LMA is not very dependent on the length \( L \) of the time window chosen if it is in a reasonable range. All the modes that appear for a given length of the time window also appear for a shorter time window. The MPMs are also very similar for all time windows. The only problem is the loss of somewhat persistent modes, particularly when more than 20 MPMs are considered for the ISO. This certainly is an inevitable problem when a window is used to isolate local features. There is thus no ideal time window length, and some tests have to be made for each application of the LMA. Note that the time evolutions for the mean period [Eq. (18)] of the PMs are actually very similar for all \( L \) except for \( L = 60 \), where the mean period is generally smaller (Fig. 3c). This is mostly due to the poor spectral resolution with such a small \( L \), especially for the harmonics corresponding to large periods. A time window shorter than 90 days has then to be avoided in order to keep sufficient spectral resolution. In the remainder of this study, a window of \( L = 120 \) days will be used because it is large enough to give a sufficient spectral resolution and small enough to avoid the loss of persistent modes due to the temporal distribution of the variance.

### b. The ISO modes

The temporal evolution of the variance percentage (Fig. 3a), the total variance (Fig. 3b), and the mean period (Fig. 3c) of the ISO show some interannual and seasonal variations. The persistence of the ISO (Fig. 3a) is larger during Northern Hemisphere (NH) winter, with 10 of the 20 MPMs occurring between January and March. As previously depicted by Lau and Chan (1988), there is also a tendency for a smaller persistence and total variance for NH winter during ENSO years of 1977–78, 1982–83, and 1986–87. Note, however, that there are persistent modes during NH winter 1991/92. The most striking feature is the strong seasonal variation of the total variance of the OLR (Fig. 3b) over the considered region, with a clear maximum between January and March. There is also a larger NH winter variance after 1984 that leads to a larger seasonal variation on the variance.

Regional maxima of the mean variance of the PMs have comparable values for NH winters and summers (Fig. 4). This is also true for both the variance of the filtered signal and the maximum regional variance of the modes. The NH winter maximum is thus not due to stronger regional perturbation of the convection but mostly to a more spatially extended perturbation with amplitude maxima located over the Indian Ocean, northern Australia, and the western Pacific. For NH summer, the ISO is active only in the northern Indian Ocean and in the northwestern Pacific Ocean. Note that a large part of the difference between the variance of the filtered signal and the mean variance of the modes is due to the Welch window weighting that divides the variance by nearly a factor of 2. The remaining part of the difference is due to the reduction of the local variance for each mode because it represents the part of the local signal that is coherent at large scale. The distribution of the variance of the filtered signal is close to the distribution of the variance of the OLR in the 20–70-day band obtained by Lau and Chan (1988) for years 1974–86. The distribution of the variance of the modes has nearly the same shape, showing that most of the variability at this timescale is related to coherent perturbation of the convection at large spatial scale. The maximum regional variance of the modes has very large values compared to the mean variance. This shows the strong variability of the amplitude and location of the convective perturbation associated to the ISO and thus the variability of the structure of the different modes. The distribution of the maximum regional variance also shows that the maximum perturbation of the convection by the ISO is not located on the equator but rather around 10°–15° in the summer hemisphere. An interesting feature is the systematically smaller variance over land for southern India as well as for the different islands of the Maritime Continent. The only exception is northern Australia during NH winter, showing the strong interaction between the ISO and the Australian monsoon.

### Table 2. Number of pairs of similar PMs between time window length \( L_1 \) and \( L_2 \) when considering the 20 MPMs.

<table>
<thead>
<tr>
<th>( L_2 )</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 = 60 )</td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>( L_1 = 90 )</td>
<td>—</td>
<td>18</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>( L_1 = 120 )</td>
<td>—</td>
<td>—</td>
<td>19</td>
<td>17</td>
</tr>
<tr>
<td>( L_1 = 150 )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>18</td>
</tr>
</tbody>
</table>

### Table 3. Number of pairs of similar PMs between time window length \( L_1 \) and \( L_2 \) when considering a number of MPMs equal to the total number of PMs (in parentheses) obtained with \( L_2 \).

<table>
<thead>
<tr>
<th>( L_2 )</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 = 60 )</td>
<td>86 (103)</td>
<td>67 (86)</td>
<td>49 (68)</td>
<td>37 (55)</td>
</tr>
<tr>
<td>( L_1 = 90 )</td>
<td>—</td>
<td>79 (86)</td>
<td>58 (68)</td>
<td>40 (55)</td>
</tr>
<tr>
<td>( L_1 = 120 )</td>
<td>—</td>
<td>—</td>
<td>63 (68)</td>
<td>43 (55)</td>
</tr>
<tr>
<td>( L_1 = 150 )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>46 (55)</td>
</tr>
</tbody>
</table>
FIG. 4. (a) Variance (W m$^{-2}$)$^{2}$ for the NH winter (Dec–May): (top) for the filtered OLR time series; (middle) average variance $A^2(x)$ [Eq. (8)] for the PM; (bottom) maximum values of $A^2(x)$ obtained for the PM. (b) As for (a) but for the NH summer (Jun–Nov).

In agreement with Anderson et al. (1984) and Cadet and Daniel (1988), there is no evident seasonal variation of the period of the oscillation in the first half of the time series (Fig. 3c). The period exhibits, however, coherent seasonal cycles after 1988, with a shorter timescale of the oscillation during NH summer. Also, in agreement with Gray (1988), Lau and Chan (1988), and Kuhnel (1989), the timescale of the oscillation tends to be smaller during ENSO years (Fig. 3c). As for the amplitude and the persistence, the 1991–92 ENSO appears as an exception when compared to the three other relatively strong ENSO events. Note that the period represented in Fig. 3c corresponds to the mean period for the coherent perturbation of the convection at a large spatial scale and is thus more representative than any local spectrum.

1) AVERAGE MODES

Average modes for each season (NH winter from December to May and NH summer from June to November) may be computed considering the mean cross-spectral matrix [Eq. (A4)] among all the time windows corresponding to the PMs (i.e., 45 PMs for winter and 41 PMs for summer). The first and second eigenvectors obtained from the seasonal mean cross-spectral matrix thus give an idealized average seasonal structure of the ISO. The LMA makes it possible to test the significance of these average modes in regard to the modes that actually succeed one another in time. To this end, the normalized distance [Eq. (A6)] between the average modes (i.e., the first and second eigenvectors of the mean analysis) and each PM (i.e., the first eigenvector representing a given PM) has been computed.

For the NH winter season, the first mean eigenvector (Fig. 5a) represents an eastward propagation that corresponds well to the mean structure of the ISO already reported in previous studies with, in particular, a phase opposition between the western Pacific Ocean and the Indian Ocean (Zhu and Wang 1993). Among the 45 PMs used to compute the mean structure, only 7 (15) modes have a normalized distance smaller than 0.3 (0.4) relative to this first eigenvector (Fig. 6a). A distance larger than 0.4 clearly means that the spatial structures are very different, as demonstrated by the two modes of winter 1989 that have equal distance of 0.4–0.5 relative to both the first and second eigenvectors. The mode of the first mean eigenvector is thus representative of, at best, 30% of the NH winter ISO modes mainly in the second half of the time series because of the larger amplitude after 1984. The second mean eigenvector for NH winter (Fig. 5b) represents an in-phase oscillation between the Pacific Ocean and Indian Ocean and is clearly the result of the orthogonality constraint. This second mean eigenvector does not represent any PM (Fig. 6a).

For the NH summer season, the first mean eigenvector (Fig. 5c) represents a phase opposition between the northwestern Pacific Ocean and the northern Indian Ocean, with a northward propagation already described by Lau and Chan (1986). As for NH winter modes, the second mean eigenvector (Fig. 5d) is clearly a result of the orthogonality constraint. Among the 41 PMs used to compute the mean structure, only 3 (9) modes have a normalized distance smaller than 0.3 (0.4) relative to the first eigenvector (Fig. 6b). The mode of the first mean eigenvector is thus representative of, at best, 20% of the NH summer ISO modes. The second eigenvector is close to only one mode in September 1984.
FIG. 5. (a) Distribution of amplitude [Eq. (4)] and relative phase [Eq. (5)] of the first eigenvector obtained when considering the mean cross spectral matrix among the 45 time windows corresponding to the PMs of the NH winter (Dec–May). The amplitude of the mode is proportional to the segment length and the relative phase of the mode is represented as the angle of the segment increasing clockwise with time (e.g., eastward propagation for a segment rotating clockwise toward the east). Segments are not drawn for an amplitude lower than 1 W m\(^{-2}\). Isolines represent the mean OLR over the 45 time windows (210 W m\(^{-2}\) dashed and 230 W m\(^{-2}\) solid). (b) As for (a) but for the second eigenvector. (c) As for (a) but for the first eigenvector over the 41 time windows corresponding to the PMs of the NH summer (Jun–Nov). (d) As for (c) but for the second eigenvector.
2) Example of Persistent Modes Not Represented by the Average Modes

The mode of November 1985 is presented here as an example of a persistent mode (Fig. 3a) that can be extracted using the LMA but that does not resemble the first or second mean eigenvector obtained for either the summer or winter season (Fig. 6). This mode is characterized by an in-phase oscillation of the deep convection between the northern Indian Ocean and the South Pacific Convergence Zone (SPCZ) and a phase quadrature with the western equatorial Pacific (Fig. 7a). This particular structure of the oscillation stands for two–three oscillations (Fig. 8) and is thus one of the most persistent modes in the series. Its mean period is smaller compared to surrounding modes and is one of the smallest periods obtained for this season (Fig. 3c). The study of the physical origin of this mode is outside the scope of this paper; however, it is interesting to note that this mode appears at the end of the Indian monsoon and thus corresponds to the transition phase between the summer and winter regime above this region.

The PM of January 1992 (Fig. 7b) corresponds to an El Niño episode (Fig. 3c) and poorly resembles the first or second mean eigenvector obtained for either the summer or winter season (Fig. 6). The convective perturbation is weak over the Indian Ocean, and the eastward propagation of the perturbation that can be seen over the Pacific just south of the equator is very irregular. This mode exhibits an in-phase variation of the convection above Australia and the central Pacific and a phase opposition with the SPCZ region in the southwestern Pacific. The phase opposition between the central and western Pacific stands for at least two oscillations (Fig. 9). Over Australia, however, the timescale of the oscillation is shorter and the phase opposition is relative to the SPCZ stands for only one oscillation. The inhomogeneity of the period of the oscillation for the three main regions of activity is well depicted by the relatively large value (7.8 days) of the deviation of the period [Eq. (20)]. This value is smaller (2.4 days) for the more homogeneous mode of November 1985.

3) The Eastward Propagation During NH Winter

Results from the LMA make it possible to inspect the propagation of the convective perturbation for the NH winter modes. This point is especially interesting because the propagative or stationary nature of the intra-seasonal convective perturbation remains controversial (Zhang and Hendon 1997). In the average NH winter modes, both tendencies are clearly present (Fig. 5a), and it is interesting to study the variability of the propagative characteristic from one mode to another. Here, we consider all the modes (26 modes) obtained between January and March that correspond to months with maximum amplitude and persistence (Fig. 3). As shown in Fig. 4, maximum perturbations are located between the...
equator and 15°S. For each longitude \( x \), the mean phase and the mean variance of each mode \( m \) over this latitude band was thus computed as

\[
\overline{\phi_m}(x) = \arg(\overline{Z_m}(x)) ; \quad \overline{V_m}(x) = |\overline{Z_m}(x)|^2,
\]

where the overbar represents the average over the 5° × 5° regions centered at latitudes 2.5, 7.5, and 12.5°S. The propagation time \( \Delta L_m(x) \) at each longitude \( x \) (with a step of 5°) is thus computed by a finite centered difference as

\[
\Delta L_m(x) = -\frac{\tau_m}{2\pi} \frac{1}{2} \left( \overline{\Phi_m(x+1)} - \overline{\Phi_m(x-1)} \right),
\]

where \( \tau_m \) is the mean period in days for the mode \( m \) [Eq. (18)]. The values \( \Delta L_m(x) \) correspond to the propagation time for 5° of longitude. The histogram of the \( \Delta L_m \) values obtained from the 26 modes at each longitude is reported in Fig. 10a (values of \( \Delta L_m \) larger than 5° are added in the first and last rows of the histogram). Values of \( \Delta L_m \) are generally between 0 and 2 days (5°) showing a preferred eastward propagation with a speed larger than 3 m s\(^{-1}\). Stable \( \Delta L_m \) are obtained over the Indian Ocean with a propagation speed between 3 and 6 m s\(^{-1}\) and over the Maritime Continent with a clear stationary perturbation between 120° and 140°E. Over the Pacific Ocean, the \( \Delta L_m \) are more variable from one mode to another with, however, a tendency for a stationary perturbation between 190° and 220°E.

Having this variability in mind, it is possible to compute an average \( \Delta L(x) \) over the 26 modes. Since the phase information is more relevant if the regional amplitude of the mode \( \overline{V_m}(x) \) is large, we have also computed the average \( \Delta L(x) \) at each longitude with two different thresholds in the regional variance \( \overline{V_m}(x) \) at 50 (W m\(^{-2}\)) and 100 (W m\(^{-2}\))\(^2\). We thus obtained three sets for the longitudinal distribution of \( \Delta L \). To give a better reading of the results, it is convenient to compute the average lag in days \( L(\ell) \) between the longitudes by integrating \( \Delta L(x) \) between the western longitude (37.5°E) of the considered region and the longitude \( \ell \):

\[
L(\ell) = \sum_{x=37.5}^{\ell} \Delta L(x).
\]

This lag has been centered relative to the lag at \( \ell = 127.5° \), that is, the lag at 127.5° is zero. Note that this normalization does not mean that \( \ell = 127.5° \) is a reference region for the computation of the lag. In fact, the lag was computed only from the local information given by Eq. (22) and the integration and normalization are performed only to give a better reading of the results.
Strong perturbation of the convection (Fig. 10b) and eastward propagation (Fig. 10c) are confined between 80° and 190°E, around the Maritime Continent. West of 60°E and east of 210°E there is a westward propagation of the convective perturbation. Even for this average over 26 modes, the propagation is not uniform between 80° and 190°E. In particular, as already shown on Fig. 10a, there is a stationary perturbation roughly between 120° and 140°E corresponding to the center of the Maritime Continent for which the mean perturbation of the convection is the highest (Fig. 10b). There are two breaks in the convective perturbation at the western and eastern limits of the Maritime Continent (110° and 140°E). Note that the stationary character roughly between 150° and 160°E is not significant because of the strong variability of the ΔL over these regions (Fig. 10a and the error bar on Fig. 10c). The lag L(ℓ) is not sensitive to the amplitude threshold over the Indian Ocean and the Maritime Continent. This shows a relative homogeneity in the propagation of either strong or weak convective perturbations for these longitudes. In contrast, the eastward propagation over the western Pacific is dependent upon the amplitude of the convective perturbation, with a propagative character farther east as the local amplitude increases and a stationary character (or even a westward propagation) (Fig. 10a) east of 150°E for the weakest modes.

These results show the importance of the Maritime Continent and Australia in the propagation and amplitude of the intraseasonal convective perturbation during NH winter. In most cases, the convection first developed over the Indian Ocean around 60°-70°E and then rapidly expanded west and east. This extended perturbation propagates eastward, has a maximum amplitude around 90°E, and decays rapidly at the longitude of the Indonesian archipelago. This perturbation triggers (or is associated with) a strong stationary fluctuation of the convection south of the Maritime Continent (Fig. 4), with
an evident relation with the Australian monsoon. A few days later, a strong convective perturbation may be generated around 160°E, which propagates eastward to 200°–210°E. However, the intraseasonal convective perturbation over the Pacific has various propagative properties, including westward propagation or stationary oscillations.

5. Summary and conclusions

This paper presents a new approach to detect and characterize different modes of intermittent oscillations. This approach, the local mode analysis (LMA), is based on the detection of different modes that succeed one another in time along a time series. As mentioned in the introduction of this paper, what we call different modes of an oscillation may also characterize different types of oscillation with different physical mechanisms. The main objective of such an approach is to identify the different structures of a given oscillation in order to better understand its physical origin or to test the applicability of different theoretical hypotheses. This approach makes it possible to detect the dates for which especially strong and persistent oscillations are present and to characterize the structure of these modes. From a geophysical point of view, results of an LMA may be first used to assess the general behavior of a parameter in a given time frequency range by inspecting the time evolution of the variance, the percentage of variance, and the period. This makes it possible to detect any trends (e.g., interannual, seasonal) in the characteristics of the phenomenon. The knowledge of the different spatial structures given by the LMA makes it possible to separate different types of oscillation. A group of modes conforming to a given criteria such as a particular phase relationship between different regions (e.g., propagation, phase opposition) may be considered for a series of case studies or a composite analysis. The LMA also makes it possible to test the representativity of a mean structure in regard to actual modes that succeed one another in time.

From a mathematical point of view, the method is based on the computation of a series of successive complex EOFs (CEOFs) on relatively short time windows of the whole time series. We have shown that the percentage of variance explained by the first eigenvector is a very useful parameter, since it characterizes the persistence of a given spatial structure of the oscillation. Selecting only the modes represented by a local maximum of the variance percentage, a subset of Principal Modes (PMs) may thus be extracted. This method appears to be robust, since the results are not very de-
Fig. 10. Longitudinal distributions of phase and variance on the average for the latitude band (0°–15°S) and for the 26 modes obtained between Jan and Mar. (a) Histograms of the propagation time ΔL; (b) mean variance; (c) average lag for all the modes and for regional variance larger than 50 (W m⁻²)² or 100 (W m⁻²)². The error bar represents the standard deviation of the ΔL for regional variances larger than 50 (W m⁻²)². Land areas in the latitude band 0°–15°S are shown in (b) and (c).

The input time series must be in a given timescale in order for the relative phase lag between regions to be interpretable. Filtering is thus often required. A spectral range that is too wide may also lead to a mixture of different processes at different timescales on a single mode. This last point may be tested on the deviation of the period (Eq. 20).

- The length of the “running” time window must be chosen in an interval that gives a sufficient spectral resolution for the lowest boundary and that avoids a mixture between modes for the largest boundary. The maximum length of the time window corresponds to...
the expected maximum duration of a given mode. The length of the time window may be adapted in this “reasonable” interval to get modes that are more or less persistent. A reduction of this length tends to add PMs that are less persistent.

Basically, the LMA detects all the time windows for which there exists a persistent spatial organization of the fluctuations of a given parameter in a given frequency range, and this is for the entire studied area. For this reason, the geographical area considered must be concentrated in the location of the phenomenon studied. If the area is too large, the central date of the PMs and the structure of the modes may be influenced by fluctuations unrelated to the studied phenomenon. Also, the LMA does not give direct information on possible superpositions of different perturbations in the same time window. Only the resulting field will be extracted, and a physical interpretation of the mode is necessary to identify a possible mixing of different perturbations. The LMA identifies well modes that succeed one another in time over the whole studied area but do not necessarily separate modes that are combined in a given time window.

The existence of a particular mode does not guarantee that its particular spatial structure is caused by a physical interaction between the different regions. It simply proves that such a spatial organization may exist with some persistence. A reproducible mode, that is, a spatial structure that appears on several PMs, is certainly more interesting than a single mode, from a meteorological point of view, as a particular mechanism to study or because it may be used to improve the forecast at the corresponding timescale. A reproducible mode may appear in a mean composite, cross spectral, or EOF analysis; however, such an average (single) mode does not always represent an oscillation that is actually present in a time series. For example, the mean ISO mode for the NH summer represents well some properties of the summer modes such as the phase opposition between the northern Indian Ocean and Pacific Ocean and the northward propagation of the convective perturbation; however, its idealized structure represents only very few actual ISO modes during NH summer.

The main finding concerning the ISO may be summarized as follows:

- Interannual and seasonal variations of the mean period and amplitude of the ISO show a tendency to have less persistent modes and a smaller period of oscillation during the El Niño years and during the summer.
- The maximum perturbation of the convection by the ISO is not located on the equator but rather around 10°–15° in the summer hemisphere. In addition, the local amplitude of the intraseasonal perturbation of the convection at these latitudes has similar values for summer and winter seasons.
- There is a strong variability of the spatial organization of the convective perturbation at the intraseasonal timescale. In particular, some persistent modes exhibit neither the phase opposition between the Indian and Pacific Oceans nor the eastward equatorial propagation that characterize the average modes. Mean modes for either summer or winter seasons are thus representative of only 20%–30% of the different local ISO modes.

The existence of even a single persistent mode that does not conform to the average mode, such as the ISO mode of November 1985, may be of special interest, since it may help to test hypotheses about the physical origin of a given quasiperiodic phenomenon. The existence of this mode does not prove that the in-phase convective perturbation found between the Indian and Pacific Oceans is due to a reproducible large-scale interaction between these two regions. This mode appears as a PM in the LMA simply because the three main areas (the Indian Ocean, the western Pacific, and the SPCZ) undergo an ISO of the convection at the same time. However, this mode is certainly of interest from a geophysical point of view because it proves that such a large-scale organization of the convection at the intraseasonal timescale may exist. If the particular phase relation between these regions is not due to a large-scale interaction, this mode has to be interpreted as independent development of convective oscillations above the Indian Ocean and the SPCZ. This suggests that some intraseasonal oscillations of the convection may be due only to local interactions between the ocean and the atmosphere.

Despite the various spatial structures of the intraseasonal convective perturbations, it is clear that there are preferred spatial organizations of these perturbations associated with the more persistent and the strongest modes. In particular, the eastward propagation over the Indian Ocean and the stationary oscillation over the Maritime Continent are strong characteristics of the ISO during the NH winter. Also, the eastward propagation over the Pacific Ocean appears only for the strongest perturbations of the convection above this region. This suggests that an ensemble of local conditions such as a particular SST distribution (related in particular to El Niño) or a phasing with the Australian monsoon are required to allow eastward propagation of a large convective perturbation from the Indian Ocean to the central Pacific. The fact that the Maritime Continent acts as a barrier for the propagation of the convective perturbation reinforces previous hypotheses, in agreement with observational studies (Jones and Weare 1996; Chen et al. 1996) and model analyses (Sperber et al. 1997), that this propagation may be due to local interactions between the ocean and the atmosphere. In this context, the tendency for a standing oscillation between the Indian and Pacific Oceans may be due to an additional large-scale interaction that exists for both winter and summer seasons.

The LMA approach presented in this paper appears to be a powerful tool to extract the main characteristics
of intermittent atmospheric oscillations in a given timescale. This approach provides information on an ensemble of modes that actually succeed one another in time and is thus more adapted than conventional EOF analysis to study oscillations that have variable spatial structures. The LMA provides not only the time evolution of the persistence, the amplitude, and the period of the oscillations but also the spatial structure of each mode and its degree of similarity compared to average modes. This makes it possible to extract atypical persistent modes that may be of importance to better understand the behavior of the atmosphere. First results obtained by using the LMA to study the ISO are very promising and appear to be a good basis for further analysis of the physical origin of this phenomenon.

Acknowledgments. The authors appreciated helpful discussions with François Vial about the mathematical justification of the LMA approach. We also thank Albert Hertzog, Robert Kandel, and Serge Janicot for their participation in the improvement of the legibility of the final version of this paper.

APPENDIX A

Alternative Computation Method of \( P_n \) and \( Z_n \)

To save computation time, if the number of regions \( N \) is larger than the number of harmonics \( k_s = k_x - k_y + 1 \), it becomes more rational to use the smaller \( k_x \times k_x \) spectral cross spectrum matrix \( \zeta' \) defined as

\[
\zeta'(k, k') = \sum_{x=1}^{N} \tilde{F}(x, k) \tilde{F}^*(x, k'). \quad (A1)
\]

The eigenvectors of \( \zeta' \) are complex spectra \( P_n(k) \) from which we can retrieve the spatial eigenvectors \( Z_n(x) \). For normalized \( P_n(k) \),

\[
\sum_{k=1}^{k_s} P_n(k) P_n^*(k) = \delta_n^o, \quad (A2)
\]

the \( Z_n \) obtained from the cross spectrum matrix \( \zeta \) [Eq. (1)] are exactly recovered by

\[
Z_n(x) = \sum_{k=1}^{k_s} \tilde{F}(x, k) P_n^*(k). \quad (A3)
\]

For \( \zeta' \), the \( P_n \) are the eigenvectors and the \( Z_n \) are the principal components. These \( Z_n \) have the properties of Eq. (3). The trace of \( \zeta' \) is equal to the trace of \( \zeta \), and the eigenvalues are also identical. The two methods used to deduce the \( P_n \) and the \( Z_n \) give identical results.

Note that it is not possible to deduce an average spatial structure from a mean spectral cross spectrum matrix \( \zeta' \) for several time windows. However, such an average mode may be deduced from the mean cross spectrum matrix \( \zeta \) defined as

\[
\zeta(x, x') = \frac{1}{M} \sum_{r=1}^{M} \sum_{k=1}^{k_s} \tilde{F}_r(x, k) \tilde{F}_r^*(x', k), \quad (A4)
\]

where \( M \) is the total number of time windows that are averaged. The average spatial structures are given simply by the eigenvectors of \( \zeta \).

APPENDIX B

Interpretation of the Percentage of Variance

To better understand the result of a CEOF analysis, it is important to extract the basic properties linking the percentage of variance, the eigenvectors \( Z_n(x) \), and the \( C_n(t) \) to the original time series. The relation between the percentage of variance and the correlation between \( C_n(t) \) and the time series \( S(x, t) \) is first deduced. Since \( S(x, t) \) and \( C_n(t) \) are centered, their complex correlation coefficient \( \rho_n(x) \) is

\[
\rho_n(x) = \frac{\sum_{r=1}^{T} \tilde{S}(x, t) C_n^*(t)}{\left( \sum_{r=1}^{T} |\tilde{S}(x, t)|^2 \right)^{1/2} \sum_{r=1}^{T} |C_n(t)|^2}^{1/2}. \quad (B1)
\]

Using Eqs. (9) and (10) this correlation coefficient becomes

\[
\rho_n(x) = \frac{Z_n(x)}{\sigma_n(x)}, \quad (B2)
\]

where

\[
\sigma_n(x) = \left( \frac{1}{2T} \sum_{r=1}^{T} |\tilde{S}(x, t)|^2 \right)^{1/2}. \quad (B3)
\]

The amplitude of the eigenvector \( Z_n(x) \) thus represents the correlation coefficient between the time evolutions of the region \( x \) and \( C_n(t) \), weighted by the standard deviation of the region \( x \). The phase of \( Z_n(x) \) is the lag of the correlation. Using Eq. (3) and the fact that the sum on \( x \) of the variance \( \sigma_n^2(x) \) is equal to the trace \( \text{Tr}(\zeta) \) of the cross spectral matrix, we thus obtain

\[
\sum_{x=1}^{N} |\rho_n(x)|^2 \sigma_n^2(x) \quad (B4)
\]

Since the first eigenvector \( Z_1(x) \) represents the largest \( u \), its amplitude and relative phase distributions maximize the spatial sum (weighted by the variance) of the correlation between \( C_n(t) \) and the time series \( S(x, t) \). Note that this relation is more straightforward if the original time series are normalized by dividing the \( S(x, t) \) by their standard deviation \( \sigma(x) \) prior to the analysis (Horel 1984). Here, with no normalization, the correlation is more strongly weighted by regions with large temporal variances and the mode extracted will thus be more representative of these regions. Also, if the signal
presents amplitude variations with time, the maximization of the correlation will emphasize the period during which the amplitude is large. The extracted mode is thus more weighted by regions with large variance for time periods when the amplitude is large.

It is also interesting to look at the relation between the percentage of variance and the spatial resemblance between the eigenvectors and the ISSs. To this end, it is appropriate to use the mean quadratic distance between two complex vectors $Z_n$ and $Z_o$ defined as

$$D^2 = \sum_{x=1}^{N} |Z_n(x) - Z_o(x)|^2$$

$$= \sum_{x=1}^{N} |Z_n(x)|^2 + |Z_o(x)|^2$$

$$- 2|Z_n(x)||Z_o(x)| \cos(\varphi_n - \varphi_o). \quad (B5)$$

Such a distance increases as the amplitude and phase differences between the vectors’ components increase. To establish the similarity between an eigenvector and the ISS ensemble, it is appropriate to compute the mean distance between the reconstructed ISS from this eigenvector and the original ISSs. From Eq. (10), the reconstructed ISS from $Z_n(x)$ are simply

$$\tilde{S}_n(x, t) = Z_n(x)C_n(t). \quad (B6)$$

The modes represented by these vectors are identical to the mode of the eigenvector $n$ because the spatial distributions of the relative phase are unchanged and the amplitude distributions are the same apart from a constant factor. From the point of view of the distance, Eq. (B6) rotates the eigenvector and weights its amplitude to minimize phase and amplitude differences relative to the ISS at time $t$. From Eqs. (3), (9), (10), and (B5), the distance between the reconstructed ISS $\tilde{S}_n(x, t)$ and the original ISS $S(x, t)$ at time step $t$ is

$$D_n^2(t) = \sum_{x=1}^{N} |\tilde{S}(x, t)|^2 - \lambda_n|C_n(t)|^2. \quad (B7)$$

The summation of $D_n^2(t)$ over all time steps thus gives

$$\frac{\sum_{t=1}^{T} D_n^2(t)}{2T \text{Tr}(\tilde{\zeta})} = 1 - v_n. \quad (B8)$$

The first eigenvector $Z_1(x)$ thus minimizes the mean distance between its reconstructed ISSs and the original ISSs. The difference between the different ISSs and the spatial structure of the first eigenvector is thus also minimized.

REFERENCES


