Spiral Bands in a Simulated Hurricane. Part I: Vortex Rossby Wave Verification

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ABSTRACT

An initially axisymmetric hurricane was explicitly simulated using the high-resolution PSU–NCAR nonhydrostatic mesoscale model (MM5). Spiral potential vorticity (PV) bands that formed in the model were analyzed. It was shown that PV bands and cloud bands are strongly coupled. The PV anomalies in and at the top of the boundary layer interact with friction to produce upward motion that gives rise to the inner cloud bands. The propagation properties of the PV bands were studied and found to be consistent with predictions of vortex Rossby wave theory.

In the control simulation with full physics, continuous generation of PV through latent heat release in the eyewall and spiral rainbands maintain a “bowl-shape” PV field. Inward transport of high PV by the vortex Rossby waves and the process of nonlinear mixing tend to increase the inner-core PV and in turn intensify the hurricane. On the other hand, frictional and PV mixing processes acted linearly to spin down the hurricane to a midlevel vortex in a dry run, which indicates that a monopolar PV structure is the asymptotic stable state in the absence of condensation.

1. Introduction

A hurricane is a highly axisymmetric rapidly rotating vortex. However, observations, especially revealed by radar imagery, indicate persistent asymmetric banded features within the symmetric circulation. The spiral rainbands are made up of organized intense convective cells embedded in widespread stratiform precipitation (e.g., Barnes et al. 1991; May 1996). Some of these bands are found to rotate around the center and propagate radially (e.g., Senn and Hiser 1959; May 1996; Gall et al. 1998; Reasor et al. 2000).

The spiral rainbands usually are associated with distinct local maxima of vorticity or potential vorticity (e.g., May et al. 1994; May and Holland 1999). For three-dimensional motions, the equation governing the Ertel potential vorticity (PV) is

\[ q = \alpha \mathbf{\eta} \cdot \mathbf{\nabla} \theta, \]

where \( D/dt \) is the material derivative, \( q \) the PV, \( \alpha \) the specific volume of air, \( \mathbf{\eta} = 2 \mathbf{\Omega} + \mathbf{\nabla} \times \mathbf{V} \) is the absolute vorticity vector, \( \mathbf{V} \) is the three-dimensional velocity vector, \( \theta \) the potential temperature, \( Q = D\theta/dt \) the diabatic heating rate, and \( \mathbf{F} \) the friction vector. The PV is conserved in the absence of friction and diabatic heating.

In a hurricane, intense convection in the eyewall and spiral bands releases huge amounts of latent heat. The PV is produced mainly by the projection of the vorticity vectors along the gradient of heating rate (Raymond and Jiang 1990; Schubert et al. 1999). The rate of PV generation is comparable to the intensification rate of a typical tropical cyclone (May and Holland 1999). Therefore, the wide extent of spiral bands may have significant effects on the intensity of hurricanes and tropical cyclones (Lewis and Hawkins 1982; Willoughby et al. 1984; Guinn and Schubert 1993; Montgomery and Enagonio 1998; May and Holland 1999; and others).

Although rainbands were detected in the earliest radar observations, their formation and their thermodynamic and dynamic role in hurricanes are still unresolved. Some theories exist to explain the formation of the spiral bands. Kurihara (1976), Willoughby (1978a,b), and others postulated the horizontally propagating internal gravity wave theory. Fung (1977) and Shapiro (1983) related the formation of the bands to boundary layer flow structures. Noticing the similarities between hurricane asymmetries and large-scale Rossby waves, MacDonald (1968), Guinn and Schubert (1993, hereafter GS93), and Montgomery and Kallenbach (1997, hereafter MK97) proposed the vortex Rossby wave theory.

Vortex Rossby waves have been studied from both observational analysis and simplified models. Using 3D wind fields derived from airborne dual-Doppler radar measurements for Hurricane Olivia (1994), Reasor et
al. (2000) found that an azimuthal wavenumber 2 feature, which dominates the asymmetry in the near-core region, shows some consistency with vortex Rossby waves. GS93 demonstrated with a shallow-water model that the bands can form by only the “slow manifold” advective mechanisms. However, this issue is still controversial (e.g., Schade 1994; Guinn and Schubert 1994). MK97 further examined the properties of the azimuthally and radially propagating vortex Rossby waves and connected the vortex Rossby wave dynamics to the axisymmetrization process. Building on fluid dynamical studies, various researchers have suggested that vortex axisymmetrization on smoothly distributed vortices is a universal process of quasi-2D vortex dynamics (Melander et al. 1987; Carr and Williams 1989; Sutyrin 1989; Smith and Montgomery 1995). Based on simulations by a quasi-linear nondivergent asymmetric balance model (Shapiro and Montgomery 1993, hereafter SM93) and a shallow-water model, MK97 hypothesized that vorticity asymmetries, which arise from asymmetric processes such as moist convective forcing, can accelerate the basic-state tangential wind. To further elucidate the physics of vortex axisymmetrization, a barotropic asymmetric balance model (Möller and Montgomery 1999, hereafter MM99), a 3D quasigeostrophic model (Montgomery and Enagonio 1998, hereafter ME), and a 3D asymmetric balance model (Möller and Montgomery 2000, hereafter MM00) were used. By adding PV anomalies into the vortex to mimic the convective forcing, these studies found that an initial barotropic or baroclinic vortex can intensify via vortex Rossby wave–mean flow and wave–wave interactions in the absence of gravity waves.

Studies to date are limited by the scarcity of observational data and the assumptions of simplified models using idealized initial conditions. Recently, Liu et al. (1997, hereafter LZY97) successfully simulated Hurricane Andrew (1992) with a 3D triply nested nonhydrostatic primitive equation model MM5. Their results (Zhang et al. 2001; Yau et al. 1999) indicate vortex wavelike structures in the simulated Andrew. Since a detailed comparison of realistic model results with vortex Rossby wave theory has not yet been undertaken, it is the purpose of this paper to fill the gap. Specifically,
new 6-km grid-size simulations will be carried out with initial and boundary conditions provided by the simulation of LZY97. However, the initial conditions are simplified to allow easy identification of asymmetric wave structures. A detailed comparison of theory and model results will be made to determine the validity of the vortex Rossby wave theory.

In section 2, we summarize the model and the initial conditions used in the present study. Section 3 presents the control simulation and a verification of the existence of vortex Rossby waves. Section 4 contains a comparison of a dry experiment with the moist control experiment. Conclusions are given in section 5.

2. Model description and initial conditions

The simulations are performed with an improved version of the nonhydrostatic Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model Version 5 (MM5; see Dudhia 1993; Grell et al. 1995). There are 24 uneven \( \sigma \) levels in the vertical, with the higher resolution in the boundary layer. The Tao–Simpson (1993) microphysics scheme, which predicts cloud water (ice), rainwater (snow), and graupel, is applied to simulate moist processes explicitly. Other physical packages include a cloud–radiative interaction scheme and the Burk–Thompson (1989) high-resolution planetary boundary layer scheme. Other model details can be found in LZY97.

A high-resolution grid mesh with 6-km grid size and \( 232 \times 169 \) grid points (1386 \( \times \) 1008 \( \text{km}^2 \) zonal–meridional extent) is used. The initial conditions and time-dependent boundary conditions are provided by the simulation in LZY97. For the sake of simplicity, the model topography is removed and the sea surface temperature is held constant at 28°C. To depict more clearly the
formation of the spiral bands, the initial conditions are modified as follows. We first compute the azimuthal averages of all variables about the center of the storm in a circle with a radius of 390 km centered on the storm. Except for the horizontal winds, the initial conditions in this circle are replaced by the azimuthally averaged values. For the horizontal winds, we superposed the steering flow (Liu et al. 1999) to the azimuthally averaged winds in the circle so that the simulated storm track is close to the observed track. To avoid discontinuity in the fields, a buffer zone is prescribed extending from a radius of 312 to 390 km from the center of the storm. Within the buffer zone, the variables are given by a linear combination of the initial conditions from LZY97 and the modified values to ensure a smooth field. The model is initialized at 1200 UTC 23 August 1992, when Hurricane Andrew was in its rapidly deepening stage, and ran for 24 h. The model retains the $\beta$ effect, although preliminary simulations suggest that its influence is not significant. The reason is that the “$\beta$ gyre” is a wavenumber 1 asymmetric feature (Fiorino and Elsberry 1989), but the scale is too large (order of 1000 km) to be of importance in the current study.

Figure 1 shows the initial conditions used in the model simulations near the vortex core. Figure 1a depicts the initial axisymmetric structures of the tangential wind. The radius of maximum wind (hereafter RMW) is at 48 km and 1 km above the sea surface, and its axis tilts outward with height. The secondary transverse circulation is weak except for the strong inflow in the boundary layer, ascent in the eyewall region and strong outflow near the tropopause. The potential temperature field shows a warm core structure, a distinct characteristic of tropical systems (Fig. 1b). Strong cross-isentrope flow in the eyewall suggests that the system is strongly diabatic. Condensations occur in nearly saturated regions especially in the eyewall (relative humidity >90%). Figure 1c also shows the dry eye associated with the weak downdraft. Unlike some of the initial conditions used by ME and MM00, with a barotropic vortex and radially monotonic decreasing PV, Fig. 1d depicts a “bowl-shape” PV field (Yau et al. 1999) or “hollow-tower” PV cited by Schubert et al. (1999, hereafter S99). Large PV associated with the eyewall tilts upward and radially outward. At a constant height level, the PV field shows a horizontal annular ring structure. The radius of the PV ring is generally smaller in the lower troposphere than aloft. S99 suggested that PV mixing occurs in the annular ring due to barotropic instability, and concluded that in the absence of convective and frictional forcing the monopolar structure is the final stable state. In our studies, we find that the generation of positive PV anomalies by the convective forcing in the annular ring overwhelm the mixing dilutions, and the moist processes maintain the PV ring structure.

3. The control simulation

a. The formation of spiral bands

A 24-h numerical simulation provides a detailed picture of the formation of the spiral bands. Modeled radar reflectivities can be estimated from the rainfall rate, which in turn is related to the contents of the model-resolved hydrometeors (see LZY97, section 4). Comparisons between the model-simulated radar reflectivities and the PV fields (graphs not shown) suggest that a spiral rainband usually corresponds to a pair of local maximum and minimum PV bands or positive and negative PV anomaly bands. This configuration of the PV anomalies is caused by the local cycloidal and anticyclonic shear in the vicinity of the midlevel jets associated with the rainband (e.g., Wingham et al. 1984; May et al. 1994). Therefore, spiral rainbands can be studied from the perspective of the PV bands.

Figure 2 depicts the time evolution of the total PV...
Fig. 4. Time evolution of PV (left column), vertical velocity $W$ (middle column) and cloud water mixing ratio (right column) in the boundary layer ($z = 0.45$ km) from 4 h 20 min (top panels) to 4 h 50 min (bottom panels) with 10-min intervals. PV are contoured at $-10,$
field in the middle troposphere ($z = 6$ km) at 6-h intervals. The initial PV field is approximately axisymmetric. Weak asymmetries arise from the asymmetric environment in addition to computational errors in the initialization processes. Negative PV patches split the near-core PV into distinct bands. Positive and negative PV anomalies rotate cyclonically with the large PV core while propagating radially outward and inward. Tangential wind shear stretches the PV anomalies into spiral bands that are wrapping around the vortex core. Note that the radial wavelengths of the spiral bands tend to decrease when the bands travel outward. MK97 suggested that the radial wavenumber $k$ is a function of time $k(t)$. From their local Wenzel–Kramers–Brillouin (WKB) analysis, $k$ increases with time (or radius) for the radially outward propagating vortex Rossby waves in the vicinity of the negative radial gradient of the basic-state angular velocity. Unlike the PV mixing process in an unforced barotropic nondivergent model (S99), which leads to a monopolar PV distribution, large PV is continually generated in a concentric ring region with a diameter less than 100 km associated with the active convection in the eyewall region (cf. Fig. 2d).

The annular PV ring structure can be seen clearly in a radius–height cross section. Figure 3 shows the mean state (i.e., axisymmetric component) of PV and tangential winds after 6 h and 45 min of simulation time. When compared to the initial fields, the hurricane appears to have intensified. The maximum increase of PV is about 20 PVU ($1 \text{ PVU} = 10^{-6} \text{ kg}^{-1} \text{ K m}^2 \text{ s}^{-1}$) in the bottom of the “bowl” and 10 PVU in the boundary layer where the associated low-level maximum tangential wind accelerates by 15 to 80 m s$^{-1}$. The tilted PV tower representing the eyewall region is related to the large cyclonic shear of the tangential winds. The annular ring structure extends throughout the whole troposphere. Negative PVs in the stratosphere are related to the upward motion and concentrated cloud water and these fields are in phase. The strong coupling suggests that the thermal and dynamic factors help each other to develop. Specifically, high PV is stripped off the outer edge of the PV ring by vortex Rossby wave dynamics. In or at the top of the boundary layer, high PV (vorticity) produces frictional convergence and upward motion (i.e., Ekman pumping), with condensation occurring simultaneously in the nearly saturated environment. The latent heat release generates positive PV anomalies below the level of maximum heating, and this process reinforces the PV bands.

The vertical structure of this spiral band is depicted in Fig. 5. The banded large vorticity (PV) slants outward with increasing height. The axis of the maximum upward motion, which is close to the axis of the maximum heating, lies above the axis of the maximum vorticity (PV). This configuration is favorable for the further development of the spiral PV bands. By applying simple quasigeostrophic-like Ekman pumping theory, the vertical motion at the top of the boundary layer $W_T$ in the vorticity band region can be estimated as $W_T = \eta h_T/2$, where $\eta$ is the vertical component of absolute vorticity and $h_T$ is the depth of the boundary layer. Using $h_T = 1$ km and $\eta = 2 \times 10^{-3}$ s$^{-1}$, $W_T$ turns out to be 1 m s$^{-1}$, which is of the same order of magnitude as the simulated ascending velocity of 2 m s$^{-1}$. Air parcels rising from the boundary layer tend to accelerate upward in the spiral band.

Figures 4 and 5 depict only one event for the for-
FIG. 5. Vertical cross section along the thick solid line AB in the middle-bottom panel in Fig. 4 for (a) vertical component of the absolute vorticity at intervals of 0.5 \(10^{-3}\) s\(^{-1}\) and (b) vertical velocity \(W\) at intervals of 1 m s\(^{-1}\), valid at 4 h and 50 min model simulation time. Negative values are depicted by dashed curves. Only the levels below 10 km are shown. The thick dashed and solid lines denote the axes of maximum absolute vorticity and maximum \(W\), respectively.

formation of an inner spiral band. The simultaneous occurrence of the formation of the PV bands and the cloud bands can be found throughout the whole simulation. Given any variable that is projected into a Fourier–azimuthal space, the wavenumber zero component in the Fourier series is identical to the symmetric or basic-state field. The radial distribution of the magnitude of the total anomaly is the square root of the sum of the square of the Fourier amplitudes for each wavenumber. Figure 6 shows the evolution of the magnitude of PV anomalies and cloud water mixing ratio anomalies from the azimuthal mean fields at the same height as Fig. 4. A bimodal PV anomaly appears frequently with the inner peak located inside the PV ring (at a radius of about 20 km) and the outer peak outside. This is an expected feature from the angular-momentum invariant constraint. On careful inspection, one can relate each maximum of cloud water mixing ratio anomaly to the appearance of a bimodal PV anomaly event.

The coupling between the PV bands and the cloud bands extends up to 7 km above the sea surface (cf. Fig. 5). The vertical scale is consistent with the observation of Gall et al. (1998). Our results indicate that forced vortex Rossby wave theory is a more plausible explanation for the formation of the spiral bands. Further analysis to determine if these bands behave consistently as vortex Rossby waves in the simulated hurricane follows.

b. Vortex Rossby wave dispersion relation and wave kinematics

Applying asymmetric balance theory (SM93) and WKB analysis, MK97 have formulated a local dispersion relation for vortex Rossby waves in a two-dimensional barotropic model and an asymmetric balanced shallow-water model. MM00 further extended the derivations to a three-dimensional stably stratified barotropic circular vortex in gradient wind balance. Assuming that monochromatic baroclinic disturbances have wavelike solutions in both the radial and the vertical directions, the local dispersion relation derived by MM00 is

\[
\omega = n\bar{\Omega} + \frac{n}{R} \frac{\overline{\tilde{q}'(R)}}{\sqrt[k]{k^2 + n^2/R^2 + (\bar{\eta}m^2)/N^2}},
\]

(2)

where \(\omega\) is the local wave frequency, an overbar sign signifies mean-state quantities, and a prime the radial derivative. The azimuthal and vertical wavenumber are denoted by \(n\) and \(m\), respectively. Also, \(k\) is the time-dependent radial wavenumber \(k(t) = k_0 - n\bar{\Omega}'(R)\), \(\bar{\Omega}\) the mean angular velocity, \(\overline{\tilde{q}}\) the mean PV, \(\bar{\eta}\) the mean absolute vorticity \((\bar{\eta} = f + \tilde{\xi})\), where \(f\) is the Coriolis parameter, and \(\tilde{\xi}\) the mean inertial parameter. The reference radius is \(R\). See MK97 section 2f and MM00 section 3 for the details of the equations.

The azimuthal phase speed is

\[
C_{\phi} = \frac{\omega}{n/R} = R\bar{\Omega} + \frac{\tilde{\xi}}{\overline{\tilde{q}}} \frac{\overline{\tilde{q}'(R)}}{\sqrt[k]{k^2 + n^2/R^2 + (\bar{\eta}m^2)/N^2}}.
\]

(3)

Analogous with the planetary Rossby wave formula, the first term on the right-hand side is the advection velocity and the second term is the intrinsic wave propagation speed, which is proportional to the radial gradient of basic-state PV.

The radial group velocity is

\[
C_r = \frac{\partial\omega}{\partial k} = -\frac{n}{R} \frac{2k\overline{\tilde{q}'(R)}}{\overline{\tilde{q}} \sqrt[k]{k^2 + n^2/R^2 + (\bar{\eta}m^2)/N^2}}.
\]

(4)

From (3) and (4), it is clear that for a negative gradient of basic-state PV, a vortex Rossby wave in the form of
Fig. 6. Time evolution of the radial distribution of the magnitude of (a) the PV anomaly and (b) the cloud water mixing ratio anomaly in the boundary layer ($z = 0.45$ km).
a trailing spiral ($k > 0$) propagates in a retrograde sense relative to the tangential winds and that the corresponding radial group velocity is directed outward. Scale analysis using the numerical model data shows that the radial wavenumber is the leading term in the total wavenumbers. The radial group velocity is then proportional to $O(k^{-3})$. The outward propagating wave packets will be slowed down by the shearing effect, which tends to decrease the wavelength of the bands (cf. Figs. 2b–d). When $k$ is large enough, the radial propagation of a wave packet will cease. A stagnation radius can thus be determined at which wave–mean flow interactions can occur (MK97).

Although the above formulas are obtained by linearizing the governing equations based on a barotropic, radially monotonic decreasing basic-state PV, it is of interest to compare them to the propagation properties of vortex Rossby waves in our baroclinic annular ring PV environment. The local Rossby number for low azimuthal wavenumber asymmetries (cf. SM93, section 2) computed from the model data is generally less than unity or of the order of unity in the near-core region (graph not shown), which indicates that the asymmetric balance theory and the vortex Rossby wave dynamics should at least furnish a first-order approximation for describing the model-simulated spiral bands.

Figure 3a suggests that the azimuthally prograde (retrograde) vortex Rossby waves may be generated inside (outside) the eyewall where the radial gradient of PV is positive (negative). In addition to unstable discrete modes that can cause PV mixing (S99), radially propagating continuous spectrum vortex Rossby waves with negative (positive) radial group velocity inside (outside) the eyewall can also exist. These continuous spectrum waves can inject eddy vorticities into the eyewall leading to the reinforcement of eyewall PV (M. Montgomery 2000, personal communication).

c. Verification of vortex Rossby waves

Figure 7 depicts the PV anomalies in an azimuth–height cross section 60 km from the center. This radius is outside the tilted maximum PV tower, thus corresponding to a negative PV gradient. The upper and lower panels are, respectively, snapshots 10 min before and after the time of Fig. 3. The PV budget calculations will show that in the lower and the middle troposphere, PV-generation bands due to diabatic heating and PV anomalies are in phase. This indicates that only the magnitudes but not the phases of PV anomalies can change by external forcing. Therefore, we may assume that the solid lines A and B trace the azimuthal phase propagation of a positive PV anomaly over the 20-min period. The azimuthal phase speed obtained from the distance between two solid lines at a height of 8 km is 35 m s$^{-1}$, which is slower than the mean-state tangential wind (50 m s$^{-1}$; cf. Fig. 3b). Since the traced asymmetry is more like a wavenumber 2 feature in the middle and upper troposphere, the wavenumber 2 phase speed can be computed from (3).

The radial wavenumber $k$ is estimated from the horizontal section of the PV anomaly by assuming that the width (23 km) of the traced PV anomaly represents the half wavelength. Similarly, the vertical wavenumber $m$ is chosen with the half wavelength given by the vertical extent (10 km) of this PV anomaly. Using 60 km as the reference radius ($R$), together with $\zeta = 2 \times 10^{-3}$ s$^{-1}$, $\tau_\zeta = 0.5 \times 10^{-3}$ s$^{-1}$, and $N^2 = 1.5 \times 10^{-4}$ s$^{-2}$ at $R$, we calculated a predicted phase speed of 40 m s$^{-1}$, which is consistent with the value obtained from the graphic evaluation of Fig. 7.

Note that the radial gradient of basic-state PV does not change very much in this vertical column (cf. Fig. 3a). Given a constant $\tau_\zeta$, (3) predicts an increasing phase speed from the tropopause down to the lower troposphere due to the vertical shear of the tangential winds. Figure 7 does show the phase speed of 31 m s$^{-1}$ at 10-km height increasing to 42 m s$^{-1}$ at 4 km. These speeds are all less than the corresponding magnitude of the tangential winds.

The dispersion relation (2) also predicts the vertical phase speed ($C_{pc} = \omega/m = nC_{ps}/mR$). A crude estimate
shows that the phase propagation in the vertical direction is one order slower than in the azimuthal direction. The $C_{ph}$ depends on the sign of $n/m$, namely, the phase tilt. Usually $C_{ph}$ is positive because of the large tangential winds; therefore, a downstream tilted PV anomaly relative to the tangential winds ($n/m < 0$) propagates downward, and vice versa for an upstream tilted anomaly. The vertical phase speed obtained from measuring the mean vertical distance between the PV anomaly phase lines is $\sim 3.9 \text{ m s}^{-1}$. By taking into consideration the mean upward vertical motion of $0.8 \text{ m s}^{-1}$, the vertical phase speed computed from the formula is $\sim 3.4 \text{ m s}^{-1}$. The upward phase propagation can be recognized for the upstream tilted PV anomaly in the northwest quadrant in Fig. 7. Specifically, the positive anomaly initially concentrated at 2–4 km extends upward to 7 km with a speed of about $3.5 \text{ m s}^{-1}$ (Fig. 7b), indicating that part of the high PV in the upper levels originates in the boundary layer. We should also mention that the upward phase propagation of PV anomalies was also observed by MM00 in their 3D asymmetric balance model simulations.

To find the prograde waves, another azimuth–height cross section of the PV anomaly field at a radius of 30 km is shown in Fig. 8. At this distance, the positive radial gradient of the basic-state PV is located in regions between 6- and 12-km height. The thick solid lines C and D trace positive phase propagation with a phase speed of $27 \text{ m s}^{-1}$ at 9 km, $7 \text{ m s}^{-1}$ faster than the corresponding symmetric tangential wind. Above the boundary layer, a downstream tilted wavenumber 1 PV anomaly, spinning vertically roughly over a circle, dominates the asymmetric features. Taking $n = 1$ in (3) and estimating the radial and vertical wavelength to be 50 and 10 km, respectively, the predicted azimuthal phase speed turns out to be $23 \text{ m s}^{-1}$. A possible explanation for the small difference between the model simulation and the prediction of the theory is that the tightly wound limit ($kR \gg 1$), an assumption in the derivation of the vortex Rossby wave dispersion relation, is not strictly valid for small $R$ and relatively long radial wavelengths (small $k$). In addition, the discrete modes, if they exist, may prevent the accurate evaluation of the phase speed of the continuous mode waves from the graphs. The vertical phase speed estimated from Fig. 8 is $\sim 3.8 \text{ m s}^{-1}$, similar to the value predicted by the formula in a background with trivial azimuthal mean vertical motion.

In this control simulation, latent heat release continually generate PV anomalies, making it difficult to distinguish the individual wave packets. The verification of the group velocity is therefore left for the dry sensitivity test.

d. Symmetric and asymmetric PV

The intensity of a hurricane or tropical cyclone can be described not only by the central pressure, tangential winds, and other usual quantities but also by the near-core PV values. An intense mature hurricane is usually characterized by a hollow tower of PV with the largest PV in the eyewall region, and relatively smaller values in the eye and outside of the eyewall. Thus hurricane intensification can also be viewed as a process that increases the near-core PV. Although the vertical propagation and advection of PV anomalies involve the horizontal redistribution of PV, we have assumed that the vertical PV flux is vertically uniform except at the surface and the tropopause; that is, there is no PV accumulation at any one level in the lower and middle troposphere. Therefore, the time evolution of the symmetric PV field at 6-km height (Fig. 9) can be considered to represent hurricane intensity changes.

The continuous growth of high PV inside the RMW depicts the hurricane’s rapidly deepening stage. The radius of maximum PV, associated with the concentric convective ring, decreases with time from 40 km down to 20 km by the end of the simulation. In the absence of forcing, S99 demonstrated that such an axisymmetric ring of high PV in the absence of forcing can be barotropically unstable and in such cases vigorous mixing relaxes the ring of PV into a stable monopolar distribution. In the present study, moist physical processes are shown to maintain the PV ring structure.

Figure 10 shows the evolution of the magnitude of PV anomaly at a height of 6 km. Initially, only weak
anomalies (less than 2 PVU) around the RMW are superposed on the basic vortex (not shown). However once the simulation starts, a pseudomode quickly emerges (graph not shown; see also Smith and Montgomery 1995; MK97). Higher-wavenumber PV anomalies are generated in regions where intense convection occurs, especially in the eyewall. These newly generated PV anomalies either propagate outward by vortex Rossby waves with positive group velocities, or inward by waves with negative group velocities. Inside the eyewall region, mixing of high PV into the eye can happen as well if unstable discrete modes appear.

e. PV budget

The PV conservation law and the invertibility are the kernel of “PV thinking,” which has been reviewed by Hoskins et al. (1985). However, in a hurricane, PV anomalies are largely produced by the nonconserved physical processes, especially by latent heat release associated with precipitation. This effect is much more marked in the regions of the eyewall and spiral bands where both the vorticity and the gradient of heating are large. Surface friction has usually been considered as a negative factor which tends to spin down the vortex in the concept of Ekman pumping. In this simulation, the area-mean net effect of the friction is actually to generate positive PV anomalies in the boundary layer (not shown). In addition, we have shown that Ekman pumping leads to the initiation of convection and contributes indirectly to the generation of PV.

Using MM4 and a “partitioned PV integration” method, Stoelinga (1996) studied the role of diabatic heating and friction in a baroclinic cyclone from a PV perspective. We simplify his methodology by calculating the partitioned PV tendencies only. From the model momentum and temperature tendencies, (1) can be calculated term by term exactly. Sequential snapshots of such PV tendencies describe the contributions to PV generation from various nonconserved physical processes. The detailed PV budget study will be presented in a future paper. An overview of the results is presented here.

Calculations show that latent heat release is the leading order effect in PV generation. Figure 11 shows a set of snapshots of the PV-generation rate due to the diabatic heating effect after 6 h and 50 min of simulation time overlapped with PV anomaly contours. Clearly, the fact that the bands of PV generation are associated with spiral rainbands suggests that spiral rainbands are regions where large PV anomalies arise. In a saturated environment, the maximum heating is located at the level of the maximum ascent. In our simulation this is
about 9 km in the eyewall and slightly lower in the spiral bands (see also LZY97, Fig. 15; May and Holland 1999, Fig. 2). With upward pointing vorticity vectors below and above this level, positive and negative PV anomalies are generated, respectively. Note that below the maximum heating level, positive (negative) PV anomalies correspond to positive (negative) PV generation. This phase relation between the PV anomalies and the PV generation bands implies that spiral bands of PV are triggered by moist processes in the middle troposphere (Figs. 11a–c). In and at the top of the boundary layer, as shown previously, convection may be initiated and triggered by the PV anomalies through the Ekman-pumping effect. Above the level of maximum ascent, PV anomaly bands tend to be suppressed (Fig. 11d). This vertical structure and an animation of the evolution of the PV fields indicate that spiral PV bands are generated in the lower and middle troposphere and then propagate upward and dissipate in the upper troposphere.

Figure 11 also shows that the instantaneous PV-generation rate is very high. However, the high generation rate is mitigated by the effects of advection, surface friction, and diffusion so that the actual rate of PV increase is slower.

4. Sensitivity test

The investigation of the full-physics simulation has shown the importance of diabatic heating in the generation of PV anomalies. Positive PV anomalies are produced in the strong convection regions and then prop-
agate as vortex Rossby waves. Since these waves can exist in a simple barotropic nondivergent unforced model (MK97), we should expect the appearance of vortex Rossby waves in our primitive equation model for a disturbed hurricane vortex even without moist processes. A “dry-run” sensitivity experiment is carried out, starting from the same initial conditions as the control run, but without the release of latent heat of condensation.

### a. Vortex spindown

The basic-state PV and tangential winds at 12 h are shown in Fig. 12. The initial hurricane becomes a significantly weakened midlevel vortex. The annular PV ring evolves into a monopolar structure with the maximum of 35 PVU at an altitude of 5 km. The low-level maximum tangential wind is much weaker when compared to the control run. Another jet, centered at a radius of 80 km, is located just outside of the midlevel PV bulb.

The theory of vortex spindown under the influence of surface friction in a neutrally stratified atmosphere, proposed by Eliassen and Lystad (1977), can be applied to hurricanes as well (Snell and Montgomery 1999; Montgomery et al. 2001). Reasor et al. (2000) demonstrated that the observed spindown rate of Hurricane Olivia (1994) is consistent with the axisymmetric predictions of Eliassen and Lystad. We use the same parameters as Reasor et al. (2000, see their section 3) to estimate the frictional spindown rate of the low-level maximum tangential wind in our simulated hurricane. At 12 h, the predicted tangential wind speed is 47 m s$^{-1}$, or 18 m s$^{-1}$ slower than the initial wind (65 m s$^{-1}$). However, the simulated low-level maximum tangential wind at 12 h is 30 m s$^{-1}$. On the basis of this estimate we conclude that processes other than frictional spindown must be in operation. Vertical shear of the horizontal winds is one candidate that has been studied extensively (e.g., Gray 1968; Merrill 1988; Shapiro 1992; Jones 1995; DeMaria 1996; Bender 1997; Frank and Ritchie 1999; and others). But in our simulations, the vertical shear is weak and the shear direction is not uniform, which should not significantly weaken the vortex. S99 demonstrated a new PV mixing mechanism for weakening the maximum tangential wind during the evolution of an initial annular PV ring to a monopolar PV vortex. Similar results have also been found in a 3D numerical simulation by M. Montgomery (2000, personal communication). To determine whether the PV mixing spindown mechanism is present in our case, another sensitivity experiment with the same configurations as the current dry run but with no boundary layer friction was performed. The results (not shown) at 12 h simulation time show a midlevel PV core similar to that in Fig. 12a. The low-level maximum tangential wind decreases 15 m s$^{-1}$ to 50 m s$^{-1}$. Since the net spindown rate by friction and by PV mixing is 33 m s$^{-1}$ (18 + 15 m s$^{-1}$), a value close to the simulated total spindown rate of 35 m s$^{-1}$ (cf. Fig. 12b), these two processes appear to act almost linearly in the spindown of our vortex.

Without the release of latent heat, the thermally direct gyres disappear and the low-level frictional inflow becomes much weaker (Frank 1984). Air that converged in the surface layer flows outward just above the boundary layer instead of near the tropopause (Willoughby 1979; Montgomery et al. 2001). The upper-level outflow observed in the control run has been replaced by an inflow. Upon reaching the vortex center, the inflow splits into two branches. The upper branch returns in the stratosphere, while the lower branch leads to outflow over a wide range of levels in the middle troposphere.

### b. PV evolution

The basic-state PV evolution confirms that the annular PV structure is unstable in the absence of diabatic heating. Although the final state is an approximate PV monopole, the adjustment time for each level is different. The stabilization process is first completed...
at 3 and 5 km, the heights corresponding to two concentric rings (cf. Fig. 1d). The two large PV patches merge together while the PV maxima in the boundary layer dissipate due to surface friction. Large PV tends to become compressed in the vertical and extended in the horizontal. The horizontal expansion also appears in S99’s simulation (see their Fig. 3). A stable midlevel vortex is established by 6 h, and it remains throughout the rest of the simulation.

Like the relaxation experiments presented in MK97 and S99, continuous-spectrum and discrete unstable vortex Rossby waves generally participate in this vortex spindown process. To avoid the complication of the pseudomode, wavenumber 2 PV anomaly fields at 5-km height are shown in Fig. 13 from 6 to 24 h at 6-h intervals. At 6 h, the radial gradient of the basic-state PV is already negative everywhere, so we expect to see outward propagating vortex Rossby waves. Similar to the control run, PV anomalies travel outward, are stretched by the shear effect, and finally split into segments at distances far from the center (>80 km).

Two other characteristics are also noteworthy. The first is the leading spiral inside the RMW (Figs. 13b–d). Close inspection of Fig. 5 of MK97 and Fig. 3 of S99 shows leading spirals in the vicinity of negative PV gradients as well. The reason for the formation of leading spirals is the shear effect. When the waves are excited in a negative radial gradient of basic-state PV, the azimuthal phase speeds are everywhere slower than the tangential winds. Inside the RMW, the propagation speeds represented by the second term of Eq. (3) are nearly equal; therefore the outer part of the waves that encounters larger tangential winds will tend to travel faster than the inner part. In the control simulation, the smaller RMW is unfavorable for the formation of the leading spirals, and the model may not resolve such a
phenomenon. Since both the radial phase speed and the group velocity are negative for the leading spirals \( k < 0 \) in the monotonically decreasing PV background \( \bar{q} < 0 \), PV anomalies can be transported into the center. The consequence of this is a concentric PV center with sharp radial gradient around. Note that these leading spirals inside the RMW correspond to a downshear tilt that may not produce large algebraic growth rates (Farrell 1987; MK97). The second interesting characteristic of Fig. 13, related to the inward propagation of the PV waves, is the large PV anomalies in the center. Since the level of 5 km shown in Fig. 13 coincides with the central location of the final midlevel vortex, vertical convergence may also contribute to the accumulation of the PV anomalies.

In the middle troposphere, the nonconservation of PV is primarily due to latent heat release. In the dry run, PV waves may be considered to be unforced free waves. It is therefore possible to trace the radial movement of individual wave packets. From their experiments, MM00 found that PV anomalies initiated in the lower level will propagate upward as vortex Rossby waves. They have provided a formula to compute the vertical group velocities (see their Eq. 3.4). In our model, the vertical group velocity evaluated from the model data is about 1.5 cm s\(^{-1}\) (not shown), while Eq. (3.4) of MM00 predicted the same magnitude for the group velocity \([O(10^{-2} \text{ m s}^{-1})]\) after taking into consideration the weak mean background upward motion. With the assumption of negligible vertical group velocities, wave packets can be identified following the method proposed by MK97 by tracing the third-order radial derivative of the Fourier PV amplitude at one level (see MK97, section 2d). Figure 14 shows such derivatives for wavenumbers 1 and 2 at an altitude of 4.2 km. The shaded regions have positive values and wave crests are identified by zeros. For the sake of clarity, several smooth curves are drawn following the wave crests. As indicated by MK97 (see their Figs. 4 and 11), outward propagating wave packets clearly exist. The group velocities of the wave packets for wavenumbers 1 and 2 at \((r, t) = (70 \text{ km}, 6 \text{ h})\) are 2 and 1.6 m s\(^{-1}\), respectively. Calculations using (4) by taking \(\xi\) of \(1.1 \times 10^{-3} \text{ s}^{-1}\), \(\bar{q}\) of \(5.5 \times 10^{-4} \text{ s}^{-1}\), \(N^2\) of \(1.7 \times 10^{-4} \text{ s}^{-2}\), \(\bar{q}\) of 6.5 PVU, and \(\bar{q}'\) of \(3.3 \times 10^{-4} \text{ PVU m}^{-1}\), and assuming a radial wavelength of 70 km and a vertical wavelength of 10 km, yield 1.7 m s\(^{-1}\) for wavenumber 2. For even higher azimuthal wavenumbers, radial propagation is still observed, but the group velocities decrease with increasing wavenumber. The dependence of group velocities on wavenumber implies that PV waves are dispersive waves. Wave packets slow down as they move outward. The curvatures of the tracks suggest the existence of stagnation radii where the wave packets cease to propagate. MK97 argued that the dynamics of vortex Rossby waves that have stagnation radii is distinct from that of freely propagating gravity–inertia waves. Figure 14b also shows the inward propagation wave packets corresponding to the leading spirals inside the RMW, which tend to maintain the intensity of the large PV core of the vortex. An analysis of the energetic effects of the vortex Rossby waves on the spindown process will be reported in the future.

5. Discussion and conclusions

Spiral rainbands characterize radar and satellite imagery of hurricanes and tropical cyclones. Outward propagating small-scale (10–20 km) bands are detected in recent radar studies (Gall et al. 1998; Reasor et al. 2000). The spiral rainbands are usually associated with distinct local PV maxima (May et al. 1994). GS93 and MK97 proposed a PV–vortex Rossby wave theory to explain the formation and propagation of the bands. With limited temporal and spatial resolution, observational studies have so far failed to show strong evidence for vortex Rossby waves. In this study, numerical simulations of a hurricane are performed using MM5, with initial and boundary conditions provided by the simulation of LZY97. The initial conditions are modified to
largely remove the initial asymmetries. Fine-grain (Rossby–Ertel) PV maps are produced from the extensive model data. Small-scale spiral PV bands in the near-core region are investigated to verify the existence of vortex Rossby waves in the full-physics primitive equation model.

In the control run with full physics, the hurricane intensifies. Spiral PV bands form naturally during this intensification process. The horizontal annular PV rings on each level become concentrated as the inner PV increases. The PV budget calculations show that the PV anomalies are continually generated by nonconservative physical processes, especially in the vicinity of the eye-wall and spiral bands where strong convection occurs. Two types of vortex Rossby waves are found as a result of the reversal of the radial gradient of the basic-state PV near the eyewall. One is the set of retrograding waves outside the PV rings, which propagate azimuthally slower than the tangential winds and have outward phase speeds and group velocities. These waves tend to transport eddy momentum inward but eddy vorticity both inward and outward (MK97; Enagion and Montgomery 2001). The other type of vortex Rossby waves, found inside the PV rings where the radial PV gradient is positive, propagate faster than the local tangential winds. These prograde waves, if they are not discrete wave modes, possess outward phase speeds for trailing spirals but inward group velocities. They can flux eddy vorticity toward the eyewall to reinforce the maximum PV. On the contrary, discrete unstable modes tend to transport large PV from the eyewall into the eye and tend to diminish the PV ring through the nonlinear PV mixing. For an intensifying hurricane, the mean PV cumulating in the eyewall by the continuous-spectrum vortex Rossby wave–mean flow interaction and the nonconserved PV generation must exceed the consumption by PV mixing.

In the absence of external forcing, S99 showed that two counterpropagating vortex Rossby waves could be phase locked. Barotropic instability and PV mixing distort an annular PV ring into a monopolar PV stable state. In reality, baroclinic and diabatic effects are important for the determination of the PV structure of the hurricane. The “bowl-shape” PV field will tend to be maintained by continuous high PV generation from latent heat release in the regions of the eyewall and spiral bands.

In the dry sensitivity run, moist processes are turned off. The initial hurricane spins down quickly to a midlevel vortex. The spindown is the combined effect of the boundary layer friction and the PV mixing. The PV evolution confirms that the monopolar PV structure with monotonically decreasing radial PV gradient is the final stable state. This suggests that latent heat release plays a primary role in the intensification of the hurricane. In addition to the outward propagating trailing spirals outside the RMW, inward propagating leading spirals are observed in PV maps.

While previous works have only examined the propagation properties of the vortex Rossby waves in relatively simple models (MK97 and MM00), this study has verified the dispersion relation for vortex Rossby waves (MK97; MM00) in a full-physics model simulation dataset. The phase speeds and group velocities for model-simulated PV anomalies show consistencies with vortex Rossby wave theory. Since vortex Rossby waves appear in both the control simulation and the dry sensitivity experiment, it suggests that PV dynamics are fundamental for the formation of the spiral rainbands. The fact that the PV bands are strongly coupled with the cloud bands implies that the thermal and dynamical structures trigger each other, which leads to the development of the spiral rainbands. The magnitude of the vertical motion estimated from Ekman pumping due to the interaction of PV anomalies with friction in the boundary layer is consistent with this view. It is further shown that the vortex Rossby wave induced cloud bands can extend up to 7 km from the surface, in agreement with the finding of Gall et al. (1998).

In the sense of wave–mean flow interaction, vortex Rossby waves are essential for the intensification of hurricanes. Willoughby (1978a,b) found two classes of gravity waves with Doppler-shifted frequencies between the local inertia frequency and the Brunt–Vaissàlia frequency in tropical cyclones. Only inward propagating waves interacting with the basic-state vortex at critical loci seem plausible, but they are probably not the radar-detected spiral bands (Willoughby et al. 1984; Willoughby 1995). Although the linear inertial gravity waves possess nonzero PV in the near-core region, they do not have critical levels (Montgomery and Lu 1997). In the absence of critical levels, the gravity waves satisfy the “non-acceleration” theory (Schubert 1985). On the contrary, model PV maps suggest that vortex Rossby waves do have critical radii, where the wave–mean flow interaction is strong (MK97; ME; MM99; MM00). The interactions between vortex Rossby waves and between waves and the basic-state vortex will be reported in the future.

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