ENERGY CHANGES IN THE MEAN ATMOSPHERE

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ABSTRACT

The kinetic, potential, and internal energies of the mean atmosphere in the northern hemisphere are computed for January and July. It is shown that, while the kinetic energy of the mean motion increases from summer to winter as the corresponding potential-plus-internal energy decreases, the variation of the former is less than two per cent of that of the latter.

1. Introduction

Sufficient climatological data for the free atmosphere are now available to permit at least a rough calculation of the mean energy budget of the atmosphere over the northern hemisphere up to about 20 km. The energy of the atmosphere is considered here to consist only of the kinetic energy of macroscopic motion, the potential energy associated with the gravitational field of the earth, and internal energy. The internal energy of the atmosphere consists principally of thermal energy, which, for an ideal gas, depends only on temperature, and the latent heat energy of water vapor. Since adequate humidity data were not available, no attempt was made to compute the latent heat energy, so that the internal energy referred to in this paper is the thermal energy of an ideal gas in the classical thermodynamic sense.

2. Kinetic energy of the mean motion in summer and winter

The kinetic energy of the mean motion was computed from mean pressure and temperature data because sufficient wind observations were not available to permit a direct calculation. For the purposes of this paper the mean monthly circulation is defined as the geostrophic circulation associated with the mean monthly pressure and temperature distribution. The kinetic energy per unit volume of the mean monthly circulation[2] is then given by

$$K_m = (2f^2\bar{\rho})^{-1}(\nabla_2\bar{p})^2, \qquad (1)$$

where f is the Coriolis parameter ($2\omega \sin \phi$), $\bar{\rho}$ is the mean monthly density, and $\nabla_2\bar{p}$ is the mean monthly horizontal pressure gradient.

The kinetic energy of the mean atmosphere in January and July was computed from (1) on the mean constant-level charts for the northern hemisphere published by the United States Weather Bureau and the Joint Meteorological Committee [6; 10]. Computations were made on each of the charts, from sea level to 19 km, at grid points determined by the intersections of ten-degree meridians and five-degree parallels. No measurements were attempted north of latitude 70°N or south of latitude 20°N, partly because of the paucity of data in these regions and partly because the assumption of geostrophic balance may not be justified in very low latitudes.

The horizontal distributions of the kinetic energy per unit volume of the mean motion in January and July averaged over all levels from sea level to 19 km above sea level are shown in figs. 1 and 2. Maxima are found in both seasons in the western Pacific and western Atlantic Oceans, although the magnitudes are much larger in January. In July maxima are also found in the western Mediterranean and near lower California, while in January secondary maxima appear over Africa and India. The regions of maximum kinetic energy of mean motion in January in the Atlantic and Pacific Oceans are of particular interest. Centers of maximum frequency of cyclogenesis in winter were found in these regions by Miller [7] and Miller and Mantis [8]. Jacobs [5] has computed the heat transferred to the atmosphere at the ocean surface in winter. The major heat sources were found in the western Atlantic and western Pacific Oceans and correspond

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[2] A distinction must be made between the kinetic energy of the mean motion and the average kinetic energy. The latter is given approximately by

$$K_{av} = K_m + \tfrac{1}{2}\bar{\rho}\overline{v'^2},$$

where v' is the deviation of the individual wind vector from the mean wind vector, v. The second term on the right represents the kinetic energy of eddy motions of all magnitudes if the local variations of density are neglected.

It is easily shown that $K_{av} \geqq S^{-1}K_m$, where S is the persistence of the wind, defined by $S = |\bar{v}|/\overline{|v|}$.

On the basis of some characteristic values of persistence given by Haurwitz and Austin [4], it was found that in the trade-wind region the average kinetic energy at sea level in January is more than 1.5 times the kinetic energy of the mean wind. In the westerlies the average kinetic energy at sea level in January is more than 9 times the kinetic energy of the mean wind. At 3 km the average kinetic energy is more than 1.5 times the kinetic energy of mean motion in January over the central part of North America.

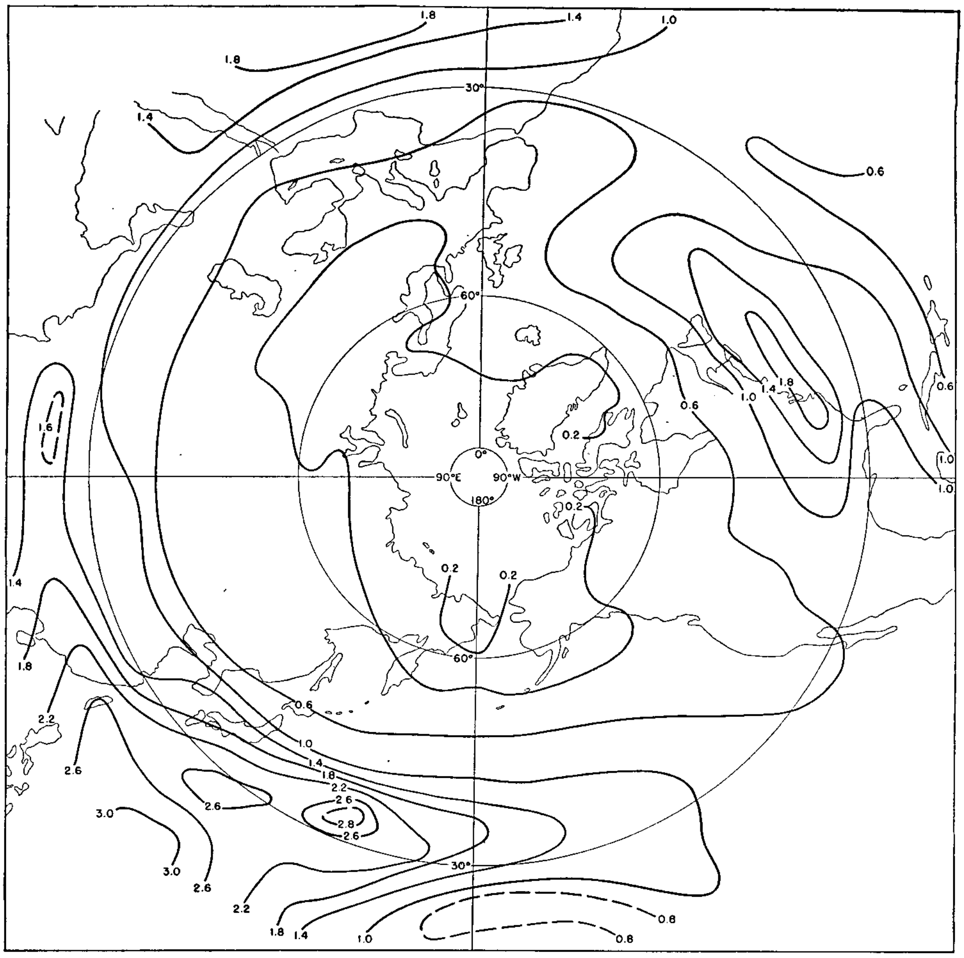

FIG. 1. Average kinetic energy per unit volume (10^3 ergs cm^{-3}) of the mean January circulation between sea level and 19 km.

closely with the maxima of kinetic energy of mean motion. Using a different method from that of Jacobs, Wexler [12] has determined the normal regions of heating and cooling in the layer from sea level to 10,000 feet in February from mean northern-hemisphere maps. His regions of heating are in good agreement with the heat sources found by Jacobs and correspond also to the regions of maximum kinetic energy of mean motion.

Figs. 3 and 4 represent meridional cross sections showing the distribution of kinetic energy of mean motion in January and July averaged over all longitudes. (The 18 values for each latitude were added and divided by 18.) Isopleths are drawn for intervals of 500 erg cm^{-3} on the January chart (fig. 3) and 100 erg cm^{-3} on the July chart (fig. 4). In January the maximum kinetic energy of mean motion is found near latitude 25°N at about 12 km. The maximum in July appears at about 8 km near latitude 45°N.

The vertical distribution of the total kinetic energy of mean motion at each level is shown in fig. 5 for the truncated hemispherical shell of air bounded by sea level, the 19-km level and 20 and 70 degrees north latitude. The total kinetic energy at each level represents the energy of a slice of air 1 cm thick and was evaluated by integrating numerically the kinetic energy of mean motion per unit volume over the area of the level surface. In both seasons the maximum kinetic energy of mean motion was found at 10 km.

The total kinetic energy of the mean circulation up to 19 km was computed from fig. 5 by graphical inte-

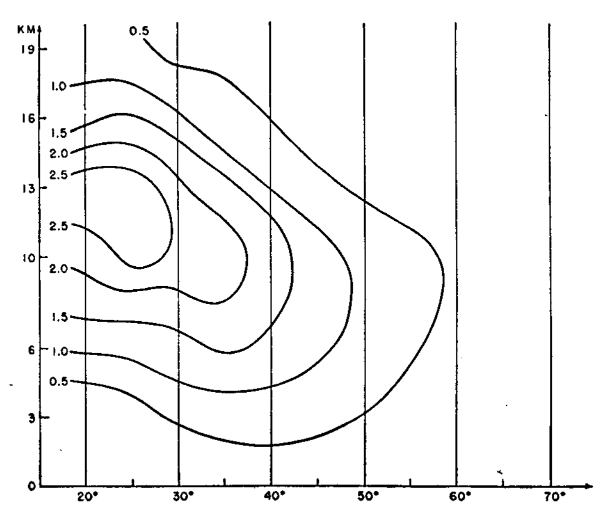

FIG. 3. Kinetic energy per unit volume (10^3 ergs cm^{-3}) of the mean January circulation averaged over all longitudes.

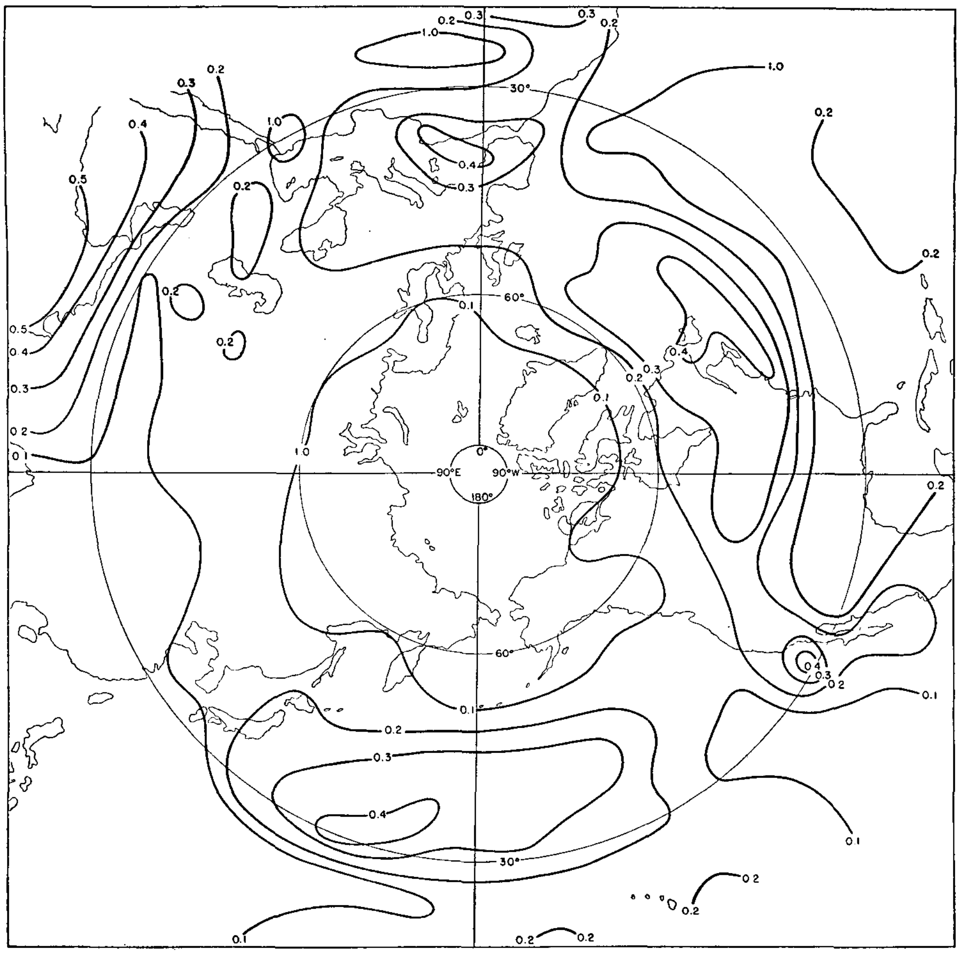

Fig. 2. Average kinetic energy per unit volume (10^3 ergs cm^{-3}) of the mean July circulation between sea level and 19 km.

gration and amounts to 56×10^{25} ergs in July and 285×10^{25} ergs in January. Brunt [1] has estimated the total kinetic energy of the mean circulation of the whole atmosphere to be of the order of 300×10^{25} ergs. In the present study slightly less than one-third of the total mass of the atmosphere was considered. The average kinetic energy of the mean motion of the whole atmosphere may be estimated under the following assumptions: (*a*) the January and July values represent extreme values of kinetic energy with a linear variation

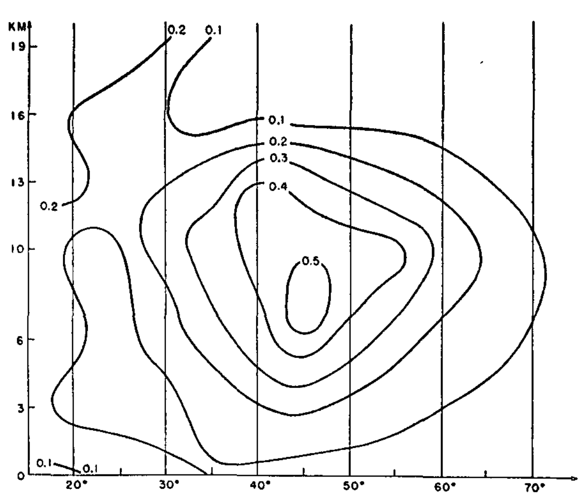

Fig. 4. Kinetic energy per unit volume (10^3 ergs cm^{-3}) of the mean July circulation averaged over all longitudes.

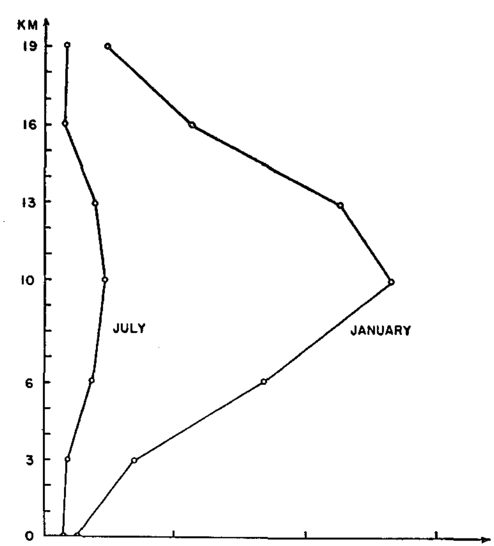

Fig. 5. Vertical distribution of the total kinetic energy of the mean circulation in January and July (10^{21} ergs cm^{-1}).

in the months between, and (*b*) the kinetic energy of the whole atmosphere is three times the kinetic energy of that portion of the atmosphere which has been considered here. The annual average kinetic energy of the mean circulation is then 510×10^{25} ergs. Brunt's rough estimate therefore gives the correct order of magnitude.

3. Potential and internal energy

The gravitational potential energy of an atmospheric column of unit cross section is given by

$$P = \int_0^\infty g\rho z \, dz. \qquad (2)$$

Upon integrating (2) under the assumption of an isothermal stratosphere, the following formula is obtained for the potential energy:

$$P = \frac{Rp_0T_0}{g + R\gamma}\left[1 + \frac{R\gamma}{g}\left(1 - \frac{\gamma\tau}{T_0}\right)^{1+g/R\gamma}\right]. \qquad (3)$$

R is the gas constant, p_0 is the sea-level pressure, T_0 is the temperature of the air at sea level, g is the acceleration of gravity, γ is the average tropospheric lapse rate, and τ is the height of the tropopause.

The potential energy per unit area in January and July of the mean northern-hemisphere atmosphere was computed from (3) for ten-degree latitude circles from the equator to the north pole. The mean sea-level pressures, tropospheric lapse rates and tropopause heights for each latitude circle were obtained from [11] and mean sea-level temperatures were obtained from [2]. The values of gravity were taken from [9]. These data, together with the computed potential energies, are shown in table 1. The total potential energy of the hemisphere was computed by numerical integration of the last column in table 1 (multiplied by the cosine of the latitude).

The internal energy of a vertical column of air extending to infinity can be shown [3] to equal $(\lambda - 1)^{-1} P$ where λ is the ratio of specific heats (1.4 for dry air). We have assumed throughout that the air is dry and of uniform composition so that λ, as well as R, is constant.

The potential, internal, and kinetic energies of the mean northern-hemisphere atmosphere are given in table 2. To obtain the kinetic energy for the hemisphere, the kinetic energy of the zone between 20°N and 70°N was multiplied by a factor of 1.53, the ratio of the mass of the whole northern-hemisphere atmosphere to that of this zone. Table 2 also contains the potential, internal and kinetic energies for the zone between 20°N and 70°N.

The difference between the January and July energies is due partly to the difference of mass which results from the seasonal transport of air across the equator.

TABLE 1. Mean sea-level gravity (g), sea-level pressure (p_0), sea-level temperature (T_0), tropopause height (τ), troposphere lapse rate (γ) and potential energy (P).

Latitude (north)	g (cm sec^{-2})	p_0 (mb)	T_0 (K)	τ (km)	γ (km^{-1})	P (10^{11} ergs cm^{-2})
			January			
0	978.0	1011.0	299.4	16.8	5.9	7.66
10	978.2	1011.8	298.8	16.2	6.1	7.62
20	978.6	1015.6	294.8	15.5	6.0	7.57
30	979.3	1019.2	287.5	13.8	5.7	7.50
40	980.2	1018.3	278.0	11.5	5.7	7.28
50	981.1	1016.2	265.9	10.1	5.0	7.11
60	981.9	1014.2	256.9	9.7	4.1	6.97
70	982.6	1012.8	246.7	9.4	3.0	6.84
80	983.1	1009.8	240.8	9.3	2.0	6.80
90	983.2	1009.8	232.0	9.2	0.8	6.97
			July			
0	978.0	1011.0	298.6	16.8	5.9	7.64
10	978.2	1010.0	299.9	17.0	5.9	7.66
20	978.6	1010.3	301.0	16.5	6.1	7.65
30	979.3	1012.3	300.3	15.2	6.3	7.63
40	980.2	1013.2	297.0	13.1	6.4	7.60
50	981.1	1011.8	291.1	10.8	6.7	7.45
60	981.9	1010.0	287.1	9.9	6.5	7.42
70	982.6	1010.0	280.3	9.2	5.8	7.38
80	983.1	1011.5	275.0	9.1	4.9	7.35
90	983.2	1011.8	272.0	9.0	4.4	7.35

If $p_{0\phi}$ and $g_{0\phi}$ are the mean sea-level pressure and mean gravity at latitude ϕ (table 1), the mass, M, of the atmosphere over a hemisphere is given by

$$M = 2\pi r^2 \int_0^1 \frac{p_{0\varphi}}{g_{0\varphi}} \, d(\sin\phi), \qquad (4)$$

where r is the mean radius of the earth. The January and July masses were computed numerically from (4) for the whole atmosphere as well as for the zone 20°N–70°N and are entered in table 2 to three significant figures.

The change of kinetic energy for the hemisphere is 1.8 per cent of the change of potential-plus-internal energy from July to January and of opposite sign.

TABLE 2. Mass–energy budget of the mean northern-hemisphere atmosphere (cgs units).

	January	July	January minus July
	0°–90°N		
Potential energy	188×10^{28}	193×10^{28}	-5×10^{28}
Internal energy	469×10^{28}	483×10^{28}	-14×10^{28}
Kinetic energy	436×10^{25}	86×10^{25}	$+350 \times 10^{25}$
Total energy	657×10^{28}	676×10^{28}	-19×10^{28}
Mass	264×10^{19}	263×10^{19}	$+1 \times 10^{19}$
Total energy per unit mass	249×10^{7}	257×10^{7}	-8×10^{7}
	20°N–70°N		
Potential energy	121×10^{28}	126×10^{28}	-5×10^{28}
Internal energy	302×10^{28}	315×10^{28}	-13×10^{28}
Kinetic energy	285×10^{25}	56×10^{25}	$+229 \times 10^{25}$
Total energy	423×10^{28}	441×10^{28}	-18×10^{28}
Mass	173×10^{19}	172×10^{19}	$+1 \times 10^{19}$
Total energy per unit mass	244×10^{7}	256×10^{7}	-12×10^{7}

Between 20°N and 70°N the change of kinetic energy is only 1.3 per cent of the potential-plus-internal energy change. Since the kinetic energy is known more accurately for the zone 20°N–70°N than for the whole hemisphere, the value 1.3 per cent may be considered the more reliable estimate of the two. It is evident that the kinetic energy constitutes a relatively small part of the total energy of the atmosphere. Even if all the January kinetic energy were completely transformed into heat, the mean temperature of the atmosphere would increase by little more than 0.2C. However, the seasonal percentile change of kinetic energy is much larger than that of the potential and internal energies. The former is about 135 per cent whereas the latter is less than 3 per cent of its mean value.

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