NOTES AND CORRESPONDENCE

On the Ability of Dry Tropical-Cyclone-like Vortices to Withstand Vertical Shear

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ABSTRACT

The ability of dry tropical-cyclone-like vortices to resist vertical shear is discussed. An idealized model calculation is presented in which a dry vortex remains nearly upright during 4 days under the influence of environmental vertical shear. It is shown that the outer portion of the vortex tilts more strongly than the inner core and that the pattern of vertical velocity is related to the vertical tilt of the outer portion of the vortex. This result is discussed with relation to observations of the location of convection in tropical cyclones. An alternative definition of the vortex center is proposed for cases in which the vertical tilt of the vortex is of importance. The average vertical shear across the center of the vortex is shown to depend on both the vortex tilt and the presence of large-scale potential vorticity asymmetries in the outer regions of the vortex. The average vertical shear is a function of time and of the area of the circle over which the averaging is carried out. Thus, the initial environmental shear may not be a reliable measure of the vertical shear felt by the vortex at later times.

1. Introduction

Vertical shear of the horizontal wind is frequently responsible for a decrease in the intensity of a tropical cyclone (Riehl and Schafer 1944; Ramage 1959; Gray 1968; Merrill 1988). DeMaria (1996) proposed that vortex tilting in the presence of vertical shear could be responsible for the observed decrease in tropical cyclone intensity. If a tropical cyclone tilts in response to environmental vertical shear, the thermal field will adjust in order for the vortex to remain in balance (Raymond 1992; Jones 1995, hereafter J95; DeMaria 1996; Jones 2000b). These changes in the thermal field result in changes in the vertical stability. DeMaria (1996) postulated that such changes in the structure of the tropical cyclone vortex could result in modified convection in the inner core of the tropical cyclone, leading to a decrease in intensity. Thus it is necessary to understand how the vertical tilt of a tropical cyclone develops in vertical shear in order to better understand tropical cyclone intensity change.

Idealized numerical calculations of tropical-cyclone-like vortices in vertical shear have shown that the vertical tilt is reduced in the presence of moist processes (Flatau et al. 1994; Wang and Holland 1996b; Frank and Ritchie 1999). These results might be misinterpreted as implying that moist processes are *necessary* in order for a tropical cyclone to withstand vertical shear. The purpose of this note is to show that, under certain conditions, the vertical tilt of a *dry* tropical-cyclone-like vortex can remain small for many days, despite the presence of vertical shear. In addition, we show how different definitions of the location of the vortex center can lead to different conclusions about the magnitude of the vertical tilt.

2. An example of a dry vortex with small vertical tilt in vertical shear

In J95 it was shown how the destructive action of vertical shear on a dry tropical-cyclone-like vortex can be retarded by a mutual rotation of the upper and lower portions of the vortex about the midlevel vortex center. If the rotation leads to an upshear tilt, then the environmental flow tends to reduce the vertical tilt of the vortex rather then increasing it. The mutual rotation is initiated when the potential vorticity (PV) anomaly[1] of the cyclonic vortex tilts away from the vertical in response to the vertical shear. The balanced flow associated with the tilted PV anomaly penetrates upward and downward. The advection of the PV at a given level by the vertically penetrating flow is responsible for the mutual rotation. The strength of the vertical penetration,

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[1] Defined relative to the PV of the environmental flow far from the vortex center.

and thus also the rotation rate, depends on the parameters that determine the Rossby penetration depth H_R given for a rapidly rotating vortex by $H_R = [(f + 2v_T/r)(f + \zeta)]^{1/2}L/N$, where f is the Coriolis parameter, v_T the tangential wind, r the radius, ζ the vertical component of relative vorticity, L the horizontal length scale, and N the Brunt–Väisälä frequency (Hoskins et al. 1985; Shapiro and Montgomery 1993). An upshear tilt is achieved sooner for higher rotation rates, that is, for higher Rossby penetration depths. Thus a stronger, broader vortex at high latitudes is better able to resist vertical shear than its weaker, smaller, low-latitude counterpart.

Smith et al. (2000) developed a two-layer analog model in which the vortex in each layer is advected by a representation of the flow associated with the vortex in the other layer and by the environmental shear flow. The vortices are not allowed to change shape. For certain combinations of shear, vortex strength, and vertical coupling (representative of the Rossby penetration depth) the upper- and lower-level vortex centers rotate about their midpoint, so that the vortex in the analog model is not torn apart by the vertical shear. In addition, Smith et al. (2000) show a calculation with a two-layer quasigeostrophic model in which the vertical tilt remains smaller than twice the radius of maximum wind of the vortex over a 24-h time period. In all of the more realistic multilevel primitive equation calculations shown in J95, the vortex tilt increased with time, albeit more slowly than would have occurred due to differential advection by the vertically sheared environmental flow.

Here we present a primitive equation calculation in which the vertical tilt of a dry vortex in vertical shear remains small for 4 days. The model configuration is that of J95 on an f plane at 25°N. The vortex is the broad vortex of J95 (see J95 Table 1) with a maximum tangential wind speed of 40 m s^{-1} at a radius of 150 km. The environmental static stability is 1×10^{-4} s^{-2}, the westerly vertical shear has a magnitude of 6×10^{-4} s^{-1}, and there is zero flow at the surface. The model domain is 5760 km in the zonal direction, x, by 4320 km in the meridional direction, y, by 12 km in the vertical direction, z, with a horizontal resolution of 15 km and 14 levels in the vertical. The track and evolution of the vertical tilt is illustrated using both the location of maximum PV (Fig. 1a) and the location of minimum perturbation geopotential (Fig. 1b) to define the vortex center at selected model levels. The perturbation geopotential is defined relative to the environmental geopotential field. Over the entire 96-h period the separation of the upper- and lower-level vortex centers does not exceed 20 km, a distance of just over one horizontal grid length. Note that, if the upper- and lower-vortex centers had simply been advected by the vertically sheared environmental shear flow, then they would have a separation of 2300 km after 96 h.

The case shown here is consistent with the results of J95, since it was demonstrated that an increase in the penetration depth leads to a decrease in the vertical tilt. For all of the vortex profiles used in J95 and Jones (2000a) the vortex will remain upright if the penetration depth is increased sufficiently. In the calculation with the broad vortex in J95 the vortex tilt increases slowly with time during the 36 h shown. However, if the J95 calculation is integrated further in time, then the vertical tilt decreases between 36 and 72 h. For the broad vortex used here, with $H = 12$ km instead of the $H = 10$ km used in J95, the vertical tilt increases with time for an environmental static stability of 1.5×10^{-4} s^{-2} and an f plane at 12.5°N.

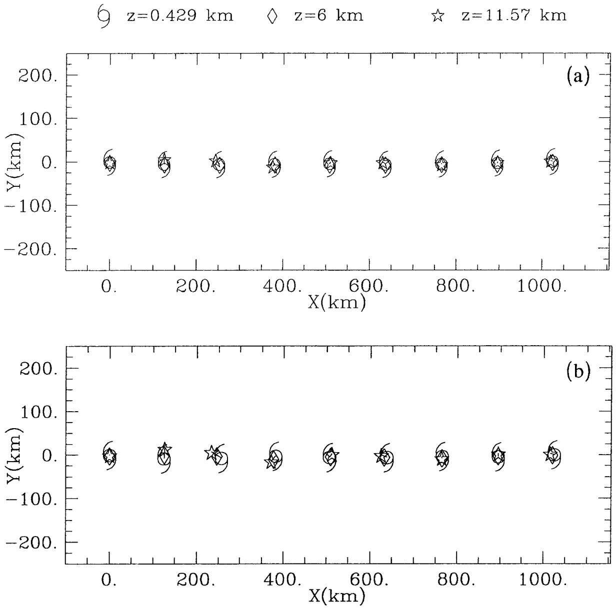

FIG. 1. Vortex tracks. Symbols show the location of (a) maximum PV and (b) minimum perturbation geopotential at selected model levels every 12 h for 96 h. The initial vortex location is at $(x, y) = (0, 0)$.

The vertical circulation for an adiabatic vortex in a stably stratified fluid is directly related to the direction of the vertical tilt, with ascent to the right of an observer looking in the downtilt direction and descent to the left (J95; Jones 2000b). Thus the absence of significant vertical tilt in Fig. 1 suggests that there should be no significant vertical circulation. In fact, a similar wavenumber-one pattern of ascent and descent to that seen for the tilted vortices in J95 is seen here also (Fig. 2). A closer examination of the PV field shows that, despite the collocation of the upper- and lower-level PV maxima, there is, in fact, a discernible vortex tilt (Fig. 3). The PV in the inner region of the vortex is distorted such that the location of maximum PV is shifted away from the geometric center of the PV anomaly. The distortion is caused by the advection of the PV by the divergent flow (J95, p. 837). Thus, at 60 h the outer regions of the vortex have a southeast–northwest tilt, consistent with the pattern of vertical velocity seen in Fig. 2.

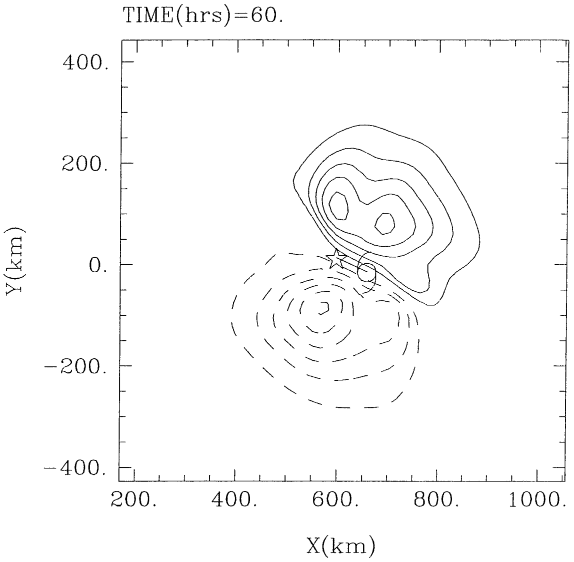

FIG. 2. Horizontal cross section at 6 km of the vertical velocity at 60 h. Contour interval is 3 cm s^{-1}, positive contours solid, negative contours dashed. The location of the center of PV (see text for details) is marked by the cyclone symbol at the lowest level and the star at the highest level.

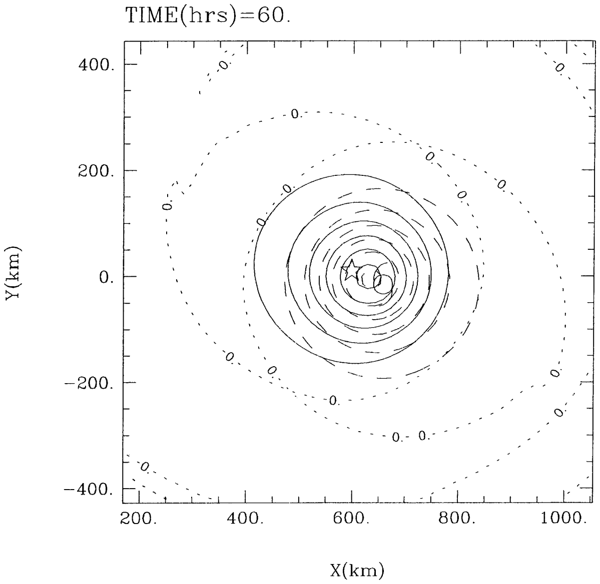

FIG. 3. Density-weighted PV at 60 h at lowest model level (dashed) and highest model level (solid). Contour interval is 0.5 × 10^{-6} K m^{-1} s^{-1}. The location of the center of PV (see text for details) is marked by the cyclone symbol at the lowest level and the star at the highest level.

For cases such as that shown here, neither the location of maximum PV nor the location of minimum perturbation geopotential give adequate information about the direction of the vertical tilt away from the inner core of the vortex. More information about the vertical tilt can be obtained by calculating a ‘‘center of PV,’’ analogous to the center of mass of a solid body. The center of PV is defined as

$$(x_{\mathrm{CPV}}, y_{\mathrm{CPV}}) = \frac{1}{\left[\displaystyle\iint q'(x, y)\,dx\,dy\right]}\iint xq'(x, y)\,dx\,dy, \iint yq'(x, y)\,dx\,dy, \tag{1}$$

where q' is the positive perturbation PV (defined relative to the PV of the environmental flow) and the double integrals are evaluated over the area within a circle of radius 500 km centered on the location of maximum PV at the given level. If the total PV is used to define the center of PV, the results agree to within one horizontal grid length. A similar definition of the vortex center was used by Reasor and Montgomery (2001), but they evaluated the integral over the entire domain. For a real tropical cyclone the radius of integration should be chosen so as to include the high PV values in the inner core of the tropical cyclone but exclude PV associated with environmental features. The location of the center of PV in Fig. 3 is indicated by the cyclone symbol at the lowest level and the star at the highest level. At each of these levels the center of PV lies closer to the geometric center of the positive PV anomaly than the location of maximum PV. The difference between the location of maximum PV and the center of PV is a result of strong PV asymmetries in the inner core of the vortex, such as seen in Fig. 3. In the presence of vertical shear such asymmetries would be enhanced by the presence of asymmetric convection (Willoughby et al. 1984; Wang and Holland 1996b; Bender 1997; Frank and Ritchie 1999, 2001; Reasor et al. 2000).

The different locations of the upper- and lower-level centers of PV give an indication of the vertical tilt of the outer region of the PV anomaly. The vortex track calculated using the center of PV is shown in Fig. 4, and the magnitude and direction of the vertical tilt calculated using the center of PV at the lowest and highest model levels is shown in Fig. 5. For comparison, the magnitude of tilt obtained using the location of maximum PV to define the center is shown in Fig. 5a. The upper- and lower-level centers of PV rotate cyclonically about their midpoint with time. The vertical tilt increases with time, reaching a maximum value of 140 km, and then decreases. Two subsequent events of increasing and

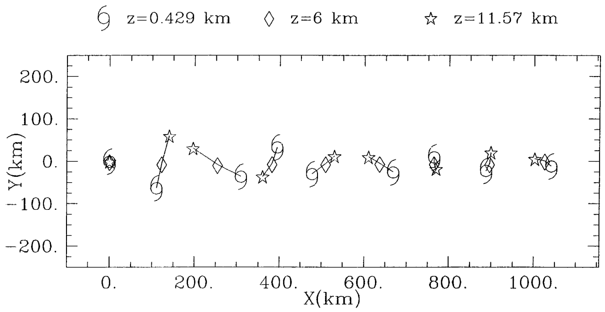

FIG. 4. As Fig. 1, but for location of center of PV (see text for details).

decreasing vertical tilts are seen. The periods of decreasing vertical tilt correspond well with the times when the vortex has an upshear component of tilt (angle of tilt between 90° and 270°). After the first episode of increasing vertical tilt the maximum magnitude of tilt reached in subsequent episodes decreases.

The impact of the vortex evolution on the environmental vertical shear can be assessed by averaging the zonal and meridional winds over a circle centered on the vortex center (thus removing the symmetric wind field) and calculating the vertical shear from these average winds (DeMaria and Kaplan 1994). The results of this calculation for a circle of 500-km radius centered on the location of minimum perturbation geopotential at the lowest model level are shown in Fig. 6a. The magnitude of the zonal vertical shear increases during the first 12 h and then decreases to almost zero at 38 h. The meridional vertical shear increases until 28 h and then decreases to a minimum at 48 h. Subsequently the magnitudes of both components of vertical shear oscillate about a value approximately one-third that of the initial environmental shear, with the zonal shear leading the meridional shear by one quarter of a period of oscillation. A comparison of Figs. 5b and 6a indicates that the oscillations are related to the changes in the vertical tilt. This is because the low-level center is used to define the center of the averaging circle so that for a tilted vortex the upper-level circulation appears as a flow asymmetry and contributes to the vertical shear. Thus, the zonal vertical shear due to the vertical tilt of the vortex is maximum (minimum) for a tilt of 90° (270°) whereas the meridional vertical shear is maximum (minimum) for a tilt of 180° (0°). The period of the oscillation depends on the change in the direction of tilt with time and thus is determined by both the Rossby penetration depth and the magnitude of the vertical tilt, as well as by the magnitude and direction of the environmental shear. For higher values of the Rossby penetration depth the period of oscillation decreases. If the separation of the upper- and lower-level centers is less than the radius of maximum wind, the period should decrease with increasing tilt whereas, if the separation exceeds the radius of maximum wind, the period should increase with increasing tilt. This statement is supported by the period

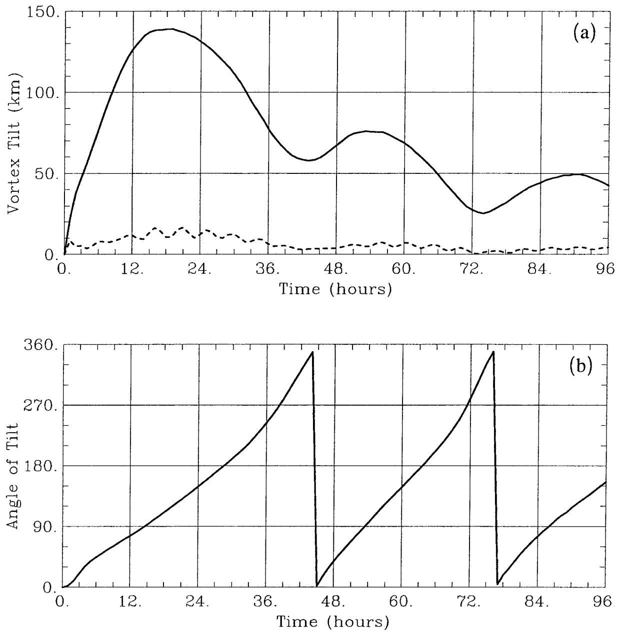

FIG. 5. (a) Magnitude and (b) angle of tilt between locations of center of PV (solid) and maximum PV [dashed in (a)] at lowest and highest model levels. Angle of tilt is defined counterclockwise from the *x* axis.

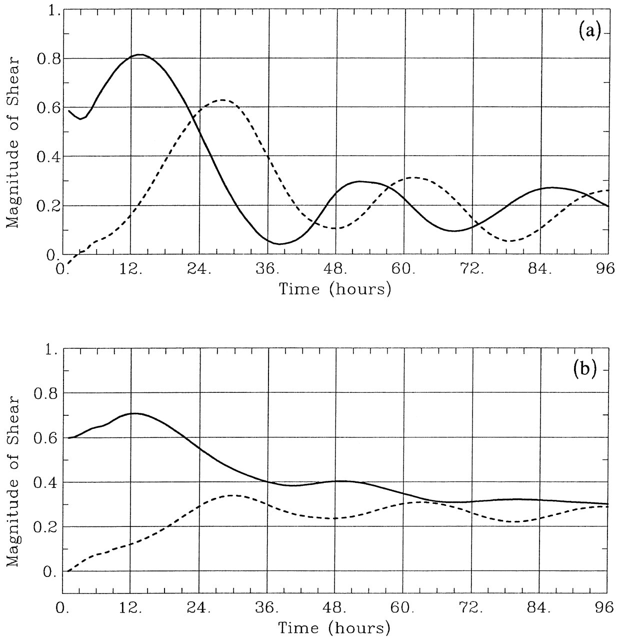

FIG. 6. Magnitude of the zonal (solid) and meridional (dashed) vertical shear calculated between the lowest and highest model levels averaged over the area within a circle of radius (a) 500 km and (b) 1000 km centered on the location of minimum perturbation geopotential at the lowest level.

of the first oscillation in Fig. 6a (during which the magnitude of the tilt was close to the radius of maximum wind) being larger than the period of subsequent oscillations.

If the averaging is carried out within a circle of radius 1000 km, the magnitude of the oscillations decreases significantly (Fig. 6b). The averaging technique removes a symmetric vortex if all of its circulation lies within the averaging area, regardless of where the center of the vortex lies. Thus, the larger the radius of the circle used for averaging, the less influence the vortex tilt has on the environmental vertical shear. The magnitude of the zonal component of vertical shear decreases with time while the meridional component increases. After 36 h both components of vertical shear have approximately the same magnitude. These changes can be attributed to the development of the large-scale PV asymmetries discussed in Jones (2000a). The flow associated with these asymmetries takes the form of large-scale anticyclones located to the southwest of the vortex center at low levels and to the northeast at upper levels, resulting in northwesterly flow across the low-level vortex center and southeasterly flow across the upper-level center. The contribution of the large-scale asymmetries to the vertical shear is a function of the radius of the circle used for averaging also, depending on how much of the anticyclonic circulation falls within the circle. The contribution is smaller for a radius of 500 km than for 1000 km (Fig. 6).

The structure of the large-scale asymmetries that develop in this calculation depends on the idealized vortex profile used (Jones 2000a). However, the results that large-scale PV asymmetries play an important role in determining the vertical shear and that the magnitude of the vertical shear is sensitive to the area over which the shear is calculated are applicable to more realistic situations. For a real tropical cyclone, large-scale asymmetries, such as β gyres that vary with height (Wang and Holland 1996a; Frank and Ritchie 2002), the PV asymmetries described by Shapiro (1992), or the downstream displacement of the upper-level anticyclone (Wu and Emanuel 1993) can contribute to the vertical shear as calculated for Fig. 6.

The center of PV could be used to define the center of a tropical cyclone vortex in cases where the PV is not maximum at zero radius. For example, in an intensifying tropical cyclone, the PV might be expected to be maximum in an annular ring around the center of the cyclone (Shapiro and Franklin 1995; Kossin and Eastin 2001). In such a case the center of PV would lie close to zero radius (or at zero radius for a symmetric ring of PV).

3. Conclusions

We have presented an example of a dry tropical-cyclone-like vortex that does not develop a large vertical tilt during many days in the presence of environmental vertical shear. The addition of this example to previous work on dry vortices in vertical shear emphasizes the wide range of possible behavior. Depending on the parameters in the Rossby penetration depth and on the strength of the shear the vortex may remain upright for many days or be torn apart by the shear within a few days. In the example given here, the inner core of the vortex remains upright while the outer regions are more strongly tilted. Hence the location of maximum PV or minimum perturbation geopotential at different levels is not a reliable indicator of vertical tilt in this case.

The dynamics of dry tropical cyclones in vertical shear can be described in terms of vortex Rossby waves (Reasor and Montgomery 2001; Schecter et al. 2002; Schecter and Montgomery 2003; Reasor et al. 2004). In this framework, the mutual rotation of a tilted vortex is attributed to an azimuthal wavenumber-one quasi mode with vertical structure described by the first internal baroclinic mode. The phase speed of the quasi mode increases with increasing Rossby penetration depth. The vortex is better able to resist vertical shear if the quasi mode is resonantly damped (Schecter and Montgomery 2003; Reasor et al. 2004). The vortex Rossby wave description is an appealing alternative to the description in terms of the penetration of PV anomalies discussed here. In the opinion of the author neither viewpoint invalidates the other, and the insight gained may depend on the particular situation under consideration. Further research is needed to compare and contrast these different ways of looking at the same problem.

We have demonstrated that a definition of the vortex center based on the center of mass of a solid body gives more information about the vertical tilt than the location of maximum PV or minimum perturbation geopotential. The center of PV definition of the vortex center may be of use in assessing the vertical tilt of tropical cyclones in which convective processes lead to maximum PV in the eyewall and not near zero radius. In addition, the analysis of vertical tilt presented here shows that care must be taken when making conclusions about the vortex tilt from a calculation of the location of maximum PV or minimum geopotential/surface pressure alone. For example, Frank and Ritchie (2001) investigated the relationship between the vertical tilt and the intensity change of moist idealized tropical cyclones in vertical shear and concluded that the mechanism of DeMaria (1996) is not important in their model calculations. However, as they define the vortex tilt using the location of minimum pressure at a given level, it is possible that they underestimate the role of the vortex tilt.

The development of large-scale asymmetries in the outer region of the tropical-cyclone-like vortex (Jones 2000a) leads to a reduction of the average environmental vertical shear across the vortex center. The changes in the average vertical shear with time presented here suggest that the initial environmental shear is not a reliable indicator of the magnitude and direction of the vertical shear felt by a vortex in model calculations.

The direction of vertical tilt as revealed by the center of PV is related to the horizontal orientation of the vertical velocity pattern, showing that the tilt of the outer region of the vortex determines the pattern of vertical velocity rather than the tilt of the inner core. Corbosiero and Molinari (2002) found that in the outer rainbands of tropical cyclones the lightning activity is maximum to the right of the shear vector and suggested that the downshear right location of lightning activity in the outer rainband might be attributed to the tropical cyclone vortex having a larger vertical tilt at the radius of the outer rainbands than in the inner core, as seen in the model calculation shown here.

The aim of this note was to show that it is not necessary to appeal to moist processes to explain why a tropical cyclone is not destroyed by vertical shear since dry vortex dynamics enable a tropical cyclone to remain upright in vertically sheared environments. However, there are considerable differences between the behavior of dry and moist vortices in vertical shear (Frank and Ritchie 1999). A more complete understanding of the interaction between environmental vertical shear and tropical cyclone structure and intensity change requires the consideration of moist processes.

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REFERENCES

Bender, M. A., 1997: The effect of relative flow on the asymmetric structure of the interior of hurricanes. *J. Atmos. Sci.,* **54,** 703–724.

Corbosiero, K. L., and J. Molinari, 2002: The effects of vertical wind shear on the distribution of convection in tropical cyclones. *Mon. Wea. Rev.,* **130,** 2110–2123.

DeMaria, M., 1996: The effect of vertical shear on tropical cyclone intensity change. *J. Atmos. Sci.,* **53,** 2076–2087.

——, and J. Kaplan, 1994: A statistical hurricane intensity prediction scheme (SHIPS) for the Atlantic basin. *Wea. Forecasting,* **9,** 209–220.

Flatau, M., W. H. Schubert, and D. E. Stevens, 1994: The role of baroclinic processes in tropical cyclone motion: The influence of vertical tilt. *J. Atmos. Sci.,* **51,** 2589–2601.

Frank, W. M., and E. A. Ritchie, 1999: Effects of environmental flow on tropical cyclone structure. *Mon. Wea. Rev.,* **127,** 2044–2061.

——, and ——, 2001: Effects of vertical wind shear on the intensity and structure of numerically simulated hurricanes. *Mon. Wea. Rev.,* **129,** 2249–2269.

——, and ——, 2002: Tropical cyclones in complex vertical shears. Preprints, *25th Conf. on Hurricanes and Tropical Meteorology,* San Diego, CA, Amer. Meteor. Soc., 315–316.

Gray, W. M., 1968: Global view of the origin of tropical disturbances and storms. *Mon. Wea. Rev.,* **96,** 669–700.

Hoskins, B. J., M. E. McIntyre, and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.,* **111,** 877–946.

Jones, S. C., 1995: The evolution of vortices in vertical shear. I: Initially barotropic vortices. *Quart. J. Roy. Meteor. Soc.,* **121,** 821–851.

——, 2000a: The evolution of vortices in vertical shear. II: Large-scale asymmetries. *Quart. J. Roy. Meteor. Soc.,* **126,** 3137–3160.

——, 2000b: The evolution of vortices in vertical shear. III: Baroclinic vortices. *Quart. J. Roy. Meteor. Soc.,* **126,** 3161–3185.

Kossin, J. P., and M. D. Eastin, 2001: Two distinct regimes in the kinematic and thermodynamic structure of the hurricane eye and eyewall. *J. Atmos. Sci.,* **58,** 1079–1090.

Merrill, R. T., 1988: Environmental influences on hurricane intensification. *J. Atmos. Sci.,* **45,** 1678–1687.

Ramage, C. S., 1959: Hurricane development. *J. Meteor.,* **16,** 227–237.

Raymond, D. J., 1992: Nonlinear balance and potential-vorticity thinking at large Rossby number. *Quart. J. Roy. Meteor. Soc.,* **118,** 987–1015.

Reasor, P. D., and M. T. Montgomery, 2001: Three-dimensional alignment and corotation of weak TC-like vortices via linear vortex Rossby waves. *J. Atmos. Sci.,* **58,** 2306–2330.

——, ——, F. D. Marks Jr., and J. F. Gamache, 2000: Low-wavenumber structure and evolution of the hurricane inner core observed by airborne dual-Doppler radar. *Mon. Wea. Rev.,* **128,** 1653–1680.

——, ——, and L. D. Grasso, 2004: A new look at the problem of tropical cyclones in vertical shear flow: Vortex resiliency. *J. Atmos. Sci.,* **61,** 3–22.

Riehl, H., and R. J. Shafer, 1944: The recurvature of tropical storms. *J. Meteor.,* **1,** 42–54.

Schecter, D. A., and M. T. Montgomery, 2003: On the symmetrization rate of an intense geophysical vortex. *Dyn. Atmos. Ocean,* in press.

——, ——, and P. D. Reasor, 2002: A theory for the vertical alignment of a quasigeostrophic vortex. *J. Atmos. Sci.,* **59,** 150–168.

Shapiro, L. J., 1992: Hurricane vortex motion and evolution in a three-layer model. *J. Atmos. Sci.,* **49,** 140–153.

——, and M. T. Montgomery, 1993: A three-dimensional balance theory for rapidly rotating vortices. *J. Atmos. Sci.,* **50,** 3322–3335.

——, and J. L. Franklin, 1995: Potential vorticity in Hurricane Gloria. *Mon. Wea. Rev.,* **123,** 1465–1475.

Smith, R. K., W. Ulrich, and G. Sneddon, 2000: On the dynamics of hurricane-like vortices in vertical shear flows. *Quart. J. Roy. Meteor. Soc.,* **126,** 2653–2670.

Wang, Y., and G. J. Holland, 1996a: The beta-drift of baroclinic vortices. Part I: Adiabatic vortices. *J. Atmos. Sci.,* **53,** 411–427.

——, and ——, 1996b: Tropical cyclone motion and evolution in vertical shear. *J. Atmos. Sci.,* **53,** 3313–3332.

Willoughby, H. E., F. D. Marks Jr., and R. J. Feinberg, 1984: Stationary and moving convective bands in hurricanes. *J. Atmos. Sci.,* **41,** 3190–3211.

Wu, C.-C., and K. A. Emanuel, 1993: Interaction of a baroclinic vortex with background shear: Application to hurricane movement. *J. Atmos. Sci.,* **50,** 62–76.