Results from homogeneous, isotropic turbulence suggest that predictability behavior is linked to the slope of a flow’s kinetic energy spectrum. Such a link has potential implications for the predictability behavior of atmospheric models. This article investigates these topics in an intermediate context: a multilevel quasigeostrophic model with a jet and temperature perturbations at the upper surface (a surrogate tropopause). Spectra and perturbation growth behavior are examined at three model resolutions. The results augment previous studies of spectra and predictability in quasigeostrophic models, and they provide insight that can help interpret results from more complex models. At the highest resolution tested, the slope of the kinetic energy spectrum is approximately $-7/3$ at the upper surface but $-3$ or steeper at all but the uppermost interior model levels. Consistent with this, the model’s predictability behavior exhibits key features expected for flow with a shallower than $-3$ slope. At the highest resolution, upper-surface perturbation spectra peak below the energy-containing scales, and the error growth rate decreases as small scales saturate. In addition, as model resolution is increased and smaller scales are resolved, the peak of the upper-surface perturbation spectra shifts to smaller scales and the error growth rate increases. The implications for potential predictive improvements are not as severe, however, as in the standard picture of flows exhibiting a finite predictability limit. At the highest resolution, the model also exhibits periods of much faster-than-average perturbation growth that are associated with faster growth at smaller scales, suggesting predictability behavior that varies with time.

1. Introduction

Despite decades of research, the predictability behavior of the atmosphere at meso- and synoptic scales is not well understood. Understanding atmospheric predictability is important from theoretical, practical, and societal perspectives. As a fundamental question, atmospheric predictability is related to key aspects of atmospheric dynamics and underlies many elements of modern weather prediction. From a practical perspective of improving weather forecasts, understanding error growth dynamics and the resulting limits to atmospheric predictability is critical for effectively designing observing and forecasting systems. From a societal perspective, understanding atmospheric predictability can help provide forecast users with important information about uncertainty in current forecasts (NRC 2006; Morss et al. 2008) and about the potential (or lack thereof) for improvements in future forecasts.

Lorenz (1969) postulated that flows with many scales of motion (such as the atmosphere) can exhibit an intrinsic finite limit of predictability. In such systems, the eddy turnover time decreases with increasing spatial scale, so that small initial condition errors initially grow with the faster time scale of the small spatial scales. In the limit of an arbitrarily wide range of scales, this means that no matter how small (except zero) one makes initial condition errors, forecast errors at a given lead time cannot be reduced beyond a certain level. Because such predictability behavior may have significant implications for prediction, it is important to understand what types of flows exhibit this behavior. Using a closure model for statistically isotropic and homogeneous turbulence, Lorenz (1969) found that, within the model’s approximations, flows whose kinetic energy (KE) spectra had slopes shallower than $-3$ would exhibit this behavior, whereas those with slopes of $-3$ or steeper would not (see also Lilly 1972).

Related work indicates that homogeneous, isotropic, three-dimensional (3D) turbulence has an energy-cascading inertial range with a $-5/3$ spectral slope that exhibits the predictability behavior described by Lorenz...
(1969), while homogeneous, isotropic, two-dimensional (2D) turbulence has an enstrophy-cascading inertial range with a $-3$ slope that does not (Leith 1971; Leith and Kraichnan 1972; Metais and Lesieur 1986; Boffetta et al. 1997). This different behavior is associated with differences in eddy turnover times as a function of spatial scale, and it is evident from examining perturbation growth curves and evolution of perturbation energy spectra [see, e.g., Metais and Lesieur (1986) and Rotunno and Snyder (2008) for results from closure models and Boffetta et al. (1997) and Boffetta and Musacchio (2001) for numerical simulations]. Following Lorenz (1969), flows whose energy spectra have slopes between $-3$ and $-5/3$ are expected to exhibit predictability behavior that is qualitatively similar to that of 3D turbulence. However, such flows are expected to have less energy at smaller scales than those with a $-5/3$ slope, leading to less rapidly growing errors at intermediate scales and a longer range of predictability (see Table 3 of Lorenz 1969).

These results suggest that at least in homogeneous, isotropic turbulence, predictability behavior is connected to the slope of the flow’s kinetic energy spectrum and to error growth as a function of scale. To test the extent to which these ideas apply to more complex flows, this article examines spectral slopes, error growth at different spatial scales, and predictability behavior in a model a step up in complexity and atmospheric realism from homogeneous, isotropic turbulence: a multilevel quasigeostrophic (QG) channel model with an atmospheric jet and evolving temperature perturbations at the upper and lower surfaces. By investigating spectral slopes and predictability in an intermediate context, the study aims to serve as a bridge between predictability paradigms from homogeneous, isotropic turbulence and predictability behavior in more complex atmospheric flows, such as full numerical weather prediction (NWP) models and general circulation models (GCMs).

One motivation for employing a QG model is that at large scales, the atmosphere is approximately governed by QG dynamics. As expected from this quasi-two-dimensional behavior, analysis of observations indicates that at scales larger than several hundred kilometers, the atmosphere’s kinetic energy spectrum has a slope close to $-3$ (Boer and Shepherd 1983). Consistent with this large-scale slope, previous QG studies of atmospheric predictability have found that perturbation spectra evolve similarly to those in the $-3$ range of 2D turbulence (Roads 1985; Straus 1989; Vannitsem and Nicolis 1997; Snyder et al. 2003). Studies of atmospheric GCMs at grid spacings insufficient to resolve the mesoscale report similar predictability behavior (Boer 1994, 2003; Tribbia and Baumhefner 2004). McWilliams and Chow (1981) also find a $-3$ spectral slope and similar predictability results for a QG simulation of the oceanic Antarctic circumpolar jet. However, the models used in these previous studies had relatively low horizontal resolution, which may have influenced the predictability behavior by inhibiting the growth of perturbations at smaller scales or truncating those scales altogether. Thus, one contribution of this article is extending previous QG predictability studies to higher horizontal resolution.

The QG model used here also has high vertical resolution and potential temperature perturbations at the upper and lower surfaces. This presents the possibility of spectral slopes and predictability behavior other than that expected from standard QG theory. QG flow in the presence of nonsentropic boundaries has distinctive dynamics. The simplest case, surface quasigeostrophic (SQG) flow, is the evolution of (lower) surface potential temperature in a uniform potential vorticity (PV) fluid that is unbounded above. As discussed by Blumen (1978), Pierrehumbert et al. (1994), and Held et al. (1995), inertial-range arguments predict that homogeneous, isotropic SQG turbulence has a kinetic energy spectrum at the surface with a $-5/3$ slope. Similar to 3D turbulence, SQG turbulence exhibits faster error growth at smaller scales and corresponding predictability behavior (Rotunno and Snyder 2008). SQG motions of a given scale decay exponentially with height away from the boundary. Thus, at sufficiently small horizontal and vertical scales in the current QG model, motions at the upper surface—similar to the tropopause (Juckes 1994), where surface friction does not play a major role—may behave as in SQG and exhibit a $-5/3$ slope. Such behavior has been found by Hoyer and Sadourny (1982) using a closure model and simulated numerically by Tulloch and Smith (2009). The model used here includes similar dynamics to those in these previous two studies, but more complex interior PV and a more atmospherically realistic geometry with a jet. Given these modifications, another contribution of this article is investigating whether increasing resolution leads the current model to exhibit SQG dynamics at the upper surface, a $-5/3$ spectral slope at small scales, and the expected associated predictability behavior.

Another motivation for this study is that the observed atmospheric kinetic energy spectrum transitions from a $-3$ slope at large scales to a $-5/3$ slope for wavelengths shorter than about 400 km (Nastrom and Gage 1985; Cho

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1 Two-dimensional turbulence also has an inertial range above the scale of forcing with an inverse energy cascade and a $-5/3$ slope, which exhibits predictability behavior similar to the $-5/3$ inertial range in 3D turbulence. Here, when discussing 2D turbulence, we focus on the $-3$ inertial range below the scale of forcing.
et al. 1999). GCMs and mesoscale models run at high resolution produce spectra with a similar transition in slope (Koshyk and Hamilton 2001; Skamarock 2004). The correspondence of these spectral slopes to those in 2D and 3D, homogeneous, isotropic turbulence has raised questions about the predictability behavior of realistic atmospheric models. Unless motions in the $-\frac{5}{3}$ range do not dynamically influence those in the $-3$ range, this transition is likely important for predictability.

The reasons for this spectral transition are not yet settled; several explanations have been proposed (e.g., Gage 1979; Lilly 1983, 1989; Dewan 1979; VanZandt 1982; Lindborg 1999, 2006; Tung and Orlando 2003; Tulloch and Smith 2009). Because the transition occurs at scales where the Rossby number is $O(1)$ and the applicability of QG dynamics is uncertain, the QG model used here likely cannot reproduce the dynamics underlying this transition. Nevertheless, Tung and Orlando (2003) and Tulloch and Smith (2006, 2009) found spectra with a similar $-3$ to $-\frac{5}{3}$ transition using high-resolution QG models. Moreover, these previous high-resolution QG and primitive equation model studies did not explicitly investigate predictability behavior, leaving questions about the predictability implications of the spectral transition unanswered. By investigating the link between spectral slopes and predictability in an intermediate context, our study seeks to help interpret the possible implications of these previously observed and modeled spectra for predictability and numerical weather prediction. While the study is motivated by the observed $-3$ to $-\frac{5}{3}$ transition, none of our conclusions depend on the model reproducing the observed spectrum. Instead, we focus primarily on studying the predictability behavior of the model, given the spectra it generates.

To investigate these issues, we examine equilibrium spectra, perturbation growth behavior, and the evolution of perturbation spectra in the QG model at different resolutions. The overall goal is to examine the link between spectral slopes and predictability behavior in a context more atmospherically realistic than homogeneous, isotropic turbulence. Section 2 describes the QG model and the experimental setup. Section 3 examines equilibrium spectra for the model at different resolutions and compares results with earlier findings from both observations and models. The results indicate that at the highest resolution tested, the model’s kinetic energy spectra have a shallower than $-3$ slope at the upper surface and uppermost interior levels, but a $-3$ or steeper slope at the lower surface and in most of the interior. Section 4 investigates the predictability behavior of such a flow, with different spectral slopes at different levels, by examining the domain- and ensemble-averaged perturbation growth behavior of the model at the highest resolution tested. Sections 5 and 6 investigate how the model’s perturbation growth behavior changes with resolution and how it varies in time. Section 7 summarizes and discusses the major results.

2. Methodology

a. Quasigeostrophic model

The model used in this study is a multilevel quasigeostrophic channel model on a midlatitude $\beta$ plane. The model is described briefly below; further details, the model equations, and discussion of the model’s behavior at lower resolution can be found in Rotunno and Bao (1996), Morss et al. (2001), and Snyder et al. (2003).

The QG model flow is governed by evolution of pseudopotential vorticity $q$ (hereafter referred to as PV, for simplicity) in the interior and potential temperature $\theta$ at rigid upper and lower surfaces. The model domain has a depth $H$ of 9 km. It is periodic in the zonal ($x$) direction, with a circumference of 16 000 km (approximately half that of the midlatitude earth) and a meridional width of 8000 km (an approximately 70° band of latitude). These horizontal dimensions correspond to 16 and 8 times the Rossby radius $NHf$, respectively. The flow is forced by relaxation to a specified baroclinic zonal mean jet. Dissipation is provided by fourth-order horizontal diffusion, along with Ekman drag at the lower surface. All model parameters are the same as those in Table 1 of Snyder et al. (2003) except the diffusion coefficient $\nu$, which changes with the model resolution as discussed below.

The standard (highest) model resolution employed in this study was 512 grid points in $x$ and 256 grid points in $y$, corresponding to a grid spacing of 31.25 km ($\frac{31}{32}$ of the Rossby radius) in both $x$ and $y$, with 32 vertical levels. To investigate the effects of model resolution and compare with results in Snyder et al. (2003) and Snyder and Hamill (2003), several experiments were also run at 62.5-km grid spacing with 16 vertical levels, and at 125-km grid spacing with 8 vertical levels (the standard resolution in these two previous studies). The nondimensional diffusion coefficient $\nu$ is $4.9 \times 10^{-2}$ for the 125-km grid spacing runs; it is reduced by a factor of 8 for the 62.5-km runs, and by a further factor of 16 for the 31.25-km runs. Experiments performed at a given resolution with the diffusion increased or decreased by a factor of 2 (see section 3) indicate that changing the diffusion level does not significantly affect the results.

To examine results without nonlinear error saturation, some experiments were performed with a tangent linear version of the QG model. The tangent linear model (TLM) approximates the evolution of sufficiently small perturbations to a reference solution using the linearized
equations, as described in Snyder et al. (2003). For errors not approaching saturation, all results shown are similar for the TLM and the full nonlinear model. Thus, results from nonlinear model and TLM runs are considered interchangeable except for saturation.

The QG model was selected to investigate predictability behavior in a model of intermediate complexity, one that is more realistic for synoptic-scale atmospheric flow than homogeneous isotropic turbulence but simpler than full NWP models or GCMs. The model exhibits turbulent flow along a meandering baroclinic jet that is concentrated in the central part of the channel and is strongest at the upper surface. Waves move along the jet and break to form cutoff eddies. Snapshots of the upper-surface temperature at the three resolutions are shown in Fig. 1. As the model grid spacing decreases, the gradients and the small-scale structure increase, and the model generates finer-scale filaments and vortices. Even at 31.25-km grid spacing, however, the model does not generate finescale “curdling” on filaments to the extent depicted by Held et al. (1995) or Tulloch and Smith (2009) for SQG flow. Higher-resolution runs are computationally expensive, but preliminary experiments at higher resolution (15.125-km grid spacing with 64 vertical levels) suggest that the model begins to exhibit more of this behavior at the upper surface.

In the interior, streamfunction \( \phi \) is derived from \( q \) using appropriate boundary conditions as described in Snyder et al. (2003). Streamfunction is extrapolated to the upper and lower surfaces using the surface \( u \). Zonal wind \( u \) and meridional wind \( v \) at all levels and \( \theta \) in the model interior are then derived from \( \phi \). Kinetic energy \( KE = u^2 + v^2 \), and total energy (nondimensionalized) \( E = u^2 + v^2 + \theta^2 \), where \( u, v, \) and \( \theta \) can be either full fields or perturbations.

b. Experimental design

For each model resolution, a reference run of 250 days was generated following a spinup period of 50 days (initialized with a perturbation at 1 wavenumber). The equilibrium spectra depicted in section 3 are averaged over 51 equally spaced times during the 250-day reference run, separated by 5 days.

Perturbation runs were generated by selecting an initial field from the reference run for a given resolution, adding five sets of random perturbations to form a five-member ensemble, and running the ensemble forward using the nonlinear model or TLM. Perturbations are “white” in the energy norm, that is, they were drawn from a multivariate normal distribution having an identity covariance matrix under the energy inner product, as described in Snyder et al. (2003). To limit perturbations to specific spatial scales, appropriate Fourier coefficients were set to zero: \((k^2 + l^2)^{1/2} > 16\) for large-scale perturbations, and \( k < 50 \) or \( l < 50 \) for small-scale perturbations (where \( k \) and \( l \) are zonal and meridional wavenumbers, respectively, in units of cycles per domain length). Most results shown are averaged over five-member ensembles initiated at 10 initial times separated by 20 days (50 perturbed runs). To generate statistics over a wider range of cases, the TLM results in sections 5 and 6 are accumulated over five-member ensembles initialized at 20 initial times separated by 10 days (100 perturbed runs).

The primary perturbation norm used for examining growth rates is the square root of the volume average of \( E \). Except for the differences noted in the text, results are similar for perturbation \( E \) at different levels (including the upper and lower surface) and for perturbation \( KE \).
Spectra are depicted in terms of the density of a quantity (such as KE) at a given model level as a function of either zonal wavenumber \( k \) or total (two dimensional) horizontal wavenumber \( K = \left( k^2 + l^2 \right)^{1/2} \), approximated as discussed in the following paragraph. Most other studies with nonisotropic, limited-area models (such as ours) examine spectra as a function of zonal wavenumber because the flow is statistically homogeneous only in \( x \). The observation-derived spectra in Nastrom and Gage (1985) are also depicted as a function of one-dimensional wavenumber, along flight tracks with different orientations. However, total horizontal wavenumber can be considered a better representation of the horizontal scales in the flow. Moreover, total horizontal wavenumber is often used to analyze results from homogeneous turbulent flows and from global models. Examining spectra in both ways allows us to compare with a variety of previous results and explore the effects of how spectra are depicted. Equilibrium spectra are depicted as a function of both \( k \) and \( K \), while the discussion of perturbation spectra focuses primarily on spectra depicted as a function of \( K \). Because perturbation energy saturates at twice the climatological level, a factor of one-half is used when depicting perturbation spectra.

Spectra as a function of total horizontal wavenumber \( K \) were generated by conducting Fourier decompositions in the zonal and meridional directions, then summing values for zonal wavenumber \( k \) and meridional wavenumber \( l \) within circular shells in \( k, l \) space bounded by \( K - \frac{1}{2} \leq \left( k^2 + l^2 \right)^{1/2} < K + \frac{1}{2} \), with \( K \) the central radius of the shells (Errico 1985). Spectra as a function of \( k \) were generated by conducting Fourier decompositions in \( k \) along zonal slices, then averaging results over multiple slices. All zonal-wavenumber spectra shown are for averages over the middle three-quarters of the meridional domain. Results are similar for single meridional slices and for other meridional averages, although the meridional averaging decreases noise, and energy tends to decrease somewhat as one moves toward the meridional boundaries.

3. Spectra for reference solution

Figure 2a depicts equilibrium spectra for the standard resolution QG model (31.25-km grid spacing) for \( u, v, \theta \), and \( \phi \) at the upper surface, as a function of zonal wavenumber. Between the planetary and dissipation scales, the spectra have a well-defined regime that obeys a power-law decay with a constant slope. (Note that this power-law regime is not an inertial range, even below the primary scale of energy injection, because the relaxation to the specified zonal mean jet at each time step removes energy at all scales.) The \( u \) and \( v \) spectra have a shape similar to that reported by McWilliams and Chow (1981, their Fig. 15a) for a three-level QG model with an oceanic configuration, except here the spectra have approximately a \( -2.3 \) rather than \( -3 \) slope. Similar to real atmospheric spectra (Nastrom and Gage 1985), the \( \theta \) spectrum has a similar slope to the \( u \) and \( v \) spectra, and the spectra peak at wavelengths around 5000–10 000 km. As expected given that \( u \) and \( v \) are first derivatives of \( \phi \), the \( \phi \) spectrum has approximately a \( -4.3 \) \( (-2.3 - 2) \) slope.

![Figure 2](image-url)
Figure 2b depicts kinetic energy spectra as a function of zonal wavenumber for the standard resolution QG model at different levels. Moving down in the model atmosphere, the spectral slope steepens from $-2.3$ at the upper surface to $-2.6$ at the top interior level to approximately $-3$ at the next-to-top interior level. At lower levels, spectra do not always exhibit a clear power-law regime, but to the extent one can define a slope, it is $-3$ or steeper. At midlevels, the spectrum transitions to a shallower slope at approximately wavenumber 15; the reason for this spectral shape is unclear, but it is not a direct parallel to the observed atmospheric energy spectrum since the slope is $-3$ or steeper at all wavenumbers and the transition to shallow slope occurs at a much larger wavelength than in the observed spectrum. Note also that the mid- and lower-level kinetic energy spectra have significantly smaller magnitude than the upper-level spectra, indicating that the model energy is dominated by upper levels (see also Snyder et al. 2003).

Figure 3 depicts upper-surface KE spectra as functions of zonal wavenumber for the QG model at different resolutions. The results for 125- and 62.5-km grid spacing are similar to those in Fig. 2 of Snyder et al. (2003). At higher model resolution, spectra have a better-defined power-law regime. In addition, higher-resolution spectra appear to have a somewhat shallower slope. A full set of runs at still higher resolution has not been performed because of computational expense. However, preliminary results with resolution doubled again (15.125-km grid spacing and 64 vertical levels; not shown) suggest that although the model exhibits more finescale curdling on filaments at the upper surface, the spectral slope remains approximately $-2.3$ and that the power-law regime simply extends to smaller scales. In other words, it appears that the model is asymptotically approaching a $-2.3$ spectral slope at the upper surface.

Figure 3 also depicts the effects of changing the diffusion coefficient at a specified model resolution. As expected, increasing (decreasing) the diffusion decreases (increases) the energy in the dissipation range, without affecting the spectrum at low–medium wavenumbers.\footnote{While decreasing the diffusion at the standard resolution produces a reasonable energy spectrum, it generates some $2\Delta x$ noise in the fields. Consequently, this level of diffusion was not selected for the standard runs.} At the standard resolution, around wavenumber 80 is where the spectrum begins to decay from the constant slope and changes in diffusion produce noticeable effects. This suggests that the effective model resolution is around $6-7\Delta x$, equivalent to around 200 km at 31.25-km grid spacing [in approximate agreement with Skamarock (2004) for the Weather Research and Forecasting (WRF) model with fourth-order diffusion].

Figure 4 shows the same kinetic energy spectra as Fig. 3, but as a function of total horizontal wavenumber rather than zonal wavenumber. The spectra in Fig. 4 have similar shapes to those in Fig. 3, and the upper-surface spectral slope at the highest resolution differs by only about 1%. Total horizontal wavenumber spectra for different fields and different model levels (not shown) are also similar to those in Fig. 2. For homogeneous, isotropic turbulence, it can be shown that if spectra depicted as a function of two-dimensional wavenumbers obey a power law, then spectra depicted as a function of one-dimensional wavenumbers also obey a power law with the same slope (Mathieu and Scott 2000, chapter 6). Although the current flow is nonhomogeneous and non-isotropic, these two ways of deriving spectra appear to generate equilibrium spectra with similar slopes.

The relaxation to a zonal mean jet in the standard model formulation removes energy at all scales. To test how this influences spectra, we reran the model at the standard resolution substituting relaxation of only the zonal average fields at each time step to the specified zonal mean jet. The resulting kinetic energy spectra (not shown) are similar to those in Figs. 3–4, but they have somewhat shallower slopes (e.g., $-2.1$ at the upper surface).

As discussed in the introduction, this model has potential to exhibit SQG behavior at the upper surface and a corresponding transition to a $-5/3$ slope as the horizontal
and vertical resolution is increased. As Figs. 1–4 illustrate, however, even at high resolution, the model does not exhibit the small-scale dynamics expected from SQG flow at the upper levels nor the anticipated energy spectrum at small scales. SQG behavior is expected for motions that are not influenced by the lower boundary, in other words, for vertical scales \( L_v \ll H \). SQG motions with horizontal wavelength \( L_h \) have \( L_v = fL_h/(2\pi N) \), where \( f \) is the Coriolis parameter and \( N \) is the buoyancy frequency. Thus, we expect a transition to SQG behavior and a \( k^{-5/3} \) spectrum for \( L_h \leq 2\pi NHf = 6000 \text{ km} \), while horizontal wavelengths as small as 200 km are well resolved by the model. This scaling analysis, as well as the preliminary results at higher resolution discussed earlier, suggests that insufficient resolution is likely not responsible for the lack of a spectral transition in this model.

Why, then, might the small scales at upper levels in our solutions not behave as predicted by SQG? One possibility is the presence of variations in the interior PV. While Tulloch and Smith (2009) demonstrate a transition to a \( k^{-5/3} \) spectrum in a QG model with variations in interior PV, they explicitly consider only the two gravest vertical modes of PV. Our solutions, in contrast, resolve much finer vertical scales. As discussed below, our solutions also differ from those of Tulloch and Smith (2009), and other QG studies, in the forcing for the flow, the meridional boundary conditions, and the presence of a single, strong jet.

At scales larger than about 500 km, kinetic energy spectra derived from observations (e.g., Boer and Shepherd 1983; Nastrom and Gage 1985) have slopes of \(-3\) or steeper, as do spectra in most previous studies with QG models (e.g., Barros and Wiin-Nielsen 1974; McWilliams and Chow 1981; Vallis 1983; Roads 1985) and primitive equation models (e.g., Koshyk and Hamilton 2001; Boer 2003). At all but the uppermost levels in this model, kinetic energy spectra have slopes steeper than \(-3\), consistent with these earlier findings. However, at and near the upper surface, the spectral slope is \(-2.3\), even at synoptic scales. This differs from earlier findings from atmospheric observations and models.

What might be responsible for this \( k^{-2.3} \) behavior at the upper surface? Compared to previous QG studies, our simulations not only employ higher vertical resolution, but they also differ in the wall-bounded channel geometry and the forcing by relaxation on boundary \( \theta \) and interior PV. As noted above, the spectral slope changes if the relaxation is applied only to the zonal mean fields, indicating the possible role of details of the forcing and geometry in determining the spectral slope.

However, it is not clear that this slope is unphysical or due solely to details of the model formulation. Qualitatively, the flow at the upper surface is dominated by a single jet and its associated region of intense horizontal gradient in \( \theta \). Under the limit that \( \theta \) has a jump and the jet a logarithmic profile, the energy spectrum would have a \( k^{-2} \) dependence (as originally noted by Charney 1971), close to what our simulations produce. Moreover, spectra from both observations and primitive equation models are based on data at varying vertical distances from the tropopause. If energy spectra in the current QG solutions are averaged over the top few levels, corresponding to the vertical averaging near the jet level in the observational or primitive equation model spectra, the slope is \(-3\) or steeper.

Since our main interest is the predictability behavior of solutions for this high-resolution QG model, we leave for future work further investigation into the dynamics that generate the particular characteristics of its energy spectra. Given its different slopes at different levels and its quasi-realistic geometry and jet, the model presents an interesting context for studying predictability behavior from both theoretical and practical perspectives.

Lorenz (1969) and other works suggest that one can infer predictability behavior from the slope of kinetic energy spectra: flows with slopes shallower than \(-3\) exhibit faster growth at smaller scales, and thus different perturbation growth behavior than flows with \(-3\) or steeper slopes. What might the current QG model’s spectra indicate about its predictability behavior? At high resolution, the spectral slope at the upper surface is noticeably shallower than \(-3\). However, all but the uppermost levels have slopes steeper than \(-3\), and it is not apparent how dynamics at the different levels will interact in determining predictability behavior. Further complicating predictability interpretations based on spectral...
slopes, previous studies have found that intermittency and coherent structures—both present in this model—can cause numerical model spectra to deviate from slopes predicted by theory (e.g., Basdevant et al. 1981; Pierrehumbert et al. 1994). Consequently, the implications of these spectra for the model’s predictability behavior are unclear. Thus, next we explicitly investigate predictability behavior in this QG model by examining perturbation growth.

4. Average perturbation growth behavior at 31.25-km resolution

To investigate perturbation growth, we ran ensembles with the QG model as discussed in section 2, initialized with random perturbations at different scales and different amplitudes. This section examines the domain- and ensemble-averaged growth of perturbations at the highest model resolution tested (31.25-km grid spacing). Changes in perturbation growth behavior with model resolution and deviations from the average behavior are discussed in sections 5 and 6.

Figure 5 depicts the average evolution of total perturbation energy for three different types of initial conditions, each averaged over 50 perturbed runs. Perturbations that include small spatial scales undergo an initial decay due to the model dissipation, and there is a transient period during the first 1–3 days when perturbations exhibit slightly faster growth as the ensemble properties adjust (Snyder et al. 2003). Then, the model exhibits a linear regime with a fairly constant growth rate of approximately 0.6 day$^{-1}$, corresponding to an error doubling time of 1.2 days. Once perturbations grow sufficiently large, errors begin to saturate and the growth rate decreases with time. When the error growth rate begins to decrease depends on the initial perturbation amplitude and structure; in the runs shown in Fig. 5, this begins after about 6 days. The evolution of perturbation energy at the upper surface and at other model levels (not shown) is generally similar to that shown in Fig. 5. The primary difference is that the initial decay and adjustment period lasts for a few hours at the upper surface, compared with a few days at the lower and midlevels. After this initial adjustment period, perturbations grow at similar rates at different levels.

In flows with a shallower than $-3$ spectral slope, such as SQG or 3D turbulence, the eddy turnover time decreases with decreasing spatial scale. Small-amplitude perturbations initially experience the faster growth associated with the smallest spatial scales, until perturbations grow large enough that small-scale errors begin to saturate. Progressively larger, slower-growing scales then take over the perturbation growth dynamics, so that the growth rate decreases with time (e.g., Fig. 2a in Rotunno and Snyder 2008). Flows with a $-3$ or steeper slope, such as 2D turbulence, exhibit a similar linear-growth regime for small-amplitude perturbations. However, because small scales do not grow faster, saturation of small-scale errors has little effect on the growth rate. Consequently, the growth rate does not begin to decrease until errors saturate at the large, energy-containing scales (e.g., Fig. 2b in Rotunno and Snyder 2008).

To distinguish between these two types of predictability behavior, next we look at the growth of perturbations at different scales by examining perturbation kinetic energy spectra. Figure 6 shows the average evolution of perturbation spectra (as a function of two-dimensional wavenumber) at the upper surface in the standard-resolution model. Results are shown for initial perturbations at small scales and at all scales, corresponding to the top two domain-averaged perturbation growth curves in Fig. 5. In both experiments, the upper-surface perturbations adjust in about one day to a similar spectral shape, with a broad peak around wavenumber 30 (wavelength of approximately 500 km) and decaying for low and high wavenumbers. In 3D or SQG turbulence, perturbation spectra initially peak at the smallest resolved scales, while in 2D turbulence, perturbation spectra peak at the energy-containing scales. Thus, the upper-surface...
perturbation spectra in this model are intermediate between those in the 3D and 2D turbulence paradigms.

Following the initial adjustment, the perturbation spectra in Fig. 6 grow self-similarly for approximately 6 days. As the smallest resolved scales begin to saturate, the growth rate begins to decrease (as indicated by the decreasing spacing between the perturbation spectra in Fig. 6, or by Fig. 5). The peak of the perturbation spectra then shifts to larger and larger scales, as in 3D or SQG turbulence with a $-\frac{5}{3}$ slope [e.g., Fig. 2 in Metais and Lesieur (1986) or Fig. 1a in Rotunno and Snyder (2008)]. This differs from perturbation growth in 2D turbulence with a $-3$ slope, in which saturation of small scales does not affect the error growth rate [e.g., Fig. 9 in Metais and Lesieur (1986) or Fig. 1b in Rotunno and Snyder (2008)].

Comparing the evolution of perturbation spectra in this model to the 3D/SQG and 2D turbulence paradigms is complicated by the intermediate-scale perturbation spectral peak, the broadness of the peak, and its proximity to the dissipation range. Nevertheless, fundamental aspects of this model's predictability behavior appear similar to that expected for flow with a shallower than $-3$ slope and a faster eddy turnover time at smaller scales.

Figure 7 compares perturbation spectra at different model levels, after adjustment but prior to saturation. Perturbation spectra have a broad peak near wavenumber 30 only at the upper surface and uppermost interior levels. At lower levels, perturbation spectra peak at the energy-containing scales, consistent with the $-3$ or steeper equilibrium spectral slopes at these levels. Examination of the evolution of perturbation spectra at these levels (similar to Fig. 6; not shown) indicates that the growth rate begins to slow after around 6 days, even though perturbations do not saturate until later. This suggests that the overall perturbation growth behavior is dominated by the upper surface, with its shallower than $-3$ slope and smaller-scale perturbation spectral peak. This is consistent with the fact that, as shown in Fig. 7, the perturbation energy is dominated by the upper surface and uppermost interior levels, especially at small scales.

The initial adjustment of the perturbation spectra is linked to the initial energy decay seen in Fig. 5. Energy initially decreases as the perturbations adjust from random noise to focusing around PV gradients, vortices, and other structures in the reference-state flow (Snyder
et al. 2003). As depicted in Fig. 6a, in two-dimensional wavenumber space this appears as kinetic energy gradually spreading from the initialized wavenumbers to all scales. As noted above, the perturbations take longer to adjust at lower–midlevels than at the upper surface. Snyder et al. (2003) suggest that this is because the (effective) PV gradients are substantially weaker in the interior than at the upper surface.

Perturbation spectra depicted as a function of zonal wavenumber (not shown) are nearly flat from wavenumber 1 out to wavenumber 20, then slowly decrease for larger wavenumbers. The shape of the zonal-wavenumber perturbation spectra (at equivalent grid spacing) is very similar to that shown in Snyder and Hamill (2003, their Fig. 3) for the model’s leading Lyapunov vectors for 125-km resolution. This is consistent with Snyder et al.’s (2003) findings that this model’s perturbations rapidly converge to the subspace of leading Lyapunov vectors. Thus, while these previous studies did not examine Lyapunov vectors in two-dimensional spectral space, the shape of the two-dimensional perturbation spectra shown here is likely similar to that of the model’s leading Lyapunov vector.

What are the practical predictability implications of this flow’s perturbation growth behavior? As discussed in Lorenz (1969), errors in this model grow faster when small scales are unsaturated. However, the error doubling time in the linear regime in this model is approximately 1.2 days, compared with doubling times on the order of minutes in Lorenz (1969). The slower time scale in this model is likely due in part to the lower model resolution (especially given the dissipation) and in part to the $-7/3$ rather than $-5/3$ spectral slope. As illustrated in Lorenz (1969, see his Table 3), flows with slopes between $-3$ and $-5/3$ are expected to exhibit longer ranges of predictability than those with a $-5/3$ (shallower) slope.

Errors that are initially small double with this 1.2-day time scale until they begin to saturate. Consequently, as shown in Fig. 5, if initial condition errors are already small, improving initial conditions further (by either reducing the amplitude of initial errors or confining errors to smaller scales) can significantly increase the time for forecast errors to grow to a specified amplitude. Moreover, because of the dissipation, the perturbation amplitude at small scales decreases rapidly after initialization, increasing the time required for perturbations to saturate at small scales. Together, these results indicate that in finite-resolution, dissipative models with slopes not significantly shallower than $-3$, small-scale errors may take much longer to saturate than expected in the high-resolution limit of a flow with a $-5/3$ slope. The practical predictability limitations of such models may therefore be much less severe than the finite predictability limit hypothesized by Lorenz (1969).

![Figure 8](https://example.com/figure8.png)

**Figure 8.** Comparison of upper-surface perturbation KE spectra for different model resolutions, for ensembles initialized with large-scale random perturbations. Results are shown for 31.25- (solid lines), 62.5- (dashed lines), and 125-km (dotted lines) grid spacing, for $\Delta t = 0$ (red lines), 2 days (blue lines), and 4 days (black lines). For each resolution, results are averaged over five-member ensembles at 10 initial times. The thick solid line depicts the equilibrium KE spectrum at 31.25-km grid spacing. For easier comparison, the perturbation spectra for different resolutions were rescaled by a small factor to make them the same magnitude at $\Delta t = 0$; since the spectra depicted are all in the linear regime, the rescaling does not affect the evolution.

### 5. Perturbation growth behavior at different model resolutions

Next we investigate how changing the model resolution affects perturbation growth behavior. To facilitate comparing perturbation evolution among model resolutions, we examine ensembles initialized with large-scale random perturbations $[(k^2 + l^2)^{1/2} \leq 16]$ since these are resolved at all three grid spacings. Using large-scale perturbations also minimizes the initial energy decay due to the model dissipation (Snyder et al. 2003). Further, we use perturbations of sufficiently small amplitude that we can examine perturbation growth for several days prior to saturation.

Figure 8 compares the initial (0 h), 2-, and 4-day upper-surface perturbation spectra from ensembles at 31.25-km grid spacing with parallel results for lower model resolutions. By design, the initial spectra have similar shapes. At higher resolution, the adjusted (2 and 4 day) perturbation spectra have more power at smaller scales. This type of change is expected, given the increased energy at smaller scales evident in the higher-resolution equilibrium spectra (Figs. 3–4). Yet increasing the resolution also shifts the perturbation spectral peak to smaller scales. These changes are especially noticeable in the increase from intermediate to highest resolution, where the perturbation spectral peak moves from the
energy-containing scales to smaller scales. Increasing the resolution also increases the error growth rate at the upper surface, as illustrated by the increasing distance between the curves.

To examine how these changes in perturbation spectral shape with resolution look in Cartesian space, Fig. 9 depicts results for the same set of experiments as in Fig. 8, but for snapshots of fields and ensemble-averaged perturbations evolved from a single initial time out to 4 days. The 125-km grid spacing results are similar to those discussed in Snyder et al. (2003), with the perturbations focusing along $u^2(PV)$ gradients in the flow. As the resolution increases, the amplitude of the evolved perturbations increases. At higher resolution, perturbations also tend to have smaller scales and to focus around smaller-scale features in the flow, often away from the main jet. This shift in the dominant perturbation scale is especially noticeable at 31.25-km grid spacing. Further, at 31.25-km grid spacing, the perturbations become much more spatially localized. Together, Figs. 8 and 9 suggest that at higher resolution, smaller scales are more active in this QG model, and dynamics associated with coherent structures on scales of a few hundred kilometers are important for perturbation growth. In other words, the perturbation growth behavior is changing as the grid spacing is decreased to 31.25 km.

Figures 8–9 indicate that increasing resolution increases the perturbation growth rate at the upper surface. Figure 10 examines this from a different perspective by comparing the evolution of ensemble- and volume-averaged total perturbation energy at different resolutions. Results are shown for the same set of experiments as in Figs. 8–9 along with a 31.25-km TLM run (discussed further in section 6). Note that perturbation energy is very similar for the nonlinear model and TLM until about 8 days into the run, indicating that during this period errors are unsaturated and error growth is in the linear regime. The average perturbation growth rate prior to saturation increases noticeably with model resolution: from approximately 0.39 day$^{-1}$ at 125-km
sections 4 and 5 suggest that even at the highest resolution currently tested, the model exhibits the general behavior expected from turbulence theory given its shallower than $-3$ upper-level spectral slope: smaller-scale errors grow faster than larger-scale errors, reducing the model’s potential predictability.

6. Temporal variability in perturbation growth

As resolution increases, the perturbation growth curves in Fig. 10 also become less smooth. To examine this in greater detail, Fig. 11 depicts histograms of 0.2-day growth rates at the three model resolutions, derived from a larger set of TLM ensembles initialized with large-scale perturbations. Results are shown for the TLM rather than the nonlinear model to avoid potential variability in growth rate due to error saturation. As Fig. 11 illustrates, increasing model resolution increases not only the average perturbation growth rate, but also the temporal variability in growth rate. The growth rate variability (as indicated by the standard deviation and the tails of the distribution) increases most noticeably when the grid spacing decreases to 31.25 km.

The right-hand tails in Fig. 11 represent periods when, averaged across the domain, perturbations grow much faster than average. Examination of model fields indicates that these periods are associated with very fast perturbation growth in a portion of the domain, near a vortex or other spatially localized feature. The very fast growth tends to be supported for 6–24 h, followed by a period of slower-than-average growth or even domain-averaged decay (the left-hand tails in Fig. 11) as the feature associated with the fast growth evolves. Such periods occur at lower resolution (see also Roads 1985), but at the highest resolution tested, they tend to be associated with smaller-scale features (as depicted in Fig. 9) and larger fluctuations in perturbation growth.

Sections 4–5 focused on the model’s perturbation growth behavior averaged over many cases. Is the variability in growth rate at higher resolution associated with any changes in perturbation growth behavior? To investigate this, we selected the fastest-growing 0.5% and slowest-growing (fastest decaying) 0.5% of cases from the 31.25-km TLM runs—in other words, the cases from the tails of the distribution in the bottom panel of Fig. 11—and examined associated upper-surface perturbation spectra. We also examined perturbation spectra for the 0.5% of cases in the middle of the distribution. To examine how perturbations have evolved during periods with different growth rates, we calculated the difference between the perturbation spectrum at each time and that for 0.2 days earlier. For each case, this represents the change in perturbation spectrum during the period over
which the growth was calculated.\textsuperscript{4} We then averaged these difference spectra over the different sets of cases, and over all cases. Because this averaging combines perturbations evolved for 5 days with those evolved for 20 days (which have significantly larger amplitude), normalization is required so that cases later in the runs do not dominate the spectra. Thus, prior to differencing, each pair of perturbation spectra was normalized by the integral under the perturbation spectrum (total KE) for the later time.

Figure 12 compares the difference perturbation spectra averaged over the high-growth cases and low-growth cases with those averaged over the middle-growth cases and all cases. As expected, significantly more growth is occurring during the high-growth cases, and the low-growth cases are decaying overall. In the high-growth cases, the difference spectra shift to the right and have a sharper peak around wavenumbers 30–40; in other words, perturbations are growing faster than average at smaller scales. As discussed earlier, low-growth cases tend to follow high-growth cases: the average 0.2-day growth rate 10–20 h prior to the low-growth cases is 1.0 or higher, more than a standard deviation higher than the overall average 0.2-day growth rate of 0.6 (see Fig. 11). Figure 12 indicates that in the low-growth cases, perturbations are growing at wavenumbers less than 10 and decaying at higher wavenumbers; in other words, the perturbation spectra are adjusting back from the higher-wavenumber peak of the high-growth cases.

The broad peak in the perturbation spectra in Fig. 6 is due in part to accumulation over the different sets of cases, and over all cases. Because this averaging combines perturbations evolved for 5 days with those evolved for 20 days (which have significantly larger amplitude), normalization is required so that cases later in the runs do not dominate the spectra. Thus, prior to differencing, each pair of perturbation spectra was normalized by the integral under the perturbation spectrum (total KE) for the later time.

Figure 12 compares the difference perturbation spectra averaged over the high-growth cases and low-growth cases with those averaged over the middle-growth cases and all cases. As expected, significantly more growth is occurring during the high-growth cases, and the low-growth cases are decaying overall. In the high-growth cases, the difference spectra shift to the right and have a sharper peak around wavenumbers 30–40; in other words, perturbations are growing faster than average at smaller scales. As discussed earlier, low-growth cases tend to follow high-growth cases: the average 0.2-day growth rate 10–20 h prior to the low-growth cases is 1.0 or higher, more than a standard deviation higher than the overall average 0.2-day growth rate of 0.6 (see Fig. 11). Figure 12 indicates that in the low-growth cases, perturbations are growing at wavenumbers less than 10 and decaying at higher wavenumbers; in other words, the perturbation spectra are adjusting back from the higher-wavenumber peak of the high-growth cases.

The broad peak in the perturbation spectra in Fig. 6 is due in part to accumulation over the different sets of cases in Fig. 12, with their peaks at different scales. However, even in the high-growth cases, the most rapid growth is on average around wavenumber 40, rather than at the smallest resolved scales as in 3D or SQG turbulence. This could be a result of the dissipation, the $-7/3$

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\textsuperscript{4} Note that the growth rates were calculated using volume-averaged $E$, while the perturbation spectra are for upper-surface KE.
(rather than $-\frac{5}{3}$) slope, or interactions with the dynamics in the interior, where the spectral slope is steeper than $-3$ and perturbation spectra peak at larger scales. Further investigation of this issue is left for future work.

Overall, Figs. 11–12 indicate that at the highest resolution currently tested, the model exhibits intermittent, faster-than-average growth associated with faster growth at smaller scales. This fast growth is spatially localized, supported only for limited periods associated with the evolution of specific features in the flow. In other words, at this resolution the model exhibits temporal variability in perturbation growth behavior that may imply temporally varying predictability. This occasional faster growth may be an indication of a transition to smaller-scale perturbation growth that will occur more generally at higher resolution, or this variability may be a feature of nonhomogeneous, nonisotropic flows that exhibit faster growth at smaller scales.

7. Summary and discussion

Studies of two- and three-dimensional, homogeneous, isotropic turbulence and other simple flows suggest that a flow's predictability behavior is related to the slope of its kinetic energy spectrum (e.g., Lorenz 1969; Leith and Kraichnan 1972; Metais and Lesieur 1986; Rotunno and Snyder 2008). A $-\frac{5}{3}$ slope (as in 3D or SQG turbulence) is associated with a faster eddy turnover time at smaller spatial scales. In such flows, kinetic energy spectra for small-amplitude perturbations initially peak at small scales and grow with the faster time scale exhibited by these spatial scales. As small scales saturate, the perturbation spectral peak shifts to larger scales and the error growth rate slows. A $-3$ slope (as in 2D turbulence, below the scale of forcing) is associated with an eddy turnover time that is constant with scale. In such flows, perturbation spectra peak at the large, energy-containing scales, and saturation of small scales does not affect the error growth rate. Flows with spectral slopes between $-3$ and $-\frac{5}{3}$ are expected to exhibit faster eddy turnover times at smaller scales, but to a lesser extent than in flows with a $-\frac{5}{3}$ slope. Thus, while a $-3$ slope represents a transition between distinct predictability regimes (namely, between the existence or absence of a finite, intrinsic limit of predictability), finite-resolution flows with intermediate spectral slopes are expected to exhibit intermediate predictability behavior, in the sense that the error growth rates at any given scale will be intermediate between what they would be in a flow with a $-3$ or $-\frac{5}{3}$ slope.

Analysis of observations and of output from high-resolution primitive equation models suggests that the atmospheric kinetic energy spectrum has a $-3$ slope at synoptic scales transitioning to a $-\frac{5}{3}$ slope at mesoscales (e.g., Nastrom and Gage 1985; Cho et al. 1999; Koshyk and Hamilton 2001; Skamarock 2004). These slopes have potential implications for atmospheric models' predictability behavior, providing further motivation for understanding the relationship between spectral slopes, perturbation growth dynamics, and predictability behavior in various flows. Here we investigate this relationship in an intermediate context, simpler than full weather and climate models but more realistic than homogeneous, isotropic turbulence: a multilevel quasigeostrophic model. The study seeks to supplement the 2D and 3D turbulence paradigms in interpreting the implications of different spectral slopes, providing another anchor point for understanding predictability behavior in other models.

The model is run at higher resolution than most QG models used previously to study spectra and predictability. It also has temperature perturbations at the upper surface, representing the tropopause, as well as a quasi-realistic jet. Equilibrium spectra, perturbation growth, and predictability behavior are examined at 31.25-km grid spacing with 32 vertical levels and two lower resolutions.

At sufficiently high horizontal and vertical resolution, the temperature perturbations at the model’s upper surface have potential to generate SQG motion, and thus a $-\frac{5}{3}$ spectral slope at small scales. Even at 31.25-km grid spacing, however, model spectra do not exhibit a transition to a shallower slope at mesoscales. Rather, at high resolution, the model’s upper-surface kinetic energy spectra exhibit a power-law regime with a constant $-2.3$ slope. The spectral slopes are similar whether spectra are depicted as a function of zonal or total horizontal wavenumber. Kinetic energy spectra grow steeper as one moves away from the upper surface, such that spectral slopes are $-3$ or steeper at all but the top two interior levels. In the interior, this equilibrium spectral slope is broadly consistent with spectra derived from atmospheric observations and full atmospheric models, but it differs at the upper surface. Note, however, that our upper surface represents the tropopause, while observational and primitive equation model spectra are generally calculated at varying vertical distance from the tropopause. Averaging spectra among the model’s upper levels, simulating varying vertical distance from the jet, produces a $-3$ or steeper slope. Moreover, the $-2.3$ upper-surface slope is close to the $k^{-2}$ dependence that would follow if the baroclinic zone at the upper surface narrowed to a jump. Investigating this in greater detail requires further work. Given our model’s quasi-realistic atmospheric configuration at synoptic scales, its perturbation growth behavior may be relevant to more complex atmospheric models. Further, this flow, with different spectral slopes at different levels, provides an interesting context for studying predictability behavior.
To investigate predictability behavior, we performed perturbation experiments and examined perturbation growth and the evolution of perturbation spectra averaged over many cases. At the highest resolution tested, spectra for small-amplitude initial perturbations adjust to a shape that on average peaks around wavenumber 30 at the upper surface, but at the large, energy-containing scales at all but the uppermost levels. These shapes are consistent with the shallower than $-3$ equilibrium spectral slope at the uppermost levels and the $-3$ or steeper slope below. As small scales begin to saturate at the upper surface, the perturbation growth rate tends to decrease. In addition, as the grid spacing is decreased from 125 to 62.5 to 31.25 km, the upper-surface perturbation spectral peak shifts to higher wavenumbers, and the growth rate for unsaturated errors increases. These results suggest that the model’s predictability behavior is dominated by perturbation growth dynamics at the upper surface, with its shallower than $-3$ slope and larger energy at small scales.

Upper-surface perturbation spectra do not peak at the smallest resolved scales, and for sufficiently small initial condition errors, improving initial conditions can substantially improve forecasts. These differences from the 3D and SQG turbulence paradigms are likely due to some combination of the intermediate upper-surface spectral slope, the limited resolution, the dissipation, and contributions of the interior dynamics to the perturbation growth. Further investigation of the role of these various processes is left for future work.

As the resolution increases, the growth rate also becomes more temporally variable and perturbations become more spatially localized. These changes provide another indication that the model’s perturbation growth dynamics are changing with increasing resolution. At 31.25-km grid spacing, the model experiences periods during which spatially localized errors grow much faster than average, associated with specific features in the domain. During these periods, errors tend to grow faster at smaller scales. These periods are often followed by periods with slower-than-average growth or even decay, during which perturbation energy shifts back to larger scales. Averaging over many periods produces the average picture discussed above.

These results update previous findings on predictability behavior in OG models with quasi-realistic geometries, expanding earlier results from models with lower horizontal and/or vertical resolution. Unlike Tung and Orlando (2003) and Tulloch and Smith (2006, 2009), we do not find spectra that transition from a $-3$ slope at synoptic scales to a $-5/3$ slope at smaller scales. As discussed in section 3, scaling analysis and preliminary results at higher resolution suggest that this lack of spectral transition is not caused by limited horizontal resolution. Thus, at least in the present QG model, additional dynamics appear necessary to generate a $-5/3$ spectrum at mesoscales. However, dry QG dynamics alone are sufficient to generate a shallower than $-3$ slope and limited predictability.

In the present model, predictability behavior is linked to the spectral slope of the most energetic model levels, even in the presence of inhomogeneity and anisotropy. This suggests that paradigms from 2D, 3D, and SQG turbulence may be useful in understanding possible modes of perturbation growth and predictability behavior in more complex geospatial flows. However, our results also suggest that flows with intermediate spectral slopes may not fit neatly into one of the predictability paradigms provided by homogeneous, isotropic turbulence, especially when such flows are simulated by limited-resolution models with a substantial dissipation range. In such models, small-scale errors can take substantially longer to saturate than in Lorenz’s (1969) discussion, leading to potential for significant improvements in predictive skill even when errors grow faster at smaller scales. Further, our results illustrate how flows with intermittent growth and coherent structures—such as those studied here and in the real atmosphere—may exhibit different perturbation growth dynamics at different times, or in different parts of the domain. Thus, future studies of predictability may need to look beyond simple paradigms and consider time-varying perturbation growth behavior in greater detail.

Given that the current model does not reproduce the observed spectral transition from a $-3$ slope at large scales to a $-5/3$ slope at mesoscales, our results cannot be directly applied to understand the predictability behavior of high-resolution numerical weather prediction models that resolve the $-5/3$ range. However, if the $-5/3$ range is dynamically coupled with the larger scales, our results on the link between spectral slopes and predictability suggest that this spectrum will have significant implications for predictability behavior. As resolution is increased in such models, we expect errors to initially grow faster, as smaller, faster-growing scales are resolved. Results from Simmons and Hollingsworth (2002) suggest that this behavior is exhibited by the ECMWF model. Such models would then be expected to exhibit limited predictability. However, with the steeper than $-5/3$ spectral slope at larger scales, the possibilities for forecast improvement may be more favorable than that hypothesized by Lorenz (1969) for a flow with a $-5/3$ slope across scales.

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