NOTE ON HEAT TRANSFER AT THE SOIL SURFACE

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ABSTRACT

The heat balance equation for the earth's surface is evaluated for two sets of data taken at night in the Arizona desert region. A method is illustrated for determining the convective heat flux in the lowest layers of the atmosphere independently of measurements of the state of the lower atmosphere provided there is no condensation or evaporation at the surface. These data may then be used in conjunction with lapse-rate data to determine the eddy diffusivity at various levels near the ground. The diffusivities so obtained are of the correct order of magnitude and show a marked correlation with the wind speed. It is also found that the convective term in the heat balance equation can amount to as much as one-half the net radiation term; therefore, it should not be assumed negligible, as has been done in some cases.

1. Introduction

Since the direct source from which the atmosphere obtains most of its energy is the heated surface of the earth, even though the ultimate source of this energy is the sun, a study of the heat balance of the earth's surface is of basic importance to meteorology. This heat balance is expressed by the equation [1, p. 178],

$$S = Q + L + V, \tag{1}$$

where S is the net radiative heat flux, both long- and short-wave, received or given off at the surface; Q is the heat flux into or out of the soil; L the flux to or from the air by conduction and convection processes (pure conduction at the surface itself, of course); and V the flux due to evaporation or condensation of water at the surface.

The quantity L in (1) is the one of greatest direct influence on the daily weather. It represents the vertical heat flux due to turbulence near the ground and accounts for the largest part of the heating and cooling of the atmosphere. If the lapse rate and the eddy diffusivity of the atmosphere are known, L may be computed; or, as will be shown here, if L can be determined independently, it may be used to find the diffusivity.

Albrecht [1] has discussed the general problem and the instrumentation necessary to measure the quantities involved in (1); in 1941 he [2] presented the results of some very extensive measurements made in the Gobi Desert by Haude in 1931–1932. Unfortunately, it is difficult to obtain copies of the latter work, and it is still relatively unknown. It was brought to this author's attention recently by Dr. H. Lettau in connection with the data to be presented here.

In April 1947, the Electrical Engineering Research Laboratory of The University of Texas, operating under an Office of Naval Research contract, made some measurements at Gila Bend in the Arizona desert, which included continuous temperature records at 12 levels between 18 inches below the soil surface and six feet above it, using thermistor rods and a d-c amplifier [8, 12]. Six of these levels were at or below the soil surface. Simultaneously, continuous temperature and moisture soundings were made up to 200 feet on a tower, and wind speed and direction were recorded at three levels, The soil and air were very dry. No dew was ever observed, and only one brief shower occurred during the whole period of operation. The soil was mostly bare and consisted of a fine, loose, sandy silt.

2. Temperature distributions

Typical temperature distributions from the Arizona data are given in figs. 1 and 2. Fig. 1 shows the diurnal traces for several of the 12 levels and fig. 2 the corresponding set of vertical soundings in the soil at three-hourly intervals, with additional soundings for each hour from 0600–0900 MST on the morning of 8 April. Note that the maximum surface temperature shown is nearly 60C. Even in April the maximum was often near 65C.

The curves of fig. 2 correspond to what would be expected from theory [9]; *i.e.*, a temperature wave is propagated into the earth and is, in this case, almost entirely damped out at the −18-inch level. The period is, of course, 24 hours.

3. Heat balance and eddy diffusivity

Referring again to (1), it is desired to determine the heat flux L, to or from the atmosphere, from measurements or calculations of the other quantities in the equation. Fig. 2 suggests use of the soil soundings as energy diagrams to obtain Q, the heat flux to or

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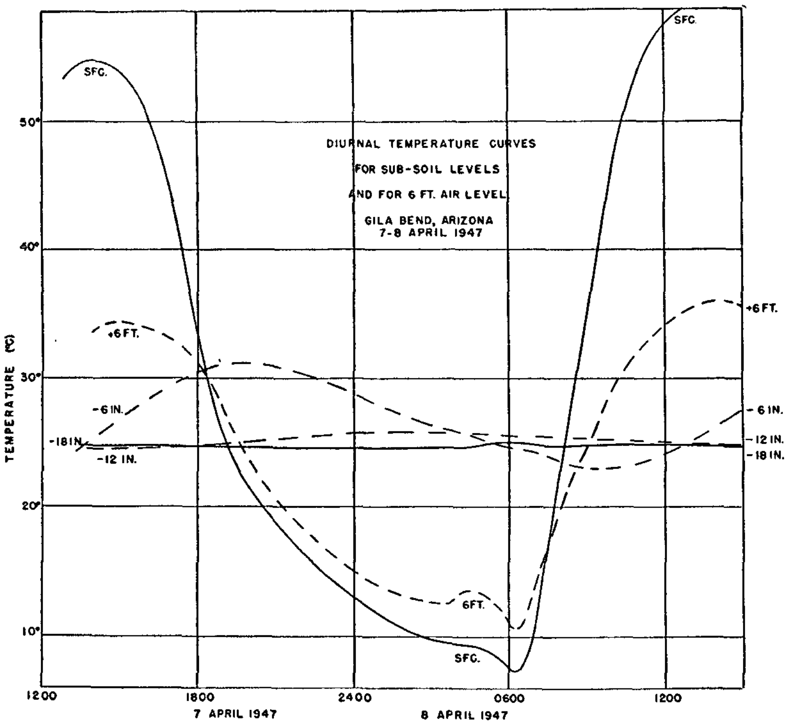

FIG. 1. Typical diurnal temperature curves for several sub-soil and air levels at Gila Bend, Arizona, 7–8 April 1947.

from the soil. Since the lapse rate is nearly zero at −18 inches, and the temperature at this level is almost constant, it appears that almost all the diurnal heat flux takes place in the top 18 inches of the soil, with only a small quantity of heat passing the −18-inch level. Neglecting this amount, and letting the temperature curve be representative of a column of soil of one square centimeter cross section, one needs only to introduce the density and specific heat of the soil for the sounding to become an energy diagram. In particular, the area between two soundings an hour apart represents Q in, say, cal cm^{-2} hr^{-1}. That is,

$$Q = \rho c \sum \Delta T \, \Delta Z, \tag{2}$$

where $\sum \Delta T \, \Delta Z$ represents the area enclosed by the two soundings, and ρ and c are the density and specific heat, respectively. For the Arizona soil, $\rho = 1.43$ gm cm^{-3} and $c = 0.20$ cal g^{-1} deg^{-1}, which may be considered constant because the soil was powder dry.

Since the soil and air were so dry, and since the air never became saturated, it appears that the condensation term V was probably zero for the Arizona data.

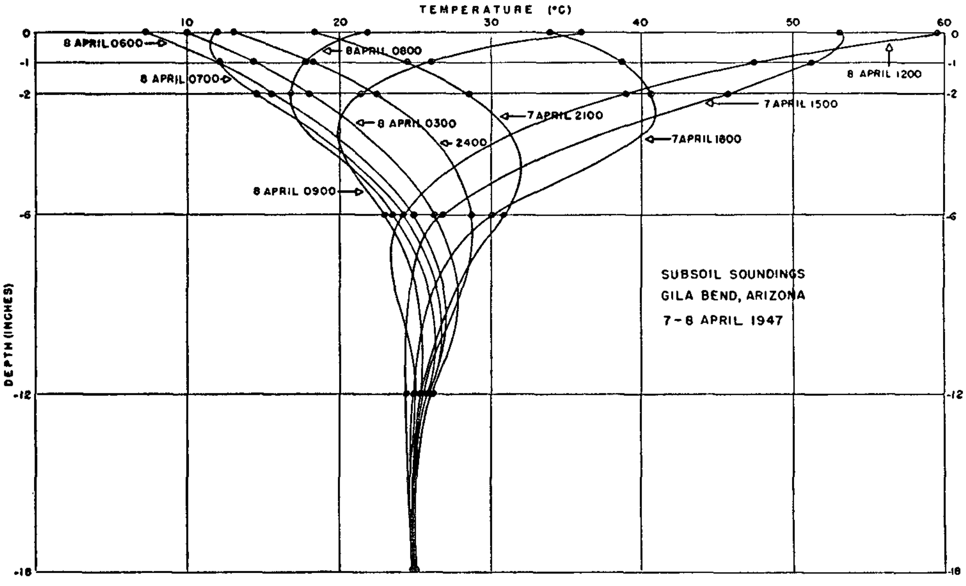

FIG. 2. Vertical temperature profiles in the soil, 7–8 April 1947, Gila Bend.

In any event, it was negligible, which greatly simplifies the problem at hand.

The other term in the equation is the net radiation. Since instruments for measuring radiation were lacking, the best approach remaining is the use of a radiation chart, such as Elsasser's [5]. The analysis of the data is therefore limited to the night hours, when the short-wave radiation is negligible.

The nearest radiosonde observations are those taken at Phoenix and Tucson, 50 miles NE and 110 miles SE, respectively, which admittedly are not necessarily representative of Gila Bend. However, by using the tower data from Gila Bend in conjunction with averaged values from Tucson and Phoenix, a reasonable estimate of the distribution of temperature and moisture over Gila Bend can be obtained. From Elsasser's chart it is found that for both data periods a conservative estimate of the net outgoing radiation at night is

$$S = 0.25\,\sigma T_s^4, \tag{3}$$

where T_s is the surface temperature and σ the Stefan constant. The factor 0.25 seems to be nearly constant, with individual determinations varying from 0.24 to 0.27. This value holds only for the present cases, of course. It is higher than would be expected for a moist climate.

Putting (2) and (3) into (1), the quantity L may be determined in cal cm^{-2} hr^{-1} for each hourly interval through the night. Knowing L and the lapse rate in the lower atmosphere, it is possible to compute the mean eddy diffusivity K from an equation given by Brunt [3] for the net flux of heat by turbulence across an isobaric surface, and which is valid without any assumption as to how K varies with height. This equation is

$$L = K\rho c_p(\partial T/\partial Z + \Gamma), \tag{4}$$

where ρ and c_p are the density and specific heat at constant pressure of air, respectively, $\partial T/\partial Z$ is the lapse rate, and Γ is the dry-adiabatic lapse rate. In making this calculation, the heat capacity of the lowest 6 feet of the atmosphere is neglected, and it is assumed that all the heat passes through this layer by eddy processes. Since the heat capacity of a column of air of one square centimeter cross section and 6 feet deep is only about 0.05 cal deg^{-1}, one calorie is sufficient to account for the total change of temperature of this column from sunset to sunrise. In view of the other probable errors in the calculations, this quantity may be safely neglected.

4. Discussion of results

General—The results of the computations for the night of 7–8 April 1947 are given in fig. 3, which shows the time variation of S, Q, L, and K at 6 feet, and windspeed at 10 feet. All values except windspeed are hourly averages plotted at the midpoint of the hour. Note that from 1900–2000 the S and Q terms just about balance each other; but as the night progresses, Q is no longer as large as S, indicating a flow of heat from the atmosphere into the soil surface, whence it is radiated outward. The L term reaches a maximum of 4.1 cal cm^{-2} hr^{-1}, half as high as the net radiation at that time. The correlation between the diffusivity and windspeed curves is quite striking, particularly since the computations are independent of the wind variation.

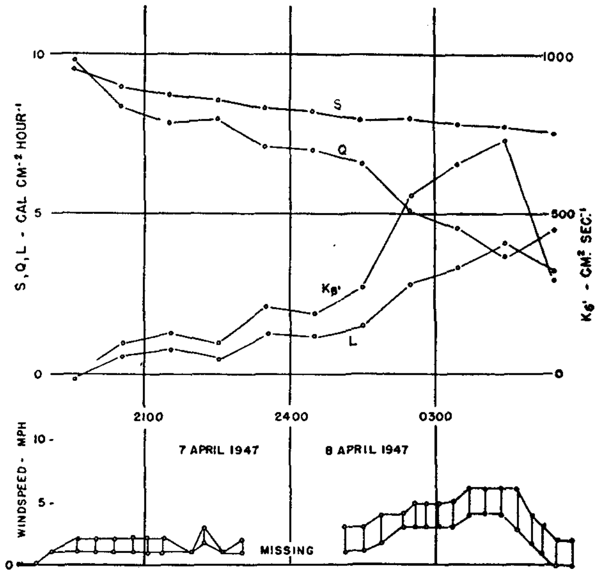

FIG. 3. Plot of heat balance terms, diffusivity at 6 ft, and windspeed at 10 ft, 7–8 April 1947, Gila Bend.

Fig. 4 shows the results for the night of 11–12 April 1947. The same close correlation between diffusivity and windspeed is again evident, with the greater magnitude of K apparently being due to the much greater windspeed and gustiness shown on the wind record. The convection curve in this case shows a steady decline from 0200 on, whereas K remains high for two more hours. This behavior seems to be due to the turbulence tending to wipe out the inversion, so that the lapse rate is much closer to the adiabatic. Early in the evening the temperature gradient at six feet was 8×10^{-3} deg C cm^{-1}; at 0200 it was 4.1×10^{-3}; and at 0300 it had dropped to 1.7×10^{-3}. Thus, even with greater diffusivity, less heat is transported downward.

The convection term L has often been neglected in the derivation of such things as minimum temperature formulae; for instance, see Brunt [3, p. 138] and Groen [7]. Also, Brunt assumes that at his zero time, which he ordinarily takes at sunset, the lapse rate in the soil is isothermal. Groen assumes a constant lapse rate. Examination of fig. 2 indicates that in the desert, at least, such assumptions are not well founded. The lapse rate is not only quite steep just below the surface, but it also changes sign in the upper 18 inches of soil. As to the neglect of the convection term, the average

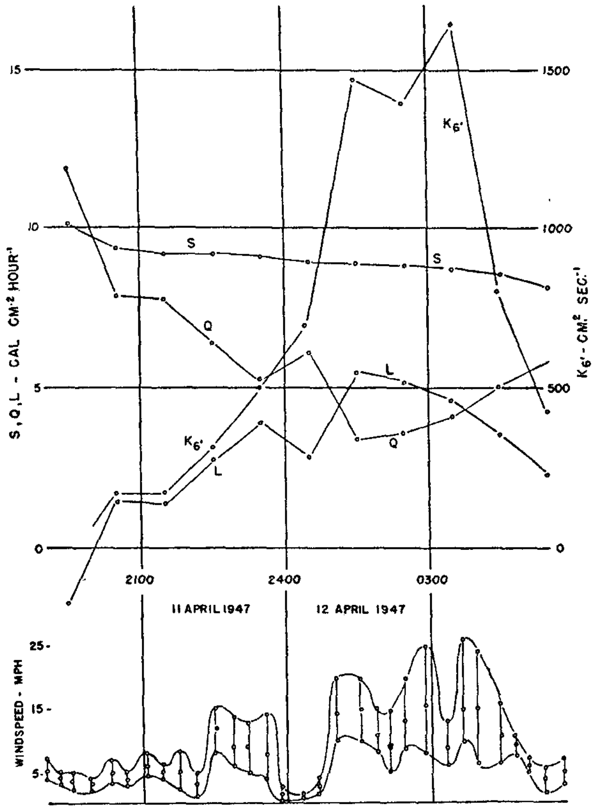

Fig. 4. Plot of heat balance, diffusivity at 6 ft, and windspeed at 10 ft, 11–12 April 1947, Gila Bend.

values of the radiation and convection terms for the two nights studied here are:

	S	L	$\%S$
7–8 April	8.4 cal/cm²hr	1.7 cal/cm²hr	20%
11–12 April	9.0 cal/cm²hr	3.6 cal/cm²hr	40%

These figures indicate that, for desert conditions, neglect of the convection term is questionable. Albrecht [2] obtained similar results.

The quantities computed for the various terms in (1) are, of course, subject to a fair amount of error, so that when differences are taken to obtain the convection component, even larger percentage errors may be present. Note, for instance, that for the period 1900–2000 on the 11 April, L is calculated to be -1.7 cal cm^{-2} hr^{-1}, which is certainly false because the ground was cooler than the air, and the lapse rate shows a definite inversion. It is obvious, therefore, that the convection figures are not absolutely correct. However, since there is no reason to expect large fluctuations in the radiation term, and the only errors in the Q term are likely to be consistent ones, such as in the values of ρ and c, it appears that the large variation of the L term must be real, particularly in view of the correlation with wind speed. The errors clearly do not affect the order of magnitude of the results.

The atmospheric inversion—There is still some doubt as to exactly how atmospheric surface inversions are formed. Panofsky [10] concludes that, in the case of inversions in warm air over cool water, convection is the primary agent in producing the inversion, at least at moderate wind speeds. For the Gila Bend data of 7–8 April 1947, a calculation was made of the heat lost by the air under the inversion. Since the inversion extended above the top sounding level, the result is only an estimate, which indicates that the air lost 1.5 to 2 times the amount of heat gained by the soil by convection processes. The difference may be due to the radiation from the air to the ground, which would be included in the net radiation term of (1), or to errors introduced by advection, which, however, appeared to be small. For the night of 11–12 April, advection was obviously influencing the data to such an extent that the calculations are of no value.

Eddy diffusivity—In addition to computing diffusivities at six feet, values were obtained for a level between the surface and one inch above the surface. The exact height of this level is unknown and probably variable, since the mean lapse rate was determined merely from the temperature differences between the levels. The values obtained for K ranged from 1.3 to 5.1 cm^2 sec^{-1}, as the temperature differences varied between 0.7 and 4.1C. Here it must be pointed out that, because of the inherent difficulty of measuring surface temperatures with a thermometer of finite dimensions placed in a region where the lapse rate is not only steep, but also changes sign at the surface itself, the probable error in surface temperature is larger than for the other levels. It is estimated that in the present case the probable error in surface temperature at night is about 0.5C. This means that the diffusivities computed above for the level below one inch are of significance only as to order of magnitude.

The diffusivities computed for the six-foot level agree as to order of magnitude with the results obtained by other investigators, who used different methods for their calculations. Schmidt [11], for example, gives values of austausch at one to two meters which correspond to a K of 1000 to 1500. For the lowest level, the values obtained seem high. Geiger [6] and Brunt [3, p. 228] both quote values of the order of 10^{-1}. However, the method by which their values were determined involves the assumption of a constant K in the solution of a second order differential equation, after which K is allowed to vary. Cowling and White [4] have shown that this procedure leads to errors even in the order of magnitude of the results. From the values obtained here for K near the surface, the conclusion is that turbulence of small but appreciable magnitude extends well below the one-inch level. The layer in which the lapse rate is steep enough such that the molecular diffusivity (0.16 cm^2 sec^{-1}) can transport the computed quantities of heat in the convection term must be very thin, certainly less than one millimeter in thickness.

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