FORECASTING BY STATISTICAL INFERENCES

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ABSTRACT

In recognition of the fact that a weather forecast is rarely 100 per cent accurate, this paper considers the value of figures for the probability of a meteorological event in meeting specified operational requirements. An objective method is presented for deciding between alternative meteorological predictors. It is emphasized that there is no essential qualitative difference between this technique and the methods normally applied in a more subjective manner.

1. Introduction

A chemical reaction between several reagents can be predicted accurately through the past performance of the same reagents; the movements of the moon and the planets can be predicted with accuracy because their past performances fit into a unified pattern. Meteorological conditions, however, have not been so easily predictable (Ertel, 1941). The temperature at any given hour may be forecast accurately on one day by one forecaster, only to be missed on another day by the same forecaster.

This paper begins with the premise that 100 per cent accuracy in weather forecasting is impossible, even though some progress may be made toward increasing the present-day efficiency. A forecaster should realize, therefore, that he might profitably express the likelihood of the accuracy or inaccuracy of his forecast. If, in a certain situation, he predicts rain for noon, one day in advance, he might state that the chance of rain, as he sees it, is p and the chance of no rain is q. In 100 similar situations, rain would be expected to occur, not in all 100 cases, but in $100p$ cases.

It is the primary purpose of this paper to obtain the maximum advantage from meteorological statistics when evolving a forecast for an operational requirement. The second purpose is to develop a criterion, or index, for choosing between alternative parameters that might be used in preparing a forecast. While the forecasting technique described in this paper is objective, it is emphasized that there is no essential qualitative difference between this technique and the methods normally applied in a more subjective manner.

2. Meeting the operational requirements

For an insight into the operational requirements, let us consider airport operations. A dispatcher, who is responsible for permitting the take-off of an airplane, usually weighs the importance of his decision against the assurances of the forecaster. Suppose that an airport is presently closed by fog. If the forecaster states that there is a 30 per cent chance that the fog will lift in six hours, the dispatcher may decide to have the airplane crew on the job to take advantage of the lifting of the fog (if it does lift). But if the forecaster states that the chance of lifting is only 15 per cent, the dispatcher may reverse his decision. Let us, therefore, define a *critical frequency*[1] (p_c) for each meteorological event. In the example just cited, the critical frequency lies between 15 per cent and 30 per cent (say 25 per cent). If the per cent chance that the event will occur is greater than this critical value, then the dispatcher makes one decision; if the chance is less than the critical value, then he makes the reverse decision. The critical frequency is determined, or partly determined, by economy, *i.e.*, by the cost of each alternative in manpower, time, or in dollars and cents; it is also affected by the importance of changes in the weather (see Appendix).

If a forecaster could state the per cent likelihood of each of all possible mutually exclusive events, he would be giving a complete answer as to his evaluation of the weather prospects (Brier, 1944). Unfortunately, the forecaster cannot be certain of the odds. What the forecaster knows, with confidence, are the relative odds. Let the operations office give the forecaster the critical frequency p_c of each event; then the forecaster might state whether the likelihood of the meteorological event is significantly greater than the critical frequency or significantly smaller.

The above two paragraphs may sound like a turnabout. Instead of the airline operator putting the meteorologist "on the spot" to commit himself as to what the weather is going to be, it is the meteorologist who should demand of the airline operator that he commit himself as to the importance of his requirements. Of course, the factors entering into airline operations are complex and multiphased; nevertheless,

[1] This expression is used in the statistical sense, not to be confused with the same expression employed in a physical sense in the study of vibrations or wave motions.

dispatchers must make decisions continually and quickly. Each dispatcher must mentally consider the tolerable limits, that is, the critical frequencies.

3. Notation and terminology

For further discussion of the forecasting problem, it is convenient to introduce two terms (Gringorten, 1949). The *predictand* is that which is to be forecast, such as the occurrence of fog or rainfall. The *predictors* are those attributes of the weather that are used in preparing the forecast, such as present and past observations, tendencies, weather-map analyses, extrapolations, and derived or computed quantities. The following treatment is presented in the notation of Yule and Kendall (1940).

The symbol X denotes the predictand, say rainfall. $A, B, C, \cdots$ denote the predictor attributes, say weather-map analysis, humidity, wind direction, *etc.*

The days of each year can be classified according to the predictand. Thus, $X_0, X_1, X_2, \cdots, X_k, \cdots$ denote the classes of days on which certain events developed. The class X_0 may consist of all days with no rain; X_1 all days with a trace of rain; X_2 with light rain, *etc.* The symbol X_k is a general symbol for the predictand-class.

Likewise, the days might be classified according to the anteceding predictors. Thus, $A_1, A_2, A_3, \cdots$ denote the predictor-classes. The class A_1 might consist of those days with a continental outbreak over a city like New York; the class A_2, those days characterized by an advance of tropical air from the Gulf of Mexico; and A_3 might represent those days on which moist air has approached from the Atlantic.

$B_1, B_2, B_3, \cdots$ represent the classification of the days by predictor attribute B.

$C_1, C_2, C_3, \cdots$ represent the classification of the days by predictor attribute C, *etc.*

$A_1B_3C_2\cdots$ denote a sub-class of days before which the indicated combination of conditions prevailed.

$(A_1B_3C_2\cdots)$ denotes the number of $A_1B_3C_2\cdots$'s in the period of years that have been studied.

$(A_1B_3C_2\cdots; X_k)$ denotes the number of $A_1B_3C_2\cdots$'s that are characterized further by the subsequent event X_k.

Each predictor is not necessarily an isolated observation or parameter. A single predictor may, in itself, be the end result of a sequence of operations, either numerical or graphical, that involves many steps and many parameters. In Brier's scheme for forecasting rainfall in the Tennessee Valley Authority Basin (1946), two predictors would suffice, one for "map type" and the other for his "W."

Naturally,

$$\frac{(A_1B_3C_2\cdots; X_k)}{(A_1B_3C_2\cdots)} \leq 1. \tag{1}$$

4. The forecaster's dilemma

The difficulties of forecasting can be stated in terms of (1). A value of unity would imply perfect forecasting. But it is not easy to select the "right" predictors $A, B, C, \cdots$ for the task of forecasting X. Furthermore, the meteorological records are not sufficiently long to establish a true value for this ratio. Again, there may be a trend in the weather, which may cause the ratio to undergo a change from one time period to another.

To elaborate further, the "right" predictors are a matter of the personal judgment of a forecaster, based on his experience and physical intuition. Whether subjective or objective, forecasting is performed by inference. Let us assume that the ratio (1) is exactly 95 per cent, as established over a long period; that is, in 95 days out of 100, when cP air invades the New York area, with dewpoint 30 to 40F and winds from the northwest, there would be no rain during the following 24 hours. If, then, a forecaster were to forecast "no rain" for a day on which the same conditions recur, he would be implying that these conditions comprise the best classification of that day's weather for the forecasting of rain. The forecaster would be issuing an incomplete statement if he were to say, "The probability of no rain, as of this date, is 95 per cent"; it is only for his classification of today's weather that the probability of "no rain" is 95 per cent. Yet, if he is to make a forecast at all, the forecaster must content himself with the tools (predictors) at his disposal and accept the statistical results of previous investigations.

The relatively short period during which meteorological records have been collected aggravates the above-mentioned problem. It is not even possible to give the *true* probability that one event will follow a set of well-defined conditions. If more predictors are used to define a day's conditions, then there will be fewer days in the past that have been characterized by the same conditions. And the smaller the sample of days out of the past, the less reliable is the observed frequency of an event. For this reason, the limits of confidence are carefully studied in the following sections.

In the following paragraphs, it is assumed that the secular trend, if it exists, affects the frequency of the predictors as much as the frequency of the predictands, and thus does not affect the ratio (1).

5. Probabilities and confidence limits

Let U_m denote a sub-class of predictors, say $A_1B_3C_2$. Then U_mX_k denotes the sub-class of U_m which is characterized further by the event X_k. Also, let $n = (U_m)$ denote the size of the observed sample of sub-class U_m. Let $s = (U_mX_k)$ represent the number of days of X_k that have followed U_m. Then s/n is the observed frequency of the consequence X_k with antecedent U_m.

If

$$\lim_{n \to \infty} s/n = p,$$

p is the true probability that X_k will follow U_m.

Let $q = 1 - p$ be the true probability that X_k will not follow U_m.

The value of s/n, which is found in a sample of size n (oftentimes small), is only an approximation to p (Bartels, 1949). It is advisable, therefore, to set confidence limits which may be expected to include the true frequency.

The chance that a sample of n days will have an apparent X_k-frequency equal to s/n, instead of the true frequency p, is given by

$$\frac{n!}{(n-s)!s!} p^s q^{n-s},$$

which is the $(s+1)$st term of the binomial expansion of $(q+p)^n$ (Kenney, 1939, Theorem VII). The chance that a sample of size n will have an apparent X_k-frequency equal to or less than s/n is

$$\sum_{i=0}^{s} \frac{n!}{(n-i)!i!} p^i q^{n-i} = P.$$

The chance that a sample of size n will have an apparent X_k-frequency equal to or greater than s/n is

$$\sum_{i=s}^{n} \frac{n!}{(n-i)!i!} p^i q^{n-i} = P'.$$

It is customary to prescribe limiting values of 2.5 per cent for P and P', known as the "95 per cent confidence limits." If $P < 0.025$, it is arbitrarily decided not to consider s/n as a random fluctuation of p, but rather as significantly smaller than p. If $P' < 0.025$, s/n is considered significantly larger than p.

This treatment yields a method for finding p_u, the upper confidence limit of p, and p_l, the lower confidence limit of p.

If, historically, there have been s days of X_k out of n days of U_m, then the upper 95 per cent confidence limit of the true frequency of X_k in the sub-class U_m is given by p_u in the equation

$$\sum_{i=0}^{s} \frac{n!}{(n-i)!i!} p_u^i (1 - p_u)^{n-i} = 0.025. \tag{2}$$

The lower 95 per cent confidence limit is given by p_l in the equation

$$\sum_{i=s}^{n} \frac{n!}{(n-i)!i!} p_l^i (1 - p_l)^{n-i} = 0.025. \tag{3}$$

If, for instance, the sample consists of three days, on each of which the event X_k took place, $s = 3$ and $n = 3$, wherefore $s/n = 100$ per cent. The upper limit of confidence is, naturally, 100 per cent; but the lower limit, as given by (3), is 29.2 per cent. That is, it has been decided (by arbitrary acceptance of the 95 per cent confidence limit) that the true probability of X_k following U_m is not less than 29 per cent.

If the sample consists of three days, on two of which the event X_k took place, but not on the third, $s = 2$ and $n = 3$, wherefore $s/n = 66.7$ per cent. By (2) and (3), $p_u = 0.9916$, $p_l = 0.094$. That is, by acceptance of the 95 per cent confidence limits, it is deemed unlikely that the true probability of X_k is less than nine per cent or higher than 99.2 per cent.

Likewise, the upper and lower confidence limits can be found for any observed frequency and for a sample of any size. However, as the sample size becomes large, the computation of the confidence limits becomes prohibitively great, so that it is necessary to resort to approximate solutions (Wilks, 1949). Fig. 1 gives the lower 95 per cent confidence limits for observed frequencies in samples that range in size from one to 1000 cases. For samples of one to nine cases, the confidence limits have been determined by (3); for samples of sizes 10, 15, 20, 30, 50, 100, 250, and 1000, the limits have been taken from Snedecor (1946). Fig. 1 shows continuous lines for the lower limits, 1, 5, 10, 20, $\cdots$ 95 per cent; likewise, a figure can be drawn for the upper limits.

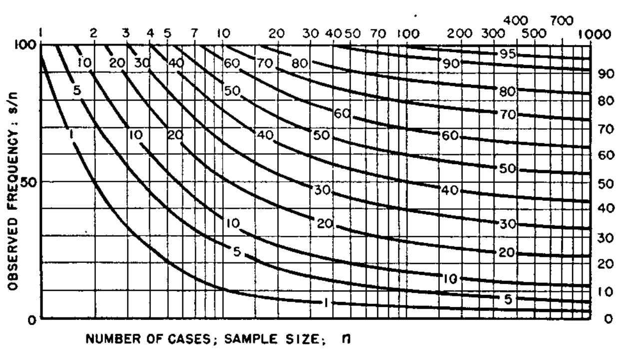

FIG. 1. Chart giving the lower 95 per cent confidence limit of the true probability of an event for each sample size n and each observed frequency s/n.

6. Predictor-combinations applied to forecasting

If there were a perfect predictor-combination, it would always be followed by one single value of the predictand. Stated symbolically, where U_m^* represents the perfect predictor-combination, and X_k the predictand value,

$$(U_m^* X_k) = (U_m^*).$$

But the forecaster faces the regrettable fact that there are no perfect predictors or combinations of predictors. The forecaster's problem, therefore, is to select the best of the imperfect predictors or combinations, *i.e.*, to decide upon what meteorological factors to base his forecast. If a result

$$\lim_{n\to\infty} \frac{(U_m X_k)}{(U_m)} = p$$

is found experimentally in a *large* sample, X_k can be expected to occur with frequency p for the same antecedent conditions U_m. But to use this frequency as a basis for forecasting X_k, or its reverse, is to imply that one has more faith in the efficiency of this predictor-combination than in the efficiency of any other predictor-combination.

A few rules can be stated for an objective selection of a predictor-combination. First, the frequency of occurrence of the event with t predictors is preferred to the frequency with $(t - 1)$ predictors that are contained among the original t predictors. That is, the combination ABC must be given prior consideration to AB. But the sample from the sub-class $A_j B_k C_l$ may be too small, yielding confidence limits for p that are too broad for forecasting purposes. Another rule, therefore, for selecting the best combination is to compare the confidence limits for the frequency p with the critical frequency p_c discussed above. If, for the predictor-combination U_m,

$$p_u > \frac{(U_m X_k)}{(U_m)} > p_l > p_c,$$

the positive event should be forecast. But if

$$p_u > \frac{(U_m X_k)}{(U_m)} > p_c > p_l,$$

the positive event cannot be forecast with confidence, and another combination of predictors should be sought.

Example.—To illustrate the principles of this and the previous section, an example is chosen from an experimental project that is being conducted at the Air Force Cambridge Research Laboratories. The problem has been to forecast the minimum conditions at Randolph Field, Texas, 16 hr in advance of the morning period 0430 to 1030 CST. The example, illustrated in fig. 2, is for 29 January 1950 when the antecedent conditions were:

a. Air mass: advancing Tropical Gulf air, moist in the turbulent layer but dry above 800 mb.
b. Relative humidity at 500 m: more than 90 per cent.
c. Wind direction: from Gulf of Mexico, as shown by surface chart.
d. Ceiling height: 1000 to 2000 ft (or alternative predictor). Height of turbulence inversion: 1000 to 1500 m.
e. Present weather: no precipitation.
f. Persistence: airport was on "instruments" the previous night.
g. Wind speed (geostrophic): 21 to 25 mi hr^{-1}.

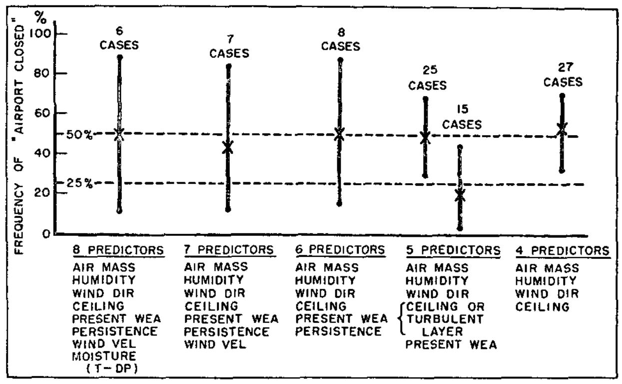

FIG. 2. Diagrammatic presentation of samples, taken from a 10-year period, of sub-classes of days to which 29 January 1950 belongs. The predictors are conditions observed at Randolph Field, Texas, at, or before, 1530 CST 28 January 1950. Lower and upper confidence limits of the true probability of "airport closed" are given by the ends of the solid vertical lines.

h. Moisture content: temperature–dew-point spread 6 to 10F.

A sample of six days, similar in these eight respects, was found in a 10-year period. The frequency of subsequent "closed" airport conditions among those six days was 50 per cent; but with only six cases, the confidence limits are 11 per cent and 89 per cent. Dropping one of the predictors, a larger sample was found with narrower limits of confidence; dropping still another predictor, a still larger sample was found, and so on.

The forecast that is prepared from these results depends upon the weight attached to each consequence. If it is three times as important to forecast correctly a "closed" condition than "not closed," then the critical frequency is 25 per cent. It can be seen from fig. 2 that a competent forecast cannot be prepared from the 8-predictor, 7-predictor, and 6-predictor samples because the critical frequency (25 per cent) is included between their confidence limits. But a forecast of "closed" can be based on the 5-predictor sample of 25 days in which the experimental frequency is 48 per cent with confidence limits of 28 per cent and 68 per cent. The forecast could also be based upon the 4-predictor sample of 27 days; but it is always preferable to base the forecast upon as many predictors as possible. If it is equally important to forecast "closed" and "not closed," then the critical frequency is 50 per cent. A competent forecast of "not closed" can be based upon the 5-predictor sample of 15 days (which differs from the other 5-predictor sample by the substitution of *ceiling* by *turbulent layer* as a predictor).

The value of these forecasts is demonstrated by what actually happened. On the morning of 29 January 1950, the ceiling lowered to 500 ft, the visibility to one mile, accompanied by fog and drizzle. While the airport was not "closed," it was a borderline case when the meteorologist should have *alerted* the operations office to the possible "closing" of the airport.

7. Index of efficiency of predictor-combinations

It is natural to inquire which are the best predictors. Can the forecasting efficiency of an air-mass analysis be compared with the efficiency of a collection of observed data, such as ceiling height, moisture content and wind direction? The predictors should be given careful qualitative consideration. But, in addition, there should be some objective evaluation of the predictors. For this purpose, a modification of Tschuprow's coefficient of contingency (Yule, 1940, p. 70) is presented in the following paragraphs.

The comparison of two single predictors.—Let N be the size of an overall sample. (In ten winters, N would represent about 900 days.) (X_k) is the number of days during which there was an event X_k, such as moderate rain. $(X_k)/N$ is the climatological frequency of X_k. (A_m) is the number of days that were preceded by the condition A_m. $(A_mX_k)/(A_m)$ is the frequency with which the event X_k followed the condition A_m (equal to zero or one for perfect forecasting).

The difference between $(A_mX_k)/(A_m)$ and $(X_k)/N$ is a measure of the efficiency of A_m for the prediction of X_k. To obtain an index of the efficiency of A as a predictor of X, $(I_{A;X})$, the difference between $(A_mX_k)/(A_m)$ and $(X_k)/N$ is squared to eliminate the sign, and divided by (X_k) to obtain a magnitude comparable to other similar differences. After each squared difference is weighted by the sample size (A_m), all such differences are added, first over all classes of A and secondly over all events of X. This procedure gives

$$I_{A;X} = \frac{1}{K-1}\sum_k\sum_m \frac{(A_m)}{(X_k)}\left[\frac{(A_mX_k)}{(A_m)} - \frac{(X_k)}{N}\right]^2, \qquad (4)$$

where K is the number of classes into which the predictand is divided, or the number of possible events. (The reason for the factor $(K-1)^{-1}$ is shown below.) The index of efficiency of another predictor, B, is given by a similar formula.

If $I_{A;X} > I_{B;X}$, A, as a predictor of X, is preferred to B.

The efficiency of two predictors in combination.—Just as in (4),

$$I_{AB;X} = \frac{1}{K-1}\sum_k\sum_m\sum_r \frac{(A_mB_r)}{(X_r)}\left[\frac{(A_mB_rX_k)}{(A_mB_r)} - \frac{(X_k)}{N}\right]^2.$$

Taking the difference between $I_{AB;X}$ and $I_{A;X}$ and realizing that

$$\sum_r (A_mB_r) = (A_m) \quad \text{and} \quad \sum_r (A_mB_rX_k) = (A_mX_k),$$

we find

$$I_{AB;X} - I_{A;X} = \frac{1}{K-1}\sum_k \frac{1}{(X_k)} \times \sum_m \left\{\sum_r \frac{(A_mB_rX_k)^2}{(A_mB_r)} - \frac{[\sum_r (A_mB_rX_k)]^2}{\sum_r (A_mB_r)}\right\}.$$

It can be proved (Hardy, 1934, p. 61, problem 35) that the term in the curled brackets is positive. Therefore, $I_{AB;X} \geqslant I_{A;X}$. Similarly, it can be shown that the index $I_{U;X}$ is monotonically increasing with each additional predictor in the combination U. This result might have been anticipated, since the efficiency of a combination of predictors should increase with the inclusion of additional predictors in the combination.

The perfect predictor combination.—Suppose that there exists a perfect predictor combination U^*. Then the classes $U_m{}^*$ can be divided into two mutually exclusive groups, $U_{m1}{}^*$ and $U_{m2}{}^*$, such that $(U_{m1}{}^*X_k) = (U_{m1}{}^*)$ and $(U_{m2}{}^*X_k) = 0$. But

$$\sum_m (U_m{}^*X_k) = (X_k).$$

Therefore,

$$\sum_{m1} (U_{m1}{}^*X_k) + \sum_{m2} (U_{m2}{}^*X_k) = (X_k).$$

Therefore,

$$\sum_{m1} (U_{m1}{}^*) = (X_k),$$

from which

$$\sum_{m2} (U_{m2}{}^*) = N - (X_k).$$

Therefore, the index of efficiency for the perfect predictor-combination $I_{U^*;X}$ is given by

$$I_{U^*;X} = \frac{1}{K-1}\sum_k \frac{1}{(X_k)}\left\{\sum_{m1} (U_{m1}{}^*)\left[1 - \frac{(X_k)}{N}\right]^2 + \sum_{m2} (U_{m2}{}^*)\left[0 - \frac{(X_k)}{N}\right]^2\right\}$$

$$= \frac{1}{K-1}\sum_k \left[1 - \frac{(X_k)}{N}\right] = 1.$$

Hence, *the index of efficiency, as defined, ranges in value from zero to one.*

Correction for small sample sizes.—There remains the further difficulty that too many predictors taken in combination divide the overall sample N into sub-samples that are too small for significance. It is necessary, therefore, to weight the sub-samples according to size, thus:

$$I_{U;X} = \frac{1}{K-1}\sum_k\sum_m \frac{(U_m)\,f(U_m)}{(X_k)}\left[\frac{(U_mX_k)}{(U_m)} - \frac{(X_k)}{N}\right]^2,$$

where $f(n)$ is the fraction by which the sample size n is multiplied to give the sample a more proper weight. A value is arbitrarily assigned to $f(n)$ in the following manner: $f(n) = 100$ per cent minus average spread between lower and upper confidence limits of frequency for samples of size n. Fig. 3 is a plot of $n\,f(n)$ versus n. If n is large, this correction is negligible.

8. Subjective versus objective forecasting

By the time he issues his forecast, a forecaster has examined many aspects of the weather picture, such as the surface and upper-air weather-maps, the past history as revealed by these maps, and the hourly sequences. He usually decides to overlook some of the existing conditions while emphasizing others. For example, if the surface weather-map should fit into no particular pattern, such as well-defined pressure centers and fronts, then the forecaster may concentrate on the general trend of the upper winds, the humidity and the stability as revealed by soundings. The forecaster must depend heavily on his knowledge and experience with the past history of the weather because similar antecedent conditions yield similar consequences. The forecaster may also decide to overlook a clearly defined parameter, such as the surface pressure, because, through his experience or the experience of others, he may not find in it a decisive factor. In addition, the forecaster is influenced by the forecast requirement.

The objective procedures, herein described, are patterned after these mental processes. The antecedent conditions are recorded and compared with the previously studied predictor-combinations, either by inspection or by machine methods. If a predictor, or condition, is hard to determine, such as the air-mass analysis, it is eliminated. Finally, the forecast statement is tailored to the forecast requirement because an event is predicted if it is known (with confidence) to occur with a frequency greater than the critical frequency.

Thus it is seen that an objective system of forecasting can parallel the mental processes of a skilled and

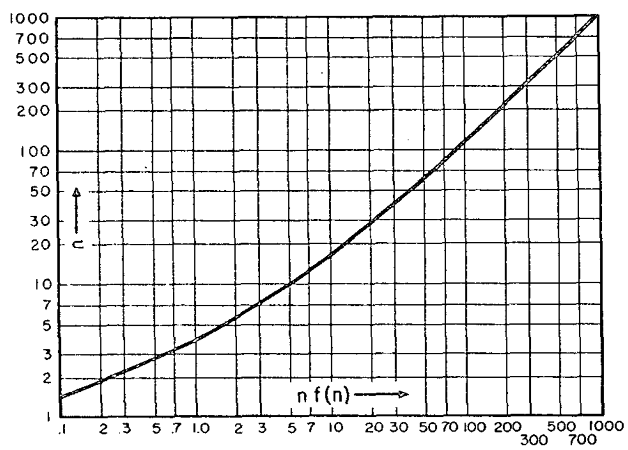

Fig. 3. Chart used in determining the value of $n\,f(n)$, the weight that is arbitrarily allotted to each term in the index of efficiency to correct for small sample size n.

experienced forecaster. The forecaster's know-how is set forth in black and white in the form of a forecast manual or deck of cards. An objective system eliminates the uncomfortable moments of hesitation that every forecaster undoubtedly has suffered when he has been pressed for a forecast during nondescript conditions. Furthermore, it meets the operational needs for the forecast through the figures for per cent likelihood of occurrence. At the same time, there are limitations. The preparation of a forecast manual is a large task, requiring the use of tabulating machines. For increased efficiency, it will also require a type of selective or sorting device to isolate the predictor-combinations that will yield an adequate forecast. Furthermore, a system devised to meet one or two requirements cannot answer all operational questions. A meteorologist will have to answer the more detailed questions subjectively, as they arise from hour to hour or minute to minute.

9. Concluding remarks

This paper, which began with the premise that weather forecasting cannot be 100 per cent accurate, has been written in an effort to adapt the present knowledge of forecasting to operational needs. The forecast requirements have been represented by *critical frequencies* of the important events. When the forecaster is provided with statistical frequencies of events, known as predictands, in association with the antecedent conditions, known as predictors, he is able to make his prediction by inference. It is possible to select objectively the best combination of predictors by use of an index of efficiency. While this paper has concerned itself with objective forecasting, it is hoped that it will clarify the role of all forecasters in meeting operational requirements.

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Appendix

The determination of the critical frequency of a meteorological event.—In Section 2, above, it was assumed that the critical frequency of a meteorological event was known. The determination of this critical frequency might be made in economical terms, as illustrated by the following:

Suppose that a destination is presently closed, and that an airliner is scheduled to depart for that airport in two hours. Let p be the probability that the destination will open by the scheduled time of arrival. Let T be the net profit from a successful flight. Let l be the net cost resulting from cancellation; this should be the overhead, such as the cost of rentals, maintenance of idle equipment, and salaries. Let L be the loss resulting from an incorrect decision; in this case the decision would be not to cancel, but to risk having the airliner return to the point of departure when finding the destination still closed upon arrival. The loss would be due to overhead, the salaries of the crew, cost of gasoline, and loss of prestige expressed in dollars and cents.

For the critical frequency, p_c, it is immaterial which decision the dispatcher makes, to cancel the flight or not; the average loss in similar situations will be the same for either decision. Therefore,

$$l = (1 - p_c)L - p_c T,$$

from which

$$p_c = (L - l)(L + T)^{-1}.$$

There is much logic summarized by this equation. If the loss L following an incorrect decision is great, the likelihood p of contact weather must be great in order to justify the decision to maintain the schedule. If the cost L is not much greater than the overhead l, the dispatcher should find it worthwhile to base a positive decision on even a small chance p of the airport opening. If the profit T is great, the dispatcher should try to maintain the schedule on the basis of a small chance of the airport opening.

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