SHORTER CONTRIBUTIONS

SOME POSSIBLE EFFECTS OF SOLAR-RADIATION CHANGES ON THE ANNUAL PRESSURE OSCILLATION

By Jerome Spar

New York University[1]

(Manuscript received 1 May 1950)

1. Introduction

In a recent paper [1], a relation was derived between periodic temperature and surface-pressure variations on a rotating spherical earth. The mean annual range of surface pressure was computed from the mean annual range of surface temperature. It was shown that the patterns of computed and observed pressure-range are quite similar, although certain explainable quantitative differences do appear. The theory of annual pressure variations is applied in the present paper to the problem of estimating what would be the magnitude and spatial distribution of the annual range of surface pressure if the output of solar radiation were to change in such a way as to alter the annual temperature range.

It was shown in [1] that the annual range of surface pressure p_s depends on a quantity Q, which is defined by

$$Q = \int_s^\infty gR^{-1}p_0 \int_s^z TT_0^{-2}\, dz\, dz, \qquad (1)$$

where g is gravity, R the gas constant for air, p_0 the equilibrium pressure, T the annual temperature range, and T_0 the equilibrium temperature. The quantities p_0, T, and T_0 are functions of the height z above the sea-level surface s. An empirical, quasi-linear relation between Q and T_s, the mean annual range of surface temperature, was used to evaluate Q from observed January and July mean surface temperatures.

If the distribution of Q over the surface of the earth, which is assumed to be everywhere at sea level, is represented by an infinite series of spherical harmonics of the form

$$Q = \sum_{n=0}^{\infty}\sum_{m=0}^{n} (a_n^m \cos m\lambda + b_n^m \sin m\lambda)\, P_n^m(\theta), \qquad (2)$$

the annual range of surface pressure can be computed from

$$p_s = h_0^{-1} \sum_{n=0}^{\infty}\sum_{m=0}^{n} (K_n^m - 1) \times (a_n^m \cos m\lambda + b_n^m \sin m\lambda)\, P_n^m(\theta). \qquad (3)$$

[1] This investigation was made possible through support and sponsorship extended by the Geophysical Research Directorate, Cambridge Field Station, Air Materiel Command, U. S. Air Force, under contract No. AF 19 (122)-49.

The computed pressure-range represents the difference between January and July surface pressure and is positive if the latter is larger than the former. In (3), h_0 is analogous to the height of the undisturbed ocean-surface in the Laplacian theory of tides and is defined by

$$h_0 = p_{s0}^{-1} \int_s^\infty p_0\, dz, \qquad (4)$$

where p_{s0} is the equilibrium surface-pressure, which is assumed to be constant. $P_n^m(\theta)$ is the semi-normalized associated Legendre polynomial of degree n and order m [2]. K_n^m is a function of the integers n and m and constant quantities. A table of K_n^m values is given in [1] for values of n and m up to and including 8. λ and θ are longitude and co-latitude, respectively.

The coefficients a_n^m and b_n^m were evaluated from the integral formulae (see [2])

$$\begin{Bmatrix} a_n^m \\ b_n^m \end{Bmatrix} = \delta_m \frac{2n+1}{4} \int_0^\pi \begin{Bmatrix} \alpha_m(\theta) \\ \beta_m(\theta) \end{Bmatrix} P_n^m(\theta) \sin\theta\, d\theta, \qquad (5)$$

where

$$\begin{Bmatrix} \alpha_m(\theta) \\ \beta_m(\theta) \end{Bmatrix} = \frac{1}{\pi\delta_m} \int_0^{2\pi} Q(\theta,\lambda) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} d\lambda, \qquad (6)$$

and

$$\delta_m = \begin{matrix} 2 & \text{for} & m = 0 \\ 1 & \text{for} & m > 0. \end{matrix}$$

Two simple models are investigated in this paper. In the first model it is assumed that the primary effect of a change of solar output is to alter the annual temperature range in such a way that it remains unchanged at the equator but increases between equator and poles by a factor which increases with the sine of the latitude. In the second model an attempt is made to take into account the differences between continents and oceans by assuming that the postulated temperature range is proportional to the observed temperature range. It is, of course, impossible to estimate with any accuracy what the resulting temperature range would be for any change of solar radiation because of the complex inter-relationship between insolation, cloudiness, precipitation, albedo, circulation, and temperature. However, the two models, particularly the second, do not appear physically unreasonable in view of the fact that an increase of solar radia-

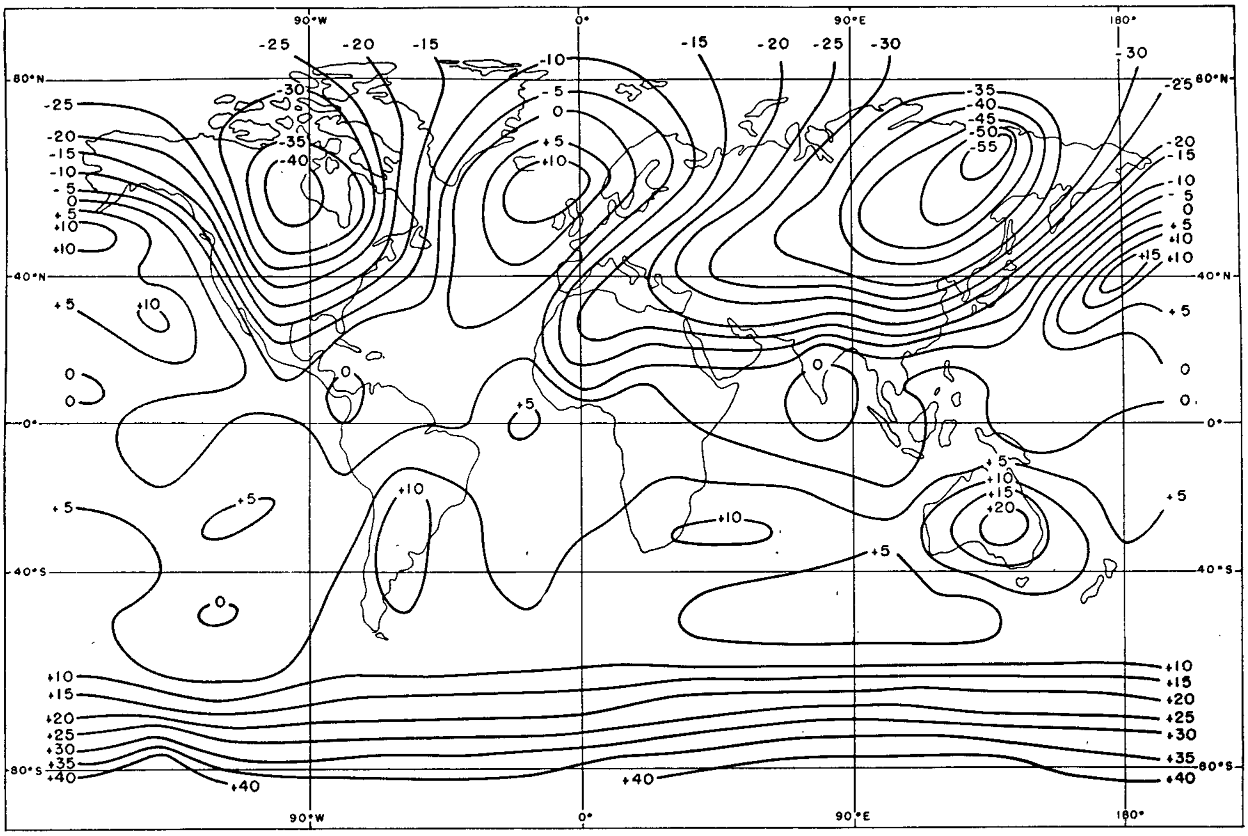

FIG. 1. Mean July surface pressure minus mean January surface pressure (mb) computed from the varied annual temperature range (first model).

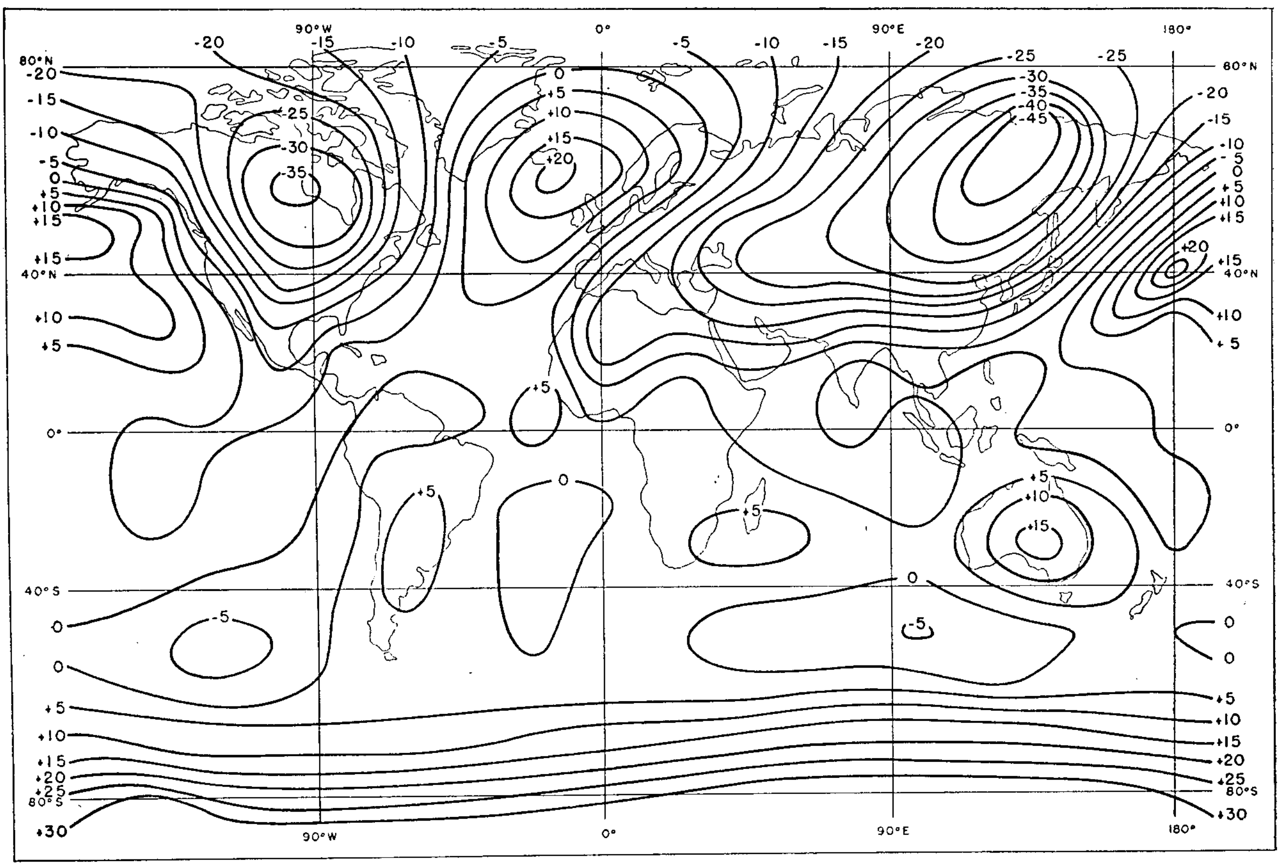

FIG. 2. Mean July surface pressure minus mean January surface pressure (mb) computed from the observed annual temperature range.

tion would probably result in a larger increase of insolation in summer than in winter everywhere but near the equator.

2. First model

If the varied state is represented by primes, the condition for the first model may be written

$$Q' = Q + H \cos \theta, \tag{7}$$

where H is the change of Q at the poles. The varied temperature-range distribution represented by (7) is obtained by superimposing on the observed temperature-range distribution a zonally symmetric wave of amplitude H with one node at the equator. The harmonic coefficients corresponding to (7) are therefore given by

$$\begin{aligned} a_1^{0\prime} &= a_1^0 + H, \\ a_n^{m\prime} &= a_n^m \quad (\text{for } m > 0;\, n \neq 1), \\ b_n^{m\prime} &= b_n^m. \end{aligned} \tag{8}$$

Upon substitution of (8) in (3),

$$p_s' = p_s + Hh_0^{-1}(K_1^0 - 1) \cos \theta, \tag{9}$$

since K_n^m does not depend on the temperature.

The varied pressure-range was computed for H equal to 2.00×10^{10} dynes cm^{-1}, which corresponds to an increase of about 20C in the annual surface temperature range at the poles. The result is shown in fig. 1 and may be compared with the pressure-range field computed from the observed temperature ranges (fig. 2).

The mean horizontal pressure-gradients in summer, at least in the Northern Hemisphere, are quite small compared with those of the winter season, except in low latitudes. Figs. 1 and 2 may therefore be considered representative of the winter pressure distribution. From this point of view, the primary effects of increased solar radiation are intensification of the continental anticyclones and weakening of the Icelandic and Aleutian lows. The geostrophic circulation is altered very little.

3. Second model

It is probably more realistic to assume that the effect of increased solar output on the annual temperature range will be greater over the continents than over the oceans. Since the variation of Q with longitude is largely a function of continentality, the condition that the effect of increased solar radiation be greater over continents than over oceans may be satisfied by assuming

$$Q' = GQ, \tag{10}$$

where G is a constant greater than one and the prime again denotes the varied condition.

From (5) and (6),

$$\left.\begin{matrix} a_n^{m\prime} \\ b_n^{m\prime} \end{matrix}\right\} = G \left\{\begin{matrix} a_n^m \\ b_n^m. \end{matrix}\right. \tag{11}$$

Upon combination of (11) and (3)

$$p_s' = Gp_s. \tag{12}$$

Thus, if the annual temperature range increases everywhere by an amount proportional to the observed temperature range, the magnitude of the annual pressure range increases by the same factor. In terms of the winter pressure distribution, this means that the continental highs and the Aleutian and Icelandic lows all increase in intensity, with resulting increase in geostrophic circulation. However, the positions of the pressure centers, as well as the isobaric configurations, are unchanged.

REFERENCES

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