Reply to “Comments on ‘How Does the Boundary Layer Contribute to Eyewall Replacement Cycles in Axisymmetric Tropical Cyclones?’”

JEFFREY D. KEPERT
Centre for Australian Weather and Climate Research, Bureau of Meteorology, Melbourne, Victoria, Australia

DAVID S. NOLAN
Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, Florida

(Manuscript received 19 December 2013, in final form 27 May 2014)

1. Introduction

Kepert (2013, hereafter K13) presented a series of idealized simulations of the boundary layer (BL) flow in axisymmetric tropical cyclones (TCs) with secondary wind maxima of various strengths using a hierarchy of diagnostic BL models. Three models, with different levels of approximation, were used because each has its own advantages. In particular, the nonlinear model of Kepert and Wang (2001, hereafter KW01) is expected to be the most accurate of these because it contains more of the relevant physics, including more realistic parameterizations of the turbulence and surface fluxes, while the simpler models of Kepert (2001) and Ooyama (1969) trade off some physical realism against the benefits of an analytical solution and the consequent ease of physical interpretation.

These models, in common with many other models of the TC BL, diagnose the steady-state boundary layer flow in response to an applied pressure field representative of a TC. That is, they assume that the BL is, to good approximation, “slaved” to the rest of the cyclone. While there are influences in the opposite direction—for instance, the frictional convergence within the BL strongly influences the distribution of convection and hence heating within the storm—the aim with these simplified models is to study one side of a two-way interaction.

Perhaps the most surprising finding of K13 was that, in a TC with concentric wind maxima of similar strength, the outer wind maximum produces a much stronger frictional updraft. Similarly, an outer wind bump—too weak to be a local maximum—can produce a frictional updraft as strong as that of the primary eyewall. These results were introduced using the nonlinear BL model, and a physical interpretation of their cause offered based on the updraft equation from the linear model

$$w_\infty = -\frac{1}{r} \frac{\partial}{\partial r} \left( C_d v_{gr} \frac{v_{gr} + 2v'(0)}{\xi_{gr} + f} \right)$$

$$= -\frac{1}{(\xi_{gr} + f)^2} \frac{\partial \xi_{gr}}{\partial r} C_d v_{gr} \frac{v_{gr} + 2v'(0)}{\xi_{gr} + f} + \frac{1}{r(\xi_{gr} + f)} \frac{\partial \left( r C_d v_{gr} \frac{v_{gr} + 2v'(0)}{\xi_{gr} + f} \right)}{\partial r},$$

where $w_\infty$ is the updraft in the limit as $z \to \infty$, $v_{gr}$ is the gradient wind and $\xi_{gr}$ is its vertical vorticity. $C_d$ is the drag coefficient, $v'(0)$ is the departure of the near-surface flow from $v_{gr}$, $r$ is radius, and $f$ is the Coriolis parameter. A similar equation applies for balanced slab models. The key element of this interpretation is that linearized frictional convergence in a TC is fundamentally different from classical Ekman pumping. K13 showed that the first term in (2) dominates, so the updraft is not proportional to the curl of the surface stress but rather approximately proportional to the radial gradient of the absolute vorticity of the gradient wind, divided by this vorticity squared, multiplied by the surface stress. Thus, the outer wind maximum produces a relatively strong updraft, even though the

Corresponding author address: Jeffrey D. Kepert, Centre for Australian Weather and Climate Research, Bureau of Meteorology, 700 Collins St., Docklands VIC 3008, Australia.

E-mail: j.kepert@bom.gov.au

DOI: 10.1175/JAS-D-14-0014.1

© 2014 American Meteorological Society
associated radial vorticity gradient is weaker and the surface stress similar to those at the inner wind maximum, because it is in an environment of low absolute gradient vorticity.

However, while the linear model is useful for physical interpretation, there are significant improvements with the nonlinear model, including the inward displacement of the frictional updraft and the development of strong supergradient flow. Both of these points are discussed in K13.

Based on these ideas, K13 proposed a mechanism by which the BL contributed to secondary eyewall formation (SEF) and eyewall replacement cycles (ERCs) through a positive feedback between the vorticity, BL convergence, and moist convection. The hypothesized mechanism will be a useful description of reality only to the extent that the updraft diagnosed from the boundary layer processes discussed in K13 matches that in nature or in more complete models. Montgomery et al. (2014, hereafter M14), in commenting on K13, have correctly identified this test as being crucial to K13’s mechanism and have compared the azimuthal-mean updraft from simulated ERCs using the Weather Research and Forecasting (WRF) Model and the Regional Atmospheric Modeling System (RAMS) model, with that calculated from (1). They find that the agreement is not good and conclude that they have shown K13’s hypothesis to be false.

We agree with M14 that this test is appropriate. However, we disagree that the comparison should be made to the linear model. Indeed, K13 extensively discussed the differences between the linearized and nonlinear models and noted immediately after introducing the positive feedback mechanism that it was important that the nonlinear BL model be used rather than the linearized model. M14 ignore this important point. In this reply, we examine a recent WRF simulation of SEF and ERC by Nolan et al. (2013, hereafter N13). We compare the azimuthal-mean updraft and boundary layer flow from that simulation with the corresponding fields diagnosed by the nonlinear boundary layer model forced with the azimuthal-mean pressure gradient from N13’s WRF simulation, and we demonstrate very good agreement. This good agreement confirms that the TC BL can be well approximated as being slaved to the gradient flow. The idea that the TC BL can be so approximated is fundamentally at variance with the “new paradigm” of TC intensification discussed by M14 because this approximation excludes “the local time derivatives of the horizontal velocity in the BL” [M14, their section 4; see also Abarca and Montgomery (2013)]. We will also present similar diagnostics of N13’s simulation using the linearized model and show that satisfactory performance can be achieved, albeit with a displacement of the eyewall updrafts to larger radii as noted by K13, if the forcing pressure field is first filtered. From these comparisons, we identify another important difference between the nonlinear and linearized models—namely, that the nonlinear model is much less sensitive to small-scale fluctuations in the gradient wind field.

2. Models and data processing

In this reply, we use a WRF simulation of a TC that includes an SEF and an ERC, prepared as a nature run for data assimilation experiments and described by N13. That simulation nested WRF from 27 km down to 1 km and covered the full life of the hurricane, although we will focus attention on the 48-h period beginning at 0000 UTC 3 August. Details of the simulation, including the choice of the nesting and initial fields, and the model setup including the physical parameterizations, are given by N13. That paper also contains a detailed description of the SEF and ERC along with a comparison to observed events.

The WRF data on the 1-km grid were azimuthally averaged and interpolated from model to height coordinates, with a vertical spacing of 100 m up to 3 km and a horizontal spacing of 3 km, every 6 min. The gradient wind \( \nu_{gr} \) was calculated from the azimuthally averaged time-filtered WRF pressure and density data on these constant height levels using the same equation as M14. Even after the azimuthal averaging, the WRF data contain some small-amplitude high-frequency noise and to remove this, and for consistency with the time averaging in M14, a low-pass filter was applied to all fields. This filter reduced the amplitude of waves with period 160 min by half and is therefore somewhat stronger than the 1-h block average used by M14 for which waves of period 110 min are halved in amplitude. We chose this cutoff to remove short-term transients without obscuring the main features of the vortex evolution and shall show later that this filtering has only a slight effect on simulations with the nonlinear BL model but is needed with the linear model.

Hovmöller diagrams of the time-filtered gradient wind at 2.2-km height, its vorticity, and the moist heating rate averaged between 1- and 3-km heights are shown in 

---

1 Some of the input data used by M14 for (1) were approximated from other model variables, perhaps since the exact data [e.g., \( \nu'(0) \) and \( C_{p}' \) were not saved with the model output.
Fig. 1. Up to about 1200 UTC 3 August, the initial eyewall is contracting and forming, accompanied by an inward contraction of radial maxima in $\zeta_{gr}$ and the moist heating. Around this time, both quantities reach temporal minima within the radius band 70–120 km. Subsequently, both $\zeta_{gr}$ and the moist heating increase broadly outside of the RMW for about 12 h, before the heating consolidates into a local maximum near 120-km radius at about 0000 UTC 4 August, which then contracts. In contrast, $\zeta_{gr}$ does not form a persistent relative maximum outside the primary eyewall until some 9 h later. Rather, an extended area of elevated, roughly constant $\zeta_{gr}$ forms, extending to about 130-km radius at 0000 UTC 4 August, and subsequently contracting and intensifying, with a persistent local maximum forming near its outer edge from about 0900 UTC 4 August. While this region of roughly constant $\zeta_{gr}$ contains numerous short-lived relative extrema, it is otherwise similar to the radially constant region of vorticity used by K13 to represent the moat region in most of his idealized profiles. Note the relatively tight radial gradient of $\zeta_{gr}$ on the outside of this contracting region and that the outer moist heating maximum is located near the outer edge of this region.

The $v_{gr}$ data at hourly intervals at 2.2-km height were extracted and provided to the nonlinear BL model used by K13 (henceforth, the KW01 model). This height was chosen because it is near the top of the domain used for the KW01 model, although other choices produce very similar results since $v_{gr}$ is nearly constant with height in this part of the simulated TC. The mass data was passed between the models in the form of $v_{gr}$ rather than pressure since the KW01 model uses the Exner function to represent pressure but has a somewhat different thermodynamic structure than WRF because it omits moisture, and so a direct conversion of the pressure to the Exner function would have led to slightly different pressure gradient force. The KW01 model calculates the Exner function at its top boundary from the $v_{gr}$ data and its potential temperature, ensuring that the pressure gradient force there is identical in the two models. The model is then run for 24 h to a quasi-steady state and the data azimuthally averaged as in K13.

Some small modifications were made to the KW01 model for this study in addition to those described in K13. The surface roughness parameterization used in N13 was implemented. The Yonsei University (YSU)-like turbulence parameterization from Kepert (2012) was used, with a fixed depth of $h = 750$ m, based on typical output values from WRF. These are smaller than typical inflow-layer depths, presumably because the method for diagnosing the boundary layer top in the YSU scheme relies also on the thermal structure, which is stable in the upper

![Hovmöller diagrams](https://example.com/fig1.png)
part of the inflow layer (Nolan et al. 2009a,b; Zhang et al. 2011). Initial simulations had greater wind shear between 1- and 2-km heights than the WRF simulation, so the parameterization was modified to give extra mixing aloft and reduce the wind shear there to better match WRF. Our first attempt implemented a first-order closure above the prescribed depth, similar to that in the YSU scheme, but did not produce sufficient mixing. Zhu et al. (2013) have argued that high-resolution simulations with this scheme contain small-scale features that provide some additional, resolved mixing and so, to represent this, we increased the diffusivity above the prescribed \( h \) to the maximum value occurring within the BL. While this tuning significantly reduced the wind shear in the upper part of the domain to better match WRF, it had only a minor effect on the distribution or strength of the frictional updraft, since most of the horizontal convergence occurs near the surface.

The diffusivity is thus defined by

\[
K = \min\left\{ k u_g z \left[ 1 - (2 - 3c) \frac{z}{h} \right] + (1 - 2c) \left( \frac{z}{h} \right)^2, K_{\text{max}} \right\},
\]

where

\[
K_{\text{max}} = k u_g z \frac{4 - 9c}{27(1 - 2c)^2}
\]

is the maximum value of \( K \) for \( 0 < z < h \). In the YSU-like formulation of Kepert (2012), \( c = 0 \) and consequently \( K = 0 \) at \( z = h \). The effect of \( c > 0 \) is that \( K = c k u_g h \) at \( z = h \); in these simulations, we use \( c = 0.1 \). A plot of the \( K \) function is shown in Fig. 2.

In summary, the WRF data are azimuthally averaged and time filtered, and \( v_{gr} \) at 2.2-km height is calculated and provided at hourly intervals to the KW01 model, which uses it to calculate the pressure gradient force. This is the only data passed between the two models. The KW01 model is run out to a quasi-steady state, and the results azimuthally averaged prior to further analysis, as in K13.

3. Results and discussion

The left panels of Fig. 3 show radius–height sections of the three azimuthally averaged time-filtered wind components from the WRF simulation at 0600 UTC August 4. At this time, the formation of a secondary wind maximum is under way, although a local wind maximum is not yet present. There are two near-surface inflow maxima in the WRF simulation: near \( r = 45 \) and 90 km. These are associated with two updraft maxima: the first near \( r = 35 \) km, immediately inside of the radius of maximum gradient winds, and a second, broader one near \( r = 75 \) km. The right panels show the same quantities from a simulation with the KW01 model forced by the azimuthal-mean pressure gradient force from 2.2-km height in the WRF simulation. This simulation is quite similar to that shown in Fig. 7 of K13, with the main difference being that the outer updraft is somewhat broader. The WRF and KW01 simulations are very similar in structure, but the primary eyewall updraft at its peak is 85% stronger, and the secondary updraft about twice as strong, in WRF compared to KW01. We believe that the main reason for this difference is that the KW01 model is dry, and therefore the updraft is purely frictionally forced, while in WRF the updraft is strengthened by the effects of latent heat release (Bui et al. 2009; Rozoff et al. 2012). Both the boundary layer inflow and azimuthal flow are up to 2 m s\(^{-1}\) stronger in WRF than in the KW01 model.

Figure 4 compares the 10-m horizontal flow and 1-km vertical velocity from four times during the ERC (1800 UTC 3 August and 0600, 1200, and 1800 UTC 4 August); the second of these times was also shown in Fig. 3. The marked similarities, and relatively small differences, between the WRF and KW01 model updrafts are similar to those examined in more detail in Fig. 3. This presentation makes another systematic difference clear, in that the 10-m inflow and azimuthal wind components tend to be a little stronger in WRF at most times. The stronger inflow in WRF is consistent, through continuity, with the
stronger updraft there. It is also consistent with the stronger azimuthal winds, since the stronger inflow will advect additional absolute angular momentum inward, accelerating the azimuthal wind. These differences are expected to be partly due to differences in the vertical mixing between the two models and the effects of azimuthal averaging but also to the absence of that component of the secondary circulation forced by latent heat release in the BL model. Nevertheless, it is clear that the diagnostic boundary layer model reproduces the main features of the boundary layer flow in the WRF simulation with a high degree of fidelity.

Hovmöller diagrams summarizing the boundary layer flow evolution from both models are presented in Fig. 5. It is clear that the strong similarity between the WRF and KW01 simulations applies throughout the 48-h period studied and that the systematic differences noted above are representative of those at other times. The only information that the boundary layer model receives from WRF is the azimuthal-mean pressure gradient force (represented by $\nu_{gr}$), which is clearly sufficient for it to reproduce the main features of the boundary layer flow throughout the simulation, including the vertical velocity. The nonlinear BL model is equally successful at reconstructing the vertical motion during the initial intensification and eyewall contraction of the storm, during the subsequent formation and contraction of the secondary eyewall and during the
further intensification of the latter eyewall after the primary eyewall has disappeared. It is apparent also from Fig. 1c and the cyan lines in Fig. 4 that the changes in $\zeta_{gr}$ outside the primary eyewall are quite modest, particularly earlier in the ERC period (from about 1200 UTC 3 August to 0600 UTC 4 August). These subtle changes are nevertheless responsible for the marked change in the frictional updraft as the secondary eyewall forms, apparent in the Hovmöller diagram of the simulation with the KW01 model from about 1800 UTC 3 August (Fig. 5f). The location and timing of this evolution closely matches that in the WRF simulation (Fig. 5c).

We carried out similar computations using the linearized model of Kepert (2001). Our initial results here were similar to those of M14 in that the vertical velocity was dominated by large-amplitude, short-wavelength fluctuations. An example of such a calculation is shown in Fig. 6a, which shows the response of the linear model to an unfiltered (but azimuthally averaged) WRF pressure field. The reason for these fluctuations follows from an examination of the $\zeta_{gr}$ data in that figure (thick gray dashed curve): the gradient vorticity field has numerous small oscillations with a wavelength of 5–10 km, which are associated with numerous changes of the sign of $\partial \zeta_{gr} / \partial r$. From (2), $w_v$ is approximately proportional to $\partial \zeta_{gr} / \partial r$, so

---

**Fig. 4.** Comparison of WRF simulation with boundary layer model diagnosed flow at (a) 1800 UTC 3 Aug, (b) 0600 UTC 4 Aug, (c) 1200 UTC 4 Aug, and (d) 1800 UTC 4 Aug. Thick red line: $v_{gr}$ (m s$^{-1}$), thin cyan line: $\xi_{gr}$ (10$^{-4}$ m s$^{-1}$), green lines: $-\mu_{10}$ (m s$^{-1}$), blue lines: $v_{10}$ (m s$^{-1}$), and magenta lines: $w_{1km}$ (10$^{-2}$ m s$^{-1}$). The WRF data are shown as dashed curves and the boundary layer model as solid curves.
Fig. 5. Hovmöller diagrams of the boundary layer flow from WRF and two diagnostic models. (a) 10-m radial flow from WRF (contour interval 2 m s⁻¹). (b) 10-m azimuthal flow from WRF (contour interval 5 m s⁻¹). (c) 1-km vertical velocity from WRF (contour interval 0.1 m s⁻¹). (d)–(f) As in (a)–(c), respectively, but for the nonlinear BL model. (g)–(i) As in (a)–(c), respectively, but for the linear BL model.
the local maxima in $\zeta_{gr}$ lead to sign changes in $w_r$, and the amplitudes of the $w_r$ fluctuations are large because the vorticity outside of the primary eyewall region is low. Fluctuations in $v_{gr}$ of similar spatial scale are also apparent in the observational analysis of Willoughby (1990).

Accordingly, we applied a spatial filter (three passes of a 1–2–1 filter) to the time-filtered gradient wind field to reduce the $\zeta_{gr}$ oscillations and repeated the $w_r$ computation. The results from the linear model applied to time- and space-filtered pressure data are shown in Fig. 6b. Inside of about 110-km radius, the solution appears well behaved and exhibits a narrow updraft near the RMW and a secondary broader updraft around 90-km radius. Outside of 110-km radius, the solution exhibits large and unphysical oscillations in $w_r$ but the linearization here is invalid because the vorticity is low (Kepert 2001; K13). Hovmöller diagrams of similar calculations for the entire time period are presented in Figs. 5g–i. While some substantial oscillations remain, especially at larger radii, the formation and contraction of the successive eyewall updrafts in the linear model is nevertheless discernable through the noise. The systematic displacement of the updraft in the linear model toward larger radius than in the KW01 model, discussed by K13, is also apparent.
One reason that M14 achieved such unsatisfactory performance with the linear model is that they have scaled the plots of w in their Fig. 2 by differing amounts: the data from RAMS are scaled by 20 and that from the approximated linear model by 5. This fourfold discrepancy is a substantial obstacle to a fair comparison. Other deficiencies in their calculation include that they did not use the same drag coefficient in their as in RAMS, that they approximated inputs to (1) in their Fig. 2 by using a surface wind factor, that they used a gradient wind from well above the BL that seems to have some significant differences to that within, and that they did not apply any spatial filtering. Unfortunately, they declined our request for the data used in their computations, so we were unable to attempt to reproduce their results or to attempt a diagnosis using the nonlinear model.

It is clear why the linear model is so sensitive to these small-scale oscillations in \( \xi_{gr} \), but why is the nonlinear model largely insensitive to them? This insensitivity is highlighted in Figs. 6c and 6d, which compare simulations with the KW01 model applied to unfiltered and time-filtered WRF data. The diffusive time scale for a BL to adjust to changed conditions is \( K \delta_0^2 \), where \( K \) is a scale for the diffusivity within the BL and \( \delta_0 \) is its depth scale. In the linear model, \( \delta_0 = \sqrt{2K/I} \), and the depth of the inflow layer in the two models is similar, so we may apply this scale to the nonlinear model and find that the adjustment time scale is \( 4I^{-1} \). This time scale for the adjustment in the TC BL has been previously proposed by Eliassen and Lystad (1977). From Fig. 3, the wavelength of the oscillations is \( O(10 \text{ km}) \) and the inflow about \( 10 \text{ m s}^{-1} \), so air passes through the \( \xi_{gr} \) oscillations in \( O(10^3 \text{ s}) \). In contrast, at \( r = 100 \text{ km} \) in these simulations, \( 4I^{-1} \sim 10^4 \text{ s} \), indicating that the inflowing air will be marginally influenced by the \( \xi_{gr} \) oscillations but will not fully adjust to them because it passes by them too quickly. The updraft in the nonlinear model is therefore influenced by larger-scale structures than these, and we expect that the scale to which it responds will vary with the inertial stability and hence with radius. In contrast, the linearization in the KW01 model means that the updraft is always determined by the local gradient wind structure, no matter how small scale.

We note, however, that this time scale is still substantially shorter than that over which TCs undergo significant structure or intensity change, which is on the order of 6–24 h. Because the BL in the core region adjusts more rapidly than the storm as a whole, it is reasonable to approximate it as being slaved to the rest of the cyclone and to diagnose the flow therein using a steady-state model.

One can see this process operating in the results in K13. For instance, in his Figs. 2 and 7, the outer updraft is 2–3 times as wide as, while the inner updraft is of similar width to, the blending zone. Compared to the inner updraft, the outer updraft is in a region of smaller inertial stability and (in K13’s Fig. 2) has stronger inflow, so it takes the inflowing air a greater distance to adjust to the changed conditions, leading to a broader updraft. This effect does not occur with the linearized model, where both updraft widths scale with the blending width (see K13’s Fig. 6). Indeed, when considering the sensitivity of the updraft to the blending width \( L_b \), K13 notes that “decreasing \( L_b \) increases the radial gradient of vorticity, and in the linear model leads to a nearly proportional increase of the updraft strength and decrease of its width through (24). In the nonlinear model, the \( u_{\text{diff}}/w \) term allows the inflow to decelerate over a longer distance and so the changes in \( w \) with \( L_b \) are much smaller” (p. 2825). Similarly, the outer updraft is substantially broader than the inner updrafts in Figs. 3 and 5 here.

This analysis of the adjustment time scale implies that perhaps one can extract a useful signal from the WRF gradient wind data using the linear model, provided that the small oscillations in \( \xi_{gr} \) are first filtered out. Indeed, that was the motivation for our radial filtering used in the calculations with the linear model presented in Fig. 5.

4. Further matters raised by M14

M14 claim in their introduction that K13 used parametric wind profiles “with two wind maxima,” which might mislead some to conclude that K13’s results apply only when a second wind maximum has already formed. While many of the simulations presented in K13 did have two maxima, the profile in K13’s Fig. 7 does not quite have an outer wind maximum, and a number of the simulations in K13’s Fig. 11 had even weaker outer vorticity \( \xi_2 \) and smaller bumps in the gradient wind profile but nevertheless produced a discrete outer updraft. We present data from another of these simulations in Fig. 7, in which it is clear that an outer vorticity bump, so weak as to be easily overlooked in the tangential wind, can nevertheless produce a significant updraft. It is clear, therefore, that the processes discussed by K13 operate well before an outer wind maximum is present.

---

\(^2\) K13 defined his vortices by two or three piecewise constant annuli of vorticity, surrounded by a skirt region. The discontinuities between adjacent annuli were smoothed over by a blending function over a blending width \( 2L_b \).
While M14 quote the section of K13 that introduces the proposed feedback mechanism accurately, the schematic in their Fig. 1 introduces some significant errors. It incorrectly identifies the BL frictional updraft with K13’s (22), ignoring K13’s clear statement as to the importance of the differences between the nonlinear and linear models. It also omits an important part of K13’s hypothesis: that some other mechanism provides the initial increment of vorticity. That component was included in recognition of the several studies that have attributed SEF to such mechanisms and also to the fact that satellite data often indicate that SEF appears to proceed via a “wrapping up” of a strong spiral rainband. Given such evidence, K13 preferred not to attribute SEF solely to a feedback involving the BL and instead to propose that feedback as a contribution toward, but not as the sole cause of, SEF.

In their section 3, M14 list four objections to K13’s hypothesis. None of them are valid. In particular,

(i) M14 claim that K13’s hypothesis cannot apply to formation or to the evolution of the system since the three boundary layer models considered do not contain time derivatives. Their claim is incorrect, since calculation of the third step of the proposed mechanism, in which increased convection leads to an increase in the low-level vorticity, would require a vorticity tendency equation, which would introduce the necessary time derivatives.

(ii) M14 complain that K13 does not specify which aspect of the deep convection he intends to increase and point out that the vertical mass flux out of the BL does not determine the diabatic heating rate within deep convection. However, K13’s hypothesis does not require a direct relationship between BL convergence and deep convection of the type found in some cumulus parameterization schemes, only that a physical link exists. That is, there is a distinction between parameterizing heating in terms of BL convergence and noting that BL convergence can plausibly localize convection. Such localization could occur through the uplift releasing conditional instability through vertical stretching reducing the stability, through moisture convergence, or through some combination of these factors. While the strength of the link may depend on other factors and may vary between storms, all that is required is that an increase in the low-level convergence leads, other things being equal, to an increase in the convection. It is evident from comparing the moist heating rate in WRF (Fig. 1c) with the diagnosed updraft data from the KW01 model (Fig. 5f) that the convection is strongly tied to the boundary layer frictional convergence. So long as some of the increase in the frictional updraft is ventilated by the convection, K13’s proposed feedback should occur.

(iii) M14 assert that the linearized BL model is the theoretical basis of the K13 hypothesis. This is untrue—each of the storms simulated by K13 is first discussed using the nonlinear model. Simulations with the linear model were then presented to demonstrate that there were significant similarities between the results from the two models, suggesting that the linear model’s analytical solution could be used to help obtain a sound physical understanding of the results from the nonlinear model. This physical interpretation was subsequently presented in section 5 of K13, which showed why the locally increased outer vorticity gradient was so efficient in producing a frictional updraft, and related the eyewall updraft(s) to the vorticity structure. In making their assertion, M14 ignore the complementary use of the models by K13 to analyze different facets of the problem, and his explicit statement in section 6 that one aspect of the nonlinear model is important to his hypothesized feedback mechanism. Under this item, M14 also note the importance of the “horizontal advective departures from gradient wind balance.” While these are eliminated from the Kepert (2001) model.
by the linearization, they are fully included in the nonlinear model (KW01), which is formally valid for vortex Rossby numbers of order $1.3$.

(iv) M14 claim that “BL theory breaks down in regions of deep convection, due to the horizontal pressure gradients induced by the convection.” Some of those authors have previously made similar claims (Smith and Montgomery 2010), although have never produced any evidence as to the magnitude of these gradients. However, we note that the BL model explains a large part of the horizontal convergence throughout the SEF and ERC in our WRF simulation, including under both eyewalls, and that the BL is clearly strongly influencing the location of the deep convection. In addition, the BL model clearly does a good job of describing the three-dimensional flow throughout the SEF and ERC (Figs. 3–5), and it does not appear to have “broken down.” We found that the total eyewall updraft is up to twice as strong as that calculated from friction alone and believe that the difference is due to the related matters of the balanced heat-induced secondary circulation and buoyant convection. The stronger updrafts in the WRF simulation merely reflect the fact that the secondary circulation there is driven by both heating and friction and in no way invalidates the boundary layer models. Finally, our calculations were based on the full pressure field from a nonhydrostatic model and therefore included convective-scale pressure perturbations, in contrast to the smoother, parametric, profiles used by K13. But we showed in section 3 that the nonlinear model was largely insensitive to these smaller-scale features.

Toward the end of their section 3, M14 complain that K13 uses the phrase “enhanced updraft” to describe the mechanism in Huang et al. (2012), when that phrase does not appear in that paper. However, Huang et al. (2012) do discuss increased frictional convergence, which is equivalent to increased updrafts through the continuity equation. K13’s choice of words is, therefore, a paraphrase.

5. Conclusions

M14 proposed a test of the hypothesized positive feedback mechanism for SEF in K13—namely, that the diagnosed updraft near the BL top from a BL model should be similar to that in simulations from a full-physics model. We agree that this is an appropriate test and have applied it to such a simulation. In contrast to their results, we find that there is very good agreement in the location of the updrafts, but that the BL model tends to underestimate the magnitude owing to the neglect of moist heating in the BL model. Since our results pass their test, we conclude that their claim, to have falsified K13’s hypothesis, is incorrect. In fact, our results strongly support K13’s hypothesis, because we have shown also that the diagnosed frictional convergence (Fig. 5f) closely matches the moist heating above the BL (Fig. 1c). In other words, frictional BL convergence localizes the convection, and buoyancy then enhances the frictional updraft.

Our results differ from those of M14 because we have used a nonlinear diagnostic BL model, while they used an approximation to a linearized one. They state in their introduction that they used the linearized model because K13 used it in his sections 5 and 6. The purpose of those discussions was to develop physical understanding of certain results from the nonlinear model, including why the frictional convergence at the outer wind maximum is relatively strong. It was not to suggest that the linearized model can substitute for the nonlinear one. Indeed, K13 expressed a clear preference for the nonlinear model in the proposed mechanism.

In justifying their use of the linearized model, M14 quote from K13 (p. 2828):

> the approximate location and strength of the updraft are determined by much simpler dynamics, namely the near balance between radial advection and the surface sink of absolute angular momentum; while there are differences of detail between the models, they can be reasonably well predicted from the gradient wind alone. It is the location of the increased vorticity outside the primary eyewall that determines where the outer frictional updraft forms and apparently interpret the words “much simpler dynamics” as implying the linearized model. This is a serious misinterpretation. Not only does K13 not specify the linear model in that passage, but the context of the passage is a discussion of both models. Moreover, the words “near balance” imply the nonlinear model, because that balance is exact in the linear model, but only approximate in the nonlinear, as discussed by K13. In fact, the analysis presented here provides strong support for the last sentence of the quoted passage.

The discussion and results herein provide an additional reason for preferring the nonlinear model—namely, its
insensitivity to small fluctuations in the gradient wind that were shown by M14 to strongly affect the linear model. The analytic $v_{gr}$ profiles used by K13 did not contain such perturbations, and so this difference between the models was not detected. The comment by M14 makes an important contribution in drawing attention to these perturbations and their consequences for the linear model. However, the existence of these perturbations does not invalidate K13’s hypothesized feedback mechanism because that feedback does not rely on the linear model.

It is perhaps helpful to compare the SEF mechanism discussed by Huang et al. (2012) to the feedback mechanism proposed by K13. This comparison is made in Table 1. On the key point of whether a diagnostic approach to the BL is sufficient to predict the distribution of frictional convergence, the evidence herein shows that the diagnostic KW01 model performs well and supports the physical arguments in K13. In this context, the attempt by Abarca and Montgomery (2013, their section 6) to study SEF through an analysis of the time-varying BL flow in response to a fixed pressure field seems to us to be fundamentally wrong because the BL adjusts to changes in the pressure field more quickly than the pressure field evolves.

Over the past few decades, improvements in computer and modeling technology have resulted in models, such as WRF and RAMS, that are capable of simulating atmospheric phenomena with a high degree of fidelity. Such simulations may reflect the result of the interaction between a large number of resolved and parameterized processes and can be correspondingly difficult to interpret in terms of cause and effect. Often, meteorologists turn to simpler models to aid such interpretation. We suggest that the work in K13 and here provides good examples of the power of a hierarchy of simplified models in helping to develop correct interpretations of complex processes in more complete models.

Acknowledgments. D. Nolan was supported in part by the NOAA Office of Weather and Air Quality (OWAQ) through its funding of the OSSE Testbed at the Atlantic Oceanographic and Meteorological Laboratory, by the NOAA Unmanned Aerial Systems (UAS) Program, and by the Hurricane Forecast Improvement Program (HFIP).

REFERENCES


Kepert, J. D., 2001: The dynamics of boundary layer jets within the tropical cyclone core. Part I: Linear theory. J. Atmos. Sci.,


