Idealized Quasi-Biennial Oscillations in an Ensemble of Dry GCM Dynamical Cores

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ABSTRACT

The paper demonstrates that quasi-biennial oscillation (QBO)-like oscillations can be simulated in an ensemble of dry GCM dynamical cores that are driven by a simple Held–Suarez temperature relaxation and low-level Rayleigh friction. The tropical stratospheric circulations of four dynamical cores, which are options in NCAR’s Community Atmosphere Model, version 5 (CAM5), are intercompared. These are the semi-Lagrangian (SLD) and Eulerian (EUL) spectral transform, finite-volume (FV), and spectral element (SE) dynamical cores. The paper investigates how the model design choices impact the wave generation, propagation, and dissipation mechanisms in the equatorial region. SLD, EUL, and SE develop spontaneous QBO-like oscillations in the upper equatorial stratosphere, whereas FV does not sustain the oscillation. Transformed Eulerian-mean (TEM) analyses reveal that resolved waves are the dominant drivers of the QBOs. However, the Eliassen–Palm flux divergence is strongly counteracted by the TEM momentum budget residual, which represents the forcing by diffusion and thermal damping. Interestingly, a reversed Brewer–Dobson circulation accelerates the downward propagation of the SLD’s QBO, whereas the EUL’s and SE’s QBOs are slowed by a mean ascent. Waves are abundant in the SLD’s, EUL’s, and SE’s tropical atmosphere despite the absence of moist convection as a typical wave trigger. Dynamic instabilities are suggested as a wave-triggering mechanism in the troposphere and wave-dissipation process in the stratosphere. In particular, there are indications that the increased occurrences of strongly negative instability indicators in SLD, EUL, and SE are related to more vigorous wave activities and higher magnitudes of the resolved wave forcing in comparison to FV.

1. Introduction

Modeling the quasi-biennial oscillation (QBO) in atmospheric general circulation models (GCMs) and understanding the QBO forcing mechanisms has been a challenge for decades. The QBO is a phenomenon in the equatorial stratosphere that occupies the region between 100 and 1 hPa. This region is characterized by a downward-propagating zonal wind regime that periodically changes from westerlies to easterlies. On average, the observed QBO period is 28 months, but it can vary substantially between 22 and 34 months. A thorough review of the QBO is provided in Baldwin et al. (2001).

The QBO is an important feature, since it not only dominates the circulation in the tropical stratosphere but also impacts the stratosphere at high latitudes, the Brewer–Dobson circulation, and the tropospheric and mesospheric flow. For example, Holton and Tan (1980) showed that the 50-hPa geopotential height at high latitudes is lower in the westerly phase of the QBO, as compared to the easterly phase. Baldwin et al. (2001) reviewed how the interaction between the QBO and the Brewer–Dobson circulation influences the mass transportation in the stratosphere and thereby impacts the distributions of chemical constituents, like ozone, and water vapor. Garfinkel and Hartmann (2011) used idealized dry model experiments with a forced QBO to discuss the impact of the QBO-induced meridional circulation on the tropospheric flow. Even the extratropical surface conditions are impacted by the QBO [as, e.g., discussed by Ebdon (1975) and Marshall and Scaife (2009)]. Furthermore, it is well documented that the QBO acts as a filtering mechanism for upward-traveling waves (e.g., see Yang et al. 2011, 2012; Krismer and Giorgetta 2014) and thereby modulates the wave composition in the mesosphere. Therefore, the ability of a GCM to simulate the QBO is an important model characteristic.

were among the first that simulated QBOs or QBO-like oscillations with GCMs. More recently, Kawatani et al. (2010), Orr et al. (2010), Xue et al. (2012), Yao and Jablonowski (2013), and Richter et al. (2014) also reported on successful QBO and QBO-like simulations. However, many aspects of the QBO drivers in GCMs still need to be understood, since models generate QBOs for very different reasons. As, for example, pointed out by Scinocca et al. (2008), a spontaneously generated QBO in any two models can result from significantly different combinations of resolved equatorial waves and parameterized gravity waves. Our paper sheds light on the resolved wave forcing. In particular, it isolates the impact of the numerical discretizations and diffusion mechanisms of GCM dynamical cores on equatorial waves and highlights their wave–mean flow interactions in the QBO region. This is accomplished via idealized dry dynamical core experiments that can expose causes and effects of the model design choices more clearly than full GCMs with physical parameterizations. Note that the phrase “dynamical core” refers to the resolved fluid flow component of GCMs, as defined in Williamson (2007). The dynamical core most often also includes explicitly applied dissipation or filtering processes needed for numerical stability (Jablonowski and Williamson 2011).

As explained by Lindzen and Holton (1968) and Holton and Lindzen (1972), the QBO is mainly generated and influenced by equatorial waves, with trapped equatorial Kelvin waves providing a westerly force and mixed Rossby–gravity (MRG) waves, providing an easterly force. However, the forcing provided by Kelvin waves and MRG waves is not enough to explain the QBO. Inertia–gravity waves and small-scale gravity waves also contribute to the QBO forcing (Dunkerton 1997). These small-scale waves have horizontal scales of about 5–500 km and are therefore too short to be fully captured by GCMs at typical climate resolutions with grid spacings between 100 and 250 km (Kim et al. 2003). However, these waves produce synoptic-scale body forces on the atmospheric flow when they dissipate. These processes have to be resolved or parameterized in GCMs in both the vertical and horizontal directions. High vertical resolutions with vertical grid spacings of about 600–1000 m in the stratosphere are therefore often required to simulate the QBO.

With the advancement of computing resources, small-scale gravity waves can now be modeled directly, for example, via high horizontal and vertical resolutions in the stratosphere, as in Kawatani et al. (2010) with $\Delta x = 60$ km and $\Delta z = 300$ m, respectively, or in the tropical channel study by Evan et al. (2012) with $\Delta x = 37$ km and $\Delta z = 500$ m. Alternatively, and more typical of today’s GCMs, a gravity wave drag (GWD) parameterization scheme can be included, as demonstrated in the middle atmosphere and QBO studies by, for example, Scaife et al. (2000, 2002), Giorgetta et al. (2002, 2006), Naoe and Shibata (2010), Orr et al. (2010), Xue et al. (2012), Krismer and Giorgetta (2014), and Richter et al. (2014). These modeling studies show that small-scale gravity and intermediate-scale inertia–gravity waves are important QBO drivers in GCMs, as also observed in nature (Ern and Preusse 2009; Ern et al. 2014; Alexander et al. 2010). As a caveat, though, the choice of the gravity wave drag scheme, as well as the strength and launch level of the gravity wave source spectrum, are highly model dependent. They significantly impact the characteristics of a QBO simulation, as, for example, documented in Giorgetta et al. (2006), Scinocca et al. (2008), Xue et al. (2012), Lott and Guez (2013), Kim et al. (2013), and Schirber et al. (2014), and can be used to empirically tune the QBO period, magnitude, and vertical extent. In our paper, we avoid such complications and reveal that QBO-like oscillations are also possible in GCM dynamical cores without resolved or parameterized small-scale gravity waves at rather coarse horizontal and vertical grid spacings of $\Delta x \approx 210$ km and $\Delta z = 1.25$ km in the stratosphere.

All of the aforementioned GCM studies except Yao and Jablonowski (2013) used model setups with realistic topography and comprehensive physical parameterization packages that include processes like moist convective parameterizations. In fact, it is widely believed that the primary source of tropical waves is latent heat release by either resolved-scale precipitation or parameterized deep cumulus convection. In general, it is difficult to isolate the main wave drivers in such GCM experiments, since all components of the model, such as the dynamical core and the individual physics processes, interact with each other. Yao and Jablonowski (2013) have presented a QBO-like simulation with a dry dynamical core that was driven by a Newtonian temperature relaxation and Rayleigh friction on a flat Earth, following Held and Suarez (1994). The current paper extends their analysis and systematically assesses the wave generation, propagation, dissipation, and forcing characteristics of stratospheric circulations in a variety of dynamical cores that are options in the Community Atmosphere Model (CAM), version 5 (Neale et al. 2010). CAMS has been jointly developed by the National Center for Atmospheric Research (NCAR) and various U.S. Department of Energy laboratories. The purpose of this paper is to examine the intrinsic ability of CAM’s four dynamical cores to simulate QBO-like oscillations when driven by the Held and Suarez (1994) forcing. As mentioned above, such an investigation sheds light on the role of the numerical schemes, their dissipation mechanisms,
and resolved wave dynamics in the simulations. Of course, we do not expect to fully capture the observed wave spectrum in such idealized model experiments, as convectively generated waves are certainly an important characteristic of the atmosphere (Kiladis et al. 2009). Rather, the investigation is a process study that reveals the sensitive link between QBO-like oscillations and the dissipation in dynamical cores that has not been documented before.

The paper is structured as follows. Section 2 provides brief descriptions of the CAM5 dynamical cores and describes the simulation setup. Section 3 documents the circulation results and characteristics of the stratospheric flows. Section 4 evaluates the wave forcing using the transformed Eulerian-mean (TEM) analysis and, furthermore, sheds light on the Brewer–Dobson circulations and diffusion properties of the dynamical cores. Section 5 examines potential wave generation mechanisms and instability indicators, while section 6 uses wavenumber–frequency analysis to analyze resolved equatorial waves. The summary and outlook are provided in section 7.

2. Model descriptions and experimental design

a. The CAM5 dynamical cores

We utilize the four CAM5 dynamical cores, which are the spectral transform semi-Lagrangian (SLD), finite-volume (FV), spectral transform Eulerian (EUL), and spectral element (SE) model. The latter dynamical core has become CAM’s new default in version 5.3 (June 2014). The numerical design choices are documented in Neale et al. (2010). Here, we only characterize them very briefly. All models are run with identical 55 vertical levels, with a model top at 0.1 hPa using a hybrid $\sigma-p$ (also known as $\eta \in [0, 1]$) vertical coordinate with variable vertical grid spacing (see also the appendix). It ranges from $\Delta \sigma \approx 0.2$ km near the surface and increases with height in the troposphere. We select a constant $\Delta \sigma = 1.25$ km in the stratosphere between 100 and 3 hPa, and slowly stretch (5% per level) the spacing above 3 hPa to reach $=2$ km at the model top. All model levels above 112 hPa are pure pressure levels, which avoids vertical interpolations in the TEM analysis along isobars. The horizontal grid spacing for all dynamical cores is about $2^\circ \times 2^\circ$ or 210–220 km in equatorial regions. The model configurations, including their dynamics time steps, dominant diffusion mechanisms, and diffusion coefficients are documented in Table 1. The associated physics time steps for the Held–Suarez forcing are 2700 s (SLD and SE), 3600 s (FV), and 720 s (EUL). All dynamical cores are built upon a hydrostatic and shallow-atmosphere equation set (the primitive equations).

The SLD dynamical core (dycore) is a two-time-level, semi-implicit, semi-Lagrangian spectral transform model. It utilizes a quadratic Gaussian transform grid with 192 × 96 grid points, which translate to a horizontal grid spacing $\sim 1.875^\circ$. The triangular truncation is T63. The SLD dycore is used without explicitly applied horizontal diffusion. Its implicit numerical diffusion due to the semi-Lagrangian interpolations provides enough dissipation to avoid a buildup of kinetic energy near the grid scale. The damping effect of the cubic SLD interpolations mimics fourth-order horizontal hyperdiffusion (McCalpin 1988).

The FV dycore uses a gridpoint-based finite-volume discretization with an explicit time stepping scheme, and it utilizes a regular $2^\circ \times 2^\circ$ latitude–longitude grid. It is built upon a 2D shallow-water approach in the horizontal (Lin and Rood 1996) and applies a vertical remapping approach to represent the vertical transport (Lin 2004). The vertical remapping algorithm conserves the total energy in the vertical column and is applied every 10 dynamics time steps. FV’s primary diffusion mechanisms are implicit numerical diffusion via limiters and explicitly applied second-order horizontal divergence damping that is explained in Whitehead et al. (2011) and Jablonowski and Williamson (2011).

The EUL dycore is a three-time-level, semi-implicit Eulerian spectral transform model in vorticity-divergence form, and, as SLD, it utilizes a T63 quadratic Gaussian transform grid with 192 × 96 grid points. The EUL dycore is run with linear, explicitly applied fourth-order horizontal hyperdiffusion, which is needed for numerical stability. A leapfrog time stepping algorithm is used with a Robert–Asselin time filter coefficient of $\alpha = 0.06$ (Asselin 1972).

The SE dycore, also known as the High-Order Methods Modeling Environment (HOMME), is based on a continuous Galerkin spectral finite-element method and has been designed for fully unstructured quadrilateral meshes.
(Taylor and Fournier 2010; Dennis et al. 2012). In particular, the cubed-sphere grid is used here. SE employs an explicit Runge–Kutta time stepping scheme. It is run at an ne16p4 horizontal resolution, which is approximately equivalent to a $1.875^\circ \times 1.875^\circ$ grid (see also Table 1 for further explanations). As with EUL, the SE dycore uses a linear fourth-order horizontal hyperdiffusion mechanism with the identical diffusion coefficient.

Neither a total energy fixer, nor explicitly added vertical diffusion, is applied in any of the four dynamical cores. In the vertical direction, EUL, SLD, and SE use a centered finite-difference vertical discretization, whereas FV is built upon a floating Lagrangian approach with periodic vertical interpolations to a hybrid ($\eta$) reference grid. The latter method is characterized by low implicit numerical diffusion in the vertical direction. All models utilize the Lorenz-type vertical grid staggering. The horizontal grid staggering in EUL, SLD, and SE is the colocated Arakawa A-grid (Arakawa and Lamb 1977), whereas the Arakawa D-grid is used in FV. The appendix provides details about the placement of the 56 model interface levels for the chosen 55-level setup.

b. Experimental setup

The idealized CAM5 simulations utilize the Held–Suarez (HS) forcing (Held and Suarez 1994), which replaces the whole CAM5 physics package. No moisture, topography, or seasonal cycles are included. The HS forcing consists of a Newtonian relaxation of the temperature field toward an analytically prescribed equilibrium state, with isothermal stratosphere and Rayleigh damping of low-level winds below $\sim 700 \text{ hPa}$ ($\eta \approx 0.7$). These processes mimic the effects of radiation and boundary layer friction. Additionally, we apply Rayleigh friction

$$\frac{\partial u}{\partial t} = -K_r u$$

(1)

to the zonal wind field in a sponge layer above 1 hPa up to the model top at 0.1 hPa to dissipate upward-traveling waves in this region. Eight model levels lie in this sponge zone. Following Boville (1986), we select a pressure-dependent Rayleigh friction coefficient, which is given by $K_r = k_0 [1 + \tanh(z - z_0)/H_0]$. The damping coefficient $k_0$ is set to 1/3 day$^{-1}$, the log-pressure height is $z = h_0 \ln(p/p_0)$, $p$ symbolizes pressure, the reference pressure $p_0$ is set to 1000 hPa, and $h_0$ stands for a scale height of 7 km. In addition, the parameter for the model top $z_0$ is set to 61 km, and the scale factor $H_0$ is 7.7 km. This leads to the damping time scales 1/$K_r$ of about 41 days at 1 hPa and 2 days at the model top. The HS damping time scale of the Newtonian temperature relaxation is 40 days in the region above $\sim 700 \text{ hPa}$ ($\eta < 0.7$) and varies according to the HS specification, with latitude and pressure at low levels between 1 $\geq \eta \geq 0.7$. The same experimental setup was also used in Yao and Jablonowski (2013), who already highlighted some characteristics of the QBO-like oscillations in the SLD model.

All experiments provide 30-day-mean data (called monthly mean herein) for 20–40 model years. One simulation year corresponds to 360 model days. In addition, 6-hourly instantaneous data are collected for at least 30 months. This high-frequency output is utilized to perform TEM, instability, and wavenumber–frequency analyses. We either use an observed (data assimilated) initial state that has been interpolated to zero topography, or spun-up initial states from an earlier Held–Suarez SLD model simulation with an established QBO-like stratospheric oscillation. These initial states already contain the QBO wind shear pattern, which helps establish QBO-like oscillations in the dynamical cores.

3. Intercomparison of the stratospheric circulations

We now compare the flow fields of the four CAM5 dynamical cores, with special focus on the stratosphere. Figure 1 shows the 20–40 yr time series of their monthly-mean zonal-mean zonal winds, which are averaged over $\pm 2^\circ$. The stratospheric circulations have very different characteristics, despite the identical experimental setups. The SLD dycore in Fig. 1a shows spontaneously generated QBO-like oscillations with an average period of about 3.6 yr, as already shown in Yao and Jablonowski (2013). The oscillation occupies the region between 1 and 50 hPa. Note that the QBO-like oscillations are not triggered or impacted by a semiannual oscillation in the upper stratosphere, since no annual cycle is included in the idealized HS forcing.

The FV dycore (Fig. 1b) cannot maintain the QBO oscillation that was present in the initial data file. The initial wind shear is diffused, and from year 6 onward, the vertical domain is occupied by two steady jets. A westerly jet lies between 3 and 30 hPa with a maximum zonal wind velocity of $\sim 10 \text{ m s}^{-1}$. An easterly jet is located below 30 hPa, with a zonal wind peak from approximately $-15$ to $-20 \text{ m s}^{-1}$.

The EUL dycore (Fig. 1c) shows zonal wind oscillations with a very long period, which seems to be greater than 13 yr. The downward-propagation speed of the westerly wind regime is not as regular as the descent in the SLD simulation. The westerly wind regime slowly travels downward between 0.3 and 7 hPa and then, relatively speaking, quickly descends to around 20 hPa. The newly formed easterly regimes near the model top start descending relatively quickly between 0.5 and 7 hPa before their descents slow considerably. This easterly branch propagates down to 10 hPa. It is unable to fully
break through the westerly regime located between 10 and 20 hPa and cannot connect to the easterly jet below 20 hPa. Most likely, the easterly wave forcing is insufficient. As a consequence, the magnitude of the westerly wind oscillates in this lower QBO domain (10–20 hPa) but never turns easterly.

The SE dycore (Fig. 1d) has very similar characteristics as EUL, with a fast descending westerly regime between 7 and 20 hPa. The figure implies that SE might have a comparable or even longer QBO period than EUL. However, the 40-yr EUL and SE simulations are too short to determine the period with certainty. The resemblance between EUL and SE is striking and suggests that they might have comparable wave generation, propagation, and absorption mechanisms. The EUL and SE QBO-like patterns also resemble the QBO simulations by Horinouchi and Yoden (1998), Scaife et al. (2000, their Fig. 1, middle panel), and Scinocca et al. (2008, their Fig. 6f). These groups similarly observed that their easterly jets could not break through the westerly regimes in the lower QBO domain. In contrast to their studies, though, the QBO patterns in EUL and SE already stall higher up at around 30 hPa and fail to descend to the lower stratosphere (70–100 hPa). Scaife et al. (2002, their Fig. 13) showed that enhanced horizontal diffusion can cause the descending westerlies to stall early. In addition, Giorgetta et al. (2006) found that their QBO stalled at higher altitudes as soon as their vertical grid spacing was increased from 700 m to 1 km and 1.4 km. The fact that the QBO, and especially its
westerly branch, does not propagate deeply enough into the lower stratosphere is a common problem in GCM simulations and was also shown in, for example, Kawatani et al. (2011), Xue et al. (2012), and Evan et al. (2012). It suggests that the wave forcing is insufficient to bring the QBO down to the tropical tropopause (100 hPa), where the largest atmospheric densities occur in the QBO region. Other aspects, like the strength of the updrafts in the Brewer–Dobson circulation, that oppose the QBO descent, also play a role and are discussed further in section 4b.

We note that the positions of the low-lying equatorial easterly jets in Fig. 1 are very different in EUL, SE, and FV in comparison to SLD, despite the identical HS forcing. The easterly jets in EUL, SE, and FV reach their peak magnitude at around 40–50 hPa, whereas the SLD jet lies lower, at around 70–80 hPa. Since easterly waves get filtered in easterly jets, this difference is likely to contribute to the differences in the QBO-like oscillations higher up. In general, equatorial shear zones are a consequence of vertical wave propagation, as explained in Dunkerton (2000). Once formed, the shear zones concentrate the breaking or absorption of a broad spectrum of waves into a shallow layer, which strengthens the shear and accelerates the flow field.

Overall, all QBO-like patterns in the HS experiments (SLD, EUL, and SE) lie too high in the stratosphere in comparison to observations [1–100 hPa; e.g., see Naujokat (1986)] or reanalysis data (Pawson and Fiorino 1998; Pascoe et al. 2005). In addition, all QBO periods are too long, which is not surprising given the idealized HS setup without moist convection as a typical wave trigger. However, the minimum–maximum zonal wind peaks are rather realistic and range between −35 and 10–15 m s⁻¹. This phase asymmetry can arise internally because of nonlinear dynamic interactions between the QBO and its meridional circulation (Dunkerton 1991; Scaife et al. 2002). However, it might also indicate that the dynamical cores generate waves that are asymmetric between the westerly and easterly phase speeds, as in observations. There are also internal variations in the length and magnitude of each QBO cycle, as it is typical for the QBO. For simplicity, we refer to the QBO-like oscillations in the HS experiments as “QBOs” herein.

Figure 2 shows the 4.9-hPa time–latitude cross sections of the monthly-mean zonal-mean zonal wind for all dycores. This model level is closest to the maximum wind amplitude. The QBO in SLD, EUL, and SE is confined to the tropical region of ±35°, with a slightly too narrow latitudinal width of the westerly phase (±12°), when compared to observations (Pawson and Fiorino 1998; Giorgetta et al. 2006). The onset of the westerly phase occurs first at the equator, whereas the onset of the easterly phase is spread out more evenly over the equatorial region, as in observations. The strong easterly winds (u < −15 m s⁻¹) in the EUL and SE simulations are narrower than the strong easterlies in the SLD simulation. We also observe that the approximate duration of the westerlies only varies by a factor of two between SLD (2 yr) and EUL/SE (3.5–4 yr), whereas the duration of the easterlies is very different (1.5–2 vs. 11–16 yr). In the FV simulation, a westerly jet occupies the tropical region of ±10°, and two easterly jets are located in the subtropical/midlatitudinal region centered around ±30° (Fig. 2b).

Figure 3 shows the pressure–latitude cross sections of the monthly-mean zonal-mean zonal wind field for a single month. In the troposphere, two westerly jets of identical strengths appear in the midlatitudes in all four dynamical cores, as is typical for the HS test. The positions of the tropopause, as indicated by the overlaid blue contours, are also very similar. This suggests that the mean tropospheric circulations are comparable in all four models. They also compare well to other tropospheric HS assessments, such as Wan et al. (2008) or Jablonowski (1998). In the equatorial stratosphere, we observe vertically stacked westerlies and easterlies in the QBO simulations with SLD, EUL, and SE, surrounded by weak easterlies at higher latitudes. However, the stratospheric flow in FV (Fig. 3b) is very different and dominated by two strong easterly jets in midlatitudes. These jets are closely connected to warmer polar temperatures in the region between 100 and 3 hPa (not shown), when compared to the other dycores. The consequently enhanced temperature gradients between the relatively warm polar and cold equatorial stratosphere in the FV dycore cut off the westerly tropospheric jets at a low location in the stratosphere at around 50 hPa and cause the strong easterlies via the thermal wind relationship. This again exemplifies that the general circulation in the stratosphere reacts very differently to the identical HS forcing, which has not been documented for the stratosphere before.

Note that the presence of the stratospheric easterlies at higher latitudes greatly hinders upward-traveling midlatitudinal Rossby waves from entering the equatorial stratosphere so that their contributions to the QBOs in SLD, EUL, and SE are presumably small. However, nonstationary Rossby waves with easterly zonal phase speeds c < 0 m s⁻¹ (typical for low-wavenumber midlatitudinal Rossby modes) are capable of propagating upward through weak mean easterlies (π < 0 m s⁻¹) if the condition 0 < π − c < Uc is met. The quantity Uc denotes the Rossby critical velocity, as shown in Charney and Drazin (1961) or Holton (2004, his chapter 12.3). This opens up the possibility that laterally propagating
midlatitudinal Rossby waves might impact the QBO region above 30 hPa, where the subtropical zonal-mean monthly-mean easterlies (see Figs. 3a,c,d) are generally weak in SLD, EUL, and SE and lie around −5 to −7 m s$^{-1}$. Instantaneous snapshots of these subtropical zonal winds above 30 hPa exhibit even weaker easterlies (even close to zero) on an occasional basis (not shown). This can allow the intermittent intrusion of the midlatitudinal planetary waves into the tropical stratospheric regions near the westerly QBO branch.

4. The forcing of the QBO

a. Transformed Eulerian mean analysis

To understand the zonal wind forcing mechanisms in the dynamical cores, we utilize 6-hourly instantaneous data and apply a transformed Eulerian mean analysis. As an aside, 3-hourly instantaneous data lead to almost identical TEM results. The TEM analysis decomposes the zonal wind acceleration into four components, as shown in Eq. (2). These are the mean meridional advection and mean vertical advection by the residual circulation, the resolved wave–mean flow interaction expressed by the divergence of the Eliassen–Palm (E–P) flux vector, and the momentum budget residual $X$. The latter captures the forcing by numerical and explicitly added diffusion, as well as the impact of the HS temperature relaxation on the zonal wind. The TEM analysis is applied along constant pressure levels upward of 112 hPa, following Andrews et al. (1983). The zonally averaged zonal momentum equation is given by
FIG. 3. Pressure–latitude cross sections of the monthly-mean zonal-mean zonal wind for (a) SLD, (b) FV, (c) EUL, and (d) SE. A single month is depicted. The blue line indicates the position of the tropopause; the zero wind line is enhanced.
\[
\frac{\partial u}{\partial t} = \bar{u}' \left[ f - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\bar{u} \cos \phi) \right] - \bar{u} \frac{\partial \bar{u}}{\partial p} \]
\[
+ \frac{1}{a \cos \phi} \left[ \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (F_{\phi} \cos \phi) + \frac{\partial}{\partial p} F_p \right] + X. \tag{2}
\]

The overbar symbolizes the zonal mean, \(u\) indicates the zonal wind, \(t\) is time, \(p\) denotes pressure, \(f\) stands for the Coriolis parameter, \(a\) indicates Earth’s radius, \(\phi\) is the latitude, and \((F_{\phi}, F_p)\) expresses the components of the E–P flux vector in pressure coordinates. They are defined by

\[
F_{\phi} = a \cos \phi \left( \frac{\nu' \Theta'}{\partial \Theta / \partial p} - \bar{u}' \right) \quad \text{and} \quad \tag{3}
\]
\[
F_p = a \cos \phi \left( \frac{\nu' \Theta'}{\partial \Theta / \partial p} \left[ f - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} (\bar{u} \cos \phi) \right] - \bar{u}' \omega \right). \quad \tag{4}
\]

where \(\Theta\) is the potential temperature, \(\nu\) and \(\omega\) denote the meridional and vertical pressure velocities, respectively, and the prime stands for the perturbation from a zonal mean. The quantities \((\bar{u}', \bar{v}')\) are the mean meridional and vertical pressure velocities of the residual circulation, defined by

\[
\bar{u}' = \bar{u} - \frac{\partial}{\partial \phi} \left( \frac{\nu' \Theta'}{\partial \Theta / \partial p} \right) \quad \text{and} \quad \tag{5}
\]
\[
\bar{v}' = \bar{v} + \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\nu' \Theta'}{\partial \Theta / \partial p} \right). \quad \tag{6}
\]

Analogous to the 50-month TEM–SLD analysis in Yao and Jablonowski (2013), Fig. 4 shows the time–pressure cross sections of the TEM analysis for the EUL simulation. The analysis is based on 100 months of 6-hourly data from year 20 onward. This time period captures the first descent of the westerly QBO branch in the region 7–20 hPa and the initial descent of the easterly jet near 1 hPa, as seen in Fig. 1. It approximately covers more than half of the EUL–QBO cycle and is representative of the forcing signatures during a full period. Figure 4 shows the time series of the monthly-mean zonal-mean (Fig. 4a) zonal wind acceleration, (Fig. 4b) mean vertical advection, (Fig. 4c) E–P flux divergence, which depicts the resolved wave forcing, and (Fig. 4d) momentum budget residual \(X\) [see Eq. (2)]. The mean meridional advection by the residual circulation is not shown because of its small magnitudes between ±0.03 m s\(^{-1}\) day\(^{-1}\). All of these panels are overlaid by contours of the monthly-mean zonal wind, mean vertical advection, E–P flux divergence, and momentum budget residual \(X\). The plots are overlaid by the monthly-mean zonal wind (white contour) with contour spacing of 8 m s\(^{-1}\). Solid lines are positive; dashed lines are negative. In (b)–(d), only the enhanced zero wind contour is shown.
zonal-mean zonal wind (in white). During the easterly phase (upper QBO branch between ~1 and 7 hPa), the mean vertical advection (Fig. 4b) almost exclusively provides a westerly acceleration, whereas the E–P flux divergence (Fig. 4c) provides most of the easterly acceleration. In addition, the E–P flux divergence supplies a westerly forcing in the two westerly shear zones ($\bar{u}w/\partial z > 0$), especially around the zero wind lines where easterlies transition to westerlies. The E–P flux divergence is clearly the main driver of the QBO. The mean vertical advection mostly counteracts the E–P flux divergence, except near the zero wind lines in the westerly shear zones, which was also shown in the QBO–TEM analyses by Hamilton et al. (2001), Giorgetta et al. (2006), Kawatani et al. (2011), and Xue et al. (2012). To close the zonal momentum budget in EUL, a strong momentum budget residual term remains (Fig. 4d). It opposes most of the forcing by the resolved waves (Fig. 4c) with similar magnitudes. This momentum budget residual is of special interest and, therefore, is further analyzed in section 4c.

Overall, the TEM analysis for EUL shows similar results to the TEM–SLD analysis in Yao and Jablonowski (2013), which is not repeated here. An interesting observation is that the peak values of the monthly-mean zonal-mean E–P flux divergence, vertical advection, and the momentum budget residual term are about 50% higher in EUL than in SLD, suggesting stronger wave activities in EUL. However, the resulting zonal wind acceleration in EUL (Fig. 4a) is much weaker than the acceleration in SLD [Fig. 3a in Yao and Jablonowski (2013)], because of the very strong cancellation effect between the TEM components. The TEM analysis for SE shares very similar characteristics with the EUL simulation and is, therefore, not shown. As mentioned above, it suggests that EUL and SE have almost identical wave generation, propagation, and dissipation properties despite their very different numerical designs and computational grids.

The TEM analysis for the FV simulation (applied during the steady-state period) exhibits much weaker accelerations when compared to the EUL simulation and is, therefore, only briefly described here. The monthly-mean zonal-mean E–P flux divergence in FV has a maximum value of $-0.15 \text{ m s}^{-2} \text{ day}^{-1}$ around the level where the background wind speed is $0 \text{ m s}^{-1}$ (at about 30 hPa; see Fig. 1b). This position coincides with the position of the peak values of the monthly-mean zonal-mean momentum budget residual term $X$ (around $-0.15 \text{ m s}^{-2} \text{ day}^{-1}$). The mean vertical advection term has a range around $\pm 0.05 \text{ m s}^{-1} \text{ day}^{-1}$, and the mean meridional advection by the residual circulation is even lower. The resulting monthly-mean zonal-mean zonal wind acceleration in FV is very weak and just big enough to maintain the steady stratospheric jets. The TEM analyses suggest that the resolved wave activity in FV is significantly lower, in comparison to SLD, EUL and SE, which is a possible, but not necessarily the only, cause for the absence of the QBO oscillation in the idealized FV simulation shown here. The wave activity in all models is analyzed in-depth in section 6.

b. Residual circulation

As briefly mentioned in section 3, the descent of the QBO phases is opposed by the mean ascent in the equatorial stratosphere that is due to the Brewer–Dobson circulation (BDC). This was, for example, discussed by Dunkerton (1997), who argued that the vertical momentum fluxes of the observed upward-traveling Kelvin and MRG waves are insufficient to drive the observed QBO cycle and that small-scale gravity waves must contribute to the QBO forcing to sustain the speed of the QBO descent against the mean equatorial upwelling. The impact of the mean upwelling on the QBO period and its vertical extent was also clearly demonstrated by Kawatani et al. (2011, 2012). They showed that increased equatorial upwelling in climate change simulations contributes to a slowdown of the QBO descent and the stalling of the westerlies higher up in the QBO domain.

The ascent due to the Brewer–Dobson circulation can be assessed via the mean residual vertical pressure velocity $\bar{\sigma}_* [\text{Eq. (6)}]$. In particular, $\bar{\sigma}_*$ just above the tropical tropopause ($\sim 100 \text{hPa}$) can be used to assess the mass flux entering the stratosphere. In or near the QBO domain, the mean BDC ascent is furthermore modulated by the locally induced secondary meridional circulation (Choi et al. 2002; Flury et al. 2013). The latter is responsible for maintaining the thermal wind balance between the QBO signals in the temperature and zonal wind fields. In easterly shear zones ($\bar{u}w/\partial z < 0$), the secondary meridional circulation triggers equatorial upwelling ($\bar{\sigma} < 0$) and therefore opposes the descent of the QBO. In westerly shear zones ($\bar{u}w/\partial z > 0$), the secondary meridional circulation triggers equatorial downdrafts ($\bar{\sigma} > 0$), thereby reducing the BDC opposition to the descending QBO.

As an example, Fig. 5 shows the close correspondence between the equatorial zonal wind (in blue, left axis) and the vertical pressure velocity (in red, right axis) in the lower QBO domain at 27 hPa in SLD. This location lies slightly below or near SLD’s stalling westerly QBO branches. Therefore, the monthly-mean zonal-mean zonal wind oscillates between $\sim 0$ and about $-16 \text{ m s}^{-1}$ over the depicted 20-yr time series. The corresponding vertical wind shear is mostly westerly, as seen in Fig. 1a, except during the periods when the easterly QBO branches break through and connect to the steady easterly jet below 50 hPa. These periods exhibit brief
and relatively weak easterly vertical wind shears. Figure 5 demonstrates that the vertical pressure velocity exactly follows the zonal wind and its vertical wind shear pattern, as also shown in Flury et al. (2013). The westerly wind shear phases (before the zonal wind dips down to its minimum) are characterized by strong downdrafts ($\omega > 0$), whereas the weak easterly wind shear phases let the vertical velocity tend toward zero or mild updrafts. However, the vertical velocity reversal from downdrafts to updrafts is not present in all QBO cycles at 27 hPa. Locations higher up show the reversal more clearly, but we picked 27 hPa in order to reuse this location for the BDC intercomparison in Fig. 6.

Figure 6 depicts the area-weighted zonal-mean residual velocities $\omega^* \cos \phi$ of all four dynamical cores, which can be interpreted as proxies for the strength of the overturning meridional mass circulation. The figure shows the latitudinal profiles of 30-month means near the equatorial tropopause at 94 hPa and in (or slightly below) the QBO domains at 27 hPa. Since negative $\omega^*$ indicates upward motion, the y axis is reversed to allow for an easier interpretation. The 30-month averaging periods cover the years 7.1–9.6 (SLD), 10–12.5 (FV), 19.75–22.25 (EUL), and 12.25–14.75 (SE). At 94 hPa below the QBO domains, the effect of the secondary meridional circulation is negligible, and all models show the mean ascent of the BDC at the equator. The magnitudes of $\omega^* \cos \phi$ in SLD, EUL, and SE are very similar. However, at 27 hPa the averaging periods are dominated by westerly shear zones that are related to the QBO (Fig. 1). This triggers downdrafts that overlay the mean ascent of the BDC at the equator. In SLD, the strong descent of the secondary meridional circulation even reverses the direction of the equatorial BDC, whereas it only creates local minima of the tropical BDC ascent in EUL and SE (Fig. 6b). It means that the downward propagation of the QBO in EUL and SE is hindered by the mean upwelling of the BDC, whereas SLD’s downward-propagating QBO is sped up, since the BDC reverses its sign at the equator. This reversal is unrealistic in comparison to observations. The mean upwelling in EUL and SE is a contributing factor to the very long QBO periods in comparison to SLD.

We note that the peak equatorial amplitudes of the mean residual vertical pressure velocity $\omega^*$ at 94 and 27 hPa in Fig. 6 translate to equatorial velocities of about $w^* \approx 0.27$ and $\pm 0.08$ mm s$^{-1}$, respectively. These estimates utilize the approximate relationship $\omega^* \approx -H \omega^* p^{-1}$ with an assumed stratospheric-scale height.
of \( H = 5.85 \text{ km} \) that corresponds to the HS equilibrium temperature of 200 K in the stratosphere. Estimates of the equatorial \( \bar{w} \) near 70 hPa range from 0.1 to 0.4 mm s\(^{-1} \) in observations and ERA-Interim (Rosenlof 1995; Schoeberl et al. 2008; Seviour et al. 2012; Osprey et al. 2013). Evan et al. (2012) showed vertical profiles of the tropical (10\(^{\circ}\)S–10\(^{\circ}\)N) annual-mean ERA-Interim \( \bar{w} \) that are 1.3 mm s\(^{-1} \) at 100 hPa (16 km), 0.2–0.3 mm s\(^{-1} \) at 70 hPa (18.5 km), and 0.2 mm s\(^{-1} \) at 30 hPa (24 km). The QBO simulations by Giorgetta et al. (2006) gave tropical (5\(^{\circ}\)S–5\(^{\circ}\)N) annual-mean \( \bar{w} \) values around 0.8 and 0.2 mm s\(^{-1} \) at 100 and 30 hPa, respectively. A recent version of their model led to 0.6 and 0.3–0.35 mm s\(^{-1} \) at these two locations (Krismer and Giorgetta 2014). Scaife et al. (2002) reported on similar monthly-mean 32-hPa \( \bar{w} \) values between 0.2 and 0.4 mm s\(^{-1} \) for the region 20\(^{\circ}\)S–20\(^{\circ}\)N. Furthermore, Kawatani et al. (2010) found simulated tropical (10\(^{\circ}\)S–10\(^{\circ}\)N) annual-mean \( \bar{w} \) values of 0.55 mm s\(^{-1} \) (100 hPa) and 0.18 mm s\(^{-1} \) (30 hPa) and judged that they were on the low side. Despite the observational uncertainties, it seems that the residual vertical velocities in our idealized QBO experiments are weaker than observed and are, therefore, not the primary reason for the longer-than-observed QBO periods in SLD, EUL, and SE.

**c. Diffusion and the TEM residual term**

All dynamical cores employ dissipation mechanisms, either implicitly in the numerical schemes or explicitly via, for example, horizontal hyperdiffusion. The diffusion is needed to avoid numerical noise, like Gibbs ringing in spectral models, and to keep the model computationally stable. The choice of the diffusion coefficient is often empirical and can even be considered a model tuning parameter in order to yield a reasonably straight tail in the kinetic energy (KE) spectra in the upper troposphere. In spectral transform models, like EUL with linear fourth-order horizontal hyperdiffusion, the e-folding damping time scale \( \tau \) of the diffusion is connected to the \( K_4 \) diffusion coefficient and the maximum resolved wavenumber \( n_0 \) (smallest wavelength) via

\[
\tau = \frac{1}{K_4} \left[ \frac{a^2}{n_0(n_0 + 1)} \right]^2. \tag{7}
\]

In gridpoint models like SE with quasi-uniform horizontal grid spacings \( \Delta x = \Delta y \), this relationship is given by

\[
\tau = \frac{(\Delta x)^4}{32K_4}, \tag{8}
\]

as documented in Jablonowski and Williamson (2011, their chapter 13.3.4). The selected EUL and SE coefficient of \( K_4 = 5 \times 10^{15} \text{ m}^4 \text{ s}^{-1} \) at the given resolutions \( n_0 = 63 \) (EUL) and \( \Delta x = 208 \text{ km} \) (SE) leads to damping time scales for the smallest waves of 5.6 and 2.8 h, respectively. A similar damping time scale of about 7 h for the spectral T63 resolution was also chosen by Boville (1991) and Krismer (2014). However, Krismer and Giorgi (2014) and Krismer (2014) applied an eighth-order hyperdiffusion, which is more scale selective.

As pointed out by Scaife et al. (2002), diffusion is even able to directly influence the large-scale QBO characteristics if the period of each QBO oscillation branch exceeds the diffusion time scale. This is especially true for the relatively narrow westerly phase of the QBO that only has a maximum meridional width of about 24\(^{\circ}\), as shown in Fig. 2. As reviewed in Jablonowski and Williamson (2011), a spectral transform model with horizontal fourth-order hyperdiffusion damps a feature with a size of approximately wavenumber \( n \), with the scale-dependent time scale of

\[
\tau_n = \frac{1}{K_4} \left[ \frac{a^2}{n(n+1)} \right]^2. \tag{9}
\]

Using the estimate that SLD’s implicit numerical diffusion acts like a horizontal fourth-order hyperdiffusion scheme with an assumed coefficient of about \( K_4 = 5 \times 10^{15} \text{ m}^4 \text{ s}^{-1} \) (Yao and Jablonowski 2013), this leads to the e-folding diffusion time scale \( \tau_n \) of about 66 days for wavenumber \( n = 360/24^\circ = 15 \). However, the actual strength of SLD’s nonlinear implicit numerical diffusion varies. Therefore, this diffusion time scale needs to be interpreted as a rough estimate and might be somewhat short. Nevertheless, as shown in Fig. 1, the period of each SLD QBO branch is about 22 months, which is far longer than \( \tau_{15} \). The same argument applies to the EUL dycore with \( K_4 = 5 \times 10^{15} \text{ m}^4 \text{ s}^{-1} \). In addition, the assessment for the fourth-order hyperdiffusion in the gridpoint model SE is analogous to Scaife et al. (2002) and given by

\[
\tau_L = \frac{1}{K_4} \left( \frac{L_y}{2\pi} \right)^4, \tag{10}
\]

where \( L_y \) symbolizes the meridional width of the atmospheric feature. For \( L_y = \pi \times 24^\circ / 180^\circ = 2669 \text{ km} \) and the identical \( K_4 \), the SE diffusion time scale is 75 days. Therefore, as also stated in Scaife et al. (2002), it is likely that horizontal diffusion directly damps the QBO winds by a significant amount, especially during the westerly phase toward the end of its descent, when it is narrowest. Support for this argument is presented by Yao (2014), who showed that an increase in the horizontal diffusion
The strength of the diffusion and therefore the diffusion coefficient greatly influences the steepness of the kinetic energy decay as demonstrated in Fig. 7. The figure intercompares the 6-month-mean 250-hPa KE spectra of the four dycores. The analysis is based on 6-hourly instantaneous data. The EUL, SE, and FV kinetic energy spectra decay considerably faster with increasing spherical wavenumber $n$ than the SLD spectrum. More specifically, the EUL and SE kinetic energy levels drop off faster from wavenumber $n=20–25$ onward, and SE damps the smallest scales the greatest, which is connected to its short damping time scale. The results suggest that SLD is less diffusive for intermediate- and small-scale waves with wavelengths below 1600 km ($n = 25$), in comparison to EUL and SE. It is interesting that FV shows reduced tropospheric KE levels at even larger scales (e.g., in the presumably resolved wavenumber range 2–20). This finding could be connected to the relatively low resolved wave activity in FV’s equatorial stratosphere, which is discussed later in section 6.

As mentioned in section 4a, the TEM residual $X$ captures the zonal wind forcing by nonlinear implicit and linear explicit diffusion, as well as the effects of the thermal HS damping, and plays an important role in the zonal momentum balance. It is computed via Eq. (2) after all terms except $X$ are assessed based on 6-hourly datasets. In Yao and Jablonowski (2013), we compared the momentum budget residual $X$ with SLD’s implicit numerical diffusion. As mentioned before, the latter was approximated via linear fourth-order horizontal hyperdiffusion (McCalpin 1988) and an estimated diffusion coefficient of $K_4 = 5 \times 10^{13} \text{m}^4 \text{s}^{-1}$. This coefficient is the same as was used in EUL and SE. Figure 8 overlays the forcing by the fourth-order horizontal hyperdiffusion (colored) and the TEM residual $X$ (contoured) for (Fig. 8a) a 50-month SLD period from year 7 onward [reproduced from Yao and Jablonowski (2013)] and (Fig. 8b) a 100-month EUL period from year 20 onward. In both plots, the hyperdiffusion tendencies have very similar patterns to the TEM residual. In addition, even the forcing magnitudes match rather well so that the damping via the thermal relaxation and other (e.g., nonlinear) effects might be of secondary importance. The analogous analysis for SE resembles the EUL results very closely (not shown). The diffusion mostly explains the momentum budget residual $X$ and counteracts the resolved wave forcing.

The results do not imply that diffusion must be avoided in QBO simulations. While it is acknowledged that too much explicitly added diffusion degrades QBO simulations (Takahashi 1996; Scaife et al. 2002), some dissipation is needed to diffuse the strong QBO shear zones, enable their descent, and provide a switching mechanism for the alternating QBO phases. The dissipation might be comprised of horizontal or vertical diffusion, or even both. Furthermore, thermal damping can help degrade the QBO shear zones and might mimic the effects of vertical diffusion, as stated in Haynes (1998). The role of diffusion for the QBO was also discussed in the mechanistic studies by Plumb (1977) and Plumb and Bell (1982) and will be further analyzed in a companion paper. As Plumb and Bell (1982) state, “the effects of viscosity are clearly of paramount importance at the equator in counteracting the wave driving and thus limiting the jet magnitudes.”

In our experiments, diffusion serves as a crucial QBO forcing mechanism. In fact, it is the delicate balance between the wave forcing and diffusion that determines the faith of the QBO in the HS experiments. As an aside, the role of explicit diffusion in GCM simulations was also analyzed by Hamilton et al. (2001), who showed that diffusive tendencies in the QBO domain can reach considerable magnitudes. However, Krismer (2014, chapter 3) states that diffusion is two orders of magnitude smaller than other terms in the QBO–TEM balance (in particular, compared to their parameterized wave drag) and is therefore negligible. We hypothesize that this apparent controversy is likely connected to the use (Krismer 2014) or nonuse (Hamilton et al. 2001; Yao and Jablonowski 2013) of...
GWD parameterization schemes. Most often, the zonal wind forcing by parameterized gravity waves becomes so dominant that other processes could be overshadowed or suppressed. Our idealized HS studies can reveal such sensitivities without judging what the more realistic scenario in nature is. Unfortunately, most QBO–TEM studies in the literature do not discuss their TEM residual $X$ term and therefore leave the budget open. In a companion study, we will report on HS simulations with an added GWD scheme and prescribed gravity wave sources (Lindzen 1981) and reevaluate the composition of the zonal momentum balance. The results highlight that QBOs in GCMs can be driven by very different forcing mechanisms. Our process study helps disentangle the composition of the forcing and sheds light on causes and effects.

![Time–pressure cross sections of the monthly-mean zonal-mean acceleration by fourth-order horizontal hyperdiffusion (color), with $K_4 = 5 \times 10^{15}$ m$^4$ s$^{-2}$ (averaged over $\pm 2^\circ$) for (a) SLD (50 months) and (b) EUL (100 months). The contours of the TEM residual $X$ are overlaid with contour spacing of 0.1 m s$^{-2}$ day$^{-1}$. Negative contours are dashed. The zero contour of the monthly-mean zonal-mean zonal wind is overlaid in white.](image)
5. Wave generation and instability indicators

Resolved waves and their wave–mean flow interactions are important drivers of the QBO in the idealized HS experiments. In nature, a broad spectrum of planetary-scale waves and small-scale gravity waves are generated via many different mechanisms, such as topography, land–sea temperature contrasts, convection, wind shear, dynamic instabilities, geostrophic adjustments of unbalanced flows in the neighborhood of jet streams and frontal zones, and nonlinear wave–wave interactions [see, e.g., the review by Fritts and Alexander (2003) or Plougonven et al. (2003)]. The term wave–wave interactions refers to wave superpositions that have been observed to be a source of middle-atmosphere gravity waves. Free-traveling planetary (Rossby) modes or global normal modes are also present in the middle atmosphere that might potentially be linked to barotropic and baroclinic instability, as suggested by Andrews et al. (1987, chapters 4.4 and 5.5). Furthermore, midlatitudinal Rossby waves have been observed to trigger equatorial MRG and Kelvin waves (Magaña and Yanai 1995; Straub and Kiladis 2003). A comprehensive analysis of all wave generation mechanisms and how they might relate to the QBOs in the SLD, EUL, and SE dynamical cores is beyond the scope of this paper. This topic is an active research area and is not fully understood. However, we can narrow down the potential wave generation mechanisms by noting that topography, land–sea temperature contrasts, and convection can be ruled out because of the HS experimental setup. In particular, we now explore whether dynamic instability indicators show systematic differences in the dynamical cores.

Instabilities can serve two purposes. They can either act as a source for waves, or, alternatively, they can trigger local turbulence and mixing, thereby acting as a wave-dissipation mechanism and a preventer of very large vertical wind shears. We explore both routes. In particular, we assess the following instability indicators that are necessary, but not sufficient conditions for the occurrence of barotropic, inertial, baroclinic, static, and Kelvin–Helmholtz (KH) instabilities. These are

\[ f(f + \zeta) < 0 \]  
\[ \beta^* = \frac{\partial(f + \zeta)}{\partial \phi} < 0 \]  
\[ \frac{\partial PV}{\partial \phi} < 0 \]  
\[ \text{Ri} = \frac{N^2}{(\partial u/\partial z)^2} < 0 \]

where \( \zeta \) symbolizes the relative vorticity, PV stands for Ertel’s potential vorticity, \( \beta^* \) is defined as the meridional gradient of the absolute vorticity, and Ri is the definition of the Richardson number that relates the squared Brunt–Väisälä frequency \( N^2 \) and the vertical shear \( \partial u/\partial z \) of the zonal wind. The use of \( \zeta \) in Eqs. (11) and (12) instead of the often-used meridional gradient of the zonal-mean zonal flow \( \pi_x \) captures zonal asymmetries, as suggested by O’Sullivan and Hitchman (1992). The Richardson number serves two purposes. A negative Ri number indicates static instability, which triggers overturning circulations. In case of \( \text{Ri} < 0.25 \) the criterion for the KH shear instability is fulfilled.

Inertial instability is a parcel instability that occurs frequently in equatorial regions where the meridional shear of the zonal wind is strong (Andrews 1987). In case of inertial instability, meridional circulations arise in order to redistribute the angular momentum imbalance. Baroclinic instability is most dominant in the mid- and high latitudes. It fosters the development of extratropical waves, which have been observed to trigger Kelvin waves in the equatorial region (Straub and Kiladis 2003; Yang et al. 2007). Furthermore, the sign reversal of the meridional PV gradient [the Charney and Stern (1962) stability criterion] is also present in tropical latitudes and indicates the possible presence of baroclinic and/or barotropic instability as a local wave trigger. We also assess the condition for barotropic instability independently via \( \beta^* \) that is of central importance in the tropics. The necessary condition is that \( \beta^* \) must switch sign.

a. Instabilities in the upper troposphere

It is widely known that upward-traveling waves with tropospheric origins can transport energy and momentum into the tropical middle atmosphere. As suggested in Nissen et al. (2000), dynamic instabilities might serve as a trigger for such tropical waves. In particular, Nissen et al. (2000) linked an increased number of PV gradient sign reversals in the upper equatorial troposphere to an increased Kelvin wave activity in their model. To check for such occurrences Fig. 9 depicts selected 30-day Hovmöller diagrams in the upper atmosphere (at 300 hPa near the equator) for all instability indicators. The shaded areas are negative and indicate where the instability criterion is satisfied. The KH criterion \( \text{Ri} < 0.25 \) is marked in red in the rightmost column.

The figure shows that the conditions for KH and static instabilities are rarely satisfied at 300 hPa. However, this does not generally exclude KH instability as a possible wave generation process in the tropics, since the excitation of large-scale gravity waves has been documented for unstable shear layers in zonal jets (Fritts 1982, 1984). The persistent easterly jets above the tropical tropopause
(between 80 and 40 hPa; Fig. 3) could therefore be candidates for KH instabilities, which will be investigated further in the future. All other instability indicators at 300 hPa show intermittent rather organized patterns that mostly travel westward. The barotropic and baroclinic instability indicators are most often fulfilled by the SLD dynamical core. The shaded areas of the inertial instability indicators seem to be comparable in all four dycores. The density of the shaded areas for all five stability indicators in FV, EUL, and SE does not reveal any systematic differences. However, the magnitudes of the FV indicators are generally a lot lower than the magnitudes of the indicators in SLD, EUL, and FV (not shown, but see a second example for inertial instability in Fig. 10, where magnitudes differ by a factor of 10). The higher instability indicator magnitudes in SLD, EUL, and SE lead most likely to a more effective triggering of instabilities and tropospheric waves.

b. Instabilities in the stratosphere

While tropospheric instabilities might serve as wave triggers, stratospheric instabilities are most likely connected to upward-propagating breaking waves, which trigger turbulence and mixing (Dunkerton 1981; Fritts and Rastogi 1985; Hayashi et al. 2002). Gravity wave drag schemes are often built upon this principle, such as the scheme by Lindzen (1981). It dissipates parameterized gravity waves once the conditions for convective overturning (static instability) or Kelvin–Helmholtz instability are fulfilled according to a “saturation” theory. Inertial instability in the upper atmosphere occurs in the form of a “pancake structure” that appears in the divergence of the meridional wind field. As discussed by O’Sullivan and Hitchman (1992), if a parcel in a rotating field is displaced by a meridional distance $\delta y$ it accelerates at the rate...
FIG. 10. Latitude–pressure cross sections of the zonal-mean divergence of the meridional wind for an instantaneous data snapshot for (a) SLD, (b) FV, (c) EUL, and (d) SE.
\[ \frac{d^2 \delta y}{dt^2} = -f(f + \xi)\delta y \]  \hspace{1cm} (15)

if \( f(f + \xi) < 0 \). Since the acceleration is in the direction of \( \delta y \), local divergence results. Once the parcel passes its neutral state, deceleration leads to local convergence. The divergence and convergence zones alternate in the horizontal and are vertically stacked (the pancake). The disturbance growth rate is largest at small vertical scales for which size depends on the diffusive properties of the model (Dunkerton 1981). Hayashi et al. (2002) showed that pancake structures occasionally occur in observations near the stratopause and in the mesosphere, where they last for about two weeks. In our dynamical core simulations we observe rather persistent pancake structures in the stratosphere near the QBO domains.

Figure 10 shows the zonal-mean divergence \( [(a \cos \phi)^{-1} \partial(\theta \cos \phi) / \partial \phi] \) of the meridional wind for each dycore, using a single instantaneous data snapshot. In SLD, the pancake structure is present upward of about 50 hPa and reaches a peak magnitude of \( 2 \times 10^{-6} \) s\(^{-1} \). The occupied latitudinal region expands with increasing height and extends to 30°N/S near the model top. This meridional width is typical and was also found by Hitchman et al. (1987) and O’Sullivan and Hitchman (1992). In the FY dycore (Fig. 10b), the divergence pattern is an order of magnitude weaker and barely visible, suggesting that mixing processes via inertial instability might not be important in FY’s stratosphere. In EUL and SE (Figs. 10c,d), the peak values are around \( 5 \times 10^{-6} \) s\(^{-1} \), and the structure expands slightly beyond 30°N/S. The pancake patterns in all models propagate downward over time, which was also observed by Hitchman et al. (1987). Hitchman et al. (1987) reported on correlations between the tropical pancake structure and midlatitudinal Rossby waves. However, in our simulations it is difficult for midlatitudinal Rossby waves to propagate into the upper equatorial region because of the persistent easterlies that surround the QBO domain (see Fig. 3). We therefore suggest that the inertial instability patterns in Fig. 10 could be connected to resolved-scale breaking gravity waves in the tropics with upward-oriented energy transports (upward group speeds), for which the corresponding phase speeds point downward. Support for this link between the pancake pairs and gravity waves comes from the fact that the pancake patterns above 30 hPa have approximate vertical extents between 4 and 6 km. This leads to an estimate of the equatorial zonal phase speeds \( c_z \) of the gravity waves via the relationship \( c_z = Nm^{-1} \), where \( N = 0.022 \) s\(^{-1} \) is the stratospheric Brunt–Väisälä frequency in the HS simulations, \( m = 2\pi L_z^{-1} \) symbolizes the vertical wavenumber, and \( L_z \) is the vertical wavelength. This approximation of the zonal gravity wave speed neglects the Coriolis parameter, which is justified close to the equator, and also assumes that the meridional wavenumber at the equator is about zero. Using the vertical extents \( L_z = 4\)–6 km of the pancake pairs leads to zonal gravity wave phase speeds along the equator between 14 and 21 m s\(^{-1} \). This range of phase speeds is similar to the modeled equatorial zonal speeds of the pancake pairs in the simulations (not shown).

In addition to inertial instability, barotropic instability is also commonly observed in the QBO domain because of the rather narrow westerly equatorial jets (Hamilton 1984). The instability happens persistently around \( \pm 15 \) in observations and lasts for multiple months, which was, for example, shown by Shuckburgh et al. (2001). They demonstrated that mixing in the tropics and subtropics is a consequence of both breaking planetary waves and barotropic instability. Shuckburgh et al. (2001) also showed that negative \( \beta^* \) regions were correlated with increasing E–P flux divergence and occurred predominantly in easterly QBO regions that flank the westerly jet. This pattern was also reported by Hamilton et al. (2001) for their QBO simulations.

Figure 11 depicts instantaneous latitude–pressure snapshots of negative \( \beta^* \) (striped), which overlay the zonal wind (colored) at a selected longitude. As also seen by Hamilton et al. (2001) and Shuckburgh et al. (2001), the regions with negative \( \beta^* \) in Fig. 11 are mostly located in the easterly wind zone and flank the westerly jet in all four dycores. FY shows the fewest occurrences of negative \( \beta^* \), which is also obvious at other times (not shown). The characteristics of the negative \( \beta^* \) distributions in EUL and SE are similar and seem to occupy wider areas in the stratosphere in comparison to SLD. As an aside and related to the earlier tropospheric discussion, Fig. 11 also shows intermittent negative \( \beta^* \) throughout the tropical troposphere, where barotropic instability could serve as a source of tropical waves.

6. Wave analysis

To compare the resolved wave activity in the four dynamical cores, a wavenumber–frequency analysis is performed following the analysis method by Wheeler and Kiladis (1999). The analysis uses 6-hourly instantaneous temperature data for 30 months, from 15°N to 15°S at \( \sim 22 \) hPa. The wavenumber–frequency spectra are based on many successive overlapping (by 30 days) 96-day data segments that are averaged over the 30 months. The corresponding simulation years for each dycore are the same as listed in section 4b.

Yang et al. (2011) showed how the wavenumber–frequency spectra are impacted by the Doppler shift in
the presence of different zonal background velocities. From linear shallow-water theory, the background wind modifies the dispersion curves according to $\omega = \omega_0 + \pi k$, where $\omega$ and $\omega_0$ are the wave frequencies with and without the Doppler shift, $\pi$ denotes the zonal background wind, and $k$ stands for the zonal wavenumber. In the wavenumber–frequency analyses (Fig. 12), we overlay the dispersion curves with background wind speed $\pi = 0 \text{ m s}^{-1}$ (solid lines) and the Doppler-shifted dispersion curves with background wind speed $\pi = -7 \text{ m s}^{-1}$ (dashed lines) for the equivalent depths $h$ of 12, 50, and 200 m. These equivalent depths correspond to the zonal wave speeds $c$ of about 11, 22, and 44 m s$^{-1}$ with gravity $g$ and approximate vertical wavelengths $L_z$ of 2.8, 6.2, and 12.6 km, which we assess via the squared vertical wavenumber equation $m^2 = N^2/gh - 1/4H$. These linear estimates take into account that the squared stratospheric Brunt–Väisälä frequency $N^2$ in the tropics and stratospheric-scale height $H$ are given by $N^2 \approx 5 \times 10^{-4} \text{ s}^{-2}$ and $H \approx 5.85 \text{ km}$ in the idealized simulations. This range of phase speeds and vertical wavenumbers encapsulates the typical range of observed equatorially trapped waves in the stratosphere like Kevin and MRG waves. Therefore, our analysis sheds light on their presence in idealized simulations that omit important wave generation mechanisms, like moist convection.

Figure 12 shows the log$_{10}$ of the raw power spectrum of the temperature wavenumber–frequency analyses for (Fig. 12a,e) SLD, (Fig. 12b,f) FV, (Fig. 12c,g) EUL, and (Fig. 12d,h) FV. The first row is the antisymmetric component, and the second row is the symmetric component. In the antisymmetric plots, we depict the dispersion curves for MRG, $n = 0$ equatorial inertia–gravity waves (EIG) and $n = 2$ equatorial Rossby waves (ER). In the symmetric plots, we show the dispersion curves for Kelvin waves and $n = 1$ ER. The symbol $n$ is the meridional mode number. As, for example, discussed in Tindall et al. (2006), the temperature waves are symmetric when $n$ is odd and antisymmetric when $n$ is even. Negative (positive) zonal wavenumbers indicate easterly (westerly) waves.

The antisymmetric plots show spectral peaks for $n = 2$ ER. The spectral peaks for ER mainly lie around wavenumbers $-1$ to $-5$, with frequencies $\leq 0.2$ cycles per day. Spectral peaks for MRG at wavenumber 4 are apparent with periods of 3–4 days. Moreover, spectral peaks for $n = 0$ EIG waves are detected in the antisymmetric plots. In the symmetric plots, strong peaks for Kelvin waves and $n = 1$ ER are detected with small wavenumbers and low frequencies. However, these wave signals are highly influenced by the background red noise. A wavenumber–frequency plot for the SLD simulation without the background signal was shown in Yao and Jablonowski (2013). It revealed that the most

**Fig. 11.** Latitude–pressure cross sections of $\beta^*$ (negative areas are striped) and the zonal wind (colored) for an instantaneous data snapshot at a selected longitude for (a) SLD, (b) FV, (c) EUL, and (d) SE.
dominant peaks in the Kelvin wave regime lie around wavenumbers 1–2 with 15–30-day periods. The wave signatures follow the Doppler-shifted dispersion curves rather closely.

In Fig. 12, we do not remove the background spectrum, since it carries important information about the overall resolved wave activity in the four dycores. The reduced occurrence of the strong (yellow- and red-colored) power range in FV indicates a reduced wave activity in both the antisymmetric and symmetric regimes. This is also true at different pressure levels. The SLD simulation shows moderate wave activity. EUL and SE exhibit the strongest resolved wave activities, which are of comparable strengths. The wave activity in all models correlates well with their corresponding TEM analyses (section 4) and instability discussions (section 5). It shows that equatorial wave activities are abundant in the QBO models SLD, EUL, and SE. These models have strong forcings by the E–P flux divergence and high potential for dynamic instabilities, which possibly served as wave triggers. On the other hand, the model with the fewest resolved waves, lowest E–P flux divergence, weakest potential for dynamic instabilities, and no QBO is FV. This raises the question whether the resolved wave forcing needs to exceed a threshold, as suggested by the Yoden and Holton (1988) QBO bifurcation theory. The results do not mean that the weaker wave activity in the FV dynamical core is wrong. In fact, an alternative interpretation could be that the waves in SLD, EUL, and SE might be overly active and a result of numerical artifacts, which could be connected to the vertical discretization method. An indication for this argument is the observation that an alternative configuration of the SE dynamical core with a floating Lagrangian vertical coordinate, like the one in FV, lets the QBO vanish (not shown). This topic needs further exploration.

As vertically propagating waves grow their amplitudes with height, they start to break and transfer momentum and energy to the background wind field. The waves are strongly absorbed near their critical levels when their phase speeds approach the speeds of the background flow. The QBO therefore works as a wave filter. This is demonstrated in Fig. 13, which shows the temperature wavenumber–frequency analyses at 181, 94, 22, and 2 hPa for the SE dynamical core as an example. The 30-month analysis period covers years 12.25–14.75 of the SE simulation, which are characterized by weak easterly equatorial flows at 181 and 94 hPa, the transition from easterlies to the westerly QBO jet at 22 hPa, and mostly westerly conditions at 2 hPa, as seen in Figs. 1d and 3d. The wave power in the temperature field at 181 hPa is low. This is typical, as upward-traveling
temperature waves have small amplitudes in the troposphere due to their strong dependence on static stability (which is small in the equatorial troposphere).

Figures 13b and 13f show that the tropospheric waves have gained strength as a result of the decrease in density and increase in static stability around the tropopause level (94 hPa), especially the $n = 2$ ER and MRG waves in the antisymmetric field. In addition, Kelvin waves are prominent at these levels. The easterly background flow in the region 94–22 hPa with jet speeds between 0 and $-22$ m s$^{-1}$ (Fig. 1d) absorbs easterly waves of matching phase speeds, corresponding to the equivalent depths below $h = 50$ m. This absorption can best be seen in the $n = 1$ and $n = 2$ ER waves. Between 94 and 22 hPa, the strengths of the slow ER waves with small wavenumbers are diminished, and their peak magnitudes are shifted upright toward faster waves with higher equivalent depths (Fig. 13c). This shift is also somewhat visible in the MRG regime, although the absorption of slow MRG modes is incomplete during their upward propagation, which contributes to the lack of easterly wave driving in SE. Kelvin waves are unaffected by the easterly background flow below 22 hPa and propagate through it. However, between 22 and 2 hPa the mostly westerly background jet reaches magnitudes between 0 and 15 m s$^{-1}$ and filters slow Kelvin waves with matching phase velocities. This is evident in Figs. 13g and 13h, which show diminished Kelvin wave amplitudes at 2 hPa for small $h$. The amplitude distribution of the Kelvin waves becomes more upright and shifts toward faster Kelvin waves (with $h > 50$ m), which travel undisturbed through the stratosphere. Similar absorption patterns were also found in Yang et al. (2011) and Krismer and Giorgetta (2014).

7. Summary and outlook

This study demonstrated that QBO-like oscillations can be simulated in an ensemble of dry GCM dynamical cores when driven by the Held and Suarez (1994) forcing. The HS forcing replaces the complex physical parameterization package in GCMs with a simple Newtonian temperature relaxation and low-level Rayleigh friction that mimic the radiative transfer and planetary boundary layer mixing on a flat Earth. Such an idealized setup gives easier access to an improved understanding of causes and effects in dynamical cores that are otherwise difficult to isolate. The tropical stratospheric circulations in four dynamical cores that are options in CAM5 were explored. These are the SLD, FV, EUL, and SE dynamical cores, which are built upon the same hydrostatic equation set but utilize different numerical schemes, diffusion mechanisms, computational grids, and mesh staggering options. The central question was how the model design choices impact the wave generation, propagation, and dissipation mechanisms in the equatorial region and whether QBO-like oscillations are supported in the tropical stratosphere.
The TEM analyses revealed that the E–P flux divergence (the resolved waves) is the dominant driver of the QBOs. However, the resolved wave forcing is strongly counteracted by the TEM momentum budget residual $X$, which represents the forcing by implicit numerical or explicitly added diffusion and the damping via the thermal HS relaxation. The diffusive tendencies explain the gross features of the residual $X$ and therefore play a considerable role in these idealized simulations. The analysis of the Brewer–Dobson circulation showed that the downward propagation of SLD's QBO is sped up by a mean descent in the QBO domain as a result of a strong secondary meridional circulation, whereas EUL's and SE's QBOs are slowed down by a mean ascent in the tropical stratosphere.

The wave analyses showed that waves are abundant in SLD's, EUL's, and SE's tropical atmosphere despite the absence of moist convection as a typical wave trigger or a GWD scheme. The wave activity in FV and its E–P flux divergence are reduced. This raises questions about the wave generation mechanisms in dry HS experiments. We focused on dynamic instabilities as a potential wave-triggering mechanism in the troposphere and wave-dissipation process in the stratosphere. In particular, there are indications that the increased occurrences of strongly negative instability indicators in SLD, EUL, and SE are related to their more vigorous wave activities and higher magnitudes of the resolved wave forcing in the tropical stratosphere. Identical stratospheric grid spacings of about $\Delta x \approx 210 \text{ km} \text{ and } \Delta z = 1.25 \text{ km}$ were utilized with model tops at 0.1 hPa.

The study revealed that three (SLD, EUL, and SE) out of the four CAM5 dynamical cores developed spontaneous QBO-like oscillations in the upper equatorial stratosphere. The SLD dynamical core shows a tropical zonal wind oscillation that is closest to observation. However, the SLD's QBO-like oscillation has an average period of 3.6 yr, which is longer than observation, and covers the region between 1 and 50 hPa. The EUL and SE simulations closely resemble each other but differ greatly from SLD. Their QBO periods are over 13 yr, and the oscillation occupies the narrow band between 1 and 30 hPa. The QBOs lie too high in comparison to observations but have realistic zonal wind magnitudes between $-35$ and $15 \text{ m s}^{-1}$. The FV dynamical core does not sustain the oscillation. The initial wind shear is dissipated within the first six years of the simulation.

### Table A1. List of the 56 hybrid interface coefficients for the 55-level setup.

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comparison to FV. This causality needs further investigations which we plan to pursue in the future. This paper focused on the dynamical-core intercomparison aspects of the QBO investigation. The roles of the horizontal and vertical grid spacings, diffusion mechanisms and their coefficients, and sponge layer settings, as well as Brunt–Väisälä frequencies and thermal relaxation time scales in the stratosphere are partly shown in Yao (2014) and will be discussed in a companion paper. In addition, we will shed light on the effects of simplified moisture processes and added gravity wave drag schemes on the QBOs in the dynamical cores, as discussed in Yao (2014).

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APPENDIX

Vertical Grid Specification

The dynamical cores in CAM utilize a pressure-based $\eta$ coordinate (Simmons and Burridge 1981) in the vertical direction, either directly for the centered finite-difference discretizations or indirectly as a reference grid for the floating Lagrangian approach with periodic remaps. We use $N_{\text{lev}} = 55$ model levels, which are bounded by $N_{\text{lev}} + 1$ interface levels (denoted by half indices $k + 1/2$). The pressure $p$ at model interface levels is then given by

$$p_{k+1/2} = a_{k+1/2} p_0 + b_{k+1/2} p_s,$$  \hspace{1cm} (A1)

with $k = 0, 1, 2, \ldots, N_{\text{lev}}$ increasing downward. A reference pressure is symbolized by $p_0 = 1000$ hPa, and $p_s$ stands for the surface pressure. The hybrid coefficients $a$ and $b$ are height dependent. They are listed in Table A1 for the 56 model interface levels in order to make the QBO simulations reproducible. The hybrid coefficients at the full model levels $a_k$ and $b_k$ can be computed as the arithmetic means of the two surrounding model interface coefficients.

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