CORRESPONDENCE

Comments on "A Semihydrostatic Theory of Gravity-Dominated Compressible Flow"

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By extending the unified framework of Arakawa and Konor (2009), Dubos and Voitus (2014, hereafter DV14) describe the construction of a soundproof semihydrostatic model.

In this model, air parcels are constrained to remain close to their hydrostatic height. Accuracy at nonhydrostatic scales is ensured by retaining vertical acceleration, approximated as the second Lagrangian derivative of hydrostatic height, in the vertical momentum budget. The constraint imposed on the vertical position of air parcels filters out acoustic waves and gives rise to a self-adjoint problem for a Lagrange multiplier identified as a nonhydrostatic displacement—that is, the difference between the actual height of air parcels and their hydrostatic height.

In the final part of the paper, DV14 envisage an application of their model for data assimilation and initialization purposes:

> Variational data assimilation systems may benefit from such an accurate elimination of acoustic waves, reducing the optimization space. Similarly, it could be useful for initialization purposes, avoiding the transient generation of acoustic waves from an initial flow. It may also be helpful to occasionally perform this projection while integrating the fully compressible Euler equations, especially after physical parameterizations have acted, potentially triggering the transient emission of acoustic waves.

Recently, Benacchio et al. (2014, hereafter BOK14) and the related work by Klein et al. (2014) and Benacchio (2014) proposed a blended semi-implicit soundproof–compressible model, motivating the development with reference to data assimilation and testing it in reducing the amplitude of acoustic waves generated by imbalanced initial data. Discussing an extension of their work to larger scales, BOK14 referred to unified approaches regarding the correct limiting model for such an extension:

> Careful consideration will be needed to identify the correct large-scale limiting model in the light of recent suggestions of unified multiscale reduced models by Durran (2008), Arakawa and Konor (2009), and Konor (2014).

The two excerpts reported above suggest the existence of a connection between DV14 and BOK14. This comment aims at clarifying this connection for the benefit of the readers of either paper.

In BOK14, a pseudoincompressible formulation featuring the thermodynamic pressure was extended to compressible dynamics by retrieving the time derivative term in the potential temperature equation written in

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conservative form. Numerically, the extension amounts to an additional zero-order term in the elliptic equation constructed by replacing the momentum tendency into the pseudoincompressible divergence constraint. The modular character of the extension was then employed to formulate a blended soundproof–compressible numerical model, where different model configurations are seamlessly accessed by straightforward switching within a single numerical framework.

BOK14 showcased the model performance in thermal plume simulations initialized in pseudoincompressible mode, then transitioning to fully compressible dynamics. As shown in Fig. 6 of BOK14 and Fig. 5.3 of Benacchio (2014), acoustic waves generated by imbalanced initial data can be effectively damped by setting up the model in soundproof mode for some initial time steps. In soundproof mode, pressure perturbations are projected away by imposing the divergence constraint at the discrete level. By the time the model reverts to fully compressible mode, the effect of acoustic waves all but disappears as their amplitude emerges significantly reduced from the soundproof phase. Similar results were obtained with a model using Exner pressure by Klein et al. (2014).

As noted in BOK14, while acoustic waves are generally deemed insignificant in weather prediction and climate studies, their uncontrolled propagation can seriously affect predictability at the mesoscale (Hohenegger and Schär 2007). Indeed, BOK14 made the case for a joint soundproof–compressible approach, acknowledging the widespread adoption of compressible models in atmospheric dynamical cores while highlighting the merits of accessing limiting model dynamics at the relevant scales. Such an approach constitutes a sort of paradigm shift from evaluations of compressible versus soundproof model output based on normal-mode analyses in the manner of Davies et al. (2003) and also DV14. As shown, for example, by the simulations of Smolarkiewicz and Dörnbrack (2008), numerical truncation errors due to the use of different numerical schemes may outweigh the differences due to the use of different analytical formulations.

The numerical procedure outlined in DV14 to integrate the semihydrostatic equations features the solution of an elliptic problem for the nonhydrostatic displacement λ, which, together with the diagnostic vertical momentum equation, is responsible for filtering the acoustic waves. DV14 then suggest employing the semihydrostatic system in an auxiliary procedure to damp imbalanced acoustic modes within a compressible model. The elliptic equation would be used to project a given flow field onto the semihydrostatically balanced modes, so that the resulting projected field could serve as new initial data free of vertically propagating acoustic signals for the subsequent fully compressible numerical solution. An elegant way of realizing this approach numerically would follow along the lines demonstrated in BOK14 for the compressible–pseudoincompressible blending. Within a unified discrete formulation for the compressible and semihydrostatic models—which is yet to be developed—the projection onto the semihydrostatically balanced modes would be achieved straightforwardly by invoking a single time step of the associated blended model tuned to the balanced equations. One might then immediately switch to the fully compressible scheme as suggested by DV14 or, based on the insight gained by BOK14 on the benefits of a smooth transition, allow the scheme to adjust to the fully compressible model over several time steps. This adjustment would likely minimize residual acoustic noise to be expected after a sharp switch.

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