Microphysical and Polarimetric Radar Modeling of Hydrometeor Refreezing

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ABSTRACT: A unique polarimetric radar signature indicative of hydrometeor refreezing during ice pellet events has been documented in several recent studies, yet the underlying microphysical causes remain unknown. The signature is characterized by enhancements in differential reflectivity (ZDR), specific differential phase (Kdp), and linear depolarization ratio (LDR), and a reduction in copolar correlation coefficient (ρhv) within a layer of decreasing radar reflectivity factor at horizontal polarization (ZH). In previous studies, the leading hypothesis for the observed radar signature is the preferential refreezing of small drops. Here, a simplified, one-dimensional, explicit bin microphysics model is developed to simulate the refreezing of fully melted hydrometeors, and coupled with a polarimetric radar forward operator to quantify the impact of preferential refreezing on simulated radar signatures. The modeling results demonstrate that preferential refreezing is insufficient by itself to produce the observed signatures. In contrast, simulations considering an ice shell growing symmetrically around a freezing particle (i.e., emulating a thicker ice shell on the bottom of a falling particle) produce realistic ZDR enhancements, and also closely replicate observed features in ZH, Kdp, LDR, and ρhv. Simulations that assume no increase in particle wobbling with freezing produce an even greater ZDR enhancement, but this comes at the expense of reducing the LDR enhancement. It is suggested that the polarimetric refreezing signature is instead strongly related to both the distribution of the unfrozen liquid portion within a freezing particle and the orientation of this liquid with respect to the horizontal.

KEYWORDS: Freezing precipitation; Mixed precipitation; Radars/Radar observations

1. Introduction

Accurate detection and prediction of precipitation type during winter storms remains a difficult problem in operational forecast settings (e.g., Ralph et al. 2005) owing to the complexity of the formation of these precipitation types (e.g., Stewart et al. 2015). A variety of winter precipitation types occur at surface temperatures near 0°C (e.g., Thériault et al. 2010; Stewart et al. 2015), and as little as a 0.5°C difference in the lower-tropospheric vertical temperature profile can significantly alter precipitation type (e.g., Sankaré and Thériault 2016). These differences ultimately depend on whether individual hydrometeors within a distribution are melting or freezing, and on whether these processes are complete or partial. For specificity, freezing that occurs after partial or complete melting of ice-phase hydrometeors is referred to as “refreezing.”

In the case of ice pellets, both of these processes occur: snowflakes aloft melt either fully or partially within an elevated layer of wet-bulb temperature Tw > 0°C, and subsequently refreeze within a near-surface Tw < 0°C layer (e.g., Brooks 2010; Hanesiak and Stewart 1995; Zerr 1997). In general, ice pellets are infrequent, but not rare occurrences. Canada and much of the United States—specifically, the Pacific Northwest and regions east of the Rocky Mountains—typically experience ice pellets at least one day a year, with more numerous occurrences in eastern Canadian provinces (>10 annual ice pellet days) and the northeastern portion of the United States (>4 annual ice pellet days; Cortinas et al. 2004). The refreezing process required for ice pellet formation is dependent on the degree of melting, such that partially melted hydrometeors still containing ice will begin to refreeze immediately upon encountering the Tw < 0°C layer, whereas fully melted hydrometeors must first nucleate ice prior to refreezing at a lower temperature. Supercooled liquid drops that form via complete melting of ice hydrometeors freeze in the same manner as those that form via traditional collision and coalescence processes; however, fully melted hydrometeors have been observed to refreeze at higher temperatures. Polarimetric radar observations indicated such refreezing in the case of fully melted hydrometeors occurred at Tw ≲ −5°C (e.g., Kumjian et al. 2013, 2020, hereafter K13 and K20; Tobin and Kumjian 2017), yet the specific ice nucleation mechanism for these particles remains unknown. Because refreezing did not occur closer to the top of the Tw < 0°C near-surface layer, it is unlikely that any ice remained within the particles after melting. However, few aerosols are known to nucleate ice via the immersion mode at such relatively high temperatures (e.g., Murray et al. 2012). Alternatively, Hogan (1985) suggested the formation of ice pellets via collisions between supercooled liquid drops and ice crystals, as supported by observations in K20. These crystals would be locally generated via either primary ice nucleation and subsequent vapor depositional growth (e.g., Thériault and Stewart 2010; Thériault et al. 2010; K13; K20) or secondary ice production mechanisms such as rime splintering (e.g., Hallett and Mossop 1974), droplet shattering or fragmentation (e.g., Johnson and Hallett 1968; Knight and Knight 1974; Takahashi 1975), or ice ejection via bubble bursting (e.g., Lauber et al. 2018).

Beyond these uncertainties of ice nucleation for fully melted hydrometeors, unexpected polarimetric radar observations illuminated additional questions related to the microphysics of
ice pellet formation. Refreezing of fully melted particles (i.e., liquid drops) into ice pellets produces a reduction in radar reflectivity factor at horizontal polarization $Z_H$ owing to the change in the particles’ relative permittivity from that of liquid to ice. Such $Z_H$ reductions and an expected increase in the dispersion of particle orientations upon freezing would, in theory, lead to reductions in differential reflectivity $Z_{DR}$ and specific differential phase $K_{DP}$ (e.g., Ryzhkov et al. 2011). However, K13 instead observed enhancements in both $Z_{DR}$ and $K_{DP}$ within the refreezing layer (RFL) of decreasing $Z_H$ toward the ground. Those authors proposed two hypotheses for this so-called polarimetric refreezing signature: 1) preferential refreezing of smaller drops and 2) locally generated columnar ice crystals. Freezing the smallest drops first would increase the relative contribution of larger drops with intrinsically higher $Z_{DR}$ values; however, this requires that all drops nucleate ice within them at the same time, which is not consistent with the microphysical understanding that larger drops are more likely to initiate freezing in the immersion mode (e.g., Bigg 1953; Pruppacher and Klett 1997). Columnar crystals, on the other hand, have intrinsically high $Z_{DR}$ and could serve to initiate ice nucleation of drops in the RFL via the contact mode. Nagumo et al. (2019) proposed that particle deformations during freezing could produce observed $Z_{DR}$ values, yet no scattering calculations were done to support this claim, and the reported axis ratios of the deformed particles were not drastically different from those of raindrops. K20 provided fully polarimetric radar observations from an ice pellet event that support the possibility of all three hypotheses (i.e., preferential refreezing, columnar ice crystals, and particle deformations). However, the authors found that the presence of locally generated ice crystals did not substantially contribute to enhanced $Z_{DR}$ in that case, but instead promoted contact nucleation to allow for the preferential refreezing of smaller drops. Further, although particle deformations did not explain the observed $Z_{DR}$ signature, they contributed to observed features in other polarimetric variables, such as reductions in the copolar correlation coefficient, $\rho_{hv}$, and enhancements in linear depolarization ratio, LDR. A schematic summarizing typical values and height-dependent features of the polarimetric radar refreezing signature is presented in Fig. 1.

Although preferential refreezing of small drops is the favored hypothesis based on these studies, it remains unclear whether it is directly responsible for the polarimetric refreezing signature. Simple calculations in K13 revealed that this process is capable of producing the observed $Z_{DR}$ enhancement in theory, but

![Fig. 1. Schematic of the polarimetric radar refreezing signature in (a) $Z_H$, (b) $Z_{DR}$, (c) $K_{DP}$, (d) LDR, and (e) $\rho_{hv}$, based on observations documented in K13, K20, Van Den Broeke et al. (2016), and Tobin and Kumjian (2017). The refreezing layer (blue) is shown between 200 and 600 m, with typical observed values of each variable (solid lines) at S band (11.0 cm), except in (d) where observations were at Ka band (8.5 mm). Gray shadowing in (a)–(c) indicates a range of reported values. The arrow in (e) indicates that observed values are less than the peak value shown, but there is no established minimum value. The dashed line in (e) represents the true values expected, whereas the solid line represents observed values owing to ground clutter. Ground clutter can also bias other observed variables near the surface, but these effects are not shown. The relative locations of respective maximum or minimum values in (b)–(e) are qualitatively consistent with observations (i.e., $K_{DP}$ maxima above $Z_{DR}$ maxima), but are not meant to convey actual height observations.](image-url)
calculations with more rigorous treatment of hydrometeor refreezing have not yet been performed. In this study, we provide such rigorous treatment by developing and implementing a one-dimensional explicit bin microphysical model. This microphysical model is then coupled with a polarimetric forward operator to simulate the radar signatures associated with the refreezing of fully melted liquid drops. As such, this study seeks to definitively verify or refute the hypothesis of preferential refreezing of small drops as the cause of the polarimetric refreezing signature. In the case of refutation, we seek to consider and explore additional microphysical processes that correspond better with observations.

2. The microphysical model for hydrometeor refreezing

The freezing model developed here follows Kumjian et al. (2012, hereafter K12), but with important changes. Hydrometeors in the K12 model froze in a convective updraft with drops nucleating via the immersion mode. Here, no updraft is present, so hydrometeors fall into the near-surface $T_0 < 0^\circ \text{C}$ layer via sedimentation. We simulate the refreezing of fully melted hydrometeors within a steady-state, one-dimensional column of precipitation with 10-m vertical grid spacing. Because our simulated hydrometeors are assumed to have fully melted aloft prior to encountering the surface $T_w < 0^\circ \text{C}$ layer, we do not explicitly model any melting, and instead begin with liquid drops to focus only on refreezing. The steady-state assumption implies that the precipitation column is fully developed with no changes in intensity (i.e., flux) in time. This assumption is appropriate for ice pellets, which are typically associated with warm-frontal, stratiform precipitation (e.g., Stewart 1985; Hanesiak and Stewart 1995). Particle interactions such as riming, collisions, aggregation, and breakup are all neglected, as are any secondary ice production processes. Thus, we have a one-to-one correspondence between a liquid drop and an ice pellet within the model. Ice pellet aggregates have been observed in some, but not all, events (e.g., Brooks 1920; Stewart and Crawford 1995); however, the environmental conditions favoring ice pellet aggregation are poorly understood (Stewart and Crawford 1995; Carmichael et al. 2011). Further, no ice pellet aggregates were documented in cases where the polarimetric refreezing signature was observed (K13; Tobin and Kumjian 2017). Mass exchange with the environment (i.e., evaporation, condensation, sublimation, deposition) is retained in the thermal energy balance equations, but is otherwise ignored such that a particle’s mass is conserved as it falls through the column. Particle mass changes via environmental exchange have a negligibly small impact on hydrometeor sizes and on the simulated polarimetric radar variables as a result, and are thus neglected for simplicity. Thus, particle mass is conserved within Lagrangian particle bin sizes, with bin sizes defined as the equivalent-volume diameter of a solid ice pellet. More complex and complete models detailing drop freezing, particle growth, and interactions between particles do exist (e.g., Phillips et al. 2014, 2015; Ilotoviz et al. 2016); however, the assumptions herein are used to isolate and explore the first-order microphysical impacts of refreezing a distribution of fully melted hydrometeors on the resulting polarimetric radar fields.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$D$</td>
<td>Particle diameter</td>
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<tr>
<td>$D_i$</td>
<td>Ice crystal maximum dimension</td>
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<tr>
<td>$f_m$</td>
<td>Liquid water mass fraction</td>
</tr>
<tr>
<td>$f_a$</td>
<td>Ice volume fraction</td>
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<tr>
<td>$H$</td>
<td>Mean depth required for a collision between a raindrop and ice crystal</td>
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<tr>
<td>$N(D)$</td>
<td>Number concentration of particles with diameter $D$</td>
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<tr>
<td>$N_0$</td>
<td>Scale parameter of PSD</td>
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<tr>
<td>$n$</td>
<td>Ice crystal number concentration</td>
</tr>
<tr>
<td>$r$</td>
<td>Particle axis ratio</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Triple-point temperature of water ($0^\circ \text{C} = 273.15 \text{K}$)</td>
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<tr>
<td>$T_w$</td>
<td>Wet-bulb temperature</td>
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<tr>
<td>$\nu$</td>
<td>Particle fall speed</td>
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<tr>
<td>$\nu_p$</td>
<td>Ice pellet fall speed</td>
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<tr>
<td>$\gamma$</td>
<td>Density correction factor</td>
</tr>
<tr>
<td>$\epsilon_i$</td>
<td>Relative permittivity of ice</td>
</tr>
<tr>
<td>$\epsilon_{ia}$</td>
<td>Relative permittivity of an ice and water mixture (i.e., slush)</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Relative permittivity of liquid water</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Slope parameter of PSD</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Shape parameter of PSD</td>
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<tr>
<td>$\rho$</td>
<td>Air density</td>
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<tr>
<td>$\rho_0$</td>
<td>Reference air density</td>
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<tr>
<td>$\sigma$</td>
<td>Width of the canting angle distribution</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Error function parameter of $f_a$</td>
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a. Freezing drops

The freezing of drops for the model is described in greater detail in K12. Pruppacher and Klett (1997) describe freezing as a two-stage process, wherein the first stage is assumed to occur nearly instantaneously and the second stage is time dependent. A small fraction of the particle freezes during the first stage of ice nucleation, and the associated enthalpy of freezing quickly increases the particle temperature to near $T_0 = 0^\circ \text{C}$. During the second stage, freezing occurs at the ice–liquid interface between a liquid core and a growing ice shell. The enthalpy of freezing at the ice–liquid interface is balanced by thermal conduction through the growing ice shell, which, in turn, is balanced by the rates of thermal energy transfer through the air (conduction) and sublimation/deposition at the particle surface. K12 provide an expression for the growth rate of the ice shell thickness, which we multiply by the model vertical grid spacing (10 m) and divide by the particle fall speed $\nu$ to obtain the change in ice shell thickness within each model level.

b. Particle fall speeds

A $\nu$ relation for freezing drops is largely unknown due to a lack of observational data. As such, we follow K12 and assume a linear relation as a function of liquid water mass fraction, $f_m$, between the empirical fall speed equations for ice pellets and for raindrops. Ice pellet fall speeds $\nu_p$, (m s$^{-1}$) are given by K12 as an expression fitting small hail velocities in the Hebrew University Cloud Model (e.g., Khain et al. 2004, 2011):
Raindrop fall speeds \( y_r \), in \( \text{m s}^{-1} \), are given by Brandes et al. (2002) as a polynomial function fitting laboratory measurements of Gunn and Kinzer (1949) and Pruppacher and Pitter (1971):

\[
y_{r}(D) = 0.2259 + 1.5954D - 0.0405D^2,
\]

with \( D \) expressed in millimeters. Figure 2 plots the resulting fall speeds for a refreezing particle with \( f_m = 0 \) to liquid raindrops \( (f_m = 1) \). A density correction factor of \( \gamma = (\rho_\infty/\rho)^{0.4} \) is applied to the resulting \( v \) to account for reduced fall speeds closer to the ground, where \( \rho_\infty = 1.2 \text{ kg m}^{-3} \) is a reference surface air density, and \( \rho \) is the air density at the current model level (e.g., Foote and du Toit 1969; Beard 1985).

### c. Particle size distribution

The particle size distribution (PSD) corresponds with observations in Gibson et al. (2009) of ice pellet major axis lengths observed in Montreal, Quebec, on 17 January 2006. The ice pellet PSD at the surface follows a gamma distribution (e.g., Ulbrich 1983) with parameters from Gibson et al. (2009):

\[
N(D) = N_0 D^\mu \exp(-\Lambda D),
\]

where \( N(D) \) is the total number of particles per unit volume of diameter size bin \( D \) (in mm), \( N_0 = 1818 \text{ m}^{-3} \text{ mm}^{-(\mu+3)} \) is the scale parameter, \( \mu = 2 \) is the shape parameter, and \( \Lambda = 3 \text{ mm}^{-1} \) is the slope parameter (Fig. 3). This is the broader of two PSDs reported by Gibson et al. (2009) for two separate events, and was chosen because K13 determined that broader PSDs produced larger \( Z_{\text{DR}} \) enhancements for the preferential refreezing of the smallest drops. Ice pellet sizes are divided into 40 bins from 0.1 to 4.0 mm in 0.1-mm increments. The maximum particle diameter of 4.0 mm was chosen in accordance with the size range of ice pellets reported by Gibson et al. (2009).

This assumed PSD is valid for solid ice pellets at the surface, but the PSD at a given height in the model is determined through flux conservation (given the steady-state approximation used). The flux is defined by the ice pellet flux at the surface as the product of ice pellet fall speed [Eq. (1)] and number concentration [Eq. (3)], and is conserved within each bin at all model levels. This implies that the number concentration of freezing drops increases toward the surface owing to reduced fall speeds with refreezing (Fig. 2).

### 3. The polarimetric radar forward operator

Particles are modeled as either one- or two-layer oblate spheroids. The aspect ratios of these particles—defined as the ratio of the vertical (minor) to the horizontal (major) axes—are determined by the spherical particle diameter \( D \) (in mm) following the Brandes et al. (2002) relationship for raindrops:

\[
r = 0.9951 + 0.02510D - 0.03644D^2 + 0.005303D^3 - 0.0002492D^4.
\]

Though developed for raindrops, this equation is comparable to ice pellet axis ratios observed in Nagumo et al. (2019), and thus will be used for liquid, freezing, and frozen particles. For two-layer mixed-phase particles, the aspect ratio of the inner spheroid is assumed equivalent to that of the outer spheroid (e.g., Ryzhkov et al. 2011), for now.

The relative permittivities of liquid and ice \( (\epsilon_r \text{ and } \epsilon_i \text{, respectively}) \) follow Ray (1972). These calculations are performed for S band (11.0 cm), assuming the particle temperature has adjusted to the environmental temperature for raindrops and ice.
pellets, but is otherwise $T_0$ for freezing hydrometeors. Freezing hydrometeors are modeled as two-layer spheroids with an ice shell and slushy (liquid and ice) core. The relative permittivity of the slushy core, $\epsilon_{\text{sl}}$, is computed using the Maxwell-Garnett (1904) formulas for an ice–liquid mixture. The first Maxwell–Garnett equation is valid for spherical inclusions of liquid water evenly distributed within an ice matrix:

$$\epsilon_{\text{sl,1}} = \frac{1 + 2(1 - f_w)\epsilon_w - \epsilon_i}{1 - (1 - f_w)\epsilon_w + 2\epsilon_i},$$

(5)

where $f_w$ is the ice volume fraction within the slush portion of the particle. The second Maxwell–Garnett equation is valid for spherical ice inclusions within a liquid water matrix:

$$\epsilon_{\text{sl,2}} = \frac{1 + 2f_w\epsilon_i + 2\epsilon_w}{1 - f_w\epsilon_i + 2\epsilon_w}.$$ 

(6)

Both $\epsilon_{\text{sl,1}}$ and $\epsilon_{\text{sl,2}}$ produce different values for the same $f_w$, although $\epsilon_{\text{sl,1}}$ is more physically representative of high $f_w$ while $\epsilon_{\text{sl,2}}$ is better suited for low $f_w$. Ryzhkov et al. (2011) provide an approach to combine both equations:

$$\epsilon_{\text{sl}} = \frac{1}{2} [\epsilon_{\text{sl,1}}(1 + \tau) + \epsilon_{\text{sl,2}}(1 - \tau)],$$

(7)

where $\tau$ is a function of $f_w$ that has been modified to

$$\tau = \text{erf} \left[ \frac{f_w}{(1 - f_w)} - 1 \right].$$

(8)

This formulation produces $\epsilon_{\text{sl}}$ values that closely match those produced by Matrosov (2008), but $\epsilon_{\text{sl}}$ varies smoothly across all $f_w$. As freezing progresses, $f_w$ remains constant. Physically, this implies that the particle mass converted to ice during the first stage of the refreezing process is evenly distributed, and that the growing ice shell is composed of mass from both liquid and ice portions from the slushy core.

The small-particle scattering approximation used for spheroids is valid at $\text{S}$ band for the particle sizes considered here. The complex scattering amplitudes of such particles are detailed in Ryzhkov et al. (2011), and we use the corresponding equations for one- or two-layer oblate spheroids. Ryzhkov (2001) and Ryzhkov et al. (2011) describe a series of angular moments for various hydrometeor orientation assumptions, and we follow the assumptions for particles at low radar elevation angles with a two-dimensional axisymmetric Gaussian distribution of canting angles with a mean of 0°. The width of the canting angle distribution, $\sigma$, is assumed to vary linearly between rain ($\sigma = 10°$) and solid ice pellets ($\sigma = 40°$) as a function of liquid mass fraction (e.g., Ryzhkov et al. 2011; K12). These $\sigma$ values are estimates obtained from polarimetric radar observations (e.g., Ryzhkov et al. 1999; Ryzhkov 2001; Ryzhkov et al. 2002), and are thus a source of uncertainty in the model. Nagumo et al. (2019) suggest that stable orientations of deformed (i.e., bulged and nearly spherical) particles may play a role in the $Z_{\text{DR}}$ observations within the RFL. To simulate this, we perform tests where $\sigma$ is set to $10°$ for ice pellets, so there is no change in particle orientation distributions upon freezing. Finally, to compute the polarimetric radar variables, the complex scattering amplitudes and orientation distributions are combined following the formulas in Ryzhkov et al. (2011).

4. Simple refreezing tests

Before simulating refreezing using the full-physics model, simple refreezing tests are conducted to illustrate the impact of preferential refreezing of small drops on the polarimetric radar variables of $Z_{\text{H}}$, $Z_{\text{DR}}$, LDR, and $\rho_{\text{hv}}$. These tests follow K13 where the smallest drops of a liquid PSD are sequentially frozen, though here we use a realistic ice pellet PSD and compute LDR and $\rho_{\text{hv}}$ in addition to $Z_{\text{H}}$ and $Z_{\text{DR}}$ therein. Physically, this is analogous to all drops ice nucleating at the same time; thus, the smallest drops will complete refreezing prior to the larger drops (e.g., Pruppacher and Klett 1997; K12). The polarimetric radar variables are computed for the entire PSD as each subsequent particle size bin is frozen. Whereas freezing is a time-dependent and continuous process across a PSD, these calculations maximize the effect of preferential refreezing by keeping the largest particles liquid as the smaller particles are completely frozen. These tests serve as the theoretical values to compare against the full-physics simulation outputs.

Results are plotted in Fig. 4 as a function of the diameter of the largest frozen particle in the PSD. The value of $Z_{\text{H}}$ decreases with an increasing maximum frozen diameter owing to the reduction in particle relative permittivity. Complete refreezing of the entire distribution results in a 6.7-dB decrease in $Z_{\text{H}}$, as expected (e.g., Ray 1972; Smith 1984; Doviak and Zrnić 1993). Beginning at a maximum frozen diameter of ~1.0–1.5 mm, $Z_{\text{DR}}$ increases for increasing diameters, reaching a peak at 2.6 mm for the increasing-$\sigma$ simulation. The peak $Z_{\text{DR}}$ is 0.16 dB greater than the all-liquid $Z_{\text{DR}}$ value. For the constant-$\sigma$ simulation, the peak $Z_{\text{DR}}$ is 0.20 dB greater than the all-liquid $Z_{\text{DR}}$ and occurs at 2.7 mm. Freezing the smaller drops reduces their contribution to the total $Z_{\text{H}}$ and increases the relative contribution from larger drops, with their intrinsically larger $Z_{\text{DR}}$ than their smaller, frozen counterparts (K13). Similar LDR enhancements are present (1.6-dB increase at 3.1 mm for increasing $\sigma$; 1.4-dB increase at 3.0 mm for constant $\sigma$) owing to the larger drops with a nonzero $\sigma$ having intrinsically higher LDR. There is a trade-off evident: simulations with constant $\sigma$ produce greater $Z_{\text{DR}}$ but smaller LDR enhancements, whereas simulations with increasing $\sigma$ produce larger $Z_{\text{DR}}$. The small-particle scattering approximation used for spheroids is valid at $\text{S}$ band for the particle sizes considered here. The complex scattering amplitudes of such particles are detailed in Ryzhkov et al. (2011), and we use the corresponding equations for one- or two-layer oblate spheroids. Ryzhkov (2001) and Ryzhkov et al. (2011) describe a series of angular moments for various hydrometeor orientation assumptions, and we follow the assumptions for particles at low radar elevation angles.
LDR but smaller $Z_{DR}$ enhancements. Particles falling with their major axis aligned horizontally maximize $Z_{DR}$, whereas the major axis must deviate from the polarization axes to depolarize incident radiation and increase LDR. A $\rho_{hv}$ minimum occurs at 3.5 mm maximum frozen diameter for both simulations, arising from the larger apparent shape diversity between the small, frozen drops and the large, liquid drops. However, these $\rho_{hv}$ reductions are very slight and not likely observable in practice. Observed deeper $\rho_{hv}$ minima within the RFL thus must arise from shape irregularities introduced during freezing, including bulges or other particle deformations (Nagumo et al. 2019), or asymmetric liquid distribution within particles (K20).

These simple tests demonstrate that preferential refreezing of small drops can contribute to slight increases in both $Z_{DR}$ and $L_{DR}$, and subtle decreases in $\rho_{hv}$, which are qualitatively consistent with observed refreezing signatures. However, the treatment of freezing in these tests (and those of K13) is highly simplified and unrealistic. Thus, a more robust treatment of the microphysics is needed, and will be presented in the next section with the full-physics simulations.

5. Full-physics refreezing simulations

The full-physics model provides a more realistic treatment of hydrometeor refreezing. We use the thermodynamic profile from an observed ice pellet case documented in Hanesiak and Stewart (1995) to initialize the simulations (Fig. 5). Our intent is not to replicate the event, but simply to use the thermodynamic profile to produce ice pellets within our model. This profile provides a classic setup for ice pellets, including a deep near-surface $T_w < 0^\circ C$ layer with a minimum temperature $< -6^\circ C$. Similar thermodynamic profiles were documented in several studies where ice pellets and the polarimetric refreezing signature were observed concurrently (K13; Ryzhkov et al. 2016; Van Den Broeke et al. 2016; Tobin and Kumjian 2017; K20).

The specific ice nucleation mechanism within the surface $T_w < 0^\circ C$ layer is not well understood for fully melted hydrometeors. For simplicity, we begin the simulations with the assumption that all drops nucleate ice at $T_w = -5^\circ C$ ($\sim$850 m; Fig. 5). This threshold is comparable to those used in Reeves et al. (2016) and Ryzhkov and Zrnić (2019), and corresponds well with observations near the RFL top in cases with fully melted hydrometeors (e.g., K13; Tobin and Kumjian 2017; K20).

a. Control simulations

We begin with a set of control simulations using the baseline assumptions in the microphysical model. The simulated $f_{nv}$ is shown in Fig. 6. Although all drops are nucleated simultaneously at $T_w = -5^\circ C$, smaller drops complete freezing first (i.e., at higher altitudes) owing to a combination of the longer time needed for larger drops to freeze and their greater fall speeds. In fact, the largest particles considered here ($>3.5$ mm) have not yet entirely frozen when they reach the surface. This simulation reveals that a more realistic treatment of the microphysics still results in the preferential freezing of small

![Fig. 4. Impacts of sequential bin freezing of liquid drops on (a) $Z_H$, (b) $Z_{DR}$, (c) $L_{DR}$, and (d) $\rho_{hv}$ for the preferential refreezing of small drops with the standard assumption of increased-canting-angle distribution with freezing (black) and constant-canting-angle distribution width of freezing drops (blue). The $x$ axes denote the diameter of the largest frozen particle (in mm) of the particle size distribution (Fig. 3), where larger particles remain liquid. As such, the leftmost values denote an all-liquid distribution, and the rightmost values denote an ice pellet distribution.](image-url)
drops. However, despite the preferential refreezing of small drops, profiles of the simulated polarimetric radar variables assuming an increase in $s$ with freezing (Fig. 7, black lines) reveal that no enhancement in $Z_{DR}$ was produced. This differs from the simple refreezing tests because the difference in relative permittivity across the PSD is not as drastic in the case of natural refreezing, because the larger freezing drops have already formed an ice shell instead of remaining completely liquid. The presence of even a thin ice shell on the larger particles therefore is sufficient to decrease their relative permittivity enough to preclude a $Z_{DR}$ enhancement. There is also no $K_{DP}$ enhancement, whereas the other polarimetric radar variables behave as expected: $Z_{H}$ decrease throughout the RFL, LDR features a 4.67-dB enhancement, and $r_{hv}$ decreases slightly ($<0.001$).

Output from simulations where $s$ remains constant at 10$^{-8}$ during freezing are also shown in Fig. 7 (blue lines). This simulation produced a slight (0.034 dB) $Z_{DR}$ enhancement, yet this is much smaller than the 0.20-dB enhancement suggested by the simple calculations in section 4. A negligibly small ($<0.0005$ km$^{-1}$) $K_{DP}$ enhancement is also produced. The LDR enhancement is now reduced to only 0.23 dB, owing to the assumed more stable particle orientations during freezing. Thus, although stable particle orientations do produce polarimetric radar profiles of the refreezing signature that are qualitatively consistent with expectations, the magnitudes of the $Z_{DR}$, $K_{DP}$, and LDR enhancements are unobservably small. This necessitates an alternative hypothesis to explain the refreezing signature.

b. Ice shell asymmetry simulations

The polarimetric radar forward operator assumes that the inner and outer spheroid axis ratios are equivalent; however, such equivalence between the core and particle exterior is not always observed. Johnson and Hallett (1968) noted that ventilated drops have radially asymmetric thermal energy and mass exchanges around a particle, and that particle tumbling could only partially restore radially symmetric freezing rates around the particle. Thus, a freezing drop that is not tumbling as it falls may form a thicker ice shell on its bottom (upwind side) owing to increased freezing rates there. Murray and List (1972) documented this asymmetry for drops in a wind tunnel (Fig. 8); noting that, on most occasions, such internal asymmetry would not alter the overall particle shape.

Within the constraints of the analytic two-layer spheroid model used for scattering calculations, ice shell asymmetry owing to asymmetric freezing around ventilated particles can be modeled crudely by reducing the axis ratio of the inner slushy core relative to that of the entire particle. This simulates particles with a thicker ice shell on the bottom than on the sides; however, equally thick ice shells on top and bottom falls short of capturing a thin top ice shell owing to the limitations of concentric spheroids. Two axis ratio assumptions used in these simulations are depicted schematically in Fig. 9. The first assumption, which we call the “extreme” axis ratios case, maximizes asymmetry by having the inner slushy spheroid maintain contact with the exterior of the particle along the major axis (Fig. 9a), leading to exaggerated slushy core axis ratios as freezing progresses. The variation of this inner spheroid axis ratio with particle size and $f_{m}$ is shown in Fig. 10a. The second assumption, which we call the “moderate” axis ratios case, simulates by varying the inner axis ratio linearly as a function of $f_{m}$ between the extreme inner axis ratios and the exterior particle axis ratio (Fig. 9b). This allows the ice shell to grow inward along the particle’s major axis as freezing progresses. At the onset of freezing, the axis ratio is equivalent to that of the particle exterior, decreases as freezing progresses, but then increases back to the original axis ratio (Fig. 10b). Thus, the
inner axis ratio remains larger than the extreme version, and symmetry is restored as freezing progresses. This transition back to a symmetric ice shell is realistic because the rate of heat transfer through a thin ice shell is greater than through a thicker ice shell, which will partially offset asymmetric refreezing. Further, if the particles’ distribution of canting angles broadens with freezing, symmetry is partially restored because of this increased wobbling (Johnson and Hallett 1968).

The simulated polarimetric radar variables for the extreme and moderate inner axis ratio tests are shown in Fig. 11, for both increasing and constant \( \sigma \). The onset of refreezing at 850 m produces a quick reduction in \( Z_H \) with the formation of a thin ice shell on most particle sizes within the distribution, with more gradual reductions thereafter as the ice shells thicken. \( Z_H \) values are within 0.2 dB of each other for all four simulations, with slightly larger values for the more stably oriented particles (which have more mass aligned horizontally, on average). A \( Z_{DR} \) enhancement is produced for all simulations, ranging from a 0.13-dB increase for the moderate inner axis ratio and increasing-\( \sigma \) simulation, to a 0.76-dB increase for the simulation with extreme inner axis ratio and constant \( \sigma \). The altitude of the maximum \( Z_{DR} \) decreases with magnitude, ranging from 90 to 260 m below the RFL top. For simulations with increasing \( \sigma \), \( Z_{DR} \) is lower, owing to increased particle wobbling with freezing, which limits the depth over which the inner spheroid axis ratios can exert a significant effect on \( Z_{DR} \). In simulations with constant \( \sigma \), \( Z_{DR} \) is maximized with the smaller inner spheroid axis ratios, particularly for the larger particles (with larger \( f_m \)) that contribute more to \( Z_H \) versus the fully frozen smaller particles. The \( Z_{DR} \) is enhanced over a greater depth in these simulations because the largest drops do not completely refreeze.

![Figure 7](image1.png)

**Fig. 7.** Simulated vertical profiles of (a) \( Z_H \), (b) \( Z_{DR} \), (c) \( K_{DP} \), (d) LDR, and (e) \( \rho_{hv} \) for the refreezing simulation shown in Fig. 6 with the standard assumption of increased-canting-angle distribution with freezing (black) and constant-canting-angle distribution width of freezing drops (blue).

![Figure 8](image2.png)

**Fig. 8.** Photograph of a 1-mm-thick section of a partially frozen drop with a maximum diameter of 5 mm from Murray and List (1972). The liquid has been removed, and the section sits on a glass plate. The inner and outer peripheries of the ice shell are visible as the darker outlines.
All four simulations also produce $K_{DP}$ enhancements, with the smallest (0.030° km$^{-1}$ at 50 m below the RFL top) in the case of moderate inner spheroid axis ratios and increasing $\sigma$, and the largest (0.046° km$^{-1}$ at 110 m below the RFL top) for the extreme axis ratios and constant $\sigma$. $K_{DP}$ responds similarly to $Z_{DR}$ in terms of relative magnitudes, altitudes at which the maximum value occurs, and enhancement depths, for the same reasons. However, the $K_{DP}$ maxima are located above the $Z_{DR}$ maxima because these variables are weighted differently to different moments of the PSD: $K_{DP}$ for liquid drops is proportional to the fourth to fifth moments whereas $Z_{DR}$ is more closely related to greater moments (Kumjian et al. 2019).

An LDR enhancement also is observed for all simulations, ranging from 2.3 dB in the moderate axis ratio simulation with constant $\sigma$ to 8.6 dB in the extreme axis ratio simulation with increasing $\sigma$. LDR is larger for the extreme inner axis ratio cases, as more extreme axis ratios enhance depolarization for a given particle orientation. The LDR maxima are located below the $Z_{DR}$ maxima for each simulation, with a greater offset for simulations with increasing $\sigma$. As particles freeze, reductions in relative permittivity tend to reduce both $Z_{DR}$ and LDR; however, whereas $Z_{DR}$ also decreases with increasing $\sigma$, increasing $\sigma$ tends to increase LDR. For simulations with increasing $\sigma$, LDR does not recover to its original liquid-drops value, implying that nonzero $f_m$ and greater wobbling outweigh the decrease in relative permittivity.

There is a negligibly small (<0.001) increase in $\rho_{nv}$ for all four simulations at the onset of refreezing owing to a lower diversity in $Z_{DR}$ that is attributable to particles having similar inner spheroid axis ratios. $Z_{DR}$ diversity across the PSD increases as more particles freeze (i.e., $Z_{DR}$ for the smaller particles decreases while larger drops maintain larger $Z_{DR}$), reducing $\rho_{nv}$ for all simulations. Values of $\rho_{nv}$ down to 0.9926 are produced for the extreme inner axis ratio simulations because of the dispersion of liquid water fractions and inner spheroid axis ratios across the PSD, though this value is still very high. The height of minimum $\rho_{nv}$ is lower for constant-$\sigma$ simulations because this assumption maximizes the dispersion in $Z_{DR}$ as freezing progresses.

c. Truncated PSD simulations

Particles > 3.4 mm were not able to refreeze completely in these simulations, which produced a refreezing signature that extended to the ground. These simulations are now performed with a maximum binned particle diameter of 2.0 mm to determine how sensitive the simulated refreezing signatures are to the presence of the larger particles. The results (Fig. 12) reveal similar polarimetric radar variable vertical profiles as with the original PSD, but with the refreezing signature constrained to a shallower depth. Refreezing now occurs over a depth of 260 m, with all enhancements in $Z_{DR}$, $K_{DP}$, and LDR, and the reduction in $\rho_{nv}$ contained entirely within the region of decreasing $Z_{HI}$ toward the ground.

d. Equivalent raindrop fall speed simulations

The 3.4–3.9-dB decrease in $Z_{HI}$ in these previous full-physics simulations is less than the 6.7-dB decrease expected from theory (Fig. 4) and the 5–7-dB decreases observed in K13. This discrepancy is attributable to the modeled reductions in particle fall speeds with freezing, which increases the number concentration of particles owing to the assumed flux conservation. This increase in particle number density partially offsets the $Z_{HI}$ reduction from the decrease in relative permittivity. The disagreement with observations suggests that the reductions in fall speeds prescribed here may be too severe.

The density change from liquid to ice results in a small increase in hydrometeor size with freezing. Assuming that at least some of this size increase contributes to an increase in cross-sectional area, some associated reduction in fall speed can be expected. However, studies suggest that a reduction in fall speed is not always observed. Spengler and Gokhale (1972) indeed found that, although some drops had significant fall speed reductions with freezing, others had minimal changes to or even increases in fall speed with freezing. Further, Nagumo and Fujiyoshi (2015) and Nagumo et al. (2019) documented a bimodal distribution of ice pellet fall speeds, with the bulk of
ice pellets falling at speeds similar to raindrops and another group (with smaller maximum dimensions) falling at speeds similar to very small, dry hailstones or low-density ice spheres (e.g., graupel).

Simulations with a 2.0-mm maximum particle size, extreme inner axis ratio, and constant $\sigma$ were repeated to compare the original fall speed parameterization (Fig. 13, blue lines) to assuming that freezing particle fall speeds continue to follow those of equivalent-mass raindrops (Fig. 13, black lines). The simulations with particle fall speeds following those of equivalent-mass raindrops result in a slightly deeper RFL, but otherwise produce similar profiles to Fig. 12, with the exception of $Z_H$ and $K_{DP}$. The values of $Z_H$ and $K_{DP}$ are larger aloft owing to larger particle number concentrations (a consequence of assuming flux conservation and prescribing the particle flux at the surface). This also leads to a greater maximum $K_{DP}$ enhancement within the RFL, with a peak value of 0.046 km$^{-1}$.

e. Ice crystal contact nucleation simulations

Up to this point, all drops nucleated ice at a single height in the model. We now consider the impact that nucleating ice via contact with ice crystals has on the simulated polarimetric radar variables. Here, we assume that drops nucleate ice at some distance below the $T_w = -5^\circ$C level and the surface, such that this mean depth is approximated as

$$H = \frac{4}{n\pi(D + D_i)^2},$$

where $n$ is the ice crystal concentration and $D_i$ is the maximum dimension of the ice crystal.

We assume a population of columnar ice crystals of $n = 10^6$ m$^{-3}$ and $D_i = 0.1$ mm, which are values typical of pristine ice crystals (e.g., Lamb and Verlinde 2011). Equation (9) follows from Stewart et al. (1990), where the inverse of $n$ is equivalent to the volume swept out by the raindrop within $H$. We thus abandon preferential freezing of the smaller drops in favor of the larger drops nucleating first, as larger drops are expected to collide with an ice crystal over a shallower depth than smaller drops. Indeed, resulting $f_m$ for these simulations indicate that the larger particles nucleate and freeze aloft, whereas particles in the smallest two size bins remain liquid to the surface (Fig. 14). We ignore the impact of any ice–liquid collisions that would occur between the larger, freezing particles and the smaller, liquid drops, which would require more complex heat balance equations to account for the freezing of accreted liquid (e.g., Phillips et al. 2014, 2015). Further, the PSD would need to be defined above the RFL with a larger number concentration of smaller drops and smaller maximum particle dimensions to produce the observed
surface PSD used herein. Last, although the Hanesiak and Stewart (1995) profile is supersaturated with respect to ice, we ignore any depositional growth of the ice crystals, for simplicity.

We continue to assume a 2.0-mm maximum binned-particle diameter, and default back to decreasing fall speeds with freezing. The prescribed ice crystals are best described as elementary needles (Matrosov et al. 1996), and contribute negligibly to $Z_H$ ($< -45$ dBZ) following calculations from Ryzhkov et al. (1998). For simplicity, we neglect the contribution of these crystals to the polarimetric radar variables. The only difference between these simulations and those in section 5c is the ice nucleation method, so we compare the results of the current simulations (Fig. 15) to those in Fig. 12. The current simulations initiate changes in the polarimetric radar variables 40 m lower than previous simulations, owing to the height at which the first drops begin to freeze (810 vs 850 m). The salient features of the polarimetric refreezing signature are again produced, with similar $Z_H$ reductions, maximum $Z_{DR}$, $K_{DP}$, and LDR values, and minimum $\rho_{hv}$ values. A notable difference between Figs. 12 and 15 is in $Z_{DR}$ and $K_{DP}$. Earlier simulations featured gradual “bumps” of enhanced values, whereas current simulations feature “sharper” $Z_{DR}$ and $K_{DP}$ enhancements. Further, the relative heights of these two enhancements are no longer consistent with previous simulations. The LDR maxima above the $\rho_{hv}$ minima are still situated below the $Z_{DR}$ maxima; however, the $Z_{DR}$ and $K_{DP}$ maxima are now nearly collocated.

6. Discussion

The refreezing model and coupled polarimetric radar forward operator successfully reproduced many characteristics of the observed refreezing signature, including enhancements in $Z_{DR}$, $K_{DP}$, and LDR, and reductions in $\rho_{hv}$ within a region of decreasing $Z_H$ toward the ground. However, these features were only simulated by adjusting some original assumptions within the model; the control version was unable to reproduce the refreezing signature’s salient features. The primary factor we found to be important was implementing a representation of an asymmetric ice shell around freezing particles. Simulations with ice shell symmetry were insufficient to produce meaningful enhancements in $Z_{DR}$ or $K_{DP}$, even with constant-particle-canting-angle distribution with freezing.

a. Comparison to observations

Simulations with the maximum particle diameter set to 2.0 mm produced polarimetric radar variable profiles consistent with observations, where the features associated with the RFL are contained within a shallow layer of $Z_H$ decreasing toward the ground (K13; Tobin and Kumjian 2017; K20; see also Fig. 1). This suggests that a smaller maximum particle size...
may be appropriate for refreezing simulations in order for all particles to freeze completely aloft. A 6.6-dB $Z_H$ reduction, which is in agreement with the observed 5–7-dB decreases reported by K13, was realized in the truncated PSD simulation in which freezing drops maintain their raindrop-equivalent fall speeds. Simulations with decreasing fall speeds produced $Z_H$ reductions $< 4$ dB, suggesting that the prescribed fall speed reductions in K12 may be too severe.

Observed maximum $Z_{DR}$ values within the RFL are 0.5–1.0 dB greater than $Z_{DR}$ at the RFL top (e.g., K13; Tobin and Kumjian 2017; K20; Fig. 1). Realistic $Z_{DR}$ enhancements were produced for the truncated PSD simulations, except for the simulation with moderate axis ratios and increasing $\sigma$ (that simulation only produced a $< 0.5$-dB enhancement). Interestingly, the $Z_{DR}$ enhancement produced from constant $\sigma$ and moderate inner spheroid axis ratios is nearly equal to the enhancement produced by an increase in $\sigma$ and extreme inner spheroid axis ratios. These two simulations also had similar $K_{DP}$ profiles, but produced the greatest differences in the LDR maxima. Thus, LDR observations in refreezing signatures may be useful in determining whether increased particle wobbling or asymmetric freezing are more likely during refreezing. In the full PSD simulations with 4.0-mm maximum diameter, in which the largest particles did not completely refreeze, only the constant-$\sigma$ simulations produced the expected $Z_{DR}$ enhancement.

Within the RFL, K20 noted a $\sim 3$-dB enhancement in LDR over the minimum detectable limit of the radar they used ($\sim 30$ dB; Fig. 1). For the simulation with increasing $\sigma$ and maximum particle sizes of 2.0 mm, a 3.1-dB increase over this $-30$-dB limit is produced. For the full PSD simulations, enhancements of 1.2–5.8 dB over the $-30$-dB limit are produced for simulations with constant $\sigma$ and extreme inner

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**Fig. 13.** Simulated vertical profiles of (a) $Z_H$, (b) $Z_{DR}$, (c) $K_{DP}$, (d) LDR, and (e) $\rho_{hv}$ for a maximum particle size of 2.0 mm, assuming the extreme inner spheroid axis ratio and constant canting angle with freezing for fall speed decreases with freezing (blue), and with no decrease in fall speed (black).

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**Fig. 14.** Liquid water mass fraction ($f_{lw}$; shaded according to color bar) of drops freezing via contact nucleation with small columnar crystals, as discussed in section 5e. Dark gray indicates that hydrometeors are liquid drops, and light gray indicates solid ice pellets.
spheroid axis ratios, and for both simulations with increasing $\sigma$. As expected, LDR is greater for simulations with increasing $\sigma$. However, these simulations do not account for particle deformations/bulges or other causes of asymmetric liquid portions within particles, such as the propagation of dendritic growth around and into a particle from a surface nucleation site prior to the formation of an ice shell (e.g., Blanchard 1957; Spengler and Gokhale 1972) or the distribution of liquid within deformed/bulged particles. K20 speculate these factors may also play a role in the observed LDR enhancement. Thus, this comparison between the simulated and observed LDR does not imply that an increase in $\sigma$ with freezing is necessary to produce the observed refreezing signature, because these other factors were not accounted for in the model.

Among all simulations, only those with constant $\sigma$ can produce profiles consistent with observations of both the expected $Z_{\text{DR}}$ and LDR enhancements. From these results, the oft-assumed canting angle distribution width of 40° for ice pellets may be too high, and that more stable particle orientations during freezing may play a key role in producing the refreezing signature. Given the signature’s possible dependence on the distribution of the slushy core within freezing particles, as well, it is difficult to assess which $\sigma$ value may be appropriate for ice pellets.

A maximum S-band $K_{\text{DP}}$ value of 0.046° km$^{-1}$ is obtained from the full PSD simulation with extreme axis ratios and constant $\sigma$. This value is comparable to the maximum value of 0.05° km$^{-1}$ observed at S band in K13. However, the presence of anisotropic ice crystals could also produce $K_{\text{DP}}$ values comparable to observations (e.g., Matrosov et al. 1996; Kumjian et al. 2016). The magnitudes of all simulated $\rho_{hv}$ minima are far greater than observed, likely owing to the spheroidal symmetry assumed in our model and exclusion of any such anisotropic ice crystals observed to be present in some refreezing layers (e.g., K20).

The relative altitudes of the $Z_{\text{DR}}$, LDR, and $K_{\text{DP}}$ maxima, and $\rho_{hv}$ minima within the region of decreasing $Z_{\text{H}}$ are also consistent with observations. In all simulations with ice shell asymmetry, the $K_{\text{DP}}$ maxima occur above the $Z_{\text{DR}}$ maxima, which is consistent with K13 observations (Fig. 1). Maximum $Z_{\text{DR}}$ values are typically observed near the lowest temperature in the near-surface $T_w < 0^\circ$C layer (e.g., K13; Tobin and Kumjian 2017; K20), but the model results here indicate that this height can vary depending on various model parameters and choices. Based on these modeling results, we speculate that the distribution of the slushy core within particles, canting angle distributions, PSD, and ice-nucleation temperature have more influence on the height of the $Z_{\text{DR}}$ maximum than the height of the lowest temperature. The fact that the $Z_{\text{DR}}$ maxima are often observed near the lowest-temperature height may be more of a coincidence based on the environments in which ice pellets form, as well as other possible similarities among ice pellet events, such as the PSD and ice-nucleation temperatures.

Observations from K20 indicate that the LDR peak occurs beneath the $Z_{\text{DR}}$ peak, which our model reproduces. The model produces a minimum in $\rho_{hv}$ beneath the LDR maxima; however, it is unclear if this result is supported by
the observations. K20 observed the $p_{\text{rh}}$ minimum at nearly the same height as the LDR peak, yet those extrema may arise from similar underlying mechanisms (e.g., particle deformations) not accounted for in our modeling study.

Simulations where particles nucleated ice via collisions with a population of ice crystals produced similar results to those from our truncated PSD simulations. The only appreciable difference is that the simulations produced $Z_{\text{DP}}$ and $K_{\text{DP}}$ maxima at nearly the same height, which is inconsistent with observations at S band. Interestingly, K13 noted $K_{\text{DP}}$ maxima below the $Z_{\text{DR}}$ in refreezing cases observed at C band, but it is unknown whether this difference is attributed to the different radar wavelengths or microphysical differences among the cases. However, without a more rigorous treatment of the inclusion of ice crystals—we consider only a single population of nongrowing crystals and a single liquid PSD, and ignore the contributions of the crystals to the radar variables—we cannot offer any further explanation for these discrepancies.

b. New proposed hypotheses

In light of the simulation results presented here, we propose several new hypotheses to explain the observed polarimetric refreezing signature. Although previous work favored preferential refreezing of small drops (K13), and its occurrence was documented in an ice pellet case (K20), we found that it is insufficient to produce the observed $Z_{\text{DR}}$ enhancement. That is because, upon nucleating ice, the larger particles form an ice shell too quickly to create meaningful differences in relative permit-


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particles are poorly understood, requiring further investigation. With appropriate constraints on the distribution of the liquid core, canting angle distribution width, and fall speeds of freezing particles within a more complex microphysical model, the polarimetric refreezing signature can be modeled—and explained—more accurately.

7. Conclusions

A bin microphysical refreezing model was developed and coupled with a polarimetric radar forward operator to simulate hydrometeor refreezing and determine the underlying cause of the observed polarimetric-radar refreezing signature. We found that preferential refreezing of the smaller particles alone is incapable of producing the observed Z_DR enhancement. In contrast, a Z_DR enhancement can result from preferential refreezing only if the particles freeze asymmetrically with thicker ice shells on their bottoms, which would be expected for stably oriented, ventilated particles undergoing freezing (e.g., Johnson and Hallett 1968; Murray and List 1972). Simulated Z_DR and K_Dp enhancements thus result from an alignment of the oblate, unfrozen particle cores with the horizontal, whereas the wobbling of these particles is responsible for an LDR enhancement within the RFL. Comparison of Z_DR and LDR enhancement magnitudes in a large sample of ice pellet cases could shed some light onto the relative importance of these two effects.

These results are applicable to the production of ice pellets that form via the refreezing of fully melted hydrometeors. It remains unknown whether asymmetric freezing occurs during all such ice pellet events, or if there are cases where the ice shells are instead symmetric around a freezing particle, in which case no Z_DR enhancement is expected. In the case of ice pellets forming via the refreezing of partially melted hydrometeors, asymmetric ice shells are possible, albeit unlikely, only if the melting snowflakes aloft have collapsed into raindrop shapes, but are otherwise not expected to form. Further, the impact of concurrent precipitation types (e.g., rain and ice pellets) on the polarimetric radar signature is unknown, especially since all observational studies documenting the refreezing signature have been for pure ice pellet events. Despite these newly introduced questions, the identification of even a subset of ice pellet events using operational polarimetric radar can be valuable to improving the detection and forecasting of winter precipitation.

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