Transmission and Reflection of Upward-Propagating Rossby Waves in the Lowermost Stratosphere: Importance of the Tropopause Inversion Layer

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ABSTRACT: Extreme stratospheric vortex states are often associated with extreme heat flux and upward wave propagation in the troposphere and lower stratosphere; however, the factors that dictate whether an upward-directed wave in the troposphere will reach the bottom of the vortex versus being reflected back to the troposphere are not fully understood. Following Charney and Drazin, an analytical quasigeostrophic planetary-scale model is used to examine the role of the tropopause inversion layer (TIL) in wave propagation and reflection. The model consists of three different layers: troposphere, TIL, and stratosphere. It is shown that a larger buoyancy frequency in the TIL leads to weaker upward transmission to the stratosphere and enhanced reflection back to the troposphere, and thus reflection of wave packets is sensitive not just to the zonal wind but also to the TIL’s buoyancy frequency. The vertical–zonal cross section of a wave packet for a more prominent TIL in the analytical model is similar to the corresponding wave packet for observational events in which the wave amplitude decays rapidly just above the tropopause. Similarly, a less prominent TIL both in the model and in reanalysis data is associated with enhanced wave transmission and a weak change in wave phase above the tropopause. These results imply that models with a poor representation of the TIL will suffer from a bias in both the strength and phase of waves that transit the tropopause region.

KEYWORDS: Rossby waves; Stratospheric circulation; Stratosphere-troposphere coupling; Tropopause

1. Introduction

Tropospheric wave activity propagating upward into the stratosphere can subsequently influence surface climate through at least two mechanisms. First, this wave activity can be absorbed within the stratospheric polar vortex (or alternatively ducted away), modifying the strength of the vortex, and subsequently influencing tropospheric climate and weather (Chen and Robinson 1992; Baldwin and Dunkerton 1999; Polvani and Kushner 2002; Limpasuvan et al. 2004; Polvani and Waugh 2004; White et al. 2019). An anomalously weak vortex is often associated with the negative phase of the Arctic Oscillation (the surface signature of the Northern Annular Mode) in the following weeks or months (Baldwin and Dunkerton 2001; Limpasuvan et al. 2004; Polvani and Waugh 2004; Kidston et al. 2015; White et al. 2021). A second mechanism involves wave reflection: if these upward-propagating waves encounter a reflecting surface, they can boomerang back to the troposphere and impact surface climate directly (Charney and Drazin 1961; Harnik and Lindzen 2001; Perlwitz and Harnik 2003, 2004; Shaw and Perlwitz 2013; Dunn-Sigouin and Shaw 2015). However, it is unclear whether additional processes can also lead to reflection. This work proposes that a second process, namely, the temperature profile in the tropopause region, can also modify upward wave propagation and reflection.

Within both the troposphere and stratosphere the temperature variations with height are approximately linear and the buoyancy frequency is nearly constant within each layer (Fig. 1). This is not the case in the vicinity of the tropopause. Rather, the temperature rapidly increases just above the tropopause, and this abrupt temperature change is associated with a sharp local maximum in buoyancy frequency. The buoyancy frequency squared is maximum in the layer between 300 and 100 hPa and values are more than double the tropospheric value and ~20% larger than the mean stratospheric values. This is demonstrated in Fig. 1, which shows the $N^2$ profile computed from ERA-5.1 (Hersbach et al. 2020) averaged between 1979 and 2018 between 45° and 75°N.¹ This layer is known as the tropopause inversion

¹ Note that the peak $N^2$ in Fig. 1 is around $4 \times 10^{-4}$ s⁻², which is lower than the values found by Hegglin et al. (2010) and Grise et al. (2010). This difference likely arises because we are using log-pressure coordinates, so the resulting $N^2$ differs from the $N^2$ of geometric height by a factor of $(T_{ref}/T)^2$ [Eq. (1.1.12) in Andrews et al. (1987)], where $T_{ref}$ is the reference temperature included in the calculation of the scale height $H$. We elect to show the log-pressure $N^2$ here in order to maintain consistency with section 2, which also uses log-pressure coordinates. However, it is important to note that in geometric height, the rapid changes in $N^2$ on either side of the TIL are even more pronounced.

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layer (TIL) and its thickness increases with latitude from about 500 m at 33°N to about 3 km at 60°N (Birner 2006). A TIL is present and even stronger in the Southern Hemisphere as well (Fig. S1 in the online supplemental material).

The recent work by Weinberger et al. (2021) highlighted the relationship between the strong increase in buoyancy frequency just above the tropopause and events of weakened wave propagation in reanalysis data. Specifically, Weinberger et al. (2021) composited reanalysis data depending on the relative anomaly of the Eliassen–Palm (EP) flux at 100 hPa versus that at 300 hPa in the Northern Hemisphere extratropics. They showed that if the index of refraction is defined according to Charney and Drazin (1961) and not according to Matsuno (1970), events of weakened wave propagation between these levels are associated with a more negative index of refraction just above the tropopause due to a sharp local maximum in buoyancy frequency. In contrast, this effect could not be accounted for using the index of refraction of Matsuno (1970), since Matsuno’s index of refraction assumes that the buoyancy frequency is constant. Therefore, Weinberger et al. (2021)’s results demonstrate that the buoyancy frequency just above the tropopause (i.e., in the TIL) is of crucial importance for upward wave flux in the lower stratosphere in reanalysis (Sjoberg and Birner 2014). Here we build on these results and further consider the role of the TIL for the transmission and reflection of the upward-propagating wave activity originating in the troposphere.

The goal of this work is to investigate the relationship between tropospheric and stratospheric vertical EP fluxes analytically, and specifically to demonstrate that the reflection of waves is sensitive to rapid changes of the buoyancy frequency.

This paper is organized as follows: In section 2, we describe an atmospheric model with three different layers: the lowest and the highest represent the troposphere and stratosphere, respectively, and the middle layer is the tropopause inversion layer (TIL). In section 3, an upward-directed wave is imposed in the troposphere, and the subsequent reflection and transmission of this wave are analyzed as a function of the buoyancy frequency in each layer. The results are summarized and placed in the context of previous work in section 4.

2. Theory

Charney and Drazin (1961) investigate quasigeostrophic adiabatic planetary-scale waves on the β plane. Using their Eq. (3.4) for the vertical dependence of the meridional wind V (more details on the origin of this equation are provided in the appendix), we solve the governing linear eigenvalue equation:

$$\frac{d^2 V}{dz^2} - \frac{1}{H} \frac{dV}{dz} + \frac{N^2}{f_0} \beta \left( k^2 + \ell^2 \right) V = 0,$$

where $H = 8000$ m is the scale height [Holton and Hakim 2013, Eq. (7.41)], $u_0$ is the constant zonal wind, the zonal wavenumber $k = 3.14 \times 10^{-2} \text{ m}^{-1}$ corresponding to wave-1 at 60°N and the meridional wavenumber $l = 9.47 \times 10^{-2} \text{ m}^{-1}$, which corresponds to a meridional wavelength of $6.63 \times 10^3$ km or $60°$ in latitude.

The buoyancy frequency is defined following Eq. (1.1.13) of Andrews et al. (1987):

$$N^2 = \frac{R}{H} \left( \frac{dT}{dz} + \kappa T \right).$$

The full dependence for the meridional wind of such a stationary wave is of the following form:

$$u' = e^{i(kx + \ell y)} V(z).$$

We solve Eq. (1) for an atmosphere with three different layers: the troposphere, stratosphere and the tropopause inversion layer (TIL) between them (a schematic is shown in Fig. 2). We allow for a different buoyancy frequency in each layer while the zonal wind is constant at all levels. (Allowing for different zonal wind would require altering the matching conditions at the boundaries). Holding the zonal wind constant at each level explicitly removes the mechanism many previous studies have shown to lead to reflection. Assuming an incident wave propagating upward to the TIL from below, part of the wave will ultimately be transmitted to the stratosphere and will continue upward, and part will be returned to the troposphere. The final steady-state wave can be divided into upward and downward components in the bottom two layers. In contrast, the stratospheric component is assumed to be purely upward-propagating as there is no source of waves at the top of the atmosphere. After normalizing the amplitude for the upward tropospheric wave to equal one (i.e., this is a linear model, and hence this choice is arbitrary), we are left to
where $h_1$ is the top of the troposphere and $h_2$ the top of the TIL while the stratosphere extends to infinity; $U_{\text{TIL}}$ and $U_{\text{strat}}$ are the upward transmission coefficients in the TIL and stratosphere, respectively; and $D_{\text{trop}}$ and $D_{\text{TIL}}$ are the downward reflection coefficients in the troposphere and TIL, respectively. We follow the terminology of Lindzen and Tung (1978) in our definition of the transmission and reflection coefficients, which is different than the widely used convention in optics [e.g., section 9.3.2 in Griffiths (1999)]$^2$. In each layer the vertical wavenumber $n$ obeys the following relation [see Charney and Drazin 1961, Eq. (3.5)]:

$$n^2 = -\frac{1}{4H^2} \frac{N^2}{f_0^2} (k^2 + l^2 - \frac{B}{u_0})$$

(7)

Note that with this definition of the vertical wavenumber Eq. (1) transforms to the canonical form:

$$\frac{d^2 V'}{dz^2} + n^2 V' = 0.$$

(8)

Upward-directed waves are present in all three atmospheric layers, and downward-directed waves are present only in the troposphere and TIL. The (piecewise-constant) buoyancy frequency is allowed to vary between layers.

For the case of a constant background-state zonal wind ($u_0$ constant), the boundary conditions specified in Eqs. (5.1)–(5.5) of Charney and Drazin (1961) can be simplified to the statement that the meridional wind ($w'$) and vertical wind ($w$) are continuous at both interfaces.

The form of the vertical wind is $w'(x, z) = e^{ikz}W(z)$ where $W(z)$ is defined by Eq. (2.18) in Charney and Drazin (1961).

For a steady wave this definition implies the following:

$$W(z) = -\frac{f_0}{N^2} u_0 \frac{dV}{dz}.$$

(9)

Recall that the zonal wind is constant on both sides of the interface, thus the four matching conditions of $V$ and $W$ at the two interfaces are as follows:

$$V_{\text{trop}} = V_{\text{TIL}} \text{ at } z = h_1,$$

(10)

$$\frac{1}{N^2_{\text{trop}}} \frac{dV_{\text{trop}}}{dz} = \frac{1}{N^2_{\text{TIL}}} \frac{dV_{\text{TIL}}}{dz} \text{ at } z = h_1,$$

(11)

$$V_{\text{strat}} = V_{\text{TIL}} \text{ at } z = h_2,$$

(12)

$$\frac{1}{N^2_{\text{TIL}}} \frac{dV_{\text{TIL}}}{dz} = \frac{1}{N^2_{\text{strat}}} \frac{dV_{\text{strat}}}{dz} \text{ at } z = h_2.$$

(13)

Substituting Eqs. (4), (5), and (6) and rearranging, yields (following some tedious algebra) the transmission and reflection coefficients:

\[\]
\begin{align}
D_{\text{strat}} &= -e^{m_{\text{strat}} h_{\text{strat}}} \frac{(m_{\text{strat}} - m_{\text{strat}}^*)}{m_{\text{strat}}} e^{m_{\text{strat}} h_{\text{strat}}} \\
U_{\text{strat}} &= U_{\text{TIL}} e^{m_{\text{strat}} h_{\text{strat}}} + D_{\text{strat}} e^{m_{\text{strat}} h_{\text{strat}}}. 
\end{align}

Section 3 will describe in detail the dependence of the transmission and reflection coefficients on the model parameters.

The EP flux has long been used in theoretical studies of wave–mean flow interaction and in stratosphere–troposphere coupling in particular. It provides a tool for the description of wave propagation in mean zonal flows as well as for the analysis of the influence of waves on the mean flow (Eliassen and Palm 1960; Andrews and McIntyre 1976; Edmon et al. 1980; Andrews et al. 1983; Tanaka et al. 2004; Jucker 2021). For linear steady conservative waves the divergence of EP flux is zero and the EP flux is identical at all height levels [see Eq. (3.6.1) of Andrews et al. 1987].

The vertical component of the EP flux vector (\(F_z\)) is given by [see Eq. (10.20) of Holton and Hakim 2013]
\begin{equation}
F_z = \rho_0 f \mathbf{R} \mathbf{V} T / N^2 H, 
\end{equation}
where \(T\) is the temperature, the asterisk stands for the complex conjugate, and \(R = 287 \text{ K}^{-1} \text{ kg}^{-1}\).

The temperature \(T\) is given in terms of the meridional wind using the thermal wind balance [see Eq. (7.43) of Holton and Hakim 2013]:
\begin{equation}
T = -\frac{H}{R} \frac{dV}{dz}. 
\end{equation}

Using Eqs. (4), (5), and (6), the temperature in each layer is
\begin{align}
T_{\text{trop}} &= -\frac{H}{R} f k (m_{\text{trop}} e^{m_{\text{trop}} z} + m_{\text{trop}}^* D_{\text{trop}} e^{m_{\text{trop}}^* z}), \\
T_{\text{TIL}} &= -\frac{H}{R} f k (m_{\text{TIL}} U_{\text{TIL}} e^{m_{\text{TIL}} z} + m_{\text{TIL}}^* D_{\text{TIL}} e^{m_{\text{TIL}}^* z}), \\
T_{\text{strat}} &= -\frac{H}{R} f k m_{\text{strat}} U_{\text{strat}} e^{m_{\text{strat}} z}.
\end{align}

3. Results

We solve the three layer model of section 2 for linear steady conservative waves with constant buoyancy frequency in each layer. To understand how the variation of \(N^2\) between the layers modifies wave propagation, we examine how the solution coefficients [Eqs. (14)–(17)] depend on the buoyancy frequencies in the three layers. We present wave 1 in the main text, while similar figures for wave 2 are presented in the supplemental material.

Figures 3a and 3b show the transmission coefficient in the stratosphere (\(U_{\text{strat}} U_{\text{TIL}}^*\)), and the reflection coefficients in the troposphere and TIL (\(D_{\text{trop}} D_{\text{TIL}}^*\) and \(D_{\text{TIL}} D_{\text{TIL}}^*\), respectively). The stratospheric buoyancy frequency squared (\(N_{\text{strat}}^2\)) is held fixed while the buoyancy frequency in the TIL (\(N_{\text{TIL}}^2\)) is allowed to vary from the tropospheric value up to 3 times the tropospheric value. In Fig. 3a, the stratospheric buoyancy frequency equals the tropospheric value (\(N_{\text{strat}}^2 = N_{\text{trop}}^2\)) and the case of a uniform atmosphere, where the three layers are identical in stability, is shown with a gray line at \(N_{\text{TIL}}^2 = N_{\text{strat}}^2 = 1\). The value of \(|U_{\text{strat}}|^2\) is one in a uniform atmosphere and decreases with a stronger TIL by \(\sim 60\%\) as \(N_{\text{TIL}}^2 / N_{\text{strat}}^2\) increases from 1 to 3 (Fig. 3a). In Fig. 3b, the stratospheric buoyancy frequency is more realistic (\(N_{\text{TIL}}^2 / N_{\text{strat}}^2 = 2.24\), see Fig. 1). The value of \(|U_{\text{strat}}|^2\) decreases with a stronger TIL by \(\sim 15\%\) as \(N_{\text{TIL}}^2 / N_{\text{strat}}^2\) increases from 1 to 3 (Fig. 3b). Similarly, \(|D_{\text{trop}}|^2|\) and \(|D_{\text{TIL}}|^2|\) equal zero in a uniform atmosphere and increase with a stronger TIL for both values of the stratospheric buoyancy frequency shown in Fig. 3. In summary, the transmission coefficient in the stratosphere decreases, while the reflection coefficients in the troposphere and TIL increases, with a stronger TIL.

Next, we consider how the TIL modifies the wave flux reaching the stratosphere in terms of the vertical component of the EP flux (\(F_z\)). For a pure plane wave, the EP flux denotes the group propagation of wave activity density. For a combination of two pure plane waves, like we have here—an upward plus a downward component (upward and downward undulatory arrows on Fig. 2)—the EP flux denotes the net flux of wave...
activity, which is the net effect of partial cancellation between the upward and downward-propagating waves. To get a sense of the wave activity propagation, we calculate the EP flux, as well as its upward and downward components, meaning, the EP flux of the purely upward-propagating components or of the purely downward-propagating components. This is done by calculating the amount of EP flux in each layer using all terms on the right-hand side of the Equations for $V$ and $T$ [Eqs. (4)–(6), (20)–(22)] to calculate the total $F_z$ (upward and downward undulatory arrows on Fig. 2), and then only including the first term in these equations (only upward undulatory arrows on Fig. 2) to calculate the upward $F_z$, or only the second terms in the same equations for the downward $F_z$ (downward undulatory arrows). The total $F_z$ is identical in all levels (not shown) as expected for linear steady conservative waves [see Eq. (3.6.1) of Andrews et al. (1987), and section 2]. However, the flux propagating upward or downward varies, with the downward flux constituting the reflected wave activity and the upward flux the wave activity that is capable of transiting to the upper layer.

**Fig. 3.** (top) The transmission and reflection coefficients from Eqs. (14) ($|D_{TIL}|^2$, red line), (16) ($|D_{trop}|^2$, yellow line), and (17) ($|U_{trop}|^2$, blue line) as a function of $N_{TIL}^2/N_{trop}^2$. The stratospheric buoyancy frequency is fixed at (a) $N_{strat}^2/N_{trop}^2 = 1$ and (b) $N_{strat}^2/N_{trop}^2 = 2.24$ (see Fig. 1). The gray line $N_{TIL}^2/N_{trop}^2 = 1$ in (a) and (c), which corresponds to $N_{TIL} = N_{strat} = N_{trop}$, is the case of a uniform atmosphere. (bottom) The EP flux vertical component is shown as a function of buoyancy frequency on the same coordinates used in (a) and (b). A blue line corresponds to $F_z$ at 10 hPa where only upward terms for the meridional wind and temperature are included (upward undulatory arrows in Fig. 2), i.e., the first term on the right-hand sides of Eqs. (4)–(6) and (20)–(22). In red and yellow are downward $F_z$ at 500 and 250 hPa, respectively, where the second term on the right-hand side of the same equations (downward undulatory arrows in Fig. 2) are included. The TIL extends from 300 to 70 hPa for all panels.

Figures 3c and 3d show the upward EP flux in the stratosphere ($F_{z10hPa,up}$) and the downward flux in both the troposphere and TIL ($F_{z500hPa,down}$ and $F_{z250hPa,down}$ respectively). In Fig. 3c, $N_{strat}^2 = N_{trop}^2$ while in Fig. 3d, $N_{strat}^2/N_{trop}^2 = 2.24$. In Fig. 3c, $F_{z10hPa}$ decreases by ~65% as $N_{TIL}^2/N_{trop}^2$ increases from 1 to 3. In Fig. 3d, $F_{z10hPa}$ decreases by ~15% as $N_{TIL}^2/N_{trop}^2$ increases from 1.5 to 3. Similarly the downward terms in both the troposphere and TIL increase in absolute value as $N_{TIL}^2/N_{trop}^2$ increases. In other words, Figs. 3c and 3d shows that upward EP flux in the stratosphere decreases, while the downward flux in both the troposphere and TIL increases, with a stronger TIL. These dependencies on $N_{TIL}^2$ highlight the importance of the TIL for wave propagation into the lower stratosphere.

Thus far, we have considered the effect of $N_{TIL}$ for a given $N_{strat}$, and now we broaden the analysis to allow both $N_{strat}$ and $N_{TIL}$ to vary. Specifically, we allow the buoyancy frequency in the stratosphere to also vary, and then explore the effects on the transmission and reflection coefficients. Figure 4
Along the diagonal the buoyancy frequency in the stratosphere and TIL are equal, where the asterisk stands for complex conjugate. The point (1, 1), which corresponds to \( N_{\text{strat}} = N_{\text{trop}} \), is the case of uniform atmosphere. A horizontal line at \( N_{\text{TIL}}^2/N_{\text{trop}}^2 = 2.6 \) and a vertical line at \( N_{\text{strat}}^2/N_{\text{trop}}^2 = 2.24 \) correspond to realistic time-mean values (see Fig. 1).

Along the diagonal the buoyancy frequency in the stratosphere and TIL are equal.

The general form of the solution in the stratosphere (downward term exists), but in practice \( D_{\text{TIL}} \) is nearly zero along the diagonal such that the form of solution in the TIL is in fact identical to that in the stratosphere.

For \( N_{\text{TIL}}^2 > N_{\text{strat}}^2 \) (i.e., a strongly stratified TIL) the transmission coefficient in the stratosphere decreases with increasing \( N_{\text{TIL}}^2 \) (on the left of the diagonal in Fig. 4a). Similarly, the reflection in both the troposphere and TIL is larger with stronger TIL (Figs. 4b,d). Hence the results shown in Figs. 3a and 3b are robust to the value of \( N_{\text{strat}}^2 \) being considered.

Supplemental Figs. S2 and S3 explore the sensitivity of the transmission and reflection coefficients to the speed of the zonal wind. While the values of the transmission and reflection coefficients depend on the zonal wind \( u_0 \), the sensitivity to \( N_{\text{TIL}}^2 \) evident in Fig. 4 is also evident in supplemental Figs. S2 and S3. The effect of the TIL on the transmission and reflection coefficients for wave 2 is qualitatively similar to that for wave 1 (a similar structure but different values on supplemental Fig. S4).

Additional insight as to the dependence on buoyancy frequency in both the TIL and stratosphere can be gleaned by focusing on the propagation of the EP flux into the stratosphere. Figure 5 shows the dependence of the upward-directed EP flux terms in the troposphere, stratosphere, and the TIL (\( F_{\text{500 up}}, F_{\text{10 up}}, \) and \( F_{\text{250 up}} \) respectively) on \( N_{\text{TIL}}^2 \) and \( N_{\text{strat}}^2 \). In a homogeneous or close to homogeneous atmosphere, \( F_{\text{up}} \) is equal in all layers [all panels in Fig. 5, near the point (1, 1)]. In the case of equal buoyancy frequencies in the TIL and stratosphere, \( F_{\text{10 up}}, F_{\text{250 up}} \) are equal (the diagonal in Fig. 5d). \( F_{\text{500 up}} \) decreases with \( N_{\text{TIL}}^2 \) provided \( N_{\text{TIL}}^2 > N_{\text{strat}}^2 \) (left of the diagonal, Fig. 5b). This is also true for \( F_{\text{250 up}} \) relative to \( F_{\text{500 up}} \) (Figs. 5d,e): the upward wave activity flux in the stratosphere weakens as \( N_{\text{TIL}}^2 \) increases. These effects are all consistent with those shown in Figs. 3c and 3d where \( N_{\text{strat}}^2 \) was held fixed.

To complete the picture, Fig. 6 presents the downward EP flux terms, \( F_{\text{500 down}} \) and \( F_{\text{250 down}} \). (Recall that the stratospheric downward term is zero [Eq. (6)] and the corresponding panels are omitted compared to Fig. 5). The downward terms vanish identically in a homogeneous atmosphere since reflection occurs only in the presence of jumps in the buoyancy frequency (as is evident from the vanishing of the reflection coefficients in this case; Figs. 4b,d). In the case of similar buoyancy frequency in the TIL and stratosphere, \( F_{\text{250 down}} \) equals zero as there is no jump in buoyancy frequency between the TIL and the stratosphere (Fig. 6b along the diagonal, consistent with Fig. 4d). The key point is that the downward terms increase in absolute value with a stronger TIL where \( N_{\text{TIL}}^2 > N_{\text{strat}}^2 \) (Figs. 6a,c). The impact
on wave-2 EP flux is similar to that on wave-1 EP flux (supplementary Figs. S5 and S6).

We now consider the effect of the TIL on the wave structure in height both in our model and in reanalysis data. Recent work highlighted the relationship between a strong increase in buoyancy frequency just above the tropopause and events of weakened upward wave propagation in reanalysis data (Weinberger et al. 2021). Weinberger et al. (2021) composited each DJF day over the period 1979 to 2018 into one of three categories: “transmitting” and “decaying” composites and all other events. A day in which the standardized vertical EP flux component at 60°N and 100 hPa ($\hat{F}_z,100hPa$) is greater than the standardized vertical EP flux component at 300 hPa ($\hat{F}_z,300hPa$) by at least 1.5 standard deviations (STD) after a 1-day lag is composited in the transmitting composite. Similarly, the decaying composite includes every day in which $\hat{F}_z,100hPa$ is smaller than $\hat{F}_z,300hPa$ by at least 1.5 STD. Note that in absolute terms, $F_z$ decays between 300 and 100 hPa most of the time so there is substantially more $F_z$ at 300 hPa than at 100 hPa. Hence the transmitting composite includes days in which this layer leads to less decay than usual while the decaying composite includes days in which this layer leads to more decay than usual. As shown in Fig. 6 of Weinberger et al. (2021), the buoyancy frequency for these composites differ in the TIL region, with a more pronounced TIL in the decaying composite. We composite the meridional wind for these composites from the European Centre for Medium-Range Weather...
Forecasts Reanalysis version 5.1 (ERA5.1) model-level dataset over the period 1979–2018 (Hersbach et al. 2020). We interpolate the data on the native hybrid vertical coordinate to pressure levels with a resolution of 10 hPa between the levels of 70–400 hPa for a more accurate analysis at and above the tropopause as compared to earlier reanalyses. The data were downloaded at a horizontal resolution of 1.25° latitude × 1.25° longitude.

Figures 7a and 7b present the vertical–zonal cross section of wave-1 meridional wind at 60°N in the transmitting versus decaying composites. Figure 7b shows the composite mean of events with enhanced wave propagation (the transmitting composite) and Fig. 7a shows the composite mean of events with reduced wave propagation in the analytical model: there is a rapid change in phase in the TIL, and wave amplitude in the stratosphere is relatively weak (cf. Figs. 7a and 7c). The kink is lower in the analytical model than in observations; however, a deeper TIL (e.g., the upward extent reaches unrealistically to 50 hPa rather than 70 hPa) leads to a better match of the vertical extent. The kink is similar, though less pronounced, for realistic climatological values of $N^2$ in each layer (not shown).

The upward- and downward-directed waves are shown separately in supplemental Fig. S7, and these calculations confirm that the downward-directed wave has greater amplitude in the presence of an exaggerated TIL.

It is evident from Fig. 7 that the presence or absence of a TIL has a large impact on the rate of change of the phase of the vertical profile for the meridional wind, and specifically a large change in $N^2$ leads to a “kink” in the phase lines. This effect is illustrated by the thick line in Fig. 7, which is drawn along maximum meridional wind at each pressure level: both the analytical model in Fig. 7c and the reanalysis decaying composite in Fig. 7a demonstrate a shift of about 100° in phase within the TIL. We now explain how this kink arises using geometric arguments. We focus here on the upward-propagating wave.

The wavelengths in the zonal and vertical directions of a wave described by Eqs. (3) and (6) are given by $\lambda_z = 2\pi/k$ and $\lambda_z = 2\pi/n$, respectively. We define the angle $\alpha$ between the phase line and a vertical line perpendicular to the interface as

$$\tan \alpha = \frac{\lambda_z}{\lambda_z} = \frac{n}{k}.$$  

Recall that $k$ is constant in all layers while $n$ varies between layers. Thus, $\alpha$ is a function of the $n$ in each layer, which in turn is a function of the buoyancy frequency in each layer. Specifically, a larger buoyancy frequency (such as in the TIL in Fig. 7c) leads to a
larger $n$, and thus to a larger $\tan \alpha$ and a larger $\alpha$. A larger $\alpha$ implies a stronger deviation from the vertical, as is indeed evident when Fig. 7c is compared to Fig. 7d or supplemental Figs. S7a and 7b. Hence a stronger TIL leads to a more pronounced “kink” in the wave phase above the tropopause.

4. Discussion and summary

Probabilistic predictability of surface climate may extend for longer times if the factors governing the fate of an upward pulse of anomalous tropospheric wave activity could be better predicted. This anomalous wave activity can alternately reach the stratospheric polar vortex and weaken it, be ducted away from subpolar latitudes toward the subtropics, or be reflected back to the troposphere. While much has been written on the factors governing whether a wave pulse will be confined to the pole versus ducted to the subtropics (Matsuno 1970; Dickinson 1968; Chen and Robinson 1992), the only known factor that has been shown to influence reflection is vertical gradients of the zonal wind (Charney and Drazin 1961; Harnik and Lindzen 1964).

Fig. 7. The vertical–zonal cross section for wave-1 meridional wind [Eq. (3)] in m s$^{-1}$. We multiply by $e^{-z/2H}$ to remove the density dependence. (a) The “decaying” composite of Weinberger et al. (2021) at 60$^\circ$N and (b) the “transmitting” composite of Weinberger et al. (2021) at 60$^\circ$N. The bottom row is based on our analytical model results also at 60$^\circ$N. (c) Strong TIL where $N_{\text{TIL}}^2/N_{\text{trop}}^2 = 5$ with a sharp reduction of the buoyancy frequency in the stratosphere ($N_{\text{strat}}^2/N_{\text{trop}}^2 = 1.5$). (d) TIL does not exist ($N_{\text{TIL}}^2/N_{\text{trop}}^2 = N_{\text{strat}}^2/N_{\text{trop}}^2 = 2.24$). A thick line is drawn along the maximum meridional wind at each pressure level. Two horizontal lines at 300 and 70 hPa present the lower and upper boundaries of the TIL, respectively.
2001; Perlwitz and Harnik 2003, 2004; Shaw and Perlwitz 2013; Dunn-Sigouin and Shaw 2015). Nevertheless, it is known that upward-propagating wave reflection affects tropospheric wave-1 fields (Perlwitz and Harnik 2004), surface temperature and the NAO phase. Specifically, seasons with multiple wave reflection events are characterized by a positive NAO phase (Shaw and Perlwitz 2013), and wave reflection is associated with cold temperatures over much of North America (Cohen et al. 2021). Here we demonstrate that reflection of a wave packet can arise due to the presence of a tropopause inversion layer (TIL) due to a local maximum in the buoyancy frequency.

We construct an atmospheric model with three different layers: troposphere, TIL, and stratosphere. The model allows for a different buoyancy frequency in each layer while the zonal wind is constant and equal in all levels. An upward-directed wave is imposed in the troposphere, and the subsequent reflection and transmission of this wave is analyzed as a function of the buoyancy frequency in each layer. Under the boundary conditions relevant to our setup, we solve the analytical model and discuss the transmission and reflection coefficients and wave activity variations.

The transmission coefficient in the stratosphere equals one in a uniform atmosphere and decreases with a stronger TIL significantly as the buoyancy frequency in the TIL increases. Similarly, the reflection coefficients in both the TIL and stratosphere equal zero in a uniform atmosphere and increase with a stronger TIL.

The TIL also modifies the amount of upward vertical Eliassen–Palm (EP) flux (\(F_z\)) reaching the stratosphere. The total \(F_z\) is identical in all levels (due to the non-acceleration theorem); however, the flux propagating upward or downward varies, with the downward flux constituting the reflected wave activity and the upward flux the wave activity that is capable of transiting the TIL. The upward \(F_z\) in the stratosphere decreases with increasing TIL strength while the downward terms in both the troposphere and TIL increase in absolute value as the buoyancy frequency in the TIL increases. This result is also consistent with the linear three-dimensional model of Chen and Robinson (1992), who find that a bigger jump in \(N^2\) leads to weaker EP flux in the stratosphere. These results are in agreement with the reanalysis composites of Weinberger et al. (2021), who found that a stronger TIL is associated with weaker wave amplitudes in the stratosphere for a given strengthened tropospheric \(F_z\). Neither of these studies focused on wave reflection back to the troposphere, which as we show here can be caused by a strong TIL.

The downward \(F_z\) terms are zero in a homogeneous atmosphere consistent with the reflection coefficients equalling zero, as there is no reflection when there are no jumps in buoyancy frequency. In a case of similar buoyancy frequency in the TIL and stratosphere (i.e., no jump in buoyancy frequency between the TIL and the stratosphere), the downward \(F_z\) and the reflection coefficient in the TIL equal zero. The downward terms increase in absolute value with a stronger TIL. These changes in the transmission and reflection coefficients and in the vertical EP flux also are evident in the zonal–vertical cross section of wave phase. If the TIL and stratosphere are indistinguishable from one another, then the wave phase does not change at the interface. A similar effect is seen when we examine a reanalysis-based composite of events in which the TIL is relatively weak and transmission of waves between the troposphere and stratosphere is strong. In contrast, a pronounced TIL leads to a kink in wave phase, and a similar effect is evident in an observational composite of events in which the TIL is strong and wave amplitude decays strongly from the troposphere to the stratosphere.

The results in this paper are highly idealized, and two possible extensions of this work would be to explore realistically smooth vertical variations in the buoyancy frequency and to include the effects of vertically varying zonal winds. The latter is, perhaps, the more important, since an abrupt change in buoyancy frequency is associated with a potential vorticity sheet only in the presence of vertical shear. Such a sheet is likely to be important for the propagation of Rossby waves. For example, it allows the possibility of Rossby edge waves trapped at the tropopause.

The results in this paper are relevant for the vertical propagation of stationary waves, which are the dominant source of waves in the stratosphere of the Northern Hemisphere winter. Future work should consider how transient waves are affected by the presence of a TIL. These results also assume that latitudinal variations of the index of refraction are relatively weak in the region of upward wave propagation, and as shown in Fig. 1 of Weinberger et al. (2021), this assumption is true for the TIL.

Our results have implications for observational studies that typically diagnose wave reflection as events in which the total \(F_z\) is negative (Dunn-Sigouin and Shaw 2015), which corresponds to around 22% of all days (Dunn-Sigouin and Shaw 2015; Weinberger et al. 2021). Our results indicate that wave reflection can still occur even if the total \(F_z\) is positive. Our model allows us to cleanly separate the upward versus the downward flux and thus diagnose wave reflection; however, a similar clean separation is likely not possible in reanalysis data where the total flux is a superposition of the upward- and downward-directed fluxes.

Our results also have implications both for comprehensive general circulation models and reanalysis data products. Accurately capturing the magnitude of the TIL in a model or reanalysis requires high resolution, and there is evidence that even ~250 m resolution is not enough (Bell and Geller 2008). The ERA-5.1, which has 137 vertical levels and a nominal vertical resolution of 300 m above and at the tropopause, likely is more realistic than previous reanalyses, but weather and climate models are almost always run with relatively coarser resolution. This may lead to a misrepresentation of upward wave propagation and reflection in this region, with a subsequent inability to represent correctly both upward troposphere–stratosphere coupling and downward stratosphere–troposphere coupling. Specifically, a model with a too-weak TIL will likely suffer from too few reflection events, and subsequently underestimate the effect that reflection has on surface climate (Cohen et al. 2021; Shaw and Perlwitz 2013).

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Data availability statement. ERA5.1 data are freely available on the Copernicus Climate Data Store.

APPENDIX

The Origin of the Wave Equation in Eq. (1)

The goal of the appendix is to show the origin of our model wave equation [Eq. (1)]. We begin with the equation for perturbation potential vorticity linearized about a basic state with zonal wind that changes both meridionally and vertically but not zonally. The quasigeostrophic linearized potential vorticity equation, using the notation of Andrews et al. (1987) in their Eqs. (4.5.1), (4.5.2), and (4.5.4), is

\[ \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \Phi_y + \nu \nabla^2 \Phi_y = H', \]  

(A1)

where \( H' \) denotes nonconservative terms, which we take as zero,

\[ \Phi_y = \psi_{xx} + \psi_{yy} + \rho_0 \left( \frac{f^2}{N^2} \psi_z \right)_z \]  

(A2)

is the perturbation quasigeostrophic potential vorticity, and

\[ \nabla_y = \beta - \nabla_{yy} - \rho_0^{-1} \left( \frac{f^2}{N^2} \nabla_z \right)_z \]  

(A3)

is the basic-state northward quasigeostrophic potential vorticity gradient. All notations are identical to that in Andrews et al. (1987). \( N \) is constant in each layer while the zonal wind \( \bar{u} \) is fixed in the entire atmosphere \( (\bar{u} = \bar{u}_z = 0) \). The perturbation potential vorticity varies sinusoidally in \( y \). We can then define the perturbation streamfunction [analogously to Eq. (4.5.26) in Andrews et al. (1987)] as

\[ \psi' = e^{i2/H} \text{Re} \left[ \Psi(z) e^{ikx +Hy -kx} \right]. \]  

(A4)

Substitution of Eq. (A4) in Eq. (A2) leads to

\[ \psi' = \text{Re} \left[ \left( -k^2 + \frac{f^2}{H} \right)e^{i2/H} \Psi + \rho_0^{-1} \frac{f^2}{N^2} \left( e^{i2/H} \Psi \right)_z \right] e^{ikx +Hy -kx}, \]  

(A5)

and the mean-state quasigeostrophic potential vorticity [Eq. (A3)] becomes \( \bar{\nabla}_y = \beta \). Substitution of Eq. (A5) in Eq. (A1) leads to (after some algebra)

\[ 0 = \left( \bar{u} - c \right) \frac{\rho_0}{N^2} \left( e^{i2/H} \Psi \right)_z \]  

\[ - \left( \frac{\rho_0 f^2}{N^2} \right)_z + \rho_0 \frac{eta f^2}{N^2} \left( \bar{u} - c \right) \left( k^2 + \frac{f^2}{\beta} \right) - 1 \left[ e^{i2/H} \Psi \right]. \]  

(A6)

Equation (A6) is identical to the equation of Charney and Drazin (1961) at the very beginning of their section 3 if \( V \) is replaced by \( e^{i2/H} \Psi \). Charney and Drazin (1961) show that Eq. (A6) can be transformed into the canonical form

\[ n^2 = \frac{N^2}{f^2} \left[ \frac{\beta}{\bar{u} - c} \right] - \frac{1}{\bar{u} - c} \frac{f^2}{N^2} \left( \rho_0 \left( \frac{\nabla_z}{N^2} \pi \right)_z \right) - \left( k^2 + \frac{f^2}{\beta} \right). \]  

(A7)

For constant \( N, \beta, c = 0, \) and \( \rho_0 = \rho_0 \text{exp}(-z/H) \), Eqs. (A6) and (A7) become

\[ 0 = \left( e^{i2/H} \Psi \right)_z - \frac{1}{\bar{u}} \left( e^{i2/H} \Psi \right)_z \]  

\[ - \frac{N^2}{f^2} \frac{\beta}{\bar{u}} \left( k^2 + \frac{f^2}{\beta} \right) - 1 \left[ e^{i2/H} \Psi \right] \]  

(A8)

\[ n^2 = \frac{N^2}{f^2} \left( \frac{1}{\bar{u}} - \left( k^2 + \frac{f^2}{\beta} \right) \right) - \frac{1}{4H^2}. \]  

(A9)

Equation (A8) is identical to Eq. (1) in the main text and to Eq. (3.4) of Charney and Drazin (1961) if \( V \) is replaced by \( e^{i2/H} \Psi \). Equation (A9) is identical to Eq. (7) in the main text and to Eq. (3.4) in Charney and Drazin (1961).

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