THE EFFECT OF WINDS AND OCEAN CURRENTS ON THE ANNUAL VARIATION IN LATITUDE

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ABSTRACT

An elliptical excursion (mean diameter 20 feet), described annually by the pole of instantaneous rotation over the earth's surface, has been deduced from precise astronomic observations of latitude. According to Jeffreys and Rosenhead, most of this excursion can be ascribed to the seasonal variation in the distribution of matter (air, water, snow and vegetable matter). The most important single factor is the gain, in winter, of air mass over the Asiatic continent, and the corresponding loss over the Atlantic and east Pacific oceans.

In this paper, it is shown that the effect of winds may be far from negligible. Here the important factor is the pressure against the Himalayas during the monsoon winds. Ocean currents account for only 1 per cent of the excursion. It is suggested that the latitude observations, together with related astronomic observations concerning an annual change in the length of day, provide a promising method for measuring certain fundamental modes in the annual oscillation of the atmosphere.

1. Introduction

Very small annual fluctuations have been reported for all three components of the vector representing the angular velocity of the earth. The fluctuations in the x- and y-components (which determine the position, in the body of the earth, of the instantaneous axis of rotation in the meridians of Greenwich and 90°E) represent a "wobble" of the earth; they are derived from precise measurements of latitude. Fluctuations in the z component[2] (through the mean north pole) are interpreted in terms of a slight seasonal variation in the length of the day. These have been discovered more recently from precise measurements of longitude.

Possible causes fall into two categories: those due to changes in the tensor of inertia because of shifts in the distribution of matter within the earth, ocean and atmosphere; and those due to changes in the angular momentum of atmosphere and ocean (winds and currents).

Jeffreys (1915–16) has found the effect of changes in the tensor of inertia to be of the right order of magnitude to account for the observed changes in latitude. Regarding a possible effect of changes in angular momentum, he writes: "If it should subsequently be found that there are large periodic motions relative to the earth as a whole that do produce a considerable angular momentum, then these will have to be included in a more detailed treatment."

In the case of seasonal variations in the length of the day, it is found that the effect of winds is larger than that due to the tensor of inertia (Van den Dungen *et al.*, 1950; Mintz and Munk, 1951; Munk and Miller, 1950). This circumstance has led us to examine the effect of angular momentum on the wobble of the earth.

2. The Eulerian equations

Let $\mathbf{H}_P = \mathbf{H}_E + \mathbf{H}_A$ be the angular momentum of the planet Earth, made up of the angular momentum $\mathbf{H}_E$ of the solid earth and $\mathbf{H}_A$ of the atmosphere. If no torque acts on the planet,

$$d\mathbf{H}_E/dt = -\,d\mathbf{H}_A/dt. \quad (1)$$

On the other hand, we may wish to adopt the point of view that the atmosphere exerts a torque $\mathbf{L}$ on the solid earth, so that

$$d\mathbf{H}_E/dt = \mathbf{L}. \quad (2)$$

At present, (1) and (2) are merely two ways of saying the same thing. Yet, they imply different approaches to the problem that require different types of measurements.

Equations (1) and (2) refer to an absolute (nonrotating) coordinate system. The "absolute" derivative d/dt can be transformed to a derivative $(\dot{\ })$ relative to a coordinate system fixed to the rotating earth, according to

$$d/dt = \boldsymbol{\omega}\cdot\times \quad + (\dot{\ }),$$

where $\boldsymbol{\omega}(t)$ is the angular velocity of the wobbling

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[2] This is *not* the Kimura z component.

earth. Thus,

$$dH_E/dt = [\boldsymbol{\omega} \times \quad + (\dot{\ })] \int_E \rho \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r})\, dV$$

$$= \int_E \rho[r^2\dot{\boldsymbol{\omega}} - (\mathbf{r}\cdot\dot{\boldsymbol{\omega}})\mathbf{r} + (\mathbf{r}\cdot\boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega})]\, dV,$$

where the integral is taken over the whole volume of the solid earth. Here $\mathbf{r}$, the vector from the earth's center to any point of the (rigid) earth, does not vary in time; but $\boldsymbol{\omega}$ does, because of the wobble. Next,

$$dH_A/dt = [\boldsymbol{\omega} \times \quad + (\dot{\ })] \int_A \rho \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r} + \boldsymbol{v})\, dV$$

$$= \int_A \rho[(\mathbf{r}\cdot\boldsymbol{\omega})(\mathbf{r} \times \boldsymbol{\omega}) + 2\mathbf{r} \times (\boldsymbol{\omega} \times \boldsymbol{v}) + \mathbf{r} \times \dot{\boldsymbol{v}}]\, dV,$$

where the integral is taken over the atmosphere, and $\boldsymbol{v}$ designates motion relative to the earth (winds). Variations in ρ have been neglected. Here $\boldsymbol{\omega}$ has been taken as constant[3], but $\mathbf{r}$ varies with time, and $\dot{\mathbf{r}} = \boldsymbol{v}$. Finally,

$$L = \int \mathbf{r} \times \mathbf{s}\, dS, \tag{3}$$

where $\boldsymbol{s}$ is the *stress* exerted by the atmosphere on the solid earth. The integral is to be evaluated over the entire surface.

For numerical calculations, we shall need to write (1) and (2) in scalar form. Let θ designate the co-latitude, ϕ the east longitude, and $\boldsymbol{v} = u\boldsymbol{\phi}_1 + v\boldsymbol{\theta}_1$ designate the wind vector, with the components u from the west and v from the north. We neglect the effect of vertical winds. For a Cartesian coordinate system fixed in the earth, let the x-axis intersect the equator at the meridian of Greenwich, the y-axis intersect the equator at 90°E, and the z-axis point to the mean position of the north pole. Let a designate the radius, and $A, A, A(1 + 1/\tau), E, F, G$ the moments and products of inertia of the solid earth; τ is the number of sidereal days in the Newcomb period (434). The expression $1 + 1/\tau$ allows for the equatorial bulge. Then, equating x- and y-components in (1), we eventually obtain

$$\begin{aligned} l - (\tau/\omega)\dot{m} &= \lambda_i + \lambda_v + \lambda_a, \\ m + (\tau/\omega)\dot{l} &= \mu_i + \mu_v + \mu_a, \end{aligned} \tag{4}$$

where

$$l = \omega_x/\omega, \quad m = \omega_y/\omega, \quad 1,$$

are the direction cosines of the earth's instantaneous axis of rotation, with $\omega \approx \omega_z$ designating the mean angular velocity of the earth.

The component equations (4) contain lengthy integrals on the right side. It is convenient to separate these into three parts, as shown. The term

$$\lambda_i = -\frac{\tau\kappa}{A} F = -\frac{\tau\kappa}{A} a^2 \int_A \rho \sin\theta \cos\theta \cos\phi\, dV, \tag{5}$$

and a similar term[4] for μ_i, are those investigated by Jeffreys. They incorporate the effects of slight differences, at various times of the year, in the products of *inertia* E and F due to the variable distribution of mass over the earth's surface. The principal components are air, water, snow, and vegetable matter. The coefficient κ allows for the elastic yielding in the earth under the pressure of superficial matter, when E and F are to be considered due only to this matter. Rosenhead (1928) sets κ equal to 0.6.

The term[4]

$$\lambda_v = -(2\tau a/A\omega) \times \int_A \rho(u \cos\theta \cos\phi + v \cos^2\theta \sin\phi)\, dV \tag{6}$$

takes into account *velocity* relative to the earth (winds), and the term

$$\lambda_a = -(\tau a/A\omega^2) \int_A \rho(\dot{u} \cos\theta \sin\phi - \dot{v} \cos\phi)\, dV \tag{7}$$

acceleration relative to the earth. In this derivation, we have neglected all products and squares of small quantities (see Jeffreys, 1915–16).

If one proceeds in a similar fashion from the torque point of view, (2) and (3) ultimately lead to

$$\begin{aligned} l - (\tau/\omega)\dot{m} &= \lambda_t, \\ m + (\tau/\omega)\dot{l} &= \mu_t, \end{aligned} \tag{8}$$

where

$$\lambda_t = -\frac{\tau}{A\omega^2} L_y = -\frac{\tau a}{A\omega^2} \int (s_\theta \cos\phi - s_\phi \cos\theta \sin\phi)\, dS, \tag{9}$$

and to a similar[4] expression for μ. The integral is to be evaluated over the earth's surface.

There is some evidence that the fluid core of the earth does not fully participate in irregularities of rotation with periods as short as one year. We shall not be able to take this possibility into account. In the extreme case that the core does not participate at all, A should pertain only to the earth's mantle, and the computed values of λ and μ should accordingly be increased by about 15 per cent over what they are in this study.

[3] This means that, in computing the effect of atmospheric motion on the wobble of the earth, we neglect the effect of this wobble back onto the atmosphere.

[4] Expressions for μ can always be found from those for λ by replacing ϕ by $\phi - \frac{1}{2}\pi$, hence by replacing $\cos\phi$ by $\sin\phi$, and $\sin\phi$ by $-\cos\phi$.

3. Comparison of momentum and torque approaches

It is instructive to consider first the earth wobble induced by some simple, specified motion; for example, by a hula dancer of mass M standing on the Greenwich meridian at some co-latitude θ_0, moving her center of mass by a small amount $b \sin \sigma t$ in an east-west direction. Then $u = \sigma b \cos \sigma t$, $\dot{u} = -\sigma^2 b \sin \sigma t$, and the terms inducing the earth's wobble, according to (5), (6) and (7), are

$$\lambda_i = -(\tau/A)a^2 M \sin\theta_0 \cos\theta_0 \cos\phi,$$
$$\lambda_v = -2(\tau/A)(\sigma/\omega)abM \cos\theta_0 \cos\sigma t,$$
$$\mu_a = -(\tau/A)(\sigma/\omega)^2 abM \cos\theta_0 \sin\sigma t,$$

with $\mu_i = \mu_v = \lambda_a = 0$. Here $\phi = (b \sin \sigma t)/(a \sin\theta_0)$. The variable part of $\cos\phi$ is nearly $-\frac{1}{2}\phi^2$. The ratio λ_v/λ_i between the wobble induced by shift in mass over that induced by the motion is of the order $(\sigma/\omega)(a/b)$, and not necessarily small. On the other hand, λ_a/λ_v is of the order of $\sigma/\omega = 1/365$, and hence small for an annual motion.

Next, consider the stresses induced. The normal pressure on the earth's surface by virtue of her mass only can be considered as due partly to Newtonian attraction by the earth, partly as related to the centrifugal force. The former is balanced exactly by the attraction of the earth by the dancer, according to Newton's third law. The latter gives

$$s_\theta = M\omega^2 a \sin\theta_0 \cos\theta_0, \quad s_\phi = 0.$$

The Coriolis force, by virtue of her motion, yields

$$s_\theta = 2M\omega u \cos\theta_0, \quad s_\phi = 0,$$

and the acceleration induces a stress $s_\theta = 0$, $s_\phi = -M\dot{u}$.

When these three expressions for stress are substituted into (9), they lead to the above formulae for λ_i, λ_v and μ_a, respectively. This merely shows that the torque approach is consistent with our former approach. This conclusion can be extended to a quite general motion, not necessarily small.

The example is no longer trivial if we consider an irregular motion, for which we do not have the analytic expression. We may proceed in two ways: (a) by measuring the position, velocity and acceleration of the dancer, and computing λ_i, μ_i, λ_v, μ_v, λ_a and μ_a according to (5), (6) and (7); (b) by placing the dancer on a platform, and measuring by means of strain gauges the force in excess of her weight at every instant of her motion. The latter method yields $\lambda_v + \lambda_a$ and $\mu_v + \mu_a$, but not λ_i and μ_i.

In the case of the atmosphere, it is customary to resolve stress into components normal and tangential to the geoid. The normal stress has the components

$$s_\theta'' \approx (p_s \omega^2 a \sin\theta \cos\theta)/g, \quad s_\phi'' = 0, \quad s_r'' \approx -p_s,$$

where p_s is the (hydrostatic) atmospheric pressure at the surface. Substituting into (9) and allowing for elastic yield, we have

$$\lambda_t'' = -(\tau\kappa a^2/Ag) \int p_s \sin\theta \cos\theta \cos\phi \, dS. \quad (10)$$

From the point of view of angular momentum, we may regard the surface pressure as a measure of the overlying air mass, according to $p_s \approx g \int_a^\infty \rho \, dr$. Substitution of this into (5) also leads to (10). Hence $\lambda_i = \lambda_t''$, *i.e.*, the effect of the normal stress corresponds to that of the inertial terms. This means that the effect of the tangential stress (components s_θ', and s_ϕ') corresponds to the remaining terms,

$$\lambda_t' = \lambda_v + \lambda_a, \quad \mu_t' = \mu_v + \mu_a.$$

The effect of winds can therefore be found in two ways: (a) by evaluation of λ_v, μ_v, λ_a and μ_a from a (supposedly) known variation of wind and density throughout the atmosphere (section 6, below); (b) by evaluation of λ_t' and μ_t' from the (supposedly) given tangential stress over the surface (section 7, below). Henceforth, we shall neglect the acceleration terms.

The tangential stress arises, aside from molecular stress, from slight pressure differences on the windward and lee sides of obstacles (blades of grass, waves, houses, mountains). It equals $p_s \nabla\eta$, where p_s is the surface pressure, and η is the elevation above sea level. In practice, it is convenient to consider separately the stresses against major mountain-chains (derived from measurements of pressure at weather stations on both sides of the mountains), and the "skin friction" against smaller obstacles (derived from surface winds by statistical considerations).

4. Astronomic observations

From astronomic observations, one can determine the coefficients in

$$l = l_c \cos\Theta + l_s \sin\Theta, \quad m = m_c \cos\Theta + m_s \sin\Theta,$$

where Θ, the sun's longitude, increases from 0 on 21 March to 2π on 21 March of the following year. The coefficients are 0.1 sec at most. For comparison, 0.01 sec or 4.85×10^{-8} rad corresponds to 1.01 ft of actual displacement of the pole of instantaneous rotation on the surface of the earth. It follows from (4) that

$$\lambda = (l_c - m_s T) \cos\Theta + (l_s + m_c T) \sin\Theta,$$

$$\mu = (m_c + l_s T) \cos\Theta + (m_s - l_c T) \sin\Theta,$$

where $T = \tau\sigma/\omega = 1.20$ is the Newcomb period in years, and σ is the annual frequency. From a summary of astronomic observations covering the first half of the 20th century, Jeffreys[5] finds

$$\begin{aligned} \lambda &= -0.013 \cos \Theta + 0.000 \sin \Theta, \\ \mu &= 0.007 \cos \Theta + 0.044 \sin \Theta, \end{aligned} \tag{11}$$

where the coefficients are in seconds and have a standard error of 0.011. These values are to be compared with those obtained from meteorologic considerations.

5. The inertial terms

The inertial terms are the ones investigated by Jeffreys (1915–16) and Rosenhead (1929). The most important part is due to shifts in air masses[6], and these can be computed from variation in station-level (not sea-level) pressures p_s, since $\int_a^\infty \rho \, dr \approx p_s/g$. Equation (5) becomes

$$\lambda_i = -(\tau \kappa a^4/Ag) \times \int_0^{2\pi} \int_0^{\pi} p_s \sin^2 \theta \cos \theta \cos \phi \, d\theta \, d\phi. \tag{12}$$

For comparison with our later results, we shall adopt a slightly different procedure than Jeffreys'. Suppose the station-level pressure is expanded into a series of tesseral surface harmonics (Schmidt, 1935),

$$p_s = \sum_{n=0}^{N} \sum_{m=0}^{n} (C_n^m \cos m\phi + D_n^m \sin m\phi) P_n^m(\theta),$$

where $P_n^m(\theta)$ are seminormalized associated spherical harmonics for integral order, defined by

$$P_n^m(\theta) = \left[\frac{2(n-m)!}{(n+m)!}\right]^{\frac{1}{2}} \left[\frac{(2n)!}{2^n n! (n-m)!}\right] \sin^m \theta \left[\cos^{n-m} \theta - \frac{(n-m)(n-m-1)}{2(2n-1)} \cos^{n-m-2} \theta + \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2 \times 4(2n-1)(2n-3)} \times \cos^{n-m-4} \theta - \cdots \right]$$

for $m > 0$, and

$$P_n^0(\theta) = \left[\frac{(2n)!}{2^n n! n!}\right] \left[\cos^n \theta - \frac{n(n-1)}{2(2n-1)} \cos^{n-2} \theta + \frac{n(n-1)(n-2)(n-3)}{2 \times 4(2n-1)(2n-3)} \cos^{n-4} \theta - \cdots \right].$$

The coefficients C and D are functions of Θ. We find that only the term for which $m = 1$, $n = 2$, contributes to the integral in (12). This gives

$$\begin{aligned} \lambda_i &= -1.451(\tau \kappa a^4/Ag) C_2^1, \\ \mu_i &= -1.451(\tau \kappa a^4/Ag) D_2^1. \end{aligned} \tag{13}$$

Let us suppose, for the moment, that all of the observed variation in latitude is due entirely to shifts in air mass[6]. Then use of the numerical values in (11), with $\kappa = 0.6$, gives (in millibars)

$$p_s = [(0.7 \cos \Theta + 0.0 \sin \Theta) \cos \phi - (0.4 \cos \Theta + 2.3 \sin \Theta) \sin \phi] \sin 2\theta$$

for the simplest pressure-pattern to achieve the observed wobble (fig. 1). When this is compared to observed conditions, its similarity to the cell over the Asiatic continent becomes at once apparent. The northwestern quadrant, centered over America, is dominated in summer by the east Pacific and Bermuda high-pressure areas, and in winter by the Aleutian and Iceland low pressures. These features, which here overshadow a relatively weak opposite effect over the American continent itself, are again in agreement with the required pattern. In the southern hemisphere, such obvious agreement is lacking. The essential effect is therefore the loss in winter of air mass over the Atlantic and east Pacific oceans, and the corresponding gain over the Asiatic continent, as first pointed out by Spitaler (1901).

The magnitude of the required effect, a seasonal change of 4.6 mb at the very most, does not seem at all excessive. In fact, local changes over Asia go up to 20 mb. However, calculations by Rosenhead (1929) indicate that the shifts in air mass account only for two-thirds of the observed effect, and are not quite of the desired phase.

6. Winds, angular momentum approach

The atmosphere as a solid shell.—Consider the atmosphere as a rotating solid shell with its "north pole" at θ_A, ϕ_A, and with angular speed ω_A relative to the earth. It can be shown that, for this case, (6) leads to

$$\lambda_v = \frac{8\pi}{3} \frac{\tau a^4 p_s}{A g \omega} \omega_A \sin \theta_A \cos \phi_A.$$

For future reference, we note that the portion of the shell contained between 20°N and 20°S contributes only

$$\frac{\cos^3 110° - \cos^3 70°}{\cos^3 180° - \cos^3 0°} \lambda_v = 0.04 \lambda_v, \tag{14}$$

or 4 per cent of the total effect.

Let us suppose, for the moment, that all of the observed variation in latitude is due to this shell. This gives

$$a\omega_A \sin \theta_A = 13.4 \text{ cm/sec}, \qquad \Theta = 80 \text{ deg},$$

[5] Prof. Jeffreys has kindly made these revised values available to us. They are to be published in the third edition of *The earth.*

[6] The effect of the actual motion, which must accompany such changes in distribution, can be neglected. The least motion required to accomplish the observed variation in the pressure pattern is altogether negligible compared to the observed winds.

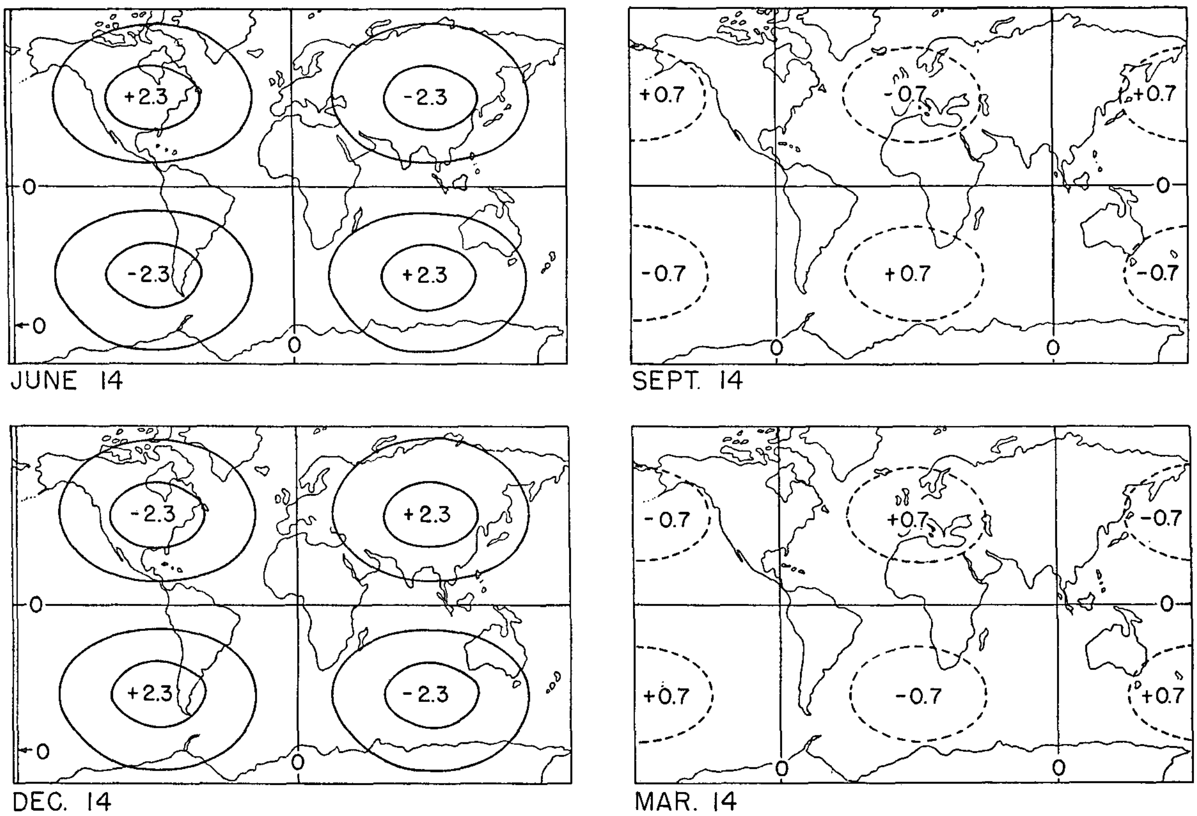

FIG. 1. Surface-pressure variations (millibars) which could account for all of observed wobble. Dates are those for which pressure variations reach maximum or minimum values. Zero isobars in left figures are at 3°E (not zero) longitude.

for the magnitude and time of the maximum winds. Suppose the north pole of the atmospheric shell remains fixed at $\theta_A = 20$ deg over the northwest corner of Siberia. Then the maximum velocity-*difference*, $2\omega_A a$, is 78 cm/sec between 14 June and 14 December. This applies only to the "equator" of the meteorologic shell. At other places, the maximum velocity-difference will be less; on the average, it equals $4\omega_A a/\pi = 50$ cm/sec. These values are so small compared to observed seasonal wind-variations that we may expect the effect of winds on the wobble to be important, but barely detectable from available data.

Geostrophic winds.—We may define a vertically averaged wind by

$$\int_a^\infty \rho v\, dr = -g^{-1}\int_{p_s}^0 v\, dp = \frac{\bar{v} p_s}{g},$$

where p_s is the atmospheric pressure at the ground. On the average, $\bar{v}$ is the wind at the 500-mb level. Except for the equatorial region, we can compute $\bar{v}$ from the geostrophic law,

$$\bar{u} = \frac{g}{2a\omega\cos\theta}\frac{\partial h}{\partial\theta}, \quad \bar{v} = -\frac{g}{2a\omega\cos\theta\sin\theta}\frac{\partial h}{\partial\phi},$$

TABLE 1. Effect of geostrophic winds on the annual variation in latitude.

n	G_n^1 (cm)		λ_v (sec) Zonal		Meridional		Total	
	cos Θ	sin Θ	cos Θ	sin Θ	cos Θ	sin Θ	cos Θ	sin Θ
2	536	−1145	.0009	−.0019	−.0160	.0341	−.0151	.0322
4	649	−1872	.0301	−.0868	−.0119	.0344	.0182	−.0524
6	−972	− 483	.0074	.0037	.0101	.0050	.0175	.0087
8	−209	759	−.0029	.0105	.0021	−.0075	−.0008	.0030
Sum			.0355	−.0745	−.0157	.0660	.0198	−.0085
n	H_n^1 (cm)		μ_v (sec) Zonal		Meridional		Total	
2	1354	− 715	.0023	−.0012	−.0403	.0213	−.0380	.0201
4	147	1572	.0068	.0729	−.0027	−.0289	.0041	.0440
6	− 498	640	.0038	−.0049	.0052	−.0067	.0090	−.0116
8	−2077	− 144	−.0287	−.0020	.0205	.0014	−.0082	−.0006
Sum			−.0158	.0648	−.0173	−.0129	−.0331	.0519

where h is the elevation of that particular pressure surface ($\approx$ 500 mb) along which the wind equals $\bar{v}$. Equation (6) now becomes

$$\lambda_v = -\frac{\tau a^2 p_s}{A\omega^2}\int_0^{2\pi}\int_0^{\pi}\left(\frac{\partial h}{\partial\theta}\sin\theta\cos\phi - \frac{\partial h}{\partial\phi}\cos\theta\sin\phi\right)d\theta\, d\phi. \quad (15)$$

We can now compute λ_v and μ_v from charts of $h(\theta, \phi)$, just as previously we computed λ_i and μ_i from charts of $p_s(\theta, \phi)$. The present calculation is, however, less reliable because (a) the 500-mb winds are a much less accurate representation of the movement of the atmosphere as a whole than the surface pressures are of the mass of the atmosphere, and (b) the elevation of the 500-mb surface is known less accurately than the surface pressures. Suppose h is expanded in tesseral harmonics,

$$h = \sum_{n=0}^{N}\sum_{m=0}^{n}(G_n^m \cos m\phi + H_n^m \sin m\phi)\, P_n^m(\theta).$$

Then only terms for which $m = 1$ contribute to (15), which becomes

$$\lambda_v = -\frac{\tau a^2 p_s \pi}{2A\omega^2}\sum_{n=1}^{N} G_n^1\left[\sqrt{2n(n+1)}\int P_n^0(\theta)\sin\theta\, d\theta - \sqrt{(n+2)(n-1)}\int P_n^2(\theta)\sin\theta\, d\theta + 2\int P_n^1(\theta)\cos\theta\, d\theta\right]. \quad (16)$$

The first two terms follow from zonal winds, the third from meridional winds. The integrals are to be evaluated over the region for which the geostrophic law applies. This excludes an equatorial strip. If this strip is symmetrical with respect to the equator, only even values of n contribute to λ_v; in particular, if we exclude the region between 20°N and 20°S latitudes (θ from 70° to 110°),

$$\lambda_v = -\frac{\tau a^2 p_s \pi}{A\omega^2}[(-0.062 + 1.109)G_2^1 + (-1.727 + 0.685)G_4^1 + (0.283 + 0.389)G_6^1 + (-0.514 + 0.367)G_8^1 + \cdots], \quad (17)$$

and a similar expression for μ_v in which the G's are replaced by H's. The first number in each parenthesis results from zonal winds, the second number from meridional winds.

If we retain only the first term[7] of the series, a comparison of (13) and (17) shows that the ratio of the effect of winds to that of inertia is given by

$$\frac{\lambda_v}{\lambda_i} = \frac{1.047\,\pi}{1.451\,\kappa}\frac{p_s}{a^2\omega^2\rho_{500}}\frac{(C_2^1)_{500}}{(C_2^1)_s} \approx 2.55\frac{(C_2^1)_{500}}{(C_2^1)_s},$$

where ρ_{500} is the mean density at 500 mb, $(C_2^1)_{500} = \rho_{500} g G_2^1$ is the pressure component at about 6 km, and $(C_2^1)_s$ is the corresponding surface-pressure component. Suppose the geostrophic winds are to account for all of the observed wobble. Then the required winds, averaged vertically, are those that would result from the pattern in fig. 1, provided the isolines are interpreted as isobars at an elevation of about 6 km and the isoline interval is changed from 1 to 1/2.55 mb. The magnitude of the required pressure change, 4.6/2.55 or 1.8 mb, is so small that it can hardly be detected from present data. This agrees substantially with the conclusion for the solid shell.

We have, nevertheless, attempted to evaluate the terms from charts of the elevation of the 500-mb surface for four seasons prepared by Brooks *et al* (1950). The method is given below in the appendix, the results in table 1. Errors in the computational scheme are smaller than those inherent in the data. The terms do not converge as rapidly with n as is desirable. The totals for λ_v are made up of two positive and two negative numbers, and the degree of cancellation makes the totals quite unreliable. There is less cancellation for μ_v. All that can be said is that the computed values resulting from the geostrophic winds are of the same order of magnitude as the astronomic observations.

Equatorial winds.—We do not believe that available data are sufficiently accurate or complete to justify the evaluation of the integrals. There are, however, several reasons why the contribution by equatorial winds can be expected to be less important than the contribution by winds elsewhere. In the first place, the trade winds vary less with the seasons than do the westerlies; in the second place, winds at the equator do not contribute to λ_v and μ_v. For the "solid shell" atmosphere, it was shown [equation (14)] that the shell between 20°N and 20°S contributes only 4 per cent of the total effect.

7. Winds, stress approach

If the components of surface stress are expanded in tesseral harmonics according to

$$s_\phi = \sum_{n=0}^{N}\sum_{m=0}^{n}(C_n^m \cos m\phi + D_n^m \sin m\phi)\, P_n^m(\theta),$$

$$s_\theta = \sum_{n=0}^{N}\sum_{m=0}^{n}(C'^m_n \cos m\phi + D'^m_n \sin m\phi)\, P_n^m(\theta),$$

it is found that only the components C_2^1, C_4^1, $\cdots$, D_2^1,

[7] For terms of higher order, C_4^1, C_6^1, $\cdots$, the winds contribute to the wobble, but the shifts in air mass do not.

$D_4^1, \cdots$, and $C_1'^1, C_3'^1, \cdots, D_1'^1, D_3'^1, \cdots$, contribute to λ_t and μ_t. The formulae are

$$(\lambda_t)_\phi = \frac{\tau a^3 \pi}{A\omega^2}(0.680\, D_2^1 + 0.155\, D_4^1 + 0.070\, D_6^1 + 0.040\, D_8^1 + \cdots),$$
$$(\lambda_t)_\theta = -\frac{\tau a^3 \pi}{A\omega^2}(1.571\, C_1'^1 + 0.241\, C_3'^1 + 0.095\, C_5'^1 + 0.051\, C_7'^1 + \cdots), \qquad (18)$$

for the effects of the meridional and zonal components, respectively. The corresponding expressions for $(\mu_t)_\phi$ and $(\mu_t)_\theta$ are found by replacing $+D$ by $-C$, and $+C'$ by $+D'$.

If we retain only the first term in each series, and ascribe all of the observed wobble first to zonal stresses only, then to meridional stresses only, the required stress-distributions are (in dynes/cm²)

$$s_\phi = -[(0.5 \cos\Theta + 3.3 \sin\Theta)\cos\phi + (1.0\cos\Theta + 0.0 \sin\Theta)\sin\phi] \sin 2\theta,$$
$$s_\theta = [(0.5\cos\Theta + 0.0\sin\Theta)\cos\phi - (0.3\cos\Theta + 1.6\sin\Theta)\sin\phi]\sin\theta,$$

respectively, as shown in figs. 2 and 3. These may be considered the simplest stress-patterns to achieve the observed wobble.

From general consideration of atmospheric circulation, we know that stresses of the order of 1 dy/cm² are representative over large portions of the earth's surface. Mintz (1951) finds that the zonal stress between 35°N and 90°N averages 1 dy/cm², of which one third to one half is due to skin friction, the remaining part to the mountain effect. Variations up to 6.6 dy/cm² in the zonal stress, as shown in fig. 2, seem rather large. Furthermore, the zonal-stress pattern does not resemble the observed pattern. The meridional stresses, however, are in phase with the monsoon winds, and their magnitude is about what would be expected.

In section 6 we had noted that annual changes in wind of less than 1 m/sec, or in pressure of only a few millibars, could account for all of the observed wobble; yet these values were so small that they were barely over the threshold of detectability. Here we find that the corresponding stress variation is by a few dynes per square centimeter, a value above the threshold of detectability. From these elementary considerations, we may conclude that the present torque method is inherently more accurate than the momentum method of section 6, and that the meridional wind-stress can be expected to contribute towards the observed wobble.

Stress over oceans.—Tables of wind stress over the North Pacific (north of 5°S) and the North and

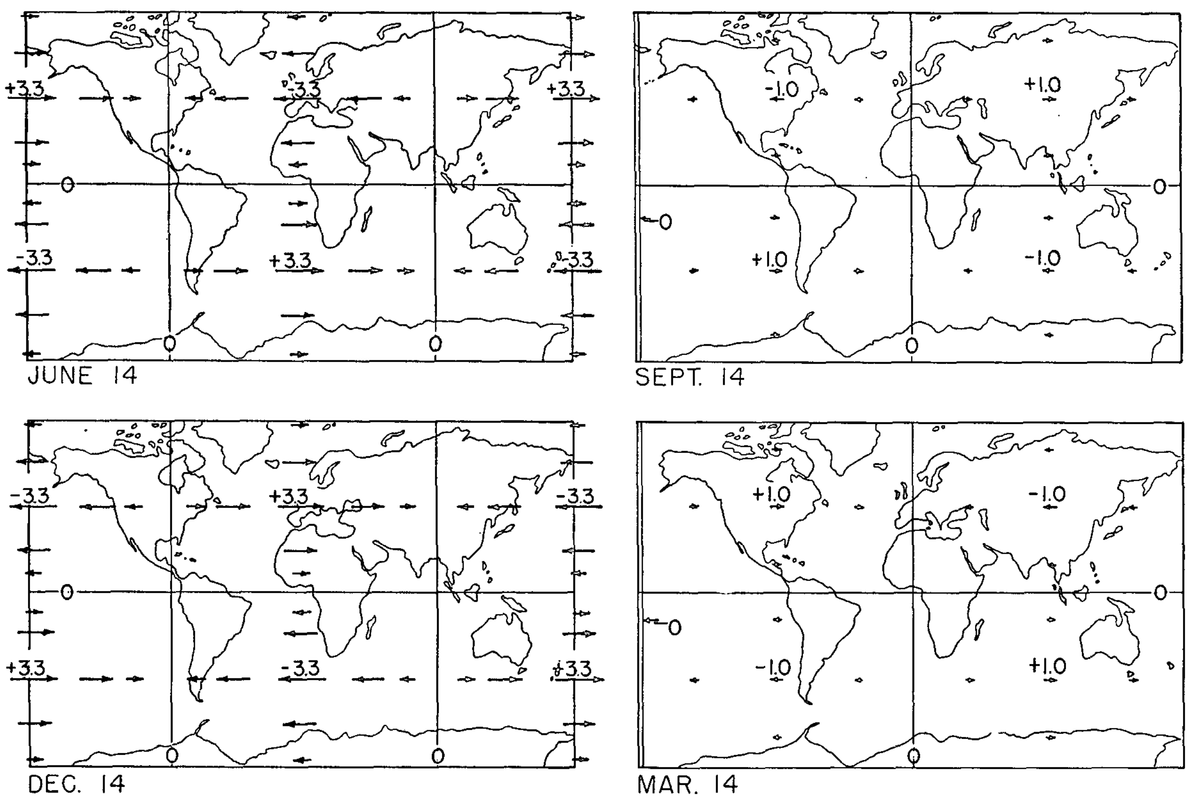

FIG. 2. Annual variation in surface stress from zonal winds, which could account for all of observed wobble. Dates are those for which stress variations reach maximum or minimum values. Length of arrows is proportional to stress. Values (dynes/cm²) of largest stress-variation are indicated. Solid lines represent zero stress.

TABLE 2. Effect of wind stress over oceans.

	λ_t (sec) cos Θ	λ_t (sec) sin Θ	μ_t (sec) cos Θ	μ_t (sec) sin Θ
Pacific Ocean:				
zonal stress	.00012	−.00005	.00020	−.00059
meridional stress	.00080	−.00067	.00022	.00062
Atlantic Ocean:				
zonal stress	−.00020	.00031	−.00019	.00042
meridional stress	−.00070	.00035	.00051	−.00054

Central Atlantic (north of 20°S), for each month and 5-deg quadrangle, have been prepared at the Scripps Institution of Oceanography.[8] We estimate that the probable error does not exceed 25 per cent. With use of these stress tables, values of λ_t and μ_t have been computed according to (9). The results are given in table 2. The amplitudes are 0.001 sec at most, and are therefore small relative to the observed values.

Actually these stresses are not transmitted directly to the solid earth, but only through the medium of the ocean currents. In section 8, below, an evaluation of the effect of ocean currents leads to the same order of magnitude.

The mountain effect.—We shall consider only the American continent and the Himalayas. The mountain effect over the American continent is due largely to pressure against the Rocky Mountains and the Sierra Nevadas. It has been shown by Mintz (1951) that we shall not be far from wrong if we consider only a wall, 3 km high, extending along 110°W between 30°N and 60°N, with a difference in pressure against its western face minus pressure against its eastern face in January exceeding that in July by 3.5 mb. This yields, approximately, in seconds,

$$\lambda_t = 0.002 \sin \Theta, \qquad \mu_t = -\,0.001 \sin \Theta,$$

for the mountain effect over America. For the Himalayas, any dependable (station level) pressure-variations are lacking but the following estimate does not seem excessive. Suppose we consider the pressure against a vertical wall along 30°N, 2 km high between 40°E and 70°E, and 4 km high between 70°E and 100°E. The difference in pressure against the northern face minus the pressure against the southern face in January exceeds that in July by 15 mb. This gives, in seconds,

$$\lambda_t = 0.004 \sin \Theta, \quad \mu_t = 0.015 \sin \Theta,$$

for the mountain effect against the Himalayas. We note that this is very roughly in phase with the observed effect [equation (11)], and has about one third the observed amplitude.

[8] *The field of mean wind stress over the North Pacific Ocean; The mean wind stress over the Atlantic Ocean.* Oceanographic Reports Nos. 14 and 21 (unpublished).

8. Ocean currents

Angular-momentum approach.—The essential elements of oceanic circulation are: (a) the antarctic circumpolar current, which is largely a zonal current; (b) the equatorial circulation; (c) the subtropical and subpolar gyres in the "confined" basins of the North Atlantic, North Pacific, South Atlantic, South Pacific and Indian Oceans. A discussion of these features can be found in Munk (1949) and Munk and Palmén (1951).

First we note that uniform zonal motion all around the earth, and motion at the equator, do not contribute to λ_v and μ_v. Accordingly, we neglect elements (a) and (b). This leaves the subtropical and subpolar gyres in the five basins. By far the most important are the subtropical gyres of the North Atlantic, including the Gulf Stream system, and of the North Pacific, including the Kuroshio systems. We shall consider these first.

The effect on the earth's wobble of the northeast-flowing Gulf Stream alone is to a large extent compensated by an equal southward flow of water on the eastern side of the Atlantic. We must therefore consider the entire effect of any gyre at one time. This can be done by expressing the vertically integrated mass-transport in terms of a stream function ψ, according to

$$\int \rho u \, dr = a^{-1}\, \partial\psi/\partial\theta,$$

$$\int \rho v \, dr = -\,(a \sin \theta)^{-1}\, \partial\psi/\partial\phi,$$

with the integration carried out from sea bottom to sea surface. In terms of this function,

$$\lambda_v = -\frac{2\tau a^2}{A\omega}\int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\left(\frac{\partial\psi}{\partial\theta}\sin\theta\cos\theta\cos\phi - \frac{\partial\psi}{\partial\phi}\cos^2\theta\sin\phi\right) d\theta\, d\phi. \quad (19)$$

The stream function can be expressed by (Munk, 1949)

$$\psi = \psi_0 \sin\left(\pi\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) f(\phi), \quad (20)$$

where $f(\phi)$ rises almost abruptly from 0 to 1 near the western boundary ϕ_1, and decreases linearly to zero at the eastern boundary ϕ_2. We may visualize the circulation, bounded by co-latitudes θ_1 and θ_2, and longitudes ϕ_1 and ϕ_2, rotating clockwise about a central point far towards the western side of the basin. The total transport across any radial line from this central point to the border equals ψ_0. For the Gulf Stream or the Kuroshio, $\psi_0 = 60 \times 10^{12}$ g/sec. Since systematic observations at fixed stations throughout the

TABLE 3. The effect of the ocean circulation in the subtropical gyres.

Location	Co-latitudes (deg) θ_1	θ_2	Longitudes (deg E) ϕ_1	ϕ_2	λ_v (sec)	μ_v (sec)
North Atlantic	45	75	285	345	$-.0005 \cos\Theta$	$+.0008 \cos\Theta$
North Pacific	45	75	130	230	$+.0014 \cos\Theta$	$-.0004 \cos\Theta$
Indian Ocean	105	135	035	115	$-.0002 \cos\Theta$	$-.0004 \cos\Theta$

year are hardly available, we depend largely on the indirect evidence of tide gauges for an estimate of the annual variation. In this manner, Iselin (1940) finds the transport in spring to be 20 per cent larger than the transport in the fall. Compilation of ships' logs (Fuglister, 1951) supports Iselin's calculations. The variable part of ψ_0 is $\Delta\psi_0 = 6 \times 10^{12} \cos\Theta$ g/sec. For the chosen expression of the stream function, the integration leads to

$$\lambda_v = \frac{2\tau a^2 \Delta\psi_0}{A\omega}\left[\beta - \frac{\beta}{2(1-4\beta^2)}(\cos 2\theta_2 + \cos 2\theta_1)\right] \times \left[\frac{\cos\phi_2 - \cos\phi_1}{\phi_2 - \phi_1} + \sin\phi_1\right], \quad (21)$$

where $\beta = (\theta_2 - \theta_1)/\pi$.

Numerical values are given in table 3. The first two lines pertain to the subtropical gyres of the North Atlantic and North Pacific. In addition, we must consider the subpolar gyres of the northern hemisphere and the subtropical gyres of the southern hemisphere for both Atlantic and Pacific. With regard to the subpolar gyres, we note that the Labrador Current is roughly in phase with the Gulf Stream, but opposite in direction. A similar relation holds between Oyashio and Kuroshio in the Pacific. The subpolar gyres will therefore *reduce* the above values somewhat. With regard to the subtropical gyres of the southern oceans, we note that the Brazil current is both opposite in phase and opposite in direction to the Gulf Stream, and so are the East Australian Current and the Kuroshio. These gyres will therefore *increase* the above values. There is, then, some cancellation. In any event, the circulation in these gyres amounts to only 10–20 per cent of the circulation in the subtropical gyres of the northern hemisphere. We shall therefore take the first two lines of table 3 as representative of the entire Atlantic and Pacific Oceans.

This leaves only the Indian Ocean. There the Agulhas Current corresponds to the Gulf Stream, but has only one-third its transport. The annual variation in sea level at Kenya and Tanganyika is also about one-third of that at Cape Hatteras. Phase and direction are both opposite to those in the Gulf Stream. Our best estimate is, then, $\Delta\psi_0 = 2 \times 10^{12} \cos\Theta$ g/sec, which leads to the values in table 3.

A comparison of these values with the observed values [equation (11)] indicates that the circulation in the Pacific Ocean alone accounts for 10 per cent of the magnitude in λ, but for only three per cent of the total observed wobble. There is, however, a good deal of cancellation between the three oceans, and the net result is less, and not reliable.

Stress approach.—The stress applied by the winds on the ocean surface is balanced largely by friction against the western boundary of the ocean basin. We can check the order of magnitude of the preceding calculation in the following manner. The western boundary is replaced by a vertical wall extending from the surface to some depth (1000 m) where the currents nearly vanish, and extending from θ_1 to θ_2 in a north-south direction. We now write for $f(\phi)$ in (20),

$$f(\phi) = -\frac{2}{\sqrt{3}} e^{-\frac{1}{2}k\xi} \cos\left(\frac{\sqrt{3}}{2}k\xi - \frac{\pi}{6}\right) + 1,$$

where $\xi = a(\phi - \phi_1)\sin\theta$ is the distance from the wall, and k is the "wave number" of the Gulf Stream [Munk, 1950, equation (20)]. Then $s_\theta = \rho K(\partial v/\partial\xi)_{\xi=0}$ is the force per unit area, and $K(\partial^2\psi/\partial\xi^2)_{\xi=0}$ is the force per unit length of wall, K being the (kinematic) eddy viscosity. The variable part of the force against the entire wall becomes

$$K\int_{\theta_1}^{\theta_2}\left(\frac{\partial^2 \Delta\psi}{\partial\xi^2}\right)_{\xi=0} a\, d\theta = \frac{2}{\pi}(\theta_2 - \theta_1)k^2 K\,\Delta\psi_0.$$

Using the previous values for $\Delta\psi_0$, θ_1 and θ_2, and setting $k = 2 \times 10^{-7}$ cm^{-1}, $K = 2.5 \times 10^7$ cm^2/sec (Munk and Carrier, 1950), we find, upon substitution into (9), values of λ_t and μ_t of the order of 0.001 sec, in agreement with our previous calculations.

9. Discussion and conclusions

Since the turn of the century, an annual motion of the axis of instantaneous rotation within the body of the earth has been deduced from measurements of latitude at observatories. With respect to this annual component, the north pole of instantaneous rotation describes nearly an ellipse. In September it is displaced, relative to its mean position, by 10 ft towards Greenland; in December by 8 ft towards Central Asia; *etc.* From the astronomic observations, the required *forcing function* with the components λ, μ has been computed by Jeffreys (table 4, line 1).

The problem is to account for this function in terms of happenings on the earth. It is convenient to treat separately the effects of (1) a variable distribution of matter over the earth and (2) a variable motion relative to the earth. The former involves integrals of $p_s(\theta, \phi, \Theta)$, the latter integrals of $\rho\bar{v}(\theta, \phi, \Theta)$, where p_s is surface pressure, ρ density, v relative velocity (the bar denotes a vertical average), θ co-latitude, ϕ longitude, and Θ is the time of year.

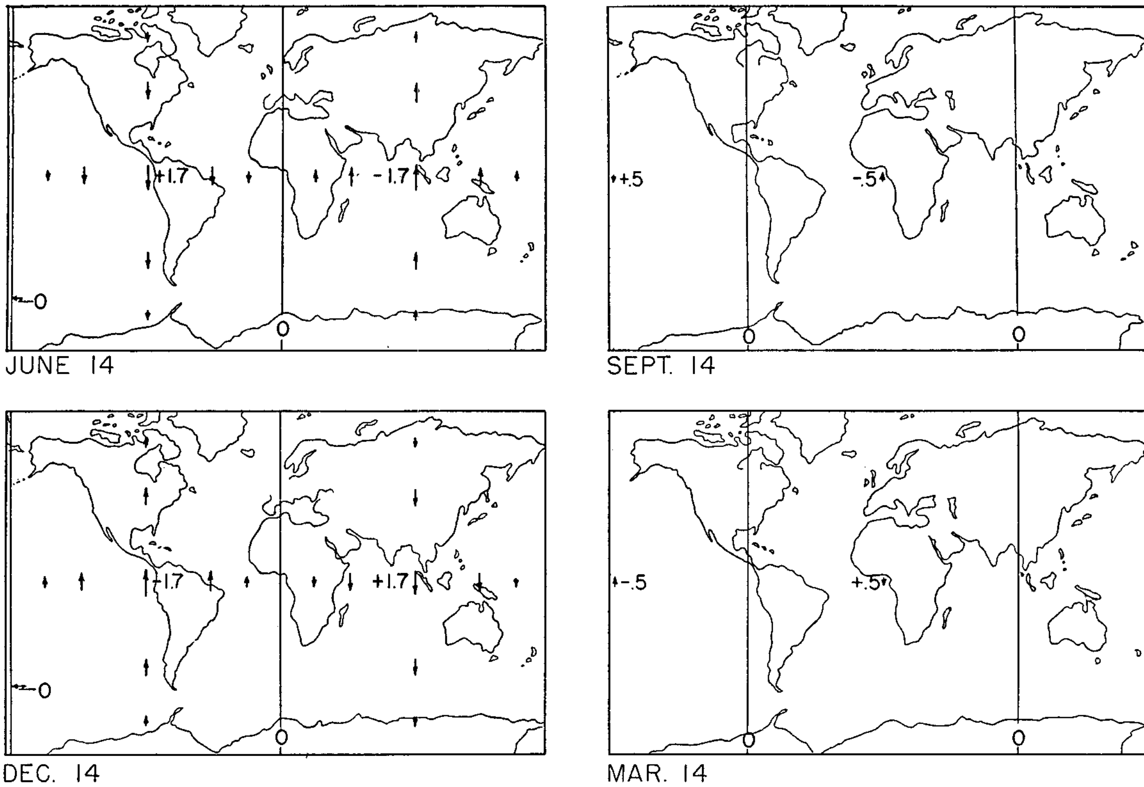

FIG. 3. Annual variation in surface stress for meridional winds. (Length and scale of stress arrows same as in fig. 2.)

In the evaluation of such integrals from meteorological data, our chief concern lies with the accuracy, even the reality, of the numerical results. Rather than to confine ourselves to somewhat inaccessible numerical compilations, we have therefore followed the procedure of first inquiring as to the simplest meteorological patterns that could account for the observed wobble. From an inspection of these patterns, the reader can tell whether they are reasonably distributed in time and geographical location, whether the required magnitudes are plausible, and if so, whether they are detectable from data now available.

Fig. 1 shows the simplest surface-pressure pattern consistent with the assumption that *all* of the observed wobble is brought about by seasonal shifts in air mass. A winter maximum of 2.3 mb over Asia does not seem unreasonable. Next, suppose the observed wobble is due entirely to winds. The prerequisite pattern can be presented either by (2a) the seasonal distribution of momentum throughout the atmosphere, or (2b) the seasonal wind-stress (skin friction and mountain effect) over the earth's surface. If in the former case we confine ourselves to geostrophic winds between 90°N and 20°N, 20°S and 90°S, the associated pressure-

TABLE 4. Summary of calculations.

	λ (sec)		μ (sec)	
	cos Θ	sin Θ	cos Θ	sin Θ
Astronomic observations	−.013	.000	.007	.044
1. Distribution of matter (according to Jeffreys,[6] with κ = 0.696):				
Atmosphere	−.001	.007	−.005	.035
Snow	−.017	.012	−.009	.006
Vegetable matter	.001	−.002	.002	−.003
Total	−.017	.017	−.012	.038
2. Motion relative to the earth:				
2a. *Angular-momentum approach*				
Winds (section 6)	.020	−.009	−.033	.052
Ocean currents (section 8)	.001	.000	.000	.000
2b. *Torque approach* (*section 7*)				
Skin friction over N. Pacific	.001	−.001	.000	.000
N. Atlantic	.001	.001	.000	.000
Mountain effect, Rocky Mts.	—	.002	—	−.001
Himalayas	—	.004	—	.015

pattern at the elevation ($\approx$ 6 km) at which the wind has its mean value is again given by fig. 1, provided the isoline interval is reduced by a factor of 2.5. The required amplitude in the annual pressure-variation aloft is $2.3/2.5 \approx 1$ mb. The corresponding variation in wind speed between winter and summer is $\frac{1}{2}$ m/sec. These values are plausible, but so small as to be hardly detectable. From the point of view of stress, the required patterns are shown in figs. 2 and 3. The required zonal stress is too large, and the distribution unfamiliar. The meridional-stress distribution, on the other hand, is in phase with the monsoon winds, and the magnitude is about what could be expected.

These examples can be summarized as follows: suppose that, relative to average conditions, there is a maximum in winter over central Asia of (1) air mass (as revealed by surface pressure), (2) anticyclonic circulation (as revealed, for example, by pressures aloft), and (3) southward stress (as revealed, for example, by excess in pressure against the northern slopes of the Himalayas as compared to pressure at corresponding levels on the southern slopes). Each of these three factors would be associated with a winter displacement c of the north pole of rotation towards Asia, a displacement in spring by $\mathrm{T}c$, or 1.2 c, towards Japan, *etc.* These circumstances are, as a whole, in agreement with what is observed. We may, therefore, expect the effect of winds to augment the effect of the shift in air mass. On the basis of numerical values, we may expect the two effects to be of the same order of magnitude.

Numerical results are summarized in table 4. The seasonal shift in air mass accounts for three fourths of the largest term, the component in $\sin \Theta$ of μ. Snow plays a lesser role, but even the effect of seasonal changes in vegetation is not wholly negligible. Under vegetable matter, Jeffreys includes the rise of sap in trees, and the formation of the annual parts in herbs and of deciduous parts in trees.

The effect of winds from the angular-momentum approach was computed with use of seasonal charts of the 500-mb surface. The totals are quite unreliable, especially for λ, on account of internal cancellation. All that can be said is that the computed values are of the same order of magnitude as the observed values. The effect of ocean currents is very much smaller.

With regard to the wind-stress approach, it is convenient to treat separately skin friction and the mountain effect. Skin friction over the ocean is negligible. Over land, it is not sufficiently well known to permit calculations. In general, the roughness there is larger, but the winds weaker. The mountain effect over the American continent, due largely to pressure differences across the Rocky Mountains and the Sierra Nevadas, is quite small. On the other hand, if we assume that during the monsoon winds the pressure against the Himalayas is larger on the upwind side than at corresponding levels on the downwind side, a reasonable estimate for the pressure difference can lead to one-third of the observed effect.

Theoretically, the angular-momentum approach and the torque approach should lead to nearly identical results. In practice, we find the former to be highly inaccurate, and the latter, at the present time, incomplete. The torque approach is potentially the more accurate[9], and there is reason to believe that the mountain effect against the Himalayas alone would give a fair account for the total effect of winds. This is because the wobble is twice as sensitive to meridional stresses as to zonal stresses, because the Himalayas are the largest east-west mountain chain, and subject to the strongest known variations of meridional winds. A quantitative estimate of the wind effect must be left to the future. The principal need is for station-level pressures from an adequate number of weather stations on the Indian and Chinese sides of the Himalayas. In the meantime, our conclusion will have to be limited to the statement that the effect of winds can be expected to be roughly in phase with the observed effect, and not necessarily small compared to the effects investigated by Jeffreys.

If the discrepancy between computed and observed values can be somewhat narrowed from what it is at present, the opportunity presents itself of approaching the problem from a reverse point of view: of using astronomic observations to measure certain fundamental modes[10] in the annual oscillation of the atmosphere. Such a method is inherently capable of far greater accuracy than one based on the interpolation of data from scattered weather-stations. It may also permit some estimate of changes, from year to year, over the last half century.[11] One difficulty would be the distinction between the effects of shift in air mass and of variable winds.

Acknowledgments.—We are greatly indebted to Prof. Yale Mintz for his help and advice.

APPENDIX

For the geostrophic winds, we computed the tesseral harmonic coefficients of the height, $h(\theta, \phi)$, of the 500-mb surface, according to

$$\begin{bmatrix} G^1_n \\ H^1_n \end{bmatrix} = \frac{2n+1}{4\pi} \int_0^{2\pi} \int_0^{\pi} h(\theta, \phi)\, P^1_n(\theta) \times \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix} \sin \theta \, d\theta \, d\phi. \quad (22)$$

[9] The opposite is true in computing the effect of winds on the length of day (Mintz and Munk, 1951).

[10] An additional mode, not considered here, might be derived from astronomic observations concerning an annual variation in the length of the day (Mintz and Munk, 1951).

[11] Some rough computations for the period 1921–1930 have been made by R. C. Cowen (*Polar wanderings and the shifting of the atmospheric mass*, M.S. thesis at Mass. Inst. Tech., 1950, unpublished).

Values of h were read off for every 20 deg long and every 5 deg lat, from 55°S to 75°N. For every latitude, values of $\sin\phi$ and $\cos\phi$ were plotted against h, and the resulting closed curves were planimetered to obtain

$$\int_{h(\theta,0)}^{h(\theta,2\pi)} \sin\phi\, dh = -\int_0^{2\pi} h(\theta,\phi)\cos\phi\, d\phi,$$

and similarly for $\cos\phi$. This integral (extrapolated to the poles) was multiplied by the successive $P_n^1(\theta)$ $\sin\theta$ at the 5-deg intervals of θ, and summed by the trapezoid rule, to obtain (22).

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