ABSORPTION OF SOLAR RADIATION BY WATER VAPOR IN THE ATMOSPHERE

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ABSTRACT

Absorption of solar radiation by water vapor in the atmosphere has been studied, with use of Fowle's observed results and application of Elsasser's transmission function. The results are compared with those of other workers. Mügge-Möller's absorption curve and Karandikar's are discussed. As a result of calculations, an absorption chart is obtained from which both the absorption of solar radiation by an air column and the rate of heating of the air can be determined. Examples of calculation with the chart are shown with use of London's data for atmospheric conditions, and the results are compared with his.

1. Introduction

Absorption of solar radiation by water vapor in the atmosphere has been measured by Fowle [7], Kimball [12], Hoelper [10], Adel and Lampland [3] and McMath and Goldberg [15]. According to Fowle, on a clear day with the sun at zenith, the total absorption of solar radiation by water vapor and carbon dioxide is 6 to 8 per cent of the solar constant. This is more than the absorption of ultra-violet radiation by ozone. However, as the water-vapor absorption takes place mainly in the lower troposphere, its heating effect on the air has generally been considered negligible. Meanwhile, Tanck [17] has shown that the diurnal change of temperature above the frictional layer can be explained by the absorption of sunlight by water vapor. Mügge and Möller [16] have shown that, within the range of 0.5 to 8 cm of precipitable water, which Fowle's measurements cover, the latter can be represented by an empirical formula which is now widely believed to be reliable; investigations of Tanck [17] and London [14] on the absorption of solar radiation by water vapor incorporated the formula. But, as will be described below, there remains some doubt as to the validity of that formula. Recently Karandikar [11] calculated the absorption of solar radiation by water vapor in the stratosphere. However, in his calculation for the range 0.9 to 2 μ, he used Hettner's [9] absorption coefficient which, having been measured on steam of 127C, seems to be too large for atmospheric conditions, and Beer's law, which is inadequate for absorption bands.

In this paper we also calculate the absorption of solar radiation on the basis of Fowle's experimental results, but with a better approximation of the absorption law and with consideration of the pressure effect. As a result, we obtain an absorption chart by which we can know both the absorption in an air column and the rate of temperature rise of the air.

2. Law of absorption

According to Elsasser [6], the absorption of a band spectrum can be approximated by that of an idealized band of equal and equidistant lines whose transmission function τ is given by

$$\tau = 1 - \sinh\beta \int_0^{Su/d \sinh\beta} \exp(-y\cosh\beta)\, J_0(iy)\, dy, \quad (1)$$

where S is the total intensity of a line, d the distance between neighboring lines, $\beta = 2\pi\alpha/d$, α the half-width, u the amount of precipitable water, and J_0 is the Bessel function of zero order with an imaginary argument. When β is small and u is large, (1) becomes

$$\tau = 1 - \phi[(S\beta u/2d)^{\frac{1}{2}}], \quad (2)$$

where ϕ is the probability integral.

Recently, however, Cowling [4] has shown that Elsasser's transmission curve given by (2) disagrees with those calculated by him for the rotation band at several wavelengths and also those obtained from measurements. Cowling attributes this discrepancy to the irregularity in strength and spacing of the absorption lines in the water-vapor band. For large values of u, this is certainly a weak point of Elsasser's transmission function. However, for small values of u Cowling's comparison is not necessarily a true criticism, because in this case we must examine the validity of (1) instead of (2). For very small values of u, the approximation of (1) becomes

$$\tau = 1 - Su/d, \quad (3)$$

which means that absorption is proportional to u and to the mean absorption coefficient S/d. This circumstance will probably be true in the actual band also. We have therefore calculated the values of (1) by graphical integration, assuming $\alpha = 0.1$ cm and $d = 2.3$ cm, which we think to be appropriate values

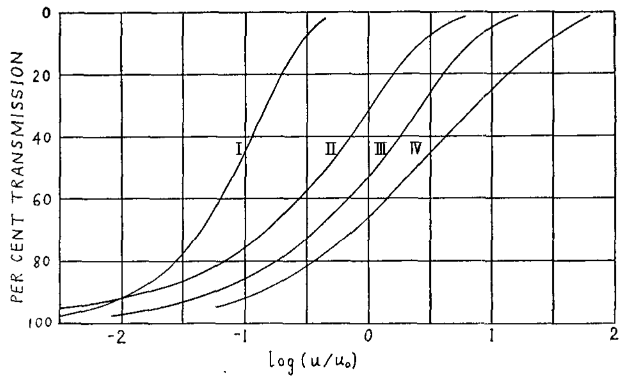

FIG. 1. Transmission curves. u_0 is arbitrary constant, with different value for each curve. I: Beer's law, exp $-ku$; II: Elsasser's law, given by (2); III: Elsasser's law, given by (1); IV: Cowling's mean curve.

for vibration bands of water vapor at normal conditions. The result is shown in fig. 1, in which Cowling's transmission curve is also shown. It will be seen that, for weak absorption, Elsasser's curve agrees with Cowling's.

Cowling himself, in another paper [5], also wrote as follows:

> Whatever law is used, a given amount of absorber produces practically complete absorption at some wavelengths, negligible absorption at others, and in a moderate range of wavelengths does the difference in law produce any appreciable effects. My own calculations of total absorption give results never differing from Elsasser's more than two or three per cent of the black body radiation, and even these differences are due partly to different assumed transparencies at the separate wavelengths.

Now, Elsasser's method has the advantage over Cowling's that absorption or transmission can be expressed analytically; this will enable us easily to introduce the pressure effect on absorption.

The pressure effect on half-width is assumed to agree with the theory of Lorentz, *i.e.*,

$$\beta = \beta_0 p/p_0, \tag{4}$$

where p is the pressure at any height, and β_0 is the value of β at standard pressure p_0.

Temperature effect on absorption is neglected.

3. Method of calculation

On a clear day, let the intensity of solar radiation of frequency ν at height z and zenith distance ζ be I_ν. Let x be the quantity of water vapor, and let k_ν be the absorption coefficient of water vapor. Then

$$I_\nu = I_{\nu\infty} \exp\left[\sec\zeta \int_\infty^z k_\nu(z)\,dz\right], \tag{5}$$

where $I_{\nu\infty}$ is the intensity at the top of the atmosphere. According to Elsasser [6], k_ν is expressed as

$$k_\nu = \frac{S}{d}\,\frac{\sinh\beta}{\cosh\beta - \cos s}, \tag{6}$$

where $s = 2\pi\nu/d$. Introducing (6) into (5), we have

$$I_\nu = I_{\nu\infty} \exp\left[\frac{S}{d}\sec\zeta \int_\infty^z \frac{x(z)\sinh\beta(z)}{\cosh\beta(z) - \cos s}\,dz\right]. \tag{7}$$

As β is small, $\sinh\beta \approx \beta$ in the numerator of the integrand and $\cosh\beta \approx \cosh\bar\beta$ in the denominator, where $\bar\beta$ is a constant corresponding to a certain height $\bar z$ $(\infty > \bar z > z)$. Then

$$I_\nu = I_{\nu\infty} \exp\left[-l_0 \frac{u(z)\sec\zeta}{\cosh\bar\beta - \cos s}\right], \tag{8}$$

where $l_0 = 2\pi\alpha_0 S/d^2$ is Elsasser's generalized absorption-coefficient at standard pressure, and

$$u(z) = -\int_\infty^z [p(z)/p_0]\,x(z)\,dz \tag{9}$$

is the effective precipitable water. Then the mean rate of absorption for a certain interval around frequency ν, $\overline{dI_\nu/du}$, is given by

$$\overline{dI_\nu/du} = \pi^{-1}\int_0^\pi (dI_\nu/du)\,ds. \tag{10}$$

Now we replace the variable s by ϕ, which is given by

$$\begin{aligned}\cos\phi &= (-1 + \cosh\bar\beta\cos s)/(\cosh\bar\beta - \cos s),\\ \sin\phi &= (\sinh\bar\beta\sin s)/(\cosh\bar\beta - \cos s).\end{aligned} \tag{11}$$

Then

$$\begin{aligned}\cosh\bar\beta - \cos s &= \sinh^2\bar\beta/(\cosh\bar\beta + \cos\phi),\\ d\phi &= \sinh\bar\beta/(\cosh\bar\beta - \cos s)\,ds,\end{aligned} \tag{12}$$

and for $s = 0$, $\phi = 0$; for $s = \frac{1}{2}$, $\pi > \phi > \frac{1}{2}\pi$; for $s = \pi$, $\phi = \pi$. Hence,

$$\begin{aligned}\overline{dI_\nu/du} &= -I_{\nu\infty} l_0 \sec\zeta \sinh^{-1}\bar\beta\\ &\quad\times \exp[-(\cosh\bar\beta/\sinh^2\bar\beta)l_0\,u(z)\sec\zeta]\\ &\quad\times \pi^{-1}\int_0^\pi \exp[-\sinh^{-2}\bar\beta\,l_0\,u(z)\\ &\quad\times \sec\zeta\cos\phi]\,d\phi\\ &= -I_{\nu\infty} l_0 \sec\zeta \sinh^{-1}\bar\beta\\ &\quad\times \exp[-(\cosh\bar\beta/\sinh^2\bar\beta)l_0\,u(z)\sec\zeta]\\ &\quad\times J_0[il_0\,u(z)\sec\zeta/\sinh^2\bar\beta].\end{aligned} \tag{13}$$

We can obtain the total rate of absorption by summing up the above values for all intervals of every absorption band.

Now, for actual calculation we must know the values of l_0 and $\bar\beta$ $[\bar\beta = \beta_0 p(\bar z)/p_0]$. We assume that $\beta_0 = 0.27$, corresponding to $p_0 = 1000$ mb and the values of α and d indicated above. $\bar\beta$ further depends on $\bar z$. As a first approximation, we assume $\bar z$ to be that value of z at which u takes half its actual value.

The values of l_0 can be determined from experimental results, if β_0 is known. Fowle [8] has carried

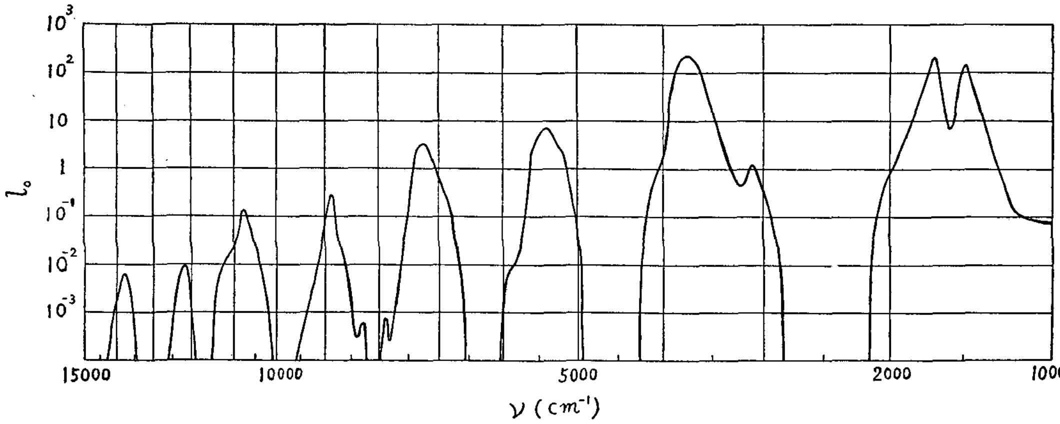

FIG. 2. Values of l_0 obtained from Fowle's measurements.

out measurements of absorption by water vapor in the range from 1 to 9 μ in the laboratory, and also has measured the absorption of solar radiation by water vapor in the atmosphere in the range from 0.66 to 2.1 μ [7]. In regions beyond 2 μ and in the central regions of the 1.4 and 1.8 μ bands, where absorption is strong, the values of l_0 were determined from Fowle's laboratory measurements; in the remaining regions, they were determined from his observations in the atmosphere. The values of l_0 thus obtained are shown in fig. 2.

In the calculations, solar radiation was assumed to be that of a black body at 6000K, and the value of the solar constant was taken as 1.94 ly/min.

4. Results and discussion

First, to find the pressure effect, the rate of absorption dA/du (A = total absorption) at normal incidence ($\zeta = 0$) was calculated for four virtual air-columns with constant pressure, namely $p = 1000$, 500, 100 and 50 mb. The results are shown in fig. 3, where, to conserve space and to represent the absorption by area on the chart, the abscissa is log u and the ordinate is $u(dA/du)$ ln 10. It is to be noted that, for $u < \approx 0.2$ cm, the rate of absorption decreases with p; for larger values of u it increases with p. The absorption, given by the area of the chart, therefore cannot be expressed solely as a function of u.

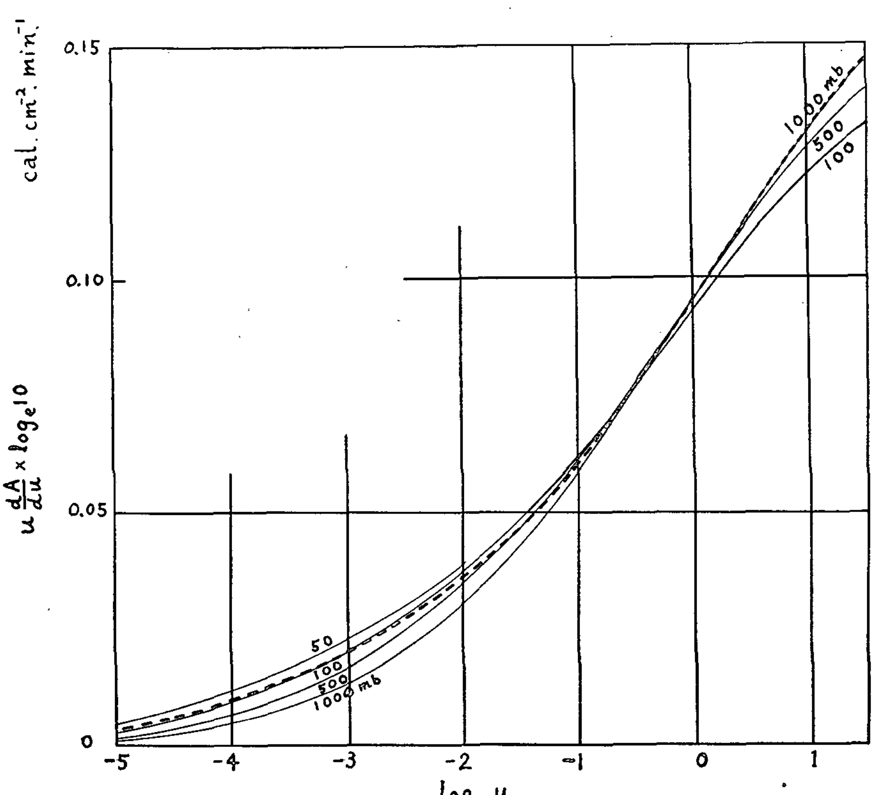

FIG. 3. Rate of absorption.

In the actual atmosphere, pressure is not constant but varies with altitude. Therefore, the rate-of-absorption curves corresponding to actual atmospheric conditions have also been calculated. These calculations were carried out for the atmospheric conditions given by London [13] as mean values for March at different latitudes. To avoid complication of the figure, the obtained curves are not shown; however, they nearly coincide with each other (within 1.5 per cent error for tropospheric conditions). A mean of the curves is shown by the broken line in fig. 3. Thus, in the actual atmosphere we may treat dA/du, and accordingly A, as a function of u only.

Next, based on the mean rate-of-absorption curve in fig. 3, absorptions for different values of ζ were obtained. They are shown in fig. 4, where the abscissa is log u and the ordinate is $A \cos \zeta$.

Now we shall compare the results with those obtained by other workers. Usually absorptions are shown as a function of precipitable water, without pressure correction; so it is necessary to express our results as functions of precipitable water. This change of variable from effective precipitable water u to actual precipitable water was carried out by assuming London's data to represent atmospheric conditions. The comparison is given in fig. 5, which shows that our absorption curve lies between Mügge-Möller's curve and Kimball-Hoelper's. Here it must be remarked that

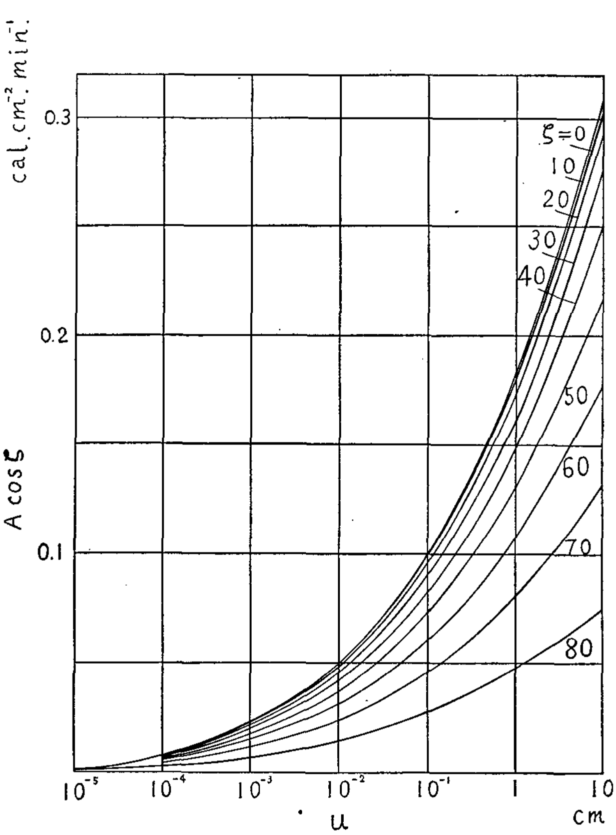

FIG. 4. A cos ζ as function of u.

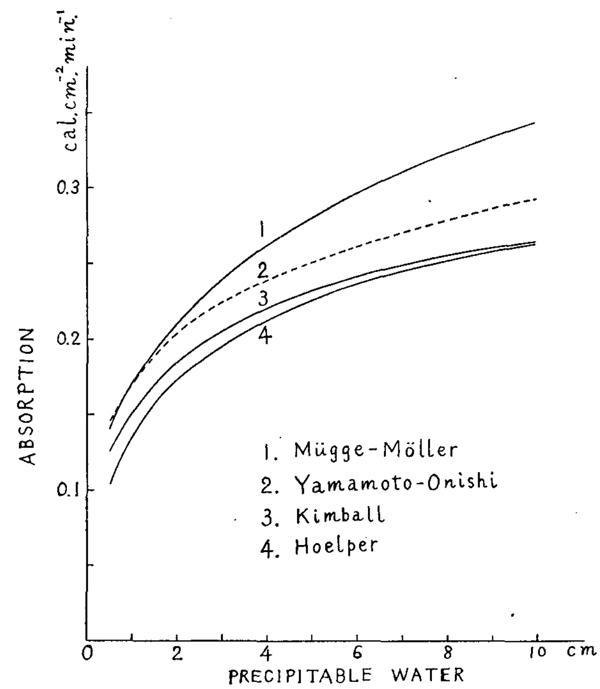

FIG. 5. Absorption of solar radiation by atmosphere.

the Mügge-Möller curve is computed by them from Fowle's atmospheric absorption, with the addition of ultra-violet and infra-red corrections given by Abbot [1] and Abbot *et al* [2]. There naturally arises the question of why there is such a discrepancy between Mügge-Möller's curve and ours, which is also calculated on the basis of Fowle's measurements. One cause is probably that they used the observed solar-energy curve, while we used the energy curve of a black body at 6000K and a solar constant of 1.94 ly/min; but the discrepancy due to this cause will be rather small. The main cause will be the ultra-violet corrections which they added and which we did not consider, because there is no absorption band of water vapor in that region.

Next, we compare our results with Karandikar's. He calculated the absorption of solar energy by a small quantity of precipitable water which will exist in the stratosphere. In his calculations, he used Hettner's [9] absorption coefficient and Beer's law for the region 0.9 to 2 μ; beyond this, to 8 μ, Fowle's [8] results were used, Elsasser's error function being applied to obtain absorptions of solar energy by small quantities of precipitable water at normal conditions for the different bands and for the total of all bands. His results can easily be extended to the cases of large values of precipitable water, by use of the data shown in Appendix I of his paper. His results, thus extended, together with our results for the same conditions, are shown in fig. 6. Under stratospheric conditions, *i.e.*, when the precipitable water is less than nearly 0.01 cm, his and our results nearly agree; but when pre-

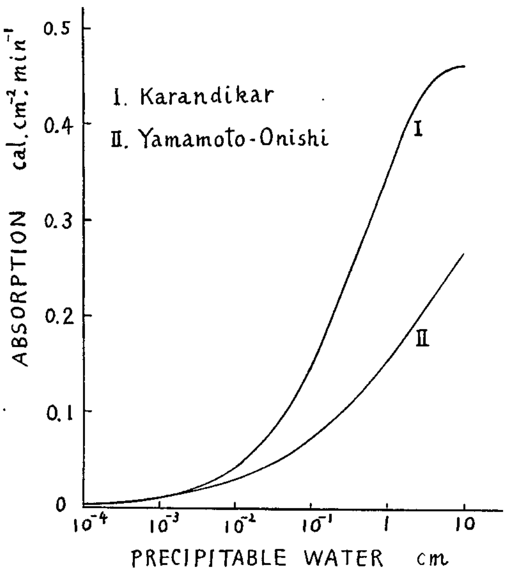

FIG. 6. Absorption by column of pressure one atmosphere.

cipitable water increases, his results far exceed our values. The precipitable water in fig. 6 is under pressure of one atmosphere, while that in fig. 5 is in the actual air column, so that the former corresponds to a slightly larger value than the latter in effect on absorption, because in the latter column the effective pressure is smaller. But this difference will not be very large, because the water vapor in the actual column is mainly concentrated in the lower layers. If we neglect this difference in the meaning of precipitable water, we can compare Karandikar's values with those of Mügge-Möller and Kimbal-Hoelper, by comparing fig. 6 with fig. 5. From this comparison, it is clear that Karandikar's values are far larger than those of Mügge-Möller and others for large values of precipitable water. Thus, we may say that Karandikar's absorption is nearly correct in the stratosphere but gives too high values in the troposphere. This character of Karandikar's absorption curve may be explained by the facts that he used, for the region 0.9 to 2μ, Hettner's absorption coefficient (which, having been measured on steam of 127C, seems to be larger than that at atmospheric conditions), and that he applied Beer's law to obtain absorptions for the same region (which will give too small absorption for small quantities of water vapor and too large absorption for large values of precipitable water). Both effects compensate each other to give nearly correct values for small quantities of water vapor, but they enhance each other to give too large values for large quantities of water vapor.

Finally, from figs. 2 and 4, we have constructed a more convenient chart, fig. 7, by means of which we

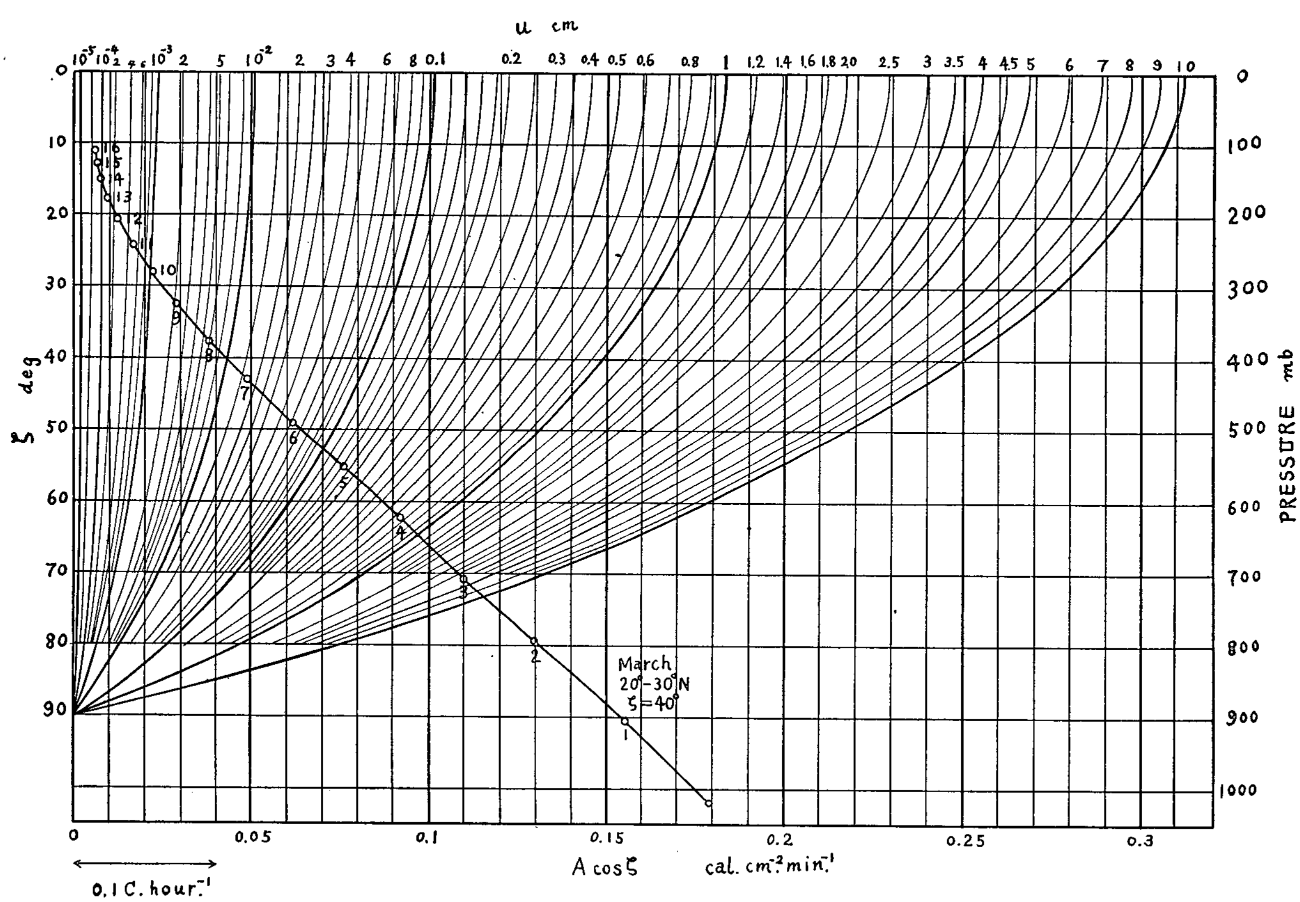

FIG. 7. Absorption chart, with example of absorption curve. Figures on curve indicate height (km).

TABLE 1. Effective precipitable water for March, 20–30°N.

Level (km)	Pressure (mb)	Δu (cm)	u (cm)
0	1017	11.1×10^{-1}	23.1×10^{-1}
1	904	60.2×10^{-2}	12.0×10^{-1}
2	802	30.2×10^{-2}	6.0×10^{-1}
3	707	14.9×10^{-2}	29.8×10^{-2}
4	623	75.6×10^{-3}	14.9×10^{-2}
5	549	39.2×10^{-3}	73.8×10^{-3}
6	486	19.5×10^{-3}	34.6×10^{-3}
7	427	8.7×10^{-3}	15.1×10^{-3}
8	373	36.9×10^{-4}	64.4×10^{-4}
9	323	15.1×10^{-4}	27.5×10^{-4}
10	279	6.2×10^{-4}	12.4×10^{-4}
11	240	2.8×10^{-4}	61.2×10^{-5}
12	204	12.3×10^{-5}	33.2×10^{-5}
13	175	6.3×10^{-5}	20.9×10^{-5}
14	149	4.1×10^{-5}	14.6×10^{-5}
15	125	2.7×10^{-5}	10.5×10^{-5}
16	107		7.8×10^{-5}

can know both the absorption of solar energy and the rate of temperature rise due to it. In fig. 7, p and ζ are independently taken as ordinates, and $A \cos \zeta$ is the abscissa; lines of equal u are entered as auxiliary lines.

From aerological data, we may determine the relation between p and u. When the incident angle of the ray is known, we can determine a point on the diagram which corresponds to given values of ζ and u. This point is then moved along the vertical to the ordinate which corresponds to the p value determined from the p,u relation. The abscissa gives the value of absorption by the vertical air-column whose base is at the given pressure (or height) and which contains the given amount of effective precipitable water. These points may be successively determined for the entire sounding, and we can then draw a curve which expresses the character of absorption of solar energy by the given air-column.

As the pressure scale is linear, the rate of temperature rise of the air is proportional to the difference in abscissa of two points on the curve which are separated by a constant pressure-interval. The scale of rate of heating which is shown in the figure refers to a pressure interval of 100 mb.

An example of an absorption curve is shown in fig. 7. It corresponds to mean atmospheric conditions given by London [13] for 20–30°N in March and shown in table 1. The value of ζ is assumed to be 40 deg.

It is seen from the curve that the absorption of solar energy increases with decreasing altitude. The rate of heating is nearly constant up to 6 km; then it gradually decreases with altitude, and beyond 12 km it becomes very small.

To compare the results given by the chart with London's [14], mean absorption of solar energy per day and mean rate of heating per day were calculated for two examples given by him. These are the mean atmospheric conditions for 0–10°N and 20–30°N in March, and we used the values of mean insolation per day given by him. Results are shown in fig. 8. For 20–30°N, our absorption curve gives smaller absorption than London's, while our rate of heating is larger in the upper troposphere and smaller in the lower troposphere than his. For 0–10°N, where precipitable water is abundant, our absorption curve gives smaller values at every level than London's; but comparison

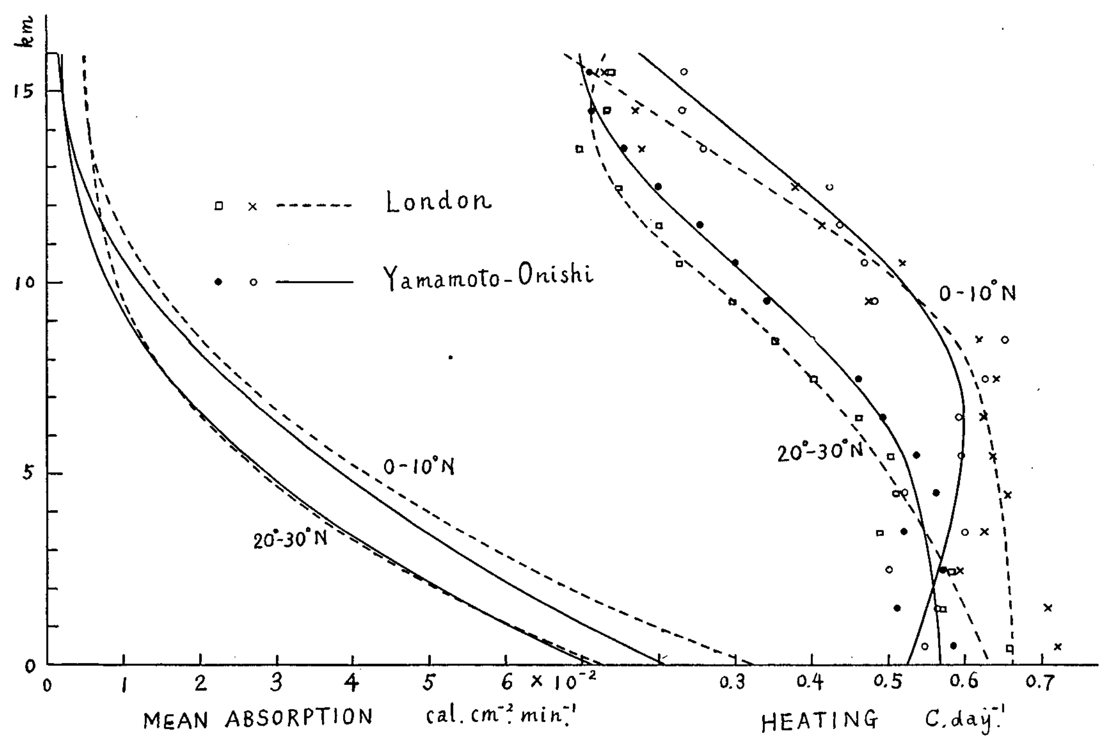

FIG. 8. Comparison of writers' results with London's for mean absorption and rate of heating.

of rate of heating shows the same tendency as before. It is to be noted that our rate of heating shows a maximum in the middle troposphere, reminiscent of the effect of ozone in the middle stratosphere. Aside from these minor differences, it may be said that London's and our results are in practical agreement.

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