

# The Relationship between Relative Humidity and the Dewpoint Temperature in Moist Air

## A Simple Conversion and Applications

BY MARK G. LAWRENCE

How are the dewpoint temperature and relative humidity related, and is there an easy and sufficiently accurate way to convert between them without using a calculator?

The relative humidity (RH) and the dewpoint temperature ( $t_d$ ) are two widely used indicators of the amount of moisture in air. The exact conversion from RH to  $t_d$ , as well as highly accurate approximations, are too complex to be done easily without the help of a calculator or computer. However, there is a very simple rule of thumb that I have found to be quite useful for approximating the conversion for moist air (RH > 50%), which does not appear to be widely known by the meteorological community:  $t_d$  decreases by about 1°C for every 5% decrease in RH (starting at  $t_d = t$ , the dry-bulb temperature, when RH = 100%):

$$t_d \approx t - \left( \frac{100 - \text{RH}}{5} \right), \quad (1)$$

or

$$\text{RH} \approx 100 - 5(t - t_d), \quad (2)$$

where  $t$  and  $t_d$  are in degrees Celsius and RH is in percent. In this article I first give an overview of the mathematical basis of the general relationship between the dewpoint and relative humidity, and consider the accuracy of this and other approximations. Following that, I discuss several useful applications of the simple conversion, and conclude with a brief perspective on the early history of research in this field.

### DEFINITIONS AND ANALYTICAL RELATIONSHIPS.

Relative humidity is commonly defined in one of two ways, either as the ratio of the actual water vapor pressure  $e$  to the equilibrium vapor pressure over a plane of water  $e_s$  (often called the “saturation” vapor pressure),

**AFFILIATIONS:** LAWRENCE—Junior Research Group, Department of Atmospheric Chemistry, Max Planck Institute for Chemistry, Mainz, Germany

**CORRESPONDING AUTHOR:** Mark G. Lawrence, Max Planck Institute for Chemistry, Junior Research Group, Department of Atmospheric Chemistry, Postfach 3060, 55020 Mainz, Germany E-mail: lawrence@mpch-mainz.mpg.de  
DOI:10.1175/BAMS-86-2-225

In final form 22 July 2004  
©2005 American Meteorological Society

*Publisher's Note:* This article was amended to correct an error in the value for coefficient  $A_1$  at the bottom of page 226. The correct value for coefficient  $A_1$  is 17.625.

$$\text{RH} = 100 \frac{e}{e_s}, \quad (3)$$

or as the ratio of the actual water vapor dry mass mixing ratio  $w$  to the equilibrium (or saturation) mixing ratio  $w_s$  at the ambient temperature and pressure:

$$\text{RH} = 100 \frac{w}{w_s}. \quad (4)$$

The two definitions are related by  $w = \epsilon e(P - e)^{-1}$  and  $w_s = \epsilon e_s(P - e_s)^{-1}$ , where  $\epsilon(0.622)$  is the ratio of the molecular weights of water and dry air, and  $P$  is the ambient air pressure. For many applications, the two definitions in Eqs. (3) and (4) are essentially equivalent, because normally  $e < e_s \ll P$ ; however, as will be shown below, in cases such as the dewpoint where exponentials are involved, the difference can become nonnegligible. The temperature to which an air parcel at initial temperature  $t$  and pressure  $P$  must be cooled isobarically to become saturated is  $t_d$  (i.e., the initial mixing ratio  $w$ , which is conserved, equals  $w_s$  at the new temperature  $t_d$ ). Normally the definition is expressed implicitly in terms of the vapor pressure

$$e_s(t_d) = e(t). \quad (5)$$

To express  $t_d$  in terms of RH, an expression for the dependence of  $e_s$  on  $t$  is needed. Over the past two centuries, an immense number of such expressions have been proposed (probably exceeding 100), both on empirical and theoretical bases. A nice review and evaluation of many of these is given by Gibbins (1990). One of the most widely used, highly accurate empirical expressions is

$$e_s = C_1 \exp\left(\frac{A_1 t}{B_1 + t}\right), \quad (6)$$

which is commonly known as the Magnus formula, although, as discussed below, this is a rather inaccurate attribution. Alduchov and Eskridge (1996) have evaluated this expression based on contemporary vapor pressure measurements and recommend the following values for the coefficients:  $A_1 = 17.625$ ,  $B_1 = 243.04^\circ\text{C}$ , and  $C_1 = 610.94$  Pa. These provide values for  $e_s$  with a relative error of  $< 0.4\%$  over the range  $-40^\circ\text{C} \leq t \leq 50^\circ\text{C}$ .

Substituting Eq. (6) in Eq. (5) yields  $t_d$  as a function of the ambient vapor pressure and temperature

$$t_d = \frac{B_1 \ln\left(\frac{e}{C_1}\right)}{A_1 - \ln\left(\frac{e}{C_1}\right)}. \quad (7)$$

Combining this with Eq. (3) then gives

$$t_d = \frac{B_1 \left[ \ln\left(\frac{\text{RH}}{100}\right) + \frac{A_1 t}{B_1 + t} \right]}{A_1 - \ln\left(\frac{\text{RH}}{100}\right) - \frac{A_1 t}{B_1 + t}}, \quad (8)$$

which is a highly accurate conversion from RH to  $t_d$ , provided that RH is defined using Eq. (3); the error that results if Eq. (4) is used is discussed below. The relationship between  $t_d$ ,  $t$ , and RH based on Eq. (8) is shown in Fig. 1, with sample values in Table 1. This conversion, broken down into multiple steps, with older coefficients (from Tetens 1930), was recently recommended for public use in a nice compilation of several humidity formulas (*USA Today*, 6 November 2000, currently available online at [www.vivoscuola.it/us/rsigpp3202/umidita/attivita/humidity\\_formulas.htm](http://www.vivoscuola.it/us/rsigpp3202/umidita/attivita/humidity_formulas.htm), or from the author on request).

A simpler, well-known analytical form for  $e_s$  can be obtained by solving the Clausius–Clapeyron equation,

$$\frac{de_s}{dT} = \frac{L e_s}{R_w T^2}, \quad (9)$$

where  $T$  is the temperature in Kelvin ( $T = t + 273.15$ ),  $R_w$  is the gas constant for water vapor ( $461.5 \text{ J K}^{-1} \text{ kg}^{-1}$ ), and  $L$  is the enthalpy of vaporization, which varies between  $L = 2.501 \times 10^6 \text{ J kg}^{-1}$  at  $T = 273.15 \text{ K}$  and  $L = 2.257 \times 10^6 \text{ J kg}^{-1}$  at  $T = 373.15 \text{ K}$ . Assuming that  $L$  is approximately constant over the temperature range encountered in the lower atmosphere allows Eq. (9) to be integrated to yield

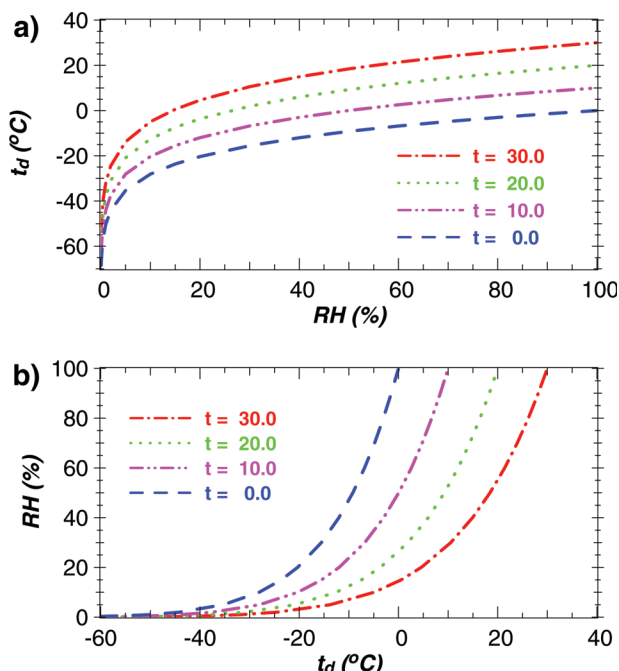
$$e_s = C_2 \exp\left(\frac{-L/R_w}{T}\right), \quad (10)$$

where  $C_2$  depends on the reference temperature for which the value of  $L$  is chosen (e.g.,  $C_2 = 2.53 \times 10^{11}$  Pa at  $T = 273.15$  K). Substituting this together with Eq. (5) into Eq. (3) and rearranging to solve for  $t_d$  gives

$$T_d = T \left[ 1 - \frac{T \ln\left(\frac{RH}{100}\right)}{L/R_w} \right]^{-1}. \quad (11)$$

This gives a good approximation to Eq. (8) when  $T$  is close to the value for which  $L$  is chosen (see Table 1), with small but nonnegligible errors away from the chosen reference temperature (see below).

**LINEAR REGRESSIONS FOR MOIST AIR.** As can be seen from Eqs. (8) and (11) and Fig. 1, the relationship between  $t_d$  and RH for any given  $T$  is nonlinear. However, for  $RH > 50\%$  the relationship becomes nearly linear; Fig. 2a shows the relationship based on Eq. (8), along with the simple approximation based on Eq. (1). A linear regression for  $RH = b - a(t - t_d)$  [the form of Eq. (2)] based on values computed with Eq. (8) for temperatures ranging from  $0^\circ$  to  $30^\circ\text{C}$  and  $RH > 50\%$  yields the slopes listed in Table 2; two cases are considered, a fixed intercept at  $RH = 100\%$ , and a free intercept (in which case the computed intercept is approximately 97.8% for all temperatures in this range). The slope  $a$  varies from 4.62 to 5.81 for the fixed intercept and from 4.34 to 5.45



**FIG. 1.** Relationship between dewpoint temperature and relative humidity for selected dry-bulb temperatures based on Eq. (8): (a)  $t_d$  as a function of RH and (b) RH as a function of  $t_d$ .

for the free intercept, which is within about  $\pm 15\%$  of the value of 5 suggested for the rule of thumb [Eq. (2)].

**MATHEMATICAL BASIS OF THE LINEAR APPROXIMATION.** The principle linearity of the relationship between RH and the dewpoint depression ( $t - t_d$ ) in moist air (Fig. 2a) has been noted pre-

**TABLE 1.** Dewpoint temperatures ( $^\circ\text{C}$ ) and absolute differences for moist air with a dry-bulb temperature of  $t = 15^\circ\text{C}$ .

RH (%)	Eq. (8)	Eq. (11)*	Eq. (1)*	Eq. (21)*	Eq. (19)*	Eq. (20)*
100.0	15.00	15.00 (0.00)	15.00 (0.00)	15.00 (0.00)	15.10 (0.10)	15.75 (0.75)
95.0	14.21	14.21 (0.00)	14.00 (-0.21)	14.26 (0.06)	14.20 (-0.01)	14.63 (0.43)
90.0	13.37	13.38 (0.00)	13.00 (-0.37)	13.46 (0.08)	13.30 (-0.07)	13.51 (0.14)
85.0	12.50	12.50 (0.00)	12.00 (-0.50)	12.58 (0.08)	12.40 (-0.10)	12.40 (-0.10)
80.0	11.58	11.58 (0.00)	11.00 (-0.58)	11.64 (0.06)	11.50 (-0.08)	11.28 (-0.30)
75.0	10.60	10.61 (0.00)	10.00 (-0.60)	10.63 (0.02)	10.60 (0.00)	10.16 (-0.44)
70.0	9.57	9.58 (0.00)	9.00 (-0.57)	9.55 (-0.02)	9.70 (0.13)	9.04 (-0.53)
65.0	8.47	8.47 (0.00)	8.00 (-0.47)	8.40 (-0.07)	8.80 (0.33)	7.93 (-0.55)
60.0	7.30	7.29 (0.00)	7.00 (-0.30)	7.19 (-0.10)	7.50 (0.20)	6.81 (-0.49)
55.0	6.03	6.02 (-0.01)	6.00 (-0.03)	5.91 (-0.12)	6.25 (0.22)	5.69 (-0.34)
50.0	4.66	4.64 (-0.02)	5.00 (0.34)	4.56 (-0.09)	5.00 (0.34)	4.57 (-0.08)

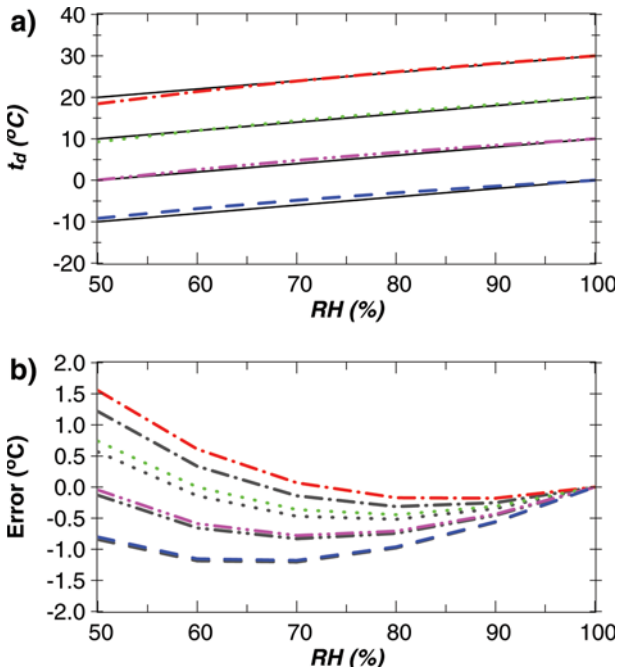
\* Absolute differences (in  $^\circ\text{C}$ ) to Eq. (8) are given in parentheses.

viously, for instance, by Sargent (1980) on an empirical basis, and theoretically by Bohren and Albrecht (1998). In particular, Bohren and Albrecht (1998) made use of this to explain why dewpoint is often preferred by meteorologists over relative humidity as an indicator of human comfort. Their approach to showing the linear nature of the curves in the moist regime begins by rearranging Eq. (11) to solve for RH (here, following their derivation, the absolute temperature  $T$  and the absolute dewpoint  $T_d$  will be used for convenience):

$$\text{RH} = 100 \exp \left[ -\frac{L}{R_w T T_d} (T - T_d) \right]. \quad (12)$$

When the exponent satisfies the condition

$$\frac{L(T - T_d)}{R_w T T_d} \ll 1, \quad (13)$$



**FIG. 2.** (a) Relationship between  $t_d$  and RH for moist air; thick colored broken lines show values based on Eq. (8) for selected dry-bulb temperatures with line styles as in Fig. (1), thin solid black lines show values based on Eq. (1); and (b) difference between  $t_d$  computed with Eq. (1) minus values from Eq. (8) for selected dry-bulb temperatures with line styles as in Fig. (1), where colored curves show values computed using Eq. (3) for RH and gray curves show values using Eq. (4) for RH assuming an air pressure of  $P = 1013$  hPa.

the exponential can be approximated by a Taylor expansion, discarding the second- and higher-order terms:

$$\text{RH} \approx 100 \left[ 1 - \frac{L}{R_w T T_d} (T - T_d) \right], \quad (14)$$

which can be rewritten in the form of Eq. (2), with the dewpoint depression expressed in degrees Celsius

$$\text{RH} \approx 100 - \beta_1 (t - t_d), \quad (15)$$

where

$$\beta_1 = \frac{100L}{R_w T T_d}. \quad (16)$$

Because  $T$  and  $T_d$  only vary by about 10% in the temperature regime that is mainly of interest (around 270–300 K),  $\beta_1$  is nearly constant, and, thus, the relationship between RH and  $t - t_d$  in Eq. (15) is nearly linear. Assuming  $T_d \approx T$ , this gives  $\beta_1 \approx 6.0$  for  $T = 300$  K and  $\beta_1 \approx 7.4$  for  $T \approx 270$  K [note that Bohren and Albrecht (1998) did not compute values for  $\beta_1$ , because they were mainly concerned with the qualitative form of the relationship]. These are somewhat larger in magnitude than the linear regression slopes in Table 2. This is because the assumption in Eq. (13) applies best near saturation; for a typical temperature of  $T = 285$  K, Eq. (13) reduces to approximately  $0.07 (T - T_d) \ll 1$ , which only holds well if  $T - T_d \lesssim$

**TABLE 2.** Slopes ( $a$ , in  $\% \text{ } ^\circ\text{C}^{-1}$ ) of the linear regression  $\text{RH} = b - a(t - t_d) = (b - at) + at_d$  for the curves in Fig. 2a, based on the values computed with Eq. (8), for a free intercept  $b$  and for a fixed intercept at  $\text{RH} = b = 100\%$ .

$t$ ( $^\circ\text{C}$ )	$b = \text{free}$	$b = 100$
30	4.34	4.62
25	4.50	4.79
20	4.67	4.97
15	4.85	5.16
10	5.03	5.37
5	5.24	5.58
0	5.45	5.81

3 K; this is about 1/3 of the overall quasi-linear range, which extends up to a dewpoint depression of about 10 K (Fig. 2a).

A more accurate expression for the slope tangent to any point in the RH versus  $t - t_d$  relationship can be obtained by dividing both sides of Eq. (11) by  $(100 - \text{RH})$  and rearranging to obtain

$$\text{RH} \approx 100 - \beta_2 (t - t_d), \quad (17)$$

where

$$\beta_2 = \frac{L(\text{RH} - 100)}{R_w T T_d \ln\left(\frac{\text{RH}}{100}\right)} = \beta_1 \left[ \frac{\left(\frac{\text{RH}}{100} - 1\right)}{\ln\left(\frac{\text{RH}}{100}\right)} \right]. \quad (18)$$

The magnitude of the slope is now seen to decrease with RH; it also decreases for higher values of  $T$  (as does  $\beta_1$ ). At RH = 50%, this gives  $\beta_2 \approx 4.3$  for  $T = 300$  K and  $\beta_2 \approx 5.4$  for  $T = 270$  K; thus, the overall slopes using Eqs. (17) and (18) are in accord with the linear regressions in Table 2. Note that it is also possible to obtain nearly the same expression as in Eqs. (17) and (18) by using the series expansion  $1(1 - x)^{-1} = 1 + x + x^2 + \dots$  directly on Eq. (11), and discarding the second- and higher-order terms, which is valid to do over a larger range of dewpoint depressions than the approximation in Eq. (13).

**ACCURACY AND OTHER APPROXIMATIONS.** How accurate is the simple conversion in Eqs. (1) and (2)? Sample values for Eq. (1) for  $t = 15^\circ\text{C}$  are given in Table 1, and the error in the rule of thumb relative to Eq. (8) is plotted in Fig. 2b for a range of temperatures. Generally, the conversion is accurate to better than  $1^\circ\text{C}$  for  $t_d$  or 5% for RH for most of the range  $0^\circ < t < 30^\circ\text{C}$  and  $50\% < \text{RH} < 100\%$ , with exceptions at the extreme temperatures. To an extent, the largest errors can be compensated for by noting the form of the error in Fig. 2b (e.g., subtracting  $1^\circ\text{C}$  for  $t \approx 30^\circ\text{C}$  and  $\text{RH} \leq 60\%$ , and adding  $1^\circ\text{C}$  for  $t \approx 0^\circ\text{C}$  and  $\text{RH} \leq 80\%$ ). When a high degree of accuracy ( $\leq 1\%$  error) is required, for example, for modeling, publication of tables or current weather reports, then this simple conversion is clearly inadequate. However, there are several applications for which this accuracy is sufficient that the rule of thumb can be very useful, as discussed in the next section.

The values in Table 1 are computed using Eq. (3) for the definition of RH. With this definition, Eq. (8) provides an accurate conversion over a wide range of temperatures, as does Eq. (11) near the chosen reference temperature (the values in Table 1 were computed with  $L = 2.472 \times 10^6 \text{ J kg}^{-1}$ , appropriate for a reference temperature of  $T = 285 \text{ K}$ ). If instead Eq. (4) is used for RH, and an air pressure of  $P = 1013 \text{ hPa}$  is assumed, then generally slightly smaller errors are computed for the rule-of-thumb conversion in Eq. (1); this is illustrated in Fig. 2b (gray curves). However, interestingly, when Eq. (4) is used, then directly using Eqs. (8) and (11) [i.e., assuming  $e_s = P$ , so that Eqs. (3) and (4) are equivalent] can lead to notable errors, compared with the accurate values computed by instead substituting Eq. (4) and  $e = (wP)(w + \epsilon)^{-1}$  into Eq. (7), particularly at the lowest values of RH considered. For RH = 50%, the error in Eq. (8) applied with Eq. (4) ranges from  $-0.04^\circ\text{C}$  at  $t = 0^\circ\text{C}$  to  $-0.34^\circ\text{C}$  at  $t = 30^\circ\text{C}$  (i.e., up to  $\sim 3\%$  of the dewpoint depression), while the error in Eq. (11) together with Eq. (4) is smallest at  $-0.14^\circ\text{C}$  for  $t = 10^\circ\text{--}15^\circ\text{C}$  (near the chosen reference temperature of  $t = 285 \text{ K}$ ), increasing in magnitude to  $-0.23^\circ\text{C}$  at  $t = 30^\circ\text{C}$  and  $-0.17^\circ\text{C}$  at  $t = 0^\circ\text{C}$ . These errors can be contrasted with the maximum error for Eq. (11) when Eq. (3) is used for RH, which is  $-0.13^\circ\text{C}$  at  $t = 0^\circ\text{C}$ . Thus, if Eq. (4) is used to define RH, and an accurate conversion is needed, then  $e$  should first be computed [using Eq. (4) and  $e = (wP)(w + \epsilon)^{-1}$ ] and then used in Eq. (7) to determine  $t_d$ , rather than directly employing Eqs. (8) or (11) with the given RH.

For cases where the rule of thumb in Eq. (1) is not adequately accurate, but a simpler conversion than Eqs. (7), (8), or (11) is desired, then various other approximations that have been proposed can be used. Sargent (1980) gives a nice overview of a wide range of approximations of various accuracies. In particular, he proposes an empirical linear fit that has the same basic form as Eq. (1):

$$t_d = t - K_0 + K_1 \text{RH}, \quad (19)$$

where  $K_0 = 17.9$  and  $K_1 = 0.18$  for  $65\% \leq \text{RH} \leq 100\%$ , and  $K_0 = 22.5$  and  $K_1 = 0.25$  for  $45\% \leq \text{RH} \leq 65\%$ . These are close to the equivalent values for the rule of thumb of  $K_0 = 20$  and  $K_1 = 0.2$  in Eq. (1), but yield a clearly more accurate conversion due to the two-part fit to the slope (see Table 1). However, Sargent's conversion is already sufficiently complex to be prohibitive for being used "on the fly." Sargent (1980) also proposed a higher-order fit, which includes a dependence on the temperature:

$$t_d = (0.198 + 0.0017t)RH + 0.84t - 19.2. \quad (20)$$

This is less accurate at higher humidities (see Table 1), but extends the range of applicability down to lower humidities, giving  $t_d$  within 1°C for 40% ≤ RH ≤ 100% and 0° <  $t$  < 30°C.

An alternative, more accurate second-order fit that removes most of the error from the rule of thumb in Eq. (1) can be obtained by 1) recognizing that the dewpoint depression ( $t - t_d$ ) depends approximately on the square of the absolute temperature  $T^2$  [see Eq. (18)], and 2) noting that the form of the remaining error (Fig. 2b) resembles a parabola centered near RH = 85%. Applying these two modifications, with coefficients for the parabola chosen to minimize the root-mean-square error for 40% ≤ RH ≤ 100% and 0° <  $t$  < 30°C (while insisting that the intercept at RH = 100% is  $t_d = t$ ) yields

$$t_d = t - \left( \frac{100 - RH}{5} \right) \left( \frac{T}{300} \right)^2 - 0.00135(RH - 84)^2 + 0.35. \quad (21)$$

Values for this conversion are also given in Table 1; the maximum error in  $t_d$  using Eq. (21) over the whole range of 50% ≤ RH ≤ 100% and 0° <  $t$  < 30°C is 0.3°C [assuming the use of Eq. (3) for RH].

At the other end of the spectrum, Sargent (1980), Gibbins (1990), and others have evaluated a wide range of complex expressions for  $e_s$ , most employing various polynomial and exponential fits for measured  $e_s$  values, such as the form of Goff and Gratch (1945, 1946); these can be used with Eqs. (3) or (4) to determine  $e$  from RH and  $e_s$ , which can then be used in Eq. (7) to yield an even greater accuracy for  $t_d$  than the analytical approximation in Eq. (8).

**APPLICATIONS.** There are several useful applications for the rule-of-thumb conversion in Eqs. (1) and (2). One that I have found to be helpful on various occasions is when maps of  $t$  and either  $t_d$  or RH (but not both) are available, and one is interested in knowing the approximate distribution of the other. In particular, weather forecast and analysis Internet sites will often provide maps of the surface air  $t$  and  $t_d$ , but not RH. In this case, using the rule of thumb in (2) allows the distribution of RH (in regions with RH > 50%) to easily be estimated. With practice, one can place maps of  $t$  and  $t_d$  next to each other and di-

rectly read off an approximate map of the RH; as long as the main interest is knowing the basic range of the RH values (e.g., ~90% versus ~70%) distributed over a geographical region, the relatively small error (< 5%) is not critical.

A related application involves comfort levels. Wallace and Hobbs (1977) noted that air with  $t_d > 20^\circ\text{C}$  is generally considered uncomfortable, and air with  $t_d > 24^\circ\text{C}$  is perceived as “sticky,” almost regardless of the actual dry-bulb temperature  $t$  and, thus, the RH. Bohren and Albrecht (1998) similarly noted that “when you ask a knowledgeable meteorologist for a forecast of how comfortable a hot summer’s day in the humid eastern United States will be, you are likely to be given a temperature and a dew point rather than a temperature and relative humidity,” and went on to provide insight into the physical basis for this by expanding on the derivation of a linear relationship discussed above. Despite this relationship between  $t_d$  and comfort, most people I knew while growing up in the southeastern United States were far more familiar with the relative humidity than the dewpoint, and if they happened to have a home weather station, it was almost certain to show  $t$ ,  $P$ , and RH, not  $t_d$ . Particularly interesting for amateur meteorologists is that in this case, the rule-of-thumb conversion makes it easy to go from RH to  $t_d$  and, thus, to have a better direct indicator of expected comfort levels.

Another use of the dewpoint is familiar to pilots and operational meteorologists: the lifting condensation level  $z_{\text{LCL}}$ , that is, the cumulus cloud-base height, is closely related to the dewpoint depression, and a first-order estimate can be obtained from the simple formula

$$z_{\text{LCL}} \approx 125(t - t_d), \quad (22)$$

where  $t$  and  $t_d$  are near-surface values and  $z$  is in meters [note that often a coefficient of 120 rather than 125 is used, though I find 125 to give more accurate values over a wider range of typical surface temperatures when Eq. (3) is used for RH]. It is interesting to note that the physical principle of this relationship was actually pointed out already over 150 yr ago by the meteorologist J. P. Espy, although he lacked an accurate value for the dry adiabatic lapse rate, so that his value for the coefficient in Eq. (22) (originally in feet and Fahrenheit) was about 1/3 too large (McDonald 1963). With modern knowledge of these parameters, Eq. (22) is now generally accurate to within about ±2% for 50% ≤ RH ≤ 100% and 0° <  $t$  < 30°C; example values are listed in Table 3, along with accurate values based on an iterative solution for a lifted,

nonentraining parcel, for which Eq. (3) is used to define RH. When Eq. (4) is instead used, notably lower values of  $z_{\text{LCL}}$  are computed for both the iterative solution and Eq. (22), by 0.5% at  $t = 0^\circ\text{C}$ , and up to 4% lower at  $t = 30^\circ\text{C}$ .

Using the rule of thumb discussed here, this relationship can be reformulated in terms of RH. Directly substituting Eq. (1) into Eq. (22) gives

$$z_{\text{LCL}} \approx 25(100 - \text{RH}). \quad (23)$$

However, as can be seen in Fig. 2b, Eq. (1) tends to underestimate  $t_d$  for lower temperatures, which results in an overestimate of the cloud-base height using Eq. (23). I have found that it is easy to adjust for this tendency by incorporating the temperature into the coefficient:

$$z_{\text{LCL}} \approx \left(20 + \frac{t}{5}\right)(100 - \text{RH}), \quad (24)$$

where  $t$  here is the dry-bulb surface temperature in degrees Celsius. This gives  $z_{\text{LCL}}$  to within  $\pm 15\%$  for the

**TABLE 3. Lifting condensation levels (m) and relative differences (%) for moist air with a surface dry-bulb temperature of  $t = 15^\circ\text{C}$  and pressure  $P = 1013 \text{ hPa}$ .**

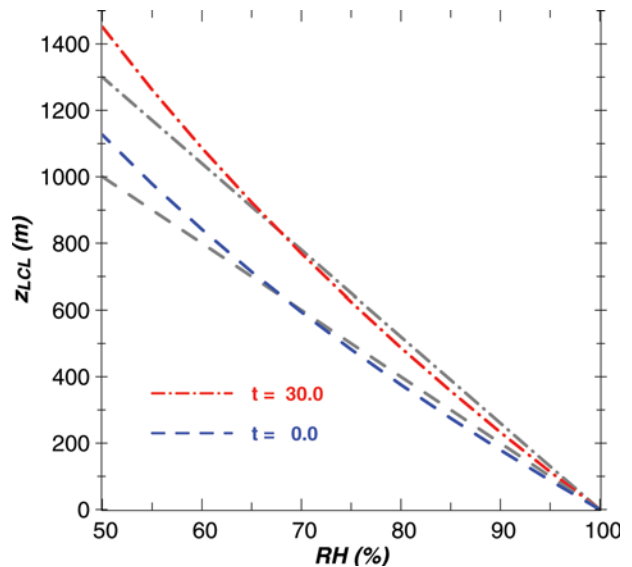
RH (%)	Iterative*	Eq. (22)**	Eq. (24)**
100.0	0.0	0.0 (0.0)	0.0 (0.0)
95.0	100.1	99.4 (-0.8)	115.0 (14.8)
90.0	204.8	203.4 (-0.6)	230.0 (12.3)
85.0	314.3	312.7 (-0.5)	345.0 (9.8)
80.0	429.3	427.8 (-0.3)	460.0 (7.1)
75.0	550.5	549.5 (-0.2)	575.0 (4.5)
70.0	678.6	678.4 (0.0)	690.0 (1.7)
65.0	814.5	815.8 (0.2)	805.0 (-1.2)
60.0	959.5	962.9 (0.4)	920.0 (-4.1)
55.0	1115.0	1121.2 (0.6)	1035.0 (-7.2)
50.0	1282.9	1292.8 (0.8)	1150.0 (-10.4)

\* Iterative solution for a nonentraining parcel lifted from the starting conditions at the surface through a layer with an ambient lapse rate of  $\Gamma_a = 6.5^\circ\text{C km}^{-1}$ , with the convergence condition that  $|wv_s^{-1} - 1| < 10^{-5}$  at the lifting condensation level.

\*\* Relative differences (%) to the iterative solution are given in parentheses.

whole range of  $50\% \leq \text{RH} \leq 100\%$  and  $0^\circ < t < 30^\circ\text{C}$ ; sample values from Eq. (24) and their relative deviations from the accurate values are listed in Table 3 and are shown in Fig. 3. If one rounds off to the nearest integer for the temperature-based coefficient, then this provides a very simple way to estimate the cumulus cloud-base height using only surface air measurements of RH and  $t$ , without needing a calculator. However, it should not be used in situations involving safety issues, such as actual flight-route planning. When greater accuracy is needed, then either Eq. (22) should be used with an accurate value for  $t_d$ , or other highly accurate approximations can be used, such as suggested by Bolton (1980).

There are also other applications beyond those discussed above. For instance, the dewpoint is useful in determining how well traditional evaporative (“swamp”) coolers will work, and it is the theoretical minimum temperature that can be achieved by modern indirect evaporative coolers. Extensive evaporative cooling is popular mainly in dry regions, such as Arizona, New Mexico, and Colorado, where the rule-of-thumb conversion does not apply (because  $\text{RH} < 50\%$ ); however, it can also be helpful in the summertime in moister regions such as Europe where air conditioning is not prevalent in homes. Then, a quick look at the humidity reading on a home weather station can give a nice indication of whether or not it is



**FIG. 3. The lifting condensation level  $z_{\text{LCL}}$  as a function of RH for two surface air temperatures; thick colored broken lines show accurate values calculated iteratively with the same nonentraining parcel model as used for Table 3, while the gray curves show approximate values using the simple formula in Eq. (24).**

worth setting up a fan and a drying rack with wet laundry in order to cool off at least a few degrees when possible, which we found to frequently be useful during the first part of the anomalously hot European summer of 2003 [as can be seen from Eq. (1), assuming at best a 50% cooling efficiency from this primitive engineering, this is only worthwhile if  $RH \leq 70\%$ ].

A final widely useful application for the simple conversion is in science education. Most students are already basically familiar with the relative humidity, and the rule of thumb provides a simple way to extend this to having a feeling for the meaning of dewpoint temperatures and dewpoint depressions. This provides a particularly nice insight when students then link this to cloud-base levels through Eq. (24), which can be put into practice outside on days with appropriate weather conditions.

**HISTORICAL PERSPECTIVE.** Practical relationships involving the humidity parameters  $t_d$  (or  $t - t_d$ ) and RH have long been of interest, and several different approximate conversions have been proposed, most notably the empirical linear fit in Eq. (19) from Sargent (1980), which is similar to Eq. (1). Nevertheless, despite a rather extensive search through recent and historical literature, as well as on the Internet, I have not yet been able to find mention of the simple rule of thumb and its applications as discussed here.

The earliest recorded careful measurements of the dewpoint that I could find were made by Dalton (1802), who was interested in understanding the process of evaporation of liquid water into moist air. He suspected that the rate of evaporation depended on the temperature, which determines the equilibrium vapor pressure ( $e_s$ ) of the liquid water, as well as the moisture content of the ambient air (which he called the “force of the aqueous atmosphere”). To quantify this moisture content, he filled a glass with cold spring water and watched to see if dew formed on the outside. If so, he poured out the water, let it warm up a bit, dried off the glass, and poured the water back in, repeating this until the first time that dew did not form; measuring the temperature of the water in the glass gave him the dewpoint (Dalton called this the “condensation point”). He then used his new table of  $e_s$  as a function of temperature (also published in the same work) to determine the vapor pressure in ambient air, reasoning (correctly) that this would be equal to  $e_s$  at the dewpoint temperature. Finally, he performed a large set of experiments in which he measured the rate of evaporation of water at different temperatures in a small tin container, using a balance to determine the change in weight, and found that the

rate of evaporation was indeed proportional to the difference between  $e_s$  at the temperature of the water in the container and the vapor pressure in the ambient air.

John Dalton’s experimental technique and his insights in this field were remarkable. His vapor pressure measurements were accurate enough to allow him to realize that  $e_s$  approximately doubled for every 22.5°F increase in temperature, and that the ratio decreased with increasing temperature (from 2.17 at 32°F to 1.59 at 212°F). This same reasoning was used by August (1828) in proposing the formula that later came to be known as the “Magnus formula” [Eq. (6)]. Gibbins (1990) also noted that G. Magnus was not the first to suggest the Magnus formula, but that this honor apparently belongs to E. F. August. I would go further and propose that Eq. (6) should properly be called the “August–Roche” or the “August–Roche–Magnus” formula (with possible coattribution to Strehlke, as well). Magnus (1844) made a very careful set of measurements of the equilibrium vapor pressure of water, which he desired to fit with a usable equation. He considered several different forms that had previously been proposed, and came to the main conclusion that the form of Eq. (6) was the best (author’s translation of the original German follows): “The form which is used by the French Academy and by Th. Young, Creighton, Southern, Tredgold, and Coriolis, and also by the authors of the entry for Steam in the Encyclopaedia Britannica [sic] . . . regardless of the exponential coefficients one may choose . . . is not as good as the form suggested by Roche, August (1828), and Strehlke, which has also been arrived at through theoretical considerations by von Wrede (1841),” which is Eq. (6). So it was clear to Magnus that attribution for the equation at least in part belonged to August (1828), the only one of the three early investigators for which Magnus gives a reference. The original form of the equation proposed by August used a base-10 logarithm, with  $A_1 = 7.9817243$ , which becomes  $A_1 = 18.3786$  when converted to the appropriate value for the form of Eq. (6); for the other coefficients he proposed that  $B_1 = 213.4878^\circ\text{C}$  and  $C_1 = 2.24208$  mm Hg. The values of  $A_1$  and  $B_1$  are not so far off from the values later recommended by Magnus (1844),  $A_1 = 17.1485$  (7.4475 in base 10) and  $B_1 = 234.69^\circ\text{C}$ , although  $C_1$  was considerably lower than Magnus’ value of 4.525 mm Hg. After Magnus (1844), and prior to more recent works like Alduchov and Eskridge (1996), the most notable update of these values was by Tetens (1930), who suggested that  $A_1 = 17.27$  (7.5 in base 10),  $B_1 = 237.3^\circ\text{C}$ , and  $C_1 = 610.66$  Pa.

So August clearly deserves at least shared attribution for this equation. Who were the others, though,



that Magnus mentions? I have not been able to find any evidence of Strehlke's work associated with this (any tips from readers would be appreciated). Von Wrede (1841) independently comes across nearly the same form, except without  $C_1$  in the equation, and, thus, quite different coefficients  $A_1$  and  $B_1$ ; he was apparently unaware of August's work, and mentions in a footnote that he had not been able to obtain a copy of Roche's work, which he had been told proposed a similar formula. The reason for this was made clearer by Magnus (1844) a few years later, who noted that (author's translation follows) "Roche had attempted to propose such a theoretical formula, yet the report for the French Academy of Sciences (1830) said that, based on the available evidence, the formula would not have the pleasure of the applause of the physicists." [The original report in French does, indeed, read rather similarly.] Nevertheless, the extensive historical account in the entry on steam in the *Encyclopaedia Britannica* (1830–1842 ed.) of that period lists about 20 equations for  $e_s(t)$ , including that of Roche, who "sent to the Academy of Sciences, in 1828, a memoir on this subject" [here they do not mention its fate; it is also difficult to determine what the coefficients Roche actually proposed were, because the equation is given differently in the *Encyclopaedia Britannica* (1830–1842 ed., s.v. "steam") and in the French Academy of Sciences (1830) report, though they are apparently significantly different from those of August and his successors]. Surprisingly, however, the authors in the *Encyclopaedia Britannica* (1830–1842 ed., s.v. "steam") do not mention the work of August (1828); this might help to explain why it was neglected, and why credit was instead later given to Magnus (1844).

Given the long history of research on atmospheric moisture, it is hard to imagine that the simple rule of thumb presented here for relating  $t_d$ ,  $t$ , and RH, as well as the simple computation of  $z_{LCL}$  from  $t$  and RH in Eq. (24), have gone unnoticed. I would certainly appreciate any information from colleagues or from science historians on whether earlier works can be found in which this rule of thumb is discussed. Nevertheless, these approximations seem to have been either generally overlooked or forgotten—at least they are not widely known in this age of the pocket calculator and laptop (perhaps already since the slide rule)—and I hope that this article will help to bring them to the attention of the various communities that could benefit from their use, including operational meteorologists, amateur meteorologists, atmospheric scientists in various specializations, and science educators.

**ACKNOWLEDGMENTS.** I would like to express my appreciation to Craig Bohren for very helpful comments throughout the development of this manuscript. The assistance of Andreas Zimmer in obtaining the historical articles is gratefully acknowledged. This work was supported by the German Ministry for Education and Research (BMBF) Project 07ATC02.

## REFERENCES

- Alduchov, O. A., and R. E. Eskridge, 1996: Improved Magnus form approximation of saturation vapor pressure. *J. Appl. Meteor.*, **35**, 601–609.
- August, E. F., 1828: Ueber die Berechnung der Expansivkraft des Wasserdunstes. *Ann. Phys. Chem.*, **13**, 122–137.
- Bohren, C., and B. Albrecht, 1998: *Atmospheric Thermodynamics*. Oxford University Press, 402 pp.
- Bolton, D., 1980: The computation of equivalent potential temperature. *Mon. Wea. Rev.*, **108**, 1046–1053.
- Dalton, J., 1802: On the force of steam or vapour from water and various other liquids, both in a vacuum and in air. *Mem. Lit. Philos. Soc. Manchester*, **5**, 550–595.
- French Academy of Sciences, 1830: Exposé des recherches faites par ordre de l'Académie royale des Sciences, pour déterminer les forces élastiques de la vapeur d'eau à de hautes températures. *Ann. Chim. Phys.*, **43**, 74–112.
- Gibbins, C. J., 1990: A survey and comparison of relationships for the determination of the saturation vapour pressure over plane surfaces of pure water and of pure ice. *Ann. Geophys.*, **8**, 859–885.
- Goff, J. A., and S. Gratch, 1945: Thermodynamic properties of moist air. *Amer. Soc. Heat. Vent. Eng. Trans.*, **51**, 125–157.
- , and —, 1946: Low-pressure properties of water from  $-160^\circ$  to  $212^\circ\text{F}$ . *Amer. Soc. Heat. Vent. Eng. Trans.*, **52**, 95–129.
- Magnus, G., 1844: Versuche über die Spannkraft des Wasserdampfs. *Ann. Phys. Chem.*, **61**, 225–247.
- McDonald, J. E., 1963: James Espy and the beginnings of cloud thermodynamics. *Bull. Amer. Meteor. Soc.*, **44**, 634–641.
- Sargent, G. P., 1980: Computation of vapour pressure, dew-point and relative humidity from dry- and wet-bulb temperatures. *Meteor. Mag.*, **109**, 238–246.
- Tetens, O., 1930: Über einige meteorologische Begriffe. *Z. Geophys.*, **6**, 297–309.
- Wallace, J. M., and Hobbs, P. V., 1977: *Atmospheric Science: An Introductory Survey*. Academic Press, 467 pp.
- von Wrede, F., 1841: Versuch, die Beziehung zwischen der Spannkraft und der Temperatur des Wasserdampfs auf theoretischem Wege zu bestimmen. *Ann. Phys. Chem.*, **53**, 225–234.