

# STOCHASTIC AND PROBABILISTIC METHODS FOR OCEAN, ATMOSPHERE, AND CLIMATE DYNAMICS

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**W**eather and climate are distinct aspects of the variability of the earth's fluid envelope. The idea that "climate is what you expect, but weather is what you get" is fundamentally probabilistic: the evolution of weather from day to day is seen as the realization of a chaotic (or stochastic) system, while climate is the probability distribution of all possible states of the weather. Understanding the connections between weather and climate is complicated by the facts that i) processes in the fluid flow result in interactions across space and time scales (so that weather and climate are dynamically coupled) and ii) there is no unique or unambiguous scale separation between fast and slow parts of the system. One person's signal is another person's noise.

Probability and stochastic processes provide natural tools for studying the dynamics of the weather-climate connection. In the summer of 2008, the University of Victoria in Victoria, British Columbia, Canada, hosted a summer school and workshop on the subject of Stochastic and Probabilistic Methods for Ocean, Atmosphere, and Climate Dynamics, bringing together graduate students, postdoctoral fellows, and senior researchers to address questions arising from such a "probabilistic perspective on climate." These questions ranged from the characterization of statistical structures in observations through fundamental issues in the development of stochastic and probabilistic tools to the application of these tools in practical problems. The summer

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**WHAT:** Graduate students, postdoctoral fellows, and senior researchers from universities and operational climate modeling centers met to discuss stochastic and probabilistic approaches to the dynamics of the atmosphere, the ocean, and the climate system

**WHEN:** 14–18 July and 21–23 July 2008

**WHERE:** Victoria, British Columbia, Canada

school presented a sequence of short (3–4 h) introductory courses: probabilistic methods for atmosphere, ocean, and climate science (by Adam Monahan, University of Victoria), information theory and statistical predictability (by Richard Kleeman, New York University), data assimilation (by Saroja Polavarapu, Meteorological Service of Canada), geophysical fluid statistical mechanics (by Xiaoming Wang, The Florida State University), parameterization in large-scale atmospheric modeling [by Norm McFarlane, Canadian Centre for Climate Modelling and Analysis (CCCma)], modern methods for transport equations (by Boualem Khouider, University of Victoria), and Monte Carlo methods (by Alexandros Sopoulos, University of North Carolina, Charlotte). The lecture notes from the summer school are available for download (at [www.pims.math.ca/scientific/summer-](http://www.pims.math.ca/scientific/summer-)

[school/summer-school-stochastic-and-probabilistic-methods-atmosphere-ocean-and-cli/](#)).

The workshop, designed to address research frontiers in the issues discussed in the summer school, involved about 40 participants, including graduate students, postdoctoral fellows, and established researchers. A number of themes emerged as contemporary directions in the application of probabilistic and stochastic approaches to atmosphere and ocean science. They are summarized here and include related details of select workshop presentations.

Stochasticity in models of geophysical fluid dynamics arises as a representation of rapidly evolving—or “turbulent”—processes that are not explicitly resolved. This fact begs the following questions: i) under what conditions is it appropriate to represent a turbulent deterministic system by an effective stochastic system and ii) when it is appropriate, what form do the effective equations take? Different approaches are presently being taken to address these questions. A first approach, the Majda–Timofeyev–Vanden–Eijnden (MTV) stochastic mode reduction strategy, makes use of a separation in time between “slow” and “fast” components of variability to arrive at effective stochastic dynamics of the slow variables. Andrew Majda (New York University) demonstrated the use of this technique for the construction of stochastic dynamical models of the low-frequency variability of the extratropical atmosphere—in particular, for the modeling of metastable regime behavior of the extratropical atmospheric flow in a “prototypical” barotropic channel model with topography. A similar approach, known as stochastic averaging (the so-called Hasselmann approach), is based on a distinct set of assumptions from that of MTV theory. Nevertheless, Joel Culina (University of Victoria) showed in the context of a two-layer

quasigeostrophic channel model that both the MTV and Hasselmann approaches can produce effective low-dimensional stochastic models capturing essential dynamical features (in this case, jet position bimodality). A third approach to the issue of effective stochastic dynamics considers systems of partial differential equations coupled to “stochastic lattice models” representing small-scale variability. Strategies for systematically averaging over the microscale dynamics to obtain either deterministic or stochastic effective resolved-scale mesoscale dynamics exist and were demonstrated by Alexandros Sopsakis (University of North Carolina, Charlotte).

Complimentary to the question of the relationship between turbulent systems and effective stochastic dynamics is the issue of how stochastic and probabilistic tools can be used to make observation-based inferences about the physics of a system. For example, while it is a basic mathematical result that linear systems driven by state-independent (i.e., “additive”) white noise display Gaussian statistics, what can be learned about a system based on observations of non-Gaussian statistics? Non-Gaussian distributions can arise as a result of either dynamical nonlinearities or the state dependence of fluctuations (so-called “multiplicative noise”). For example, Prashant Sardeshmukh (Climate Diagnostics Center, National Oceanic and Atmospheric Administration/Earth System Research Laboratory) and Philip Sura (The Florida State University) demonstrated that a linear model forced by correlated additive and multiplicative noises has a probability distribution characterized by a particular relationship between the skewness and kurtosis, which is also characteristic of observed upper-tropospheric vorticity and sea surface temperatures. Multiplicative noise can also change the mean climate; for example, Paul Williams (University of Reading) demonstrated that unresolved small-scale evaporation minus precipitation ( $E - P$ ) fluxes parameterized as a random modulation of observed moisture fluxes result in a change in the mean of other surface fluxes. Furthermore, variability in surface hydrological fluxes on subannual time scales can produce responses on interdecadal time scales in systems (such as glaciers) that are characterized by long memories (as was demonstrated by Gerard Roe, University of Washington). For any individual glacier, transient retreats of glacial extent over time scales of decades similar to the observed glacial variability can be driven by temporally uncorrelated forcing without any trends. These connections across time scales and between statistical moments of variability—that is, between “weather” and “climate”—highlight the

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care that must be taken in connecting statistics and physics in geophysical systems.

Geophysical fluid statistical mechanics represents another kind of explicit approach to probabilistic dynamics. While it has been known since the 1970s that in the absence of driving and dissipation, there is a tendency for oceanic eddies in idealized models to organize and generate “maximum entropy” mean flows steered by topography, less attention has been paid to the implications of this result for coarse-resolution ocean modeling. Greg Holloway (Institute of Ocean Sciences) and Bill Merryfield (CCCma) demonstrated, respectively, that there is evidence of these “entropic forces” in observed flows, and that these tendencies can be successfully incorporated into coarse-resolution ocean models as subgrid-scale parameterizations of unresolved eddies.

Fluctuation–dissipation theorems (FDT) provide a powerful tool for the diagnosis of the response of a dynamical system to an external perturbation in terms of the statistics of unforced variability, which can be obtained from time series of system variables, that is, for the systematic investigation of the coupling between physics and statistics. Grant Branstator (National Center for Atmospheric Research) demonstrated ways in which FDT can be used to probe the dynamics of GCMs, permitting the solution of “inverse problems” in which those forcing anomalies producing specified atmospheric flow patterns can be diagnosed. This analysis involved the assumption that the flows had quasi-Gaussian statistics, which is often reasonable at climatological time scales but is inaccurate over short time scales immediately following a change in forcing. Efforts to relax this assumption using blended algorithms are an area of active research, as Rafail Abramov (University of Illinois at Chicago) discussed.

Representation of moist convection in atmospheric models is an area in which stochastic parameterizations are particularly promising. As was noted by George Craig (Deutsches Zentrum für Luft- und Raumfahrt, Germany), there are subtle differences between two main regimes of convection: equilibrium and nonequilibrium (or triggered) convection. Craig argued that the statistical properties of equilibrium convection are highly constrained by the state of the large-scale flow, and that equilibrium statistical phys-

ics can be exploited to derive probability distributions for the upward mass flux. Cloud-resolving models can be used to look at the stochastic variability of convective mass flux in the context of the equilibrium distribution discussed by Craig; in this way, Jahanshah Davoudi (University of Toronto) has found that the statistics of the convective mass flux are in general agreement with the predicted equilibrium distribution in the lower troposphere (below 5 km) but not aloft. He suggested that this discrepancy is due to the fact that deep penetrative clouds are more intermittent in comparison to low-level/shallow clouds. An important aspect of propagating tropical convective systems is organization: low-level clouds are followed by deep convective towers and trailing stratiform anvils. This organization can be incorporated into stochastic subgrid-scale parameterizations of convective mass, moisture, and momentum transport, as was demonstrated by Boualem Khouider (University of Victoria) and Samuel Stechmann (University of California, Los Angeles).

Data assimilation is an area of immense practical significance to which probabilistic and stochastic techniques can provide real insights. Such tools are being used to study assimilation in systems with multiple interacting scales, as was discussed by John Harlim and Boris Geshgorin (both of New York University) and Nedjeljka Zagar (National Center for Atmospheric Research). As well, these tools can be used to study the structure of error covariances; examples of such analyses were presented by Nathan Arnold and Alexey Kaplan (both of Lamont-Doherty Earth Observatory).

**OUTCOME.** Recent years have seen the development and application of new probabilistic and stochastic techniques for the study of multiscale processes in the atmosphere and ocean. Through this summer school and workshop, the probabilistic toolbox of graduate students and postdoctoral fellows was expanded, and researchers in the field were able to exchange ideas. These new approaches hold the promise of important new insights into the interactions between “weather” and “climate,” with implications for improving our ability to observe, model, and understand complex fluid motions in the Earth’s atmosphere and ocean.