Quantifying the Limits of a Linear Temperature Response to Cumulative CO₂ Emissions

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ABSTRACT

Recent studies have shown that the transient climate response to cumulative carbon emissions (TCRE) of the global temperature can be well approximated by a constant value for cumulative emissions up to about 2 TtC. However, there has been little attention given in the literature to how the TCRE varies across the range of emissions rates represented by the current RCP emissions scenarios. The authors use an ensemble of simulations generated using the University of Victoria Earth System Climate Model to quantify how the temperature response to cumulative emissions varies as a function of both the total magnitude and the rate of CO₂ emissions. This study shows that the 500-yr response to a pulse CO₂ emission (1.8°C TtC⁻¹) does not depend on the magnitude of cumulative emissions up to 3 TtC. The TCRE (1.66°C TtC⁻¹), which relates to the short-term response, is relatively insensitive to constant-rate emissions up to 30 GtC yr⁻¹. This experiment shows that the formal way of estimating the TCRE—that is, at the point of CO₂ doubling in an idealized scenario with a 1% yr⁻¹ increase of the atmospheric concentration—is a highly robust measure. The authors conclude that the TCRE provides a good estimate of the temperature response to CO₂ emissions in RCP scenarios 2.6, 4.5, and 6, whereas a constant TCRE value significantly overestimates the temperature response to CO₂ emissions in RCP8.5.

1. Introduction

Since IPCC (2007), the transient climate response (TCR) and the equilibrium climate sensitivity (ECS) have been extensively used to compare the responses of different atmosphere–ocean general circulation models (AOGCMs) (Meehl et al. 2007) to changes in atmospheric CO₂ concentrations. For the current generation of earth system models (IPCC 2013), which include a global representation of the carbon cycle dynamics, the transient climate response to cumulative carbon emissions (TCRE) represents a new metric that considers additional uncertainties emerging from the response of the coupled climate–carbon system to CO₂ emissions (Collins et al. 2013).

Formally, the TCRE is defined as the global surface air temperature change per metric teraton of carbon emitted to the atmosphere (Collins et al. 2013). Although this definition is commonly accepted, it is imprecise in that it does not specify the conditions under which the TCRE has to be evaluated. To facilitate comparison among the different models and studies (Collins et al. 2013), it is helpful to calculate the value of the TCRE under standard conditions, such as at the time of CO₂ doubling in a transient simulation with 1% CO₂ increase per year, as suggested by Matthews et al. (2009) and Gillett et al. (2013). Such a calculation is therefore directly analogous to the calculation of the TCR.

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It is now well known that the global surface air temperature responds in a near-linear manner to cumulative emissions of CO$_2$ into the atmosphere (Collins et al. 2013), which implies that the TCRE is mostly independent of time and emission scenario (Allen et al. 2009; Zickfeld et al. 2012). Evidence of this linearity has been shown in observational data as well as in results from the first generation of coupled climate–carbon cycle models (Matthews et al. 2009; Allen et al. 2009) and most recently in the earth system models from phase 5 of the CMIP (CMIP5) (Gillett et al. 2013).

These studies have argued that the linearity of the TCRE emerges from two sets of compensating processes in the climate–carbon system: 1) on short time scales, the rate of saturation of carbon sinks (and hence an increase in airborne fraction of emissions) is matched approximately by the rate of saturation of radiative forcing per unit atmospheric CO$_2$ increase; and 2) on long time scales, the rate of heat uptake by the ocean is partially opposed by the rate of carbon uptake, leading to approximately stable atmospheric temperatures after CO$_2$ emissions have ceased (Goodwin et al. 2015; Solomon et al. 2009; Lowe et al. 2009; Archer 2005). Sustained CO$_2$ emissions necessarily lead to a saturation of the upper-ocean layers and the capacity of land carbon sinks, and hence to an increasing airborne fraction of emissions (Le Quéré et al. 2009). While the accumulation of CO$_2$ in the atmosphere causes the net radiative forcing to increase, there is a decrease in the radiative forcing per unit added carbon in the atmosphere due to the saturation of the strongest absorption bands of the atmosphere (Ramanathan et al. 1987). On a longer term, the ocean uptake of heat and carbon are largely governed by the vertical mixing into the deep ocean (Solomon et al. 2009; Winton et al. 2013). While the overturning circulation allows carbon and heat exchanges with the atmosphere on a multimillennial time scale (Stouffer 2004), land and upper-ocean carbon uptake have similar contributions over decades to centuries; the dynamics of both land and ocean carbon uptake are thus important factors responsible for the constancy of TCRE on decadal time scales.

According to Collins et al. (2013), the TCRE has a likely range from 0.8° to 2.5°C/TCt$^{-1}$ and remains approximately constant for emissions up to 2 TtC. This range of uncertainty reflects a fundamental underlying uncertainty in both the climate response to atmospheric CO$_2$ changes (the climate sensitivity) and response of carbon sinks (and the resulting airborne fraction of emissions) to CO$_2$ and the associated climate changes (Matthews et al. 2009). While several estimates of the TCRE have been published recently in the literature (see Collins et al. 2013), these individual studies have reported considerably different ranges for the TCRE, in part because of a lack of consistent definition of the time frame over which the TCRE is calculated (e.g., transient versus long-term warming), as well as the value of temperature change used (e.g., instantaneous versus peak warming). Additionally, there has been little attention given to how the TCRE varies as a function of the rate of CO$_2$ emissions and to the shape of the emission profile (e.g., Bowerman et al. 2011). Matthews et al. (2012) showed that the rate of global temperature change responds linearly to the rate of increase of cumulative emissions, though Krasting et al. (2014) showed that the assumption of a constant TCRE may not hold well for very high or very low emissions rates. There is therefore an urgent need to better quantify the dependence of the TCRE on the rate of emissions across a wide range of realistic emission scenarios.

The purpose of this paper is to identify and explicitly quantify the domain of emissions and emissions scenarios for which the TCRE remains approximately constant in time, considering both the total cumulative emissions and the rate of increase of cumulative CO$_2$ emissions. In addition, we highlight the difference between different definitions of the TCRE as a metric for the climate response to CO$_2$ emissions using an ensemble of 51 simulations performed with the University of Victoria (UVic) Earth System Climate Model, version 2.9 (Weaver et al. 2001; Eby et al. 2009). These runs are forced by a range of initial emission pulses and constant-rate emissions that includes the range of total cumulative emissions and emissions rates in the representative concentration pathway (RCP) scenarios 2.6, 4.5, 6, and 8.5. We first discuss the temperature response on both decadal and centennial time scales as a function of the total amount of CO$_2$ emissions (section 3a) and the rate of emissions (section 3b). We then compare these two sets of simulations, along with a third set where constant emissions were switched off at the time of CO$_2$ doubling (section 3c). Following this, we discuss the mathematical formulation of the TCRE as presented by Matthews et al. (2009) (section 3d), and use this relationship to examine the role of the carbon cycle dynamics as a key determinant of the airborne fraction of cumulative emissions and the overall stability of the TCRE (section 3e). Finally, based on the ensemble of simulations presented here, we evaluate the error involved in using the TCRE to approximate the model’s response to CO$_2$ emissions in the RCP scenarios (section 3f).

2. Model and methods

The University of Victoria Earth System Climate Model (UVic ESCM), version 2.9, consists of a three-dimensional
ocean general circulation model with 19 vertical layers and 1.81° × 3.6° of horizontal resolution. The ocean model is coupled to an energy–moisture balance atmospheric model (Weaver et al. 2001), a land surface and carbon cycle model (Meissner et al. 2003; Cox et al. 1999), and a dynamic vegetation model (Cox 2001). Other components coupled to this system include a thermodynamic–dynamic sea ice model, an ocean ecosystem biogeochemical model, an inorganic ocean carbon model, and a sediment carbon model (Eby et al. 2009; Schmittner et al. 2008).

Beginning from a stable spinup simulation under fixed boundary conditions (corresponding to the year AD 850), we then initialized the model up to the year 1850 following observed CO$_2$ concentrations (Meinshausen et al. 2011). We used the resulting year 1850 conditions to launch each simulation in the ensemble. To focus on the model’s response to CO$_2$ alone, we did not include changes in any other natural or anthropogenic external forcings during these experiments.

Figure 1 shows the cumulative carbon emissions scenarios for 27 global climate simulations covering a period of 500 years starting from 1850. This set of scenarios can be divided into four categories: pulse emissions (PE; solid lines), constant-rate emissions (CE; dashed lines), RCPs (hatched colored lines) (Meinshausen et al. 2011), and 1% idealized (hatched black line) scenario. We constructed this ensemble based on the range of cumulative carbon in the year 2350 across the four RCP scenarios, such that each RCP is matched with a pulse (from 0.63 to 5.6 TtC) again with matching constant emissions scenarios constructed by distributing the total emission over a period of 500 yr. The 1% run is a simulation where we constrained the CO$_2$ concentration to increase 1% yr$^{-1}$ until reaching CO$_2$ doubling (560 ppm) before being set to a constant value until the end of the simulation. To enable a deeper analysis of the effect of varying emissions rates on the TCRE, we also included two sets of complementary simulations. The first set consists of eight additional constant emission simulations (C7.0–C22.4) that extend the ensemble over a larger range of emissions rates up to a maximum value of 44.8 GtC yr$^{-1}$. The second group includes 16 runs labeled C2.0-OFF–C22.4-OFF. Each of these runs matches a corresponding CE simulation but with emissions switched off when atmospheric CO$_2$ concentrations reached double the preindustrial value. The 51 simulations are summarized in Table 1.

### 3. Results

#### a. Dependence on the amount of CO$_2$ emissions

The ratio of the global surface air temperature change to the cumulative CO$_2$ emissions is shown in Fig. 2a for the PE simulations. These PE runs led to relatively stable temperature values after 500 years, though the simulations with initial emissions pulses of 2.4 TtC or greater were still warming slightly, emphasizing the very long response time of the climate system to large emission pulses (Eby et al. 2009). The near stabilization of the temperature response after 500 years of simulation
approximates well the “cumulative warming commitment” (i.e., peak warming; Allen et al. 2009) associated with the initial emission pulse. The peak temperature change, however, is not reached within 500 years for most of the pulse simulations shown here, while it occurs around year 2100 for the two smallest pulses (P0.63 and P1.0, when neglecting the initial overshoot). On longer time scales, temperatures would eventually decrease slowly in all scenarios as CO$_2$ continues to be drawn out of the atmosphere by ocean and land uptake, and over the subsequent tens of thousands of years as sedimentary carbon cycle processes become significant (see, e.g., Eby et al. 2009). In the following, we refer to the 500-yr response to a pulse CO$_2$ emission as the equilibrium climate response to cumulative carbon emissions (ECRE). It is important to note that this metric does not aim at characterizing any long-term equilibrium between all components of the climate system. Rather, the ECRE terminology has been recently introduced by Frölicher and Paynter (2015) to represent the point in time when temperatures equilibrate with the contemporaneous CO$_2$ radiative forcing along with a value of ocean heat uptake that approaches zero. In the simulations analyzed here, temperatures have nearly stabilized according to CO$_2$ concentrations after 500 years of simulation, and the ocean heat uptake ranges from 7% to 16% of its initial maximum value for P0.63–P5.6 (not shown). As seen in Fig. 2, ECRE values ranged here from 1.47°C to 1.84°C TtC$^{-1}$, depending on the size of the emission pulse. Notably, for pulses larger than about 2.4 TtC, the ECRE tended to decrease with the magnitude of the emission pulse, suggesting that at these very high emissions levels, the effect of saturating CO$_2$ radiative forcing becomes more prominent than the increased airborne fraction of emissions as a result of weakened carbon sinks. In contrast, for smaller emission pulses, there is a slight increase in the ECRE as the pulse size increased from P0.63 to P2.4.

<table>
<thead>
<tr>
<th>PE</th>
<th>CE</th>
<th>RCP</th>
<th>1% idealized</th>
<th>OE</th>
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<td>C2.4</td>
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We now use this group of PE simulations to obtain a best estimate of the ECRE for the UVic model. The open circles in Fig. 2b show the year 500 ECRE values plotted as a function of the cumulative carbon emitted. The blue dashed line plotted here has a slope of \( \text{CRE} = 1.81\,\text{C TtC}^{-1} \), which we obtained from a least squares linear regression constrained at the origin and based on PE simulations from P0.63 to P3.0. We selected this particular subset of PE simulations using two criteria: the minimization of the mean-square error and the maximization of the number of data points considered in the regression. We can see here that the ECRE remains very close to this constant value of 1.81\,\text{C TtC}^{-1} emitted for any size of emission pulse up to 3 TtC, with higher pulses leading to a gradually increasing negative deviation from this linear approximation. We therefore conclude that the ECRE in this model can be reasonably treated as a near-constant value up to total emissions of about 3 TtC. It is also interesting to note that calculating the peak warming from these PE simulations would visibly lead to an even more linear relationship than the one obtained for ECRE in Fig. 2b: the peak warming would be slightly larger than ECRE for P0.63 and P1.0 (response around 2100), while it would be increasingly larger for the largest pulse simulations that reach their peak warming further in time.

b. Dependence on the rate of \( \text{CO}_2 \) emissions

Figure 3a shows the ratio of temperature change to cumulative emissions in the CE simulations. After 500 years, this ratio ranged from 1.37\,\text{C TtC}^{-1} to 1.69\,\text{C TtC}^{-1}, which was similar in spread to the ECRE measure from the PE simulations, though with slightly lower values overall. However, these simulations exhibited a different behavior over time, with the temperature change per unit emission decreasing with time, at a relatively constant rate from the year 1900 onward. Such negative deviations from a linear temperature response to emissions is a common feature of other models as well (e.g., Matthews et al. 2009; Gillett et al. 2013; Goodwin et al. 2015), which is consistent with a decreased climate response to emissions as atmospheric \( \text{CO}_2 \) concentration increases. However, Fig. 3a also shows that the larger the emissions rate, the faster the TCRE decreased as the simulation progressed. This suggests the potential for an additional limiting factor on the transient temperature response to constant emissions, which relates to the ability of the climate system to warm at the same rate as the emissions being produced. In particular, we can see here that for lower emissions rates (below about 6 GtC yr\(^{-1}\)), global temperature tracks the increase in cumulative emissions very closely, with almost no change in the temperature response to cumulative emissions over time.

Similar to the analysis of ECRE in the previous section, we now characterize the transient temperature response—that is, the TCRE—based on these CE simulations. For each run, we calculated a value for the TCRE for the year in which atmospheric \( \text{CO}_2 \) reached its doubled value (560 ppm) relative to preindustrial levels. Note that C0.63, C1.0, and C1.6 did not reach this atmospheric \( \text{CO}_2 \) concentration within 500 years of simulation because of their smaller rate of emission. Using again a linear regression constrained at the origin (same approach as for the ECRE), we obtain an estimate of the TCRE of 1.66\,\text{C TtC}^{-1}. This best estimate of
the TCRE is plotted as the green dashed line in Fig. 3b, along with all the CE curves up to C5.6.

This definition of the TCRE is consistent with Matthews et al. (2009) and Gillett et al. (2013), who recommended estimating TCRE based on the temperature change and cumulative emissions at the year of CO₂ doubling, but in a 1% yr⁻¹ CO₂ increase simulation. By using the exact same framework, the UVic model forced with a 1% scenario results in a single-point estimate of the TCRE (at the time of CO₂ doubling) that is indistinguishable from that obtained with the regression on the CE simulations. As a further comparison under the same framework, Gillett et al. (2013) reported a TCRE range (at the time of CO₂ doubling in a 1% run) from 0.8° to 2.5°C TtC⁻¹ in CMIP5 earth system models (shown as the gray area in Fig. 3b).

We now investigate explicitly how the TCRE is affected by the rate of emissions in these CE simulations, including also the extended ensemble of CE simulations up to 44.8 GtC yr⁻¹ (see Table 1). For runs C2.0–C22.4, Fig. 4 shows the single-point estimates of the TCRE at the time of CO₂ doubling (green crosses), which were used from the regression described above. The TCRE best estimate based on this regression and that calculated from the 1% run are shown at 1.66°C TtC⁻¹ (overlapping green dashed lines). The ECRE of 1.81°C TtC⁻¹ is shown at the top of the plot as the blue dashed line.

As can be seen here, the TCRE values at CO₂ doubling (green crosses) are remarkably constant for a large range of emissions rates; these estimates do not vary by more than ±0.5% for emissions rates between 9 and 30 GtC yr⁻¹. However, when calculated in this way, the TCRE value is affected by different emissions rates and different total emissions. Given that the carbon cycle response to emissions depends on the emissions rate, the cumulative emissions are not equal at 2 × CO₂ in each CE simulation; rather, faster emissions rates result in smaller cumulative emissions at 2 × CO₂ as the carbon cycle has less time to absorb these more rapid emissions.

Consequently, to isolate the effect of emissions rates from the effect of total emissions, we have also plotted in Fig. 4 (as red crosses) estimates of the TCRE values calculated at the time when total emissions reached 1 TtC. These estimates of the TCRE are much less robust to the emissions rate compared to the value at CO₂ doubling, showing variations up to 1.6% over the same range of emissions rates. Notably, it can now be clearly seen that an increased rate of emissions leads to a consistent decrease in the TCRE. We can therefore conclude that there is a sensitivity of the TCRE to the rate of emissions, though the effect is not large (less than ±0.05°C) for emissions rates up to 30 GtC yr⁻¹. Furthermore, at a given CO₂ concentration, there appears to be a compensating effect of higher emissions rates (leading to decreased TCRE) associated with lower total emissions (leading to increased TCRE). The TCRE evaluated at 2 × CO₂ is therefore very stable across the range of emissions rates considered here.

c. Comparison of 500-yr and transient responses

The 500-yr and transient temperature responses to cumulative emissions are plotted together in Fig. 5a, which shows the best estimates of the ECRE and TCRE in this model as the blue and green dashed lines, respectively. We also plot here the final (after 500 years of simulation) temperature response for both the PE (blue circles) and CE (black crosses). The values of the 500-yr response calculated from the pulse simulations were up to 10% larger than the equivalent constant-rate simulation, and this difference increased with the rate of emissions. This is consistent with the effect of the emissions rate on the TCRE described above, and also demonstrates a difference of less than 10% in this model between the instantaneous warming represented by the TCRE and the 500-yr warming associated with the same cumulative emissions (i.e., the ECRE). Frölicher and Paynter (2015) have shown that earth system models of intermediate complexity (EMICs) and low-end equilibrium climate sensitivity earth system models (ESMs) tend to simulate an ECRE that is not much larger or even smaller than the TCRE for the same amount of cumulative emissions. However, for most ESMs, the ECRE is consistently larger than the TCRE. This difference can also be seen in the analysis of Gillett et al.
(2013), who compared the TCRE calculated from the 1% simulations to that from an instantaneous CO\textsubscript{2} quadrupling experiment across 10 CMIP5 models. In all cases, the TCRE calculated from the 4 times CO\textsubscript{2} experiment (which is similar though not identical to a pulse emission and is therefore analogous to our ECRE) was either similar to or slightly larger than the TCRE at the same cumulative emissions calculated from the 1% simulations. This suggests that while the magnitude of this difference between the TCRE and ECRE is likely model specific and is likely larger for models with a higher climate sensitivity (Allen et al. 2009), we suspect nevertheless that a slightly higher ECRE compared to TCRE is a consistent feature across models.

This therefore suggests that there is some small amount of unrealized warming that could lead to up to a 10% larger (in this model) temperature increase, on a time scale of several centuries, in the case of an abrupt elimination of CO\textsubscript{2} emissions. To test this prediction, we produced a supplementary set of CE runs, but where emissions were switched off at the year where atmospheric CO\textsubscript{2} reached 2 times the preindustrial concentration (OE; see Table 1). Figure 5b shows an enlarged view of Fig. 5a over the region where the TCRE at the year of CO\textsubscript{2} doubling (green crosses) can be compared to the maximum warming that occurred in each switch-off simulation (magenta crosses). Overall, it appears that the switch-off simulations behave like pulses, with a small amount of continued warming that approaches the ECRE regression line. In all cases, the maximum warming in the switch-off simulations came very close to the ECRE predicted from the pulse simulations, consistent with the smaller pulse simulations whose ECRE values were also slightly below the regression-based estimate of 1.81°C/TtC\textsuperscript{-1}.

### d. Assessing contributions to the stability of the TCRE

As shown in Matthews et al. (2009), the TCRE can be expressed as a product of two physical quantities,

\[
\text{TCRE}(t) = \frac{\Delta T(t)}{E(t)} = \frac{\Delta T(t)}{\Delta C_A(t)} \frac{\Delta C_A(t)}{E(t)},
\]

where \(t\) is the time, \(\Delta T\) is the change in the global mean surface air temperature, \(\Delta C_A\) is the change in the atmospheric carbon level, and \(E\) is the cumulative carbon emissions. The first factor on the right-hand side of (1), \(\Delta T/\Delta C_A\), is a general measure of the climate sensitivity per unit change in the atmospheric CO\textsubscript{2} concentration. The second factor, \(\Delta C_A/E\), is the airborne fraction of total emissions (AF), which reflects the carbon cycle response to emissions and regulates the rate of increase of the CO\textsubscript{2} radiative forcing with time.

For the PE simulations, Figs. 6a and 6c show the climate sensitivity and the airborne fraction that, if multiplied together, give the climate response to cumulative emissions ratio shown in Fig. 2a. The PE simulations show an increase of the climate sensitivity with time along with a decreasing airborne fraction. The constancy of the ECRE to pulse emissions of less than 3 TtC can therefore be well explained by the compensating behavior between the climate sensitivity and the airborne fraction. On the other hand, for larger pulses, the climate sensitivity tends to saturate by the end of the simulations, while the airborne fraction is still decreasing at a rate that is independent of the size of the
pulse. This helps to explain the decrease in the temperature response to pulses larger than 3 TtC in Fig. 2a.

In general, this pattern also holds for the constant-rate emissions, with changes in the climate sensitivity (Fig. 6b) and the airborne fraction (Fig. 6d) over time mirroring each other such that the TCRE (Fig. 3a) remains relatively stable. For lower emissions rates, the climate sensitivity and the airborne fraction are relatively stable from year 2100 to the end of the simulations, with consequently no change in the value of the TCRE. As the emissions rates increased, the climate sensitivity became increasingly smaller with time, while the airborne fraction of cumulative emissions increased faster with time. The compensation of these two quantities, however, became increasingly poor for higher emissions rates leading to a general pattern of lower (and faster decreasing) TCRE with increasing emissions rates.

e. Role of the carbon sinks dynamics

Since the airborne fraction is determined by the land and ocean carbon sink dynamics, the cumulative emissions $E$ in (1) can be written as the sum of atmospheric $\Delta C_A$, ocean $\Delta C_O$, and land $\Delta C_L$ carbon anomalies: $E = \Delta C_A + \Delta C_O + \Delta C_L$. Equation (1) can therefore be expanded as

$$\frac{\Delta T(t)}{E(t)} = \frac{\Delta T(t)}{\Delta C_A(t)} \left[ 1 - \frac{\Delta C_O(t)}{E(t)} - \frac{\Delta C_L(t)}{E(t)} \right],$$

where $\Delta C_O/E$ and $\Delta C_L/E$ represent the fractional uptake from the atmosphere by the ocean and land carbon sinks, respectively.

The ocean fractional uptake of cumulative emissions is shown in Figs. 7a and 7b for pulses and constant-rate emissions runs, respectively. For the PE simulations, the ocean fractional uptake increased with time throughout all simulations (leading to the decreasing AF shown in Fig. 6c), though at a diminishing rate, owing to the decreasing level of atmospheric CO$_2$ and the consequent equilibration of deep ocean carbon concentrations. For the CE simulations, however, the ocean fractional uptake (Fig. 7b) increased throughout the simulation only for emissions rates of less than 6 GtC yr$^{-1}$ (run C3.0). At higher emissions rates, the ocean fractional uptake began to decrease with time at some point during the
simulation, with this inflection point occurring sooner for higher rates of emissions. This decrease of the ocean fractional uptake reflects a rate saturation of carbon absorption by the ocean surface layer and its transfer to the deep ocean. In our model, it appears that emissions of 6 GtC reflects a threshold above which the rate of ocean carbon uptake is unable to keep pace with the rate of emissions.

The net land carbon fractional uptake shown in Figs. 7c,d is therefore qualitatively different from that of the ocean (Figs. 7a,b). Where the ocean carbon fractional uptake generally increased with time (for all PE simulations, and for CE simulations with rates of less than 6 GtC yr\(^{-1}\)), the land fractional uptake did not. For the CE simulations (Fig. 7d), all simulations showed an early maximum land fractional uptake that was reached before the year 2000, followed by a decrease in the fractional uptake over time (with larger decreases associated with larger emissions rates). Such a maximum is also seen in the lowest pulse simulations (e.g., P0.63 and P1.0), while the largest pulses show an early stabilization followed by a slight increase toward saturation (Fig. 7b).

Finally, when ocean and land carbon uptakes are plotted as a function of cumulative emissions (Figs. 8b,c), it is interesting to note the differences in the final states of land and ocean carbon between the PE (black circles) and CE simulations (colored lines). For land carbon uptake, there is a clear pattern of increasing then decreasing land carbon uptake with increasing total emissions, but for all values of cumulative emissions, the final state of the land carbon cycle is very similar between the PE and CE simulations. This indicates that despite the very nonlinear behavior of the land carbon cycle over time, land carbon sinks nevertheless track cumulative CO\(_2\) emissions very closely (with almost no time lag). While this may carry some model dependency, this important result suggests that the land carbon uptake is purely a function of the cumulative emissions and is virtually independent of the emissions rate. This is not the case, however, for ocean carbon uptake, whereas the rate of emissions increases, the divergence between the final value of ocean carbon in the PE and CE simulations becomes ever greater. The difference in the atmospheric carbon content between the PE and CE simulations shown in Fig. 8a, therefore, reflects this ocean carbon cycle sensitivity to the rate of emissions.

**Fig. 7.** (a) Fractional ocean and (c) land uptake for the PE simulations. (b),(d) As in (a),(c), but for the CE simulations.
The dynamics of both land and ocean carbon fractional uptake shown here have an important bearing on the behavior of the airborne fraction, and therefore on the overall constancy (or lack thereof) of the TCRE. In Figs. 6c,d, the airborne fraction shows a steep decrease in both the PE and CE simulations early in the simulations, owing to the fast response and early saturation of land carbon sinks. This fast land carbon uptake is therefore a critical component of the behavior of the TCRE early in the simulations and is therefore also a key criterion of a constant TCRE on decadal time scales. By contrast, the change in ocean carbon uptake over time becomes the dominant contributor to how the airborne fraction changes in longer time scales, and is therefore the key determinant of the relative constancy of the temperature response to cumulative emissions on a time scale of decades to centuries after emissions cease.

f. How well can we approximate the climate response to CO$_2$ emissions across the RCP scenarios?

Given the constancy of both the ECRE and the TCRE across a large range of cumulative emissions and emissions rates, we can infer that both represent useful metrics for estimating the CO$_2$-induced temperature change in more realistic emissions scenarios. However, using a single constant value of either ECRE or TCRE as an estimate of the climate response to cumulative CO$_2$ emissions necessarily implies some amount of error. In the idealized simulations discussed to this point, we have shown that the error arising from a dependency on emissions scenario (i.e., the rate of increase and overall magnitude of cumulative emissions) can be characterized as a negative deviation from a linear approximation that grows with both the rate of emissions and the total cumulative emission over time. We can now assess how large this error is in the context of the range of CO$_2$ emission pathways represented by RCP scenarios.

In Fig. 9, we quantify explicitly the error associated with using ECRE or TCRE as an estimate of the CO$_2$-induced warming in the RCP scenarios 2.6, 4.5, 6, and 8.5. In general, using the TCRE (solid lines) to represent instantaneous warming over most of the simulations results in less error than using the ECRE (dashed lines), with the error relative to TCRE remaining less than $\pm 5\%$ for almost all of the RCP2.6, RCP4.5, and RCP6 scenarios. For RCP2.6 and RCP4.5, the ECRE becomes a better measure of the temperature change toward the end of the simulations, as in these simulations the majority of the cumulative emissions occurred much before the end of the simulations, and the year 2350 temperature
change is therefore comparable to the 500-yr response to a pulse emission. It is interesting to note also the slight hysteresis behavior seen in RCP2.6 associated with negative emissions after 2070, suggesting a need to more thoroughly investigate the TCRE associated with negative emissions scenarios. In the case of RCP8.5, both TCRE and ECRE give a considerable overestimate of the simulated temperature response to the very high emissions and emissions rates; this scenario therefore exceeds the limits within which the TCRE can be considered a good metric of the temperature response to cumulative emissions.

In general, this result is consistent with the 2-TtC limit for linearity given in Collins et al. (2013), though as can be seen in Fig. 9, the 2.5 TtC of emissions in RCP6 do not deviate appreciably from a linear TCRE relationship. It may also be reasonable, as discussed previously, to extend this limit upward to 3 TtC with respect to the linearity of the 500-yr (ECRE) response to emissions. Furthermore, some recent analyses have suggested that positive carbon cycle feedbacks may become more prominent under very high-emissions scenarios (Lowe et al. 2009), which would have the effect of increasing the TCRE for emissions beyond 2.5–3 TtC, leading to an even more stable TCRE relationship over a larger range of total emissions. We conclude, based on this analysis of simulations from the UVic model, that using a constant TCRE value as a measure of the climate response to cumulative emissions is a reasonable approximation across the vast majority of the range of climate-mitigation-policy-relevant emissions scenarios.

4. Discussion and conclusions

It has been previously suggested by Matthews et al. (2009) that the behavior and overall stability of the transient climate response to cumulative carbon emissions (TCRE) can be partly explained by two physical characteristics of the climate system: the climate sensitivity and the airborne fraction of cumulative emissions. In this paper, we have demonstrated that the changes in these two quantities over time tend to compensate each other for a considerably large range of emission scenarios, leading to an approximately linear temperature response to cumulative emissions that can be defined as a function of both the total magnitude and rate of increase of cumulative CO₂ emissions.

As shown in the pulse emission (PE) simulations, global surface air temperature stabilized after a few decades to centuries (depending on the size of the pulse) as the simulations approached equilibrium in terms of both a high fraction of realized warming and a small value of ocean heat uptake Frölicher and Paynter (2015). This is consistent with previous simulations of model responses to pulse emissions (e.g., Matthews and Caldeira 2008), though we also show here that there is a qualitative difference between smaller (less than about 3 TtC) and larger (greater than 3 TtC) emission pulses, whereby within the 500-yr time frame shown here, the global temperature response to larger pulses did not fully stabilize and were still warming slightly 500 years after the emission pulse.

The constant-rate emissions (CE) simulations also showed very stable TCRE values across a wide range of emissions rates. In general, however, all of the CE simulations showed a small decrease in the TCRE as the simulation progressed, with higher emissions rates leading to a faster TCRE decrease. This result is generally consistent with other recent analyses that have shown a small sensitivity of the TCRE to the rate of CO₂ emissions (e.g., Krasting et al. 2014). It is important to note, however, that there is some conflation of the response of the TCRE to high emissions rates and to high total emissions, both of which occur in simulations with high levels of sustained emissions. We have shown here that the effect of the emissions rate by itself can be isolated by calculating the TCRE at the same cumulative emissions across the CE simulations, leading to a decrease in the TCRE by 0.1 °C TtC⁻¹ as the emissions rate increases from 5 to 30 GtC yr⁻¹. However, this effect of the emissions rate appears to be smaller than the decline in TCRE in a single high CE simulation as
cumulative emissions increase. It is also notable that when the TCRE is calculated at the point of doubled CO$_2$, it varies even less as a function of emissions rates since the effect of the emissions rate at a given CO$_2$ concentration is compensated to some extent by the effect of different cumulative emissions. It is worth emphasizing also that the variation in the TCRE simulated across all the emissions scenarios shown here (1.37°–1.84°C TtC$^{-1}$) is considerably smaller than the range of TCRE simulated by the current range of CMIP5 earth system models (0.8°–2.4°C TtC$^{-1}$) (Gillett et al. 2013).

We have used here two different definitions of the temperature response to cumulative emissions: the ECRE (representing the 500-yr response to a pulse emission) and the TCRE (representing the instantaneous warming at the point of doubled CO$_2$). Given the small (but systematic) difference between these two quantities (1.81°C TtC$^{-1}$ for the ECRE and 1.66°C TtC$^{-1}$ for the TCRE), we suggest that there is value to retaining separate estimates of the 500-yr (or peak) versus instantaneous temperature response to cumulative emissions, in a manner analogous to the differences between the equilibrium climate sensitivity (ECS) and the transient climate response (TCR). Again, however, the difference between the “transient” and “500 year” values is not large—less than 10% in our model—and is certainly much less than the difference between the ECS and the TCR (Collins et al. 2013).

We also showed in this paper how the fraction of emitted CO$_2$, which remains in the atmosphere, depends on the dynamics of land and ocean carbon sinks, which in turn depend on nonlinear feedbacks within the climate system (Zickfeld et al. 2011). For all simulations presented here, the fractional uptake of emissions was roughly evenly partitioned between the ocean and land during the first century of the simulations, and then diverged in favor of higher ocean fractional uptake as land carbon sinks become saturated. This behavior of the UVic model is consistent with more complex earth system models from CMIP5 (Arora et al. 2013). While the ocean carbon uptake showed small sensitivity to the emissions rate, this was not the case for the land carbon, which appeared to depend only on the total cumulative emissions. This interesting result for land is likely to differ for models including a permafrost component, for example, which could in turn introduce some inertia to the land carbon cycle response. For the ocean fractional uptake, it began to show evidence of saturation with respect to the rate of carbon uptake for emissions rates greater than 6 GtC yr$^{-1}$, leading to a decrease over time of the ocean fractional uptake with higher emissions rates. Despite this decreased ocean uptake and corresponding increased AF, the TCRE trend over time remained negative, suggesting that 1) saturation of CO$_2$ radiative forcing remained a stronger constraint on warming despite decreased ocean carbon uptake; and/or 2) the rate of saturation of ocean carbon uptake was not balanced at this point by a corresponding constraint on the rate of ocean heat uptake.

We concluded by estimating the error associated with using either the TCRE or the ECRE as a metric for the temperature response to cumulative CO$_2$ emissions in the RCP emissions scenarios. Overall, there is a general pattern of negative deviation from a linear TCRE approximation in response to sustained emissions over time, which thus decreases the robustness of the TCRE identity for very large total and/or very rapidly increasing emissions. This is particularly evident for RCP8.5, where the error relative to TCRE exceeds ±5% long before cumulative emissions begin to approach 3 TtC. However, we do find that the TCRE is a good estimate of the transient warming for most of the RCP2.6, RCP4.5, and RCP6 simulations, and, in addition, that the ECRE provides a good estimate of the temperature change on a time scale of centuries after CO$_2$ emissions have ceased in these scenarios. In light of these results obtained using an earth system model of intermediate complexity (UVic) of medium climate sensitivity compared to the range of the CMIP5 state-of-the-art earth system models, we therefore conclude that an assumption of a constant TCRE (i.e., a linear temperature response to cumulative emissions) can be adopted with reasonable confidence for emission scenarios that fall within the range of what is likely in the context of current international climate mitigation targets.

Finally, we showed here that the compensating behavior of the climate sensitivity and the airborne fraction, as suggested by Matthews et al. (2009) to explain the near-linear nature of the TCRE, holds for a wide range of scenarios, which clearly suggests that these variables are not independent of each other. This statement carries important implications from the point of view of how uncertainties are perceived in integrated risk analyses related to global climate change. For example, the Kaya identity (Kaya 1990) has been extensively used in the IPCC Special Report on Emissions Scenarios (Nakićenović et al. 2000) and more recently by IPCC Working Group III (Blanco et al. 2014), as well as in several other studies (e.g., Li et al. 2014; Raupach et al. 2007; Friedlingstein et al. 2014), in order to relate CO$_2$ emissions to demographic, economic and technological drivers. As shown in Shlyakhter et al. (1995), the TCRE and Kaya identities can be connected into a
longer causal chain product ranging from global population to climate change impacts $\Delta h$ such as

$$\Delta h = \text{population} \times \frac{\text{energy}}{\text{person}} \times \frac{\text{CO}_2\text{emit}}{\text{CO}_2\text{atm}} \times \Delta \text{temp} \times \frac{\Delta h}{\Delta \text{temp}},$$

(3)

where factors 4 and 5 correspond to the TCRE identity. It is worth noting that the $\text{CO}_2\text{atm}$ term in the Kaya identity represents an emissions rate (e.g., Li et al. 2014), rather than cumulative $\text{CO}_2$ emissions. A near-linear relationship, however, remains between emissions and warming rates $\Delta \text{temp}$ since a given amount of $\text{CO}_2$ emitted on a yearly basis can be attributed to a value of warming (Matthews et al. 2012). While studies generally consider each factor in this equation to be independent, the conclusion that the TCRE is nearly constant in time and across a large range of scenarios could ultimately allow for a narrower estimate of the overall uncertainty in the climate impacts that result from human activities. While this complex chain of processes is being integrated into today’s most advanced state-of-the-art earth system models of intermediate complexity (EMICs) as a two-way interaction between climate and human activity (e.g., Sokolov et al. 2005), the concept of a robust and relatively constant TCRE could in turn help promote increased confidence in decision-making processes aimed at limiting cumulative emission as a mechanism for climate risk management.

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