Regional Analysis of Temperature Extremes: Spatial Analog for Climate Change?

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ABSTRACT

The statistical theory of extreme values is applied to daily minimum and maximum temperature time series in the U.S. Midwest and Southeast. If the spatial pattern in the frequency of extreme temperature events can be explained simply by shifts in location and scale parameters (e.g., the mean and standard deviation) of the underlying temperature distribution, then the area under consideration could be termed a "region." A regional analysis of temperature extremes suggests that the Type I extreme value distribution is a satisfactory model for extreme high temperatures. On the other hand, the Type III extreme value distribution (possibly with common shape parameter) is often a better model for extreme low temperatures. Hence, our concept of a region is appropriate when considering maximum temperature extremes, and perhaps also for minimum temperature extremes.

Based on this regional analysis, if a temporal climate change were analogous to a spatial relocation, then it would be possible to anticipate how the frequency of extreme temperature events might change. Moreover, if the Type III extreme value distribution were assumed instead of the more common Type I, then the sensitivity of the frequency of extremes to changes in the location and scale parameters would be greater.

1. Introduction

One of the most important aspects of climate, especially with regard to its impacts on society and the environment, is the occurrence of extreme events. For example, very high summer temperatures can cause severe damage to the U.S. corn crop (Mearns et al. 1984), and very low winter temperatures may lead to the killing of citrus trees (Miller and Glantz 1988). Along with the predicted changes in average conditions in connection with global climate change, the frequency of occurrence of such extreme events can also be expected to change. At present, general circulation models used to study the details of the enhanced greenhouse effect are unable to provide reliable information regarding these possible changes in extremes. In the absence of such information, a statistical approach could be adopted.

As a step toward anticipating how the frequency of extremes might change, the statistical theory of extreme values will be applied to study the regional pattern of extreme temperature events under the present climate in the U.S. Midwest and Southeast. Regional analysis has been applied in hydrology for many years as a way to better estimate flood frequencies (Chowdhury et al. 1991; Hosking et al. 1985a). The basic rationale for this approach is that more accurate estimates can be obtained if information is combined within an area. Although our goals are somewhat different, they entail making precisely the same assumptions about the regional characteristics of extremes. Specifically, the definition of a "region" is an area in which the spatial pattern in the frequency of extremes can be explained simply by shifts in the location and scale parameters of the underlying distribution.

Katz and Brown (1992) applied the statistical theory of extreme values to study the relationship between the frequency of extreme events and more basic statistical characteristics of climate (e.g., the mean or standard deviation). But these theoretical results are based on certain assumptions about the statistical properties of climate variables. For instance, the Type I extreme value distribution was employed as an approximation for the distribution of extreme high or low temperatures. One of the purposes of the regional analysis is to examine the adequacy of this approximation.

The statistical framework for this study, including a proposed model for differences in climate over space and a discussion of extreme value theory, is presented in section 2. Descriptive results of the regional analysis of U.S. temperature extremes are provided in section 3, with some technical details of a simulation study included in the appendix. The choice of extreme value distribution is more formally examined in section 4. Finally, the implications of these results for anticipating

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how the frequency of extreme temperature events might change as part of a changing climate are discussed in section 5.

2. Statistical framework

a. Model for regional differences in climate

Extreme temperature events defined on a daily timescale are considered. Here we denote the daily time series of a temperature variable (i.e., daily minimum or maximum temperature) over a period of length n (say, the 31 days in the month of July) by X_t, $t = 1, 2, \cdots, n$. Suppose that X_t has a distribution function (df) $F(x) = \Pr\{X_t \leqslant x\}$. For simplicity, this df F is not allowed to depend on the time index t (i.e., the X_t are identically distributed), but it is not necessarily required that the X_t be independent. We assume that the df F has a location parameter μ and a scale parameter σ; that is, the distribution of the standardized variable $(X_t - \mu)/\sigma$ does not depend on either μ or σ. Because daily temperature variables for many areas are known to have approximately normal distributions (e.g., Mearns et al. 1984), one choice of the df F would be the normal. In this special case, the location parameter μ is the mean and the scale parameter σ is the standard deviation. For other distributions, the location and scale parameters do not necessarily have such a simple correspondence to the mean and standard deviation.

This framework has been previously proposed as a convenient choice for a statistical model of temporal climate change (Katz 1993). In the present context, differences in climate over space are viewed as a combination of two different statistical operations: (i) the df F is shifted, producing a change in the location parameter μ; and (ii) F is rescaled, producing a change in the scale parameter σ. On the other hand, the shape of the df F is constrained to remain the same no matter what the site within a given region (e.g., if F were the normal df, then it would remain the normal). Many other distributions that are commonly fitted to climate variables (e.g., exponential, gamma, lognormal, squared normal, Weibull) can be seen to fall within this framework, in some instances through use of transformations or approximations (Katz 1993; Katz and Garrido 1994).

b. Definition of extreme events

One of the extreme events of interest is the temperature ever exceeding some fixed, relatively high threshold. Formally, define the extreme high temperature event, E_M say, as

$$E_M = \{M_n > c_M\}, \quad \text{where} \quad M_n = \max\{X_t: t = 1, 2, \cdots, n\},$$

where X_t denotes the daily *maximum* temperature variable, c_M a high temperature threshold, and n the length of the sequence of observations (i.e., total number of days).

Similarly, another extreme event of interest is the temperature ever falling below some relatively low threshold. Define the extreme low temperature event, E_m say, as

$$E_m = \{m_n < c_m\}, \quad \text{where} \quad m_n = \min\{X_t: t = 1, 2, \cdots, n\},$$

where X_t now denotes the daily *minimum* temperature variable and c_m a low temperature threshold.

c. Extreme value theory

Under a wide range of conditions for the time series of the climate variable X_t, it can be shown that the maximum value M_n has a limiting distribution

$$\lim_{n\to\infty} \Pr\{a_n(M_n - b_n) \leqslant x\} = G_M(x). \tag{1}$$

Here G_M is one of three possible forms of extreme value distribution (EVD) (e.g., chapter 1 of Leadbetter et al. 1983). In (1), $a_n > 0$ and b_n are normalizing constants that depend on the sequence length n and the "parent" df F (in particular, a_n and b_n depend directly on the location and scale parameters, μ and σ). Analogous limit theory holds for the distribution of the minimum of a series m_n.

The three forms of EVD $G_M(x)$ for the maximum M_n are

(i) Type I: $$G_M(x) = \exp[-\exp(-x)] \tag{2a}$$

(ii) Type II: $$G_M(x) = \exp[-(x)^{1/k}], \quad x > 0, \quad k < 0 \tag{2b}$$

(iii) Type III: $$G_M(x) = \exp[-(-x)^{1/k}], \quad x < 0, \quad k > 0. \tag{2c}$$

The Type I EVD is also known as the Gumbel distribution (Gumbel 1958), the Type II as the Cauchy or Fréchet distribution, and the Type III as the Weibull distribution (Johnson and Kotz 1970). The Type II and III EVDs include a shape parameter k.

The corresponding EVDs, $G_m(x)$ say, for the minimum m_n are

(i) Type I: $$G_m(x) = 1 - \exp[-\exp(x)] \tag{3a}$$

(ii) Type II: $$G_m(x) = 1 - \exp[-(-x)^{1/k}], \quad x < 0, \quad k < 0 \tag{3b}$$

(iii) Type III: $$G_m(x) = 1 - \exp[-(x)^{1/k}], \quad x > 0, \quad k > 0. \tag{3c}$$

The three types of EVD can be combined into a single generalized extreme value (GEV) distribution (e.g., Jenkinson 1955, 1969) for the maximum, the so-called von Mises parameterization:

$$G_M(x) = \exp[-(1 - kx)^{1/k}], \quad 1 - kx > 0, \quad k \neq 0. \tag{4}$$

This df reduces to the Type I EVD for $k = 0$, the Type II for $k < 0$, and the Type III for $k > 0$. The EVDs for the minimum can be generalized in a similar manner.

The type of EVD that is appropriate for modeling the extreme events E_M or E_m is determined by the shape of the extreme tails of the parent df F [i.e., $1 - F(x)$ for large x for the maximum; $F(x)$ for small x for the minimum]. For example, the Type I EVD is the limiting form for the maximum M_n of a sequence from a df F with a light to moderate right-hand tail (e.g., the exponential or normal distribution). The Type II arises for the maximum M_n from a df that has a relatively heavy right-hand tail and is unbounded on the right, whereas the Type III arises for an underlying df that has a relatively heavy right-hand tail but is bounded on the right. Generally, the forms of underlying df F that are conventionally fitted to meteorological variables lead to the Type I as the limiting distribution for the maximum. By symmetry, the Type I [see (3a)] also arises as the limiting distribution of the minimum m_n of a sequence from a parent normal df F.

We note that time series of daily minimum and maximum temperatures possess a substantial degree of autocorrelation. But this dependence does not necessarily invalidate the asymptotic extreme value theory (chapter 3 of Leadbetter et al. 1983). In particular, Katz and Brown (1994) found that time series models of the form usually fitted to daily temperatures to allow for this dependence do not have a substantial effect on the accuracy of extreme value approximations.

d. Parameter estimation

1) Known parent distribution

If the parent df F for the climate variable X_t is known, then the normalizing constants a_n and b_n for the maximum M_n in (1) can be determined theoretically. For example, if F is the normal df with mean μ and standard deviation σ, then G_M in (1) is the Type I EVD [see (2a)] and a_n and b_n can be expressed as

$$a_n = a_n^*/\sigma, \quad b_n = \mu + \sigma b_n^*, \tag{5a}$$

where

$$a_n^* = (2 \ln n)^{1/2}, \tag{5b}$$

$$b_n^* = a_n^* - [1/(2a_n^*)](\ln \ln n + \ln 4\pi).$$

Note that in (5b) a_n^* and b_n^* are the normalizing constants for the maximum of a standard normal variable $(X_t - \mu)/\sigma$ (chapter 1 of Leadbetter et al. 1983). Moreover, these expressions apply even when the X_t are dependent, provided that the autocorrelation function converges to zero for higher lags at a fast enough rate (chapter 4 of Leadbetter et al. 1983). Expressions for a_n^* and b_n^* are also available for other parent distributions (e.g., exponential, lognormal) and for the analogous case of the minimum m_n (chapter 1 of Leadbetter et al. 1983).

2) Empirical fit of EVDs

An alternative to theoretically determining the normalizing constants, a_n and b_n, for the maximum M_n in (1) is to introduce location and scale parameters directly into the standardized EVDs [see (2)]. The normalization in (1) implies that the maximum M_n has an EVD with location parameter $A = b_n$ and scale parameter $B = 1/a_n$. For instance, in the case of the Type I EVD,

$$\Pr\{M_n \leqslant x\} \approx \exp[-(\exp[-(x - A)/B])]. \tag{6}$$

An analogous approach works with the EVDs for the minimum m_n.

The parameters A, B, and k of the EVDs may be estimated in a number of different ways (Faragó and Katz 1990). The method of moments estimators of A and B for the Type I EVD are easy to compute but quite inefficient. Better estimators of the parameters can be obtained using the maximum likelihood approach, which requires iterative solutions (Hosking 1985). We employ these maximum likelihood estimators, making use of the computer software described in Faragó and Katz (1990). Other estimation methods include the method of probability-weighted moments (Hosking et al. 1985b).

3) Relationships among parameters

Our statistical model for spatial differences in climate (section 2a) entails changing the location and scale parameters, μ and σ, of the parent df F for the climate variable X_t. These parameters are related to the location and scale parameters, A and B, of the EVD by

$$A = \mu + \sigma b_n^*, \quad B = \sigma/a_n^*. \tag{7}$$

Here a_n^* and b_n^* are the normalizing constants in (1) that arise for the maximum of a standardized variable $(X_t - \mu)/\sigma$ [e.g., given by (5b) for F the normal df]. In our regional analysis, both μ and σ in (7) will be allowed to vary spatially.

3. Descriptive temperature statistics

a. Temperature data and basic statistics

The data analyzed here consist of relatively long series of daily temperature observations from the U.S. Midwest and Southeast. Specifically, we consider time series of daily maximum temperature for the month of July at 30 stations in the Midwest, and of daily minimum temperature for January at 28 stations in the Southeast. Figure 1 shows the geographical locations of these stations. These data are a subset of the daily surface observations archived by the U.S. National

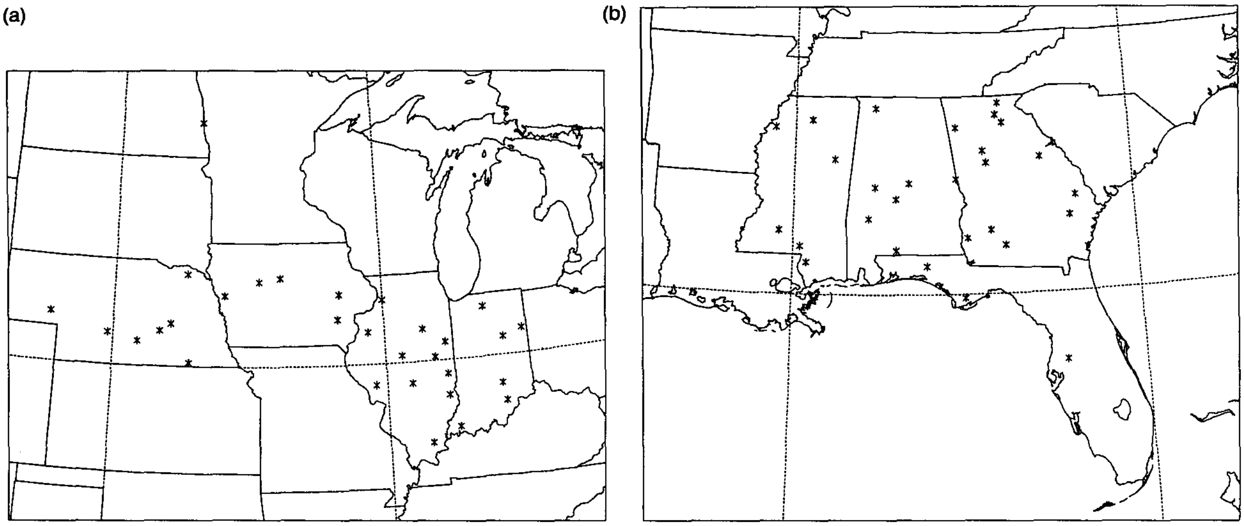

FIG. 1. Geographical locations of U.S. stations: (a) Midwest and (b) Southeast.

Climatic Data Center. The number of years of observation ranges from 41 to 88, with most stations in the Midwest having at least 75 years, and most in the Southeast having about 55–60 years (see Tables 1 and 2). These two regions are quite large, being selected to provide a substantial variation in climate characteristics as a stringent test of the ability of our statistical model (section 2a) to represent spatial patterns in extreme temperatures.

Basic statistical characteristics of the temperature data series are provided in Tables 1 and 2. The mean values for July daily maximum temperature in the Midwest (Table 1) range from about 28°C in southeast North Dakota to about 33°C in southern Illinois and southern Nebraska. The mean January daily minimum temperatures in the Southeast (Table 2) cover a wider interval, from about −4°C in northern Georgia to about 9°C in central Florida. The standard deviations range between 2.9° and 4.6°C for daily maximum temperature (Table 1) and are higher but with a similar range of between 5.5° and 7.3°C for daily minimum temperature (Table 2). The highest daily maximum temperatures ever recorded at the individual stations in the Midwest (Table 1) vary between about 41° (southern Indiana) and 48°C (northeastern Nebraska), whereas the lowest minimum temperatures ever recorded in the Southeast (Table 2) vary between about −27° (northern Georgia) and −8°C (central Florida).

b. *Regional analysis of extreme temperature events*

For simplicity, the results of the analysis of only two specific extreme temperature events are presented: (i) the daily maximum temperature in July ever exceeding 37.8°C (i.e., extreme event E_M with a threshold of $c_M = 37.8$°C), and (ii) the daily minimum temperature in January ever falling below −6.7°C (i.e., extreme event E_m with a threshold of $c_m = -6.7$°C). The observed relative frequencies for this maximum temperature event are also included in Table 1, ranging from 0.14 in east-central Illinois to 0.76 in southern Nebraska. The observed relative frequencies for this minimum temperature event (Table 2) range from 0.04 in central Florida to ~1 (i.e., 0.98) in northern Georgia.

Figure 2 shows plots of these observed relative frequencies of the extreme temperature events E_M and E_m versus the corresponding standardized threshold [i.e., $(c - \mu)/\sigma$]. Also included in the figure are the approximate sampling errors for these probability estimates. The location and scale parameters, μ and σ, of the parent df F were estimated using the long-term sample means and standard deviations of the time series of July maximum and January minimum daily temperatures for the individual stations (i.e., listed in Tables 1 and 2). Our statistical model for spatial differences in climate (section 2a) would imply that the scatterplot for event E_M (Fig. 2a) represents the right-hand tail of the standardized distribution of the highest temperature over the entire month of July (i.e., M_n). Similarly, the scatterplot for the event E_m (Fig. 2b) should represent the left-hand tail of the standardized distribution of the lowest temperature over the entire month of January (i.e., m_n).

In fact, the points in both of the scatterplots in Fig. 2 fall along reasonably smooth curves. Similar results are obtained when extreme events defined by other values for the thresholds, c_M and c_m, are analyzed. More formal tests for regional "homogeneity" have appeared in the hydrology literature (e.g., Hosking and Wallis 1993). However, these tests are inappropriate, because they are based on assumptions (especially, no spatial correlations) that are unrealistic for temperature data.

TABLE 1. Descriptive statistics of time series of daily maximum temperature for July in U.S. Midwest.

State (Number)	Station (Number)	Number of years	Mean (°C)	Std dev (°C)	Max (°C)	Relative frequency {monthly max > 37.8°C}
Illinois (11)	Harrisburg (3879)	87	33.1	3.2	45.0	0.44
Illinois (11)	Hoopeston (4198)	87	30.5	3.6	43.9	0.24
Illinois (11)	Lincoln (5079)	82	31.3	3.5	45.0	0.28
Illinois (11)	Monmouth (5768)	88	30.8	3.6	43.3	0.24
Illinois (11)	Morrison (5833)	88	30.3	3.6	44.4	0.19
Illinois (11)	Palestine (6558)	86	32.1	3.5	45.6	0.42
Illinois (11)	Pana (6579)	87	31.2	3.5	46.1	0.31
Illinois (11)	Paris Wtwks. (6610)	88	31.2	3.4	42.8	0.25
Illinois (11)	Pontiac (6910)	85	30.7	3.6	42.2	0.25
Illinois (11)	Urbana (8740)	86	30.0	3.4	42.8	0.14
Illinois (11)	White Hall 1E (9241)	85	31.7	3.5	45.0	0.33
Indiana (12)	Berne (676)	79	30.0	3.5	41.7	0.19
Indiana (12)	Columbus (1747)	88	31.3	3.4	43.9	0.25
Indiana (12)	Evansville AP (2738)	41	31.6	2.9	40.6	0.17
Indiana (12)	Marion College (5337)	88	30.0	3.6	42.2	0.20
Indiana (12)	Plymouth PS (7028)	83	30.0	3.6	42.8	0.16
Indiana (12)	Scottsburg (7875)	88	31.7	3.3	42.8	0.26
Iowa (13)	Cedar Rapids 1 (1319)	88	30.3	3.8	43.3	0.27
Iowa (13)	Onawa (6243)	86	31.3	3.7	43.3	0.38
Iowa (13)	Rockwell City (7161)	86	30.3	3.8	42.8	0.29
Iowa (13)	Washington (8688)	87	30.8	3.7	45.0	0.29
Iowa (13)	Webster City (8806)	83	30.2	3.9	42.8	0.30
Nebraska (25)	Central City (1560)	58	32.3	4.2	46.7	0.64
Nebraska (25)	Fairbury (2820)	88	33.0	4.6	45.6	0.76
Nebraska (25)	Gothenburg (3365)	87	32.3	4.2	46.7	0.69
Nebraska (25)	Grand Island AP (3395)	87	32.3	4.6	47.2	0.72
Nebraska (25)	Hartington (3630)	88	31.5	4.3	47.8	0.58
Nebraska (25)	Kearney (4335)	58	32.1	4.4	45.6	0.69
Nebraska (25)	Oshkosh (6385)	76	31.5	4.0	43.9	0.62
North Dakota (32)	Fargo AP (2859)	47	28.3	3.9	41.1	0.15

A limited simulation study was conducted to verify that the patterns obtained in Fig. 2 are indistinguishable from those attributable to sampling error for a "homogeneous" region. (See the appendix for more details.)

The results in Fig. 2 indicate a degree of spatial structure in the likelihood of extreme temperature events that is consistent with our statistical model for regional differences in climate. It is important to recall that, in this regional analysis, no assumption has been made about the particular form of parent df F, just that its shape remains the same over the entire region. Consequently, no assumption has been made about the corresponding form of the distribution of the monthly maximum temperature M_n or of the monthly minimum temperature m_n, just that its shape does not vary across the region. In particular, these distributions need not even be exactly one of the EVDs [(2) or (3)].

4. Choice of extreme value distribution

a. Parent normal distribution

As mentioned in section 2a, it is common to assume that daily maximum or minimum temperature observations have a normal distribution. Recall (section 2c) that the Type I EVD is the appropriate limiting distribution for the maximum or minimum of a sequence (i.e., M_n or m_n) when the underlying parent df F is the normal. In this case, the normalizing constants for the Type I can be theoretically determined by (5).

Figure 3 shows scatterplots of the observed relative frequencies of the same two extreme events treated in section 3b, the maximum temperature event E_M with a threshold of $c_M = 37.8°C$ and the minimum temperature event E_m with a threshold of $c_m = -6.7°C$, versus the probabilities estimated using the Type I EVD with the normalizing constants a_n and b_n determined using (5) (here $n = 31$ days). As in Fig. 2, the parameters μ and σ of the assumed normal parent df F that appear in (5) were estimated using the long-term sample means and standard deviations for the individual stations. For both maximum and minimum temperature, the frequencies of the extreme events are substantially overestimated in almost all cases. A similar degree of overestimation is obtained when extreme events defined by other threshold values are analyzed. These biases could arise from a number of different sources, as will be discussed in more detail in section 4d.

TABLE 2. Descriptive statistics of time series of daily minimum temperature for January in U.S. Southeast.

State (Number)	Station (Number)	Number of years	Mean (°C)	Std dev (°C)	Min (°C)	Relative frequency {monthly min < −6.7°C}
Alabama (1)	Brewton 3SSE (1084)	58	2.9	7.1	−16.1	0.69
Alabama (1)	Clanton (1694)	59	0.7	6.7	−20.0	0.71
Alabama (1)	Greensboro (3511)	60	2.3	6.7	−18.9	0.67
Alabama (1)	Muscle Shoals AP (5749)	48	−0.7	6.5	−23.9	0.94
Alabama (1)	Selma (7366)	56	3.7	6.5	−17.8	0.57
Alabama (1)	Thomasville (8178)	59	2.1	6.8	−18.3	0.64
Florida (8)	Apalachicola AP (211)	56	7.6	5.8	−12.8	0.14
Florida (8)	DeFuniak Spr. (2220)	56	4.7	6.9	−16.1	0.50
Florida (8)	Saint Leo (7851)	57	9.4	5.5	−7.8	0.04
Georgia (9)	Albany 3SE (140)	87	3.9	6.3	−17.2	0.52
Georgia (9)	Atlanta AP (451)	59	0.9	6.1	−22.2	0.71
Georgia (9)	Blairsville ES (969)	54	−4.0	7.3	−26.7	0.98
Georgia (9)	Blakely (979)	83	4.0	6.2	−14.4	0.48
Georgia (9)	Brooklet 1W (1266)	59	3.7	6.0	−17.2	0.47
Georgia (9)	Dahlonega (2475)	59	−0.8	6.3	−24.4	0.88
Georgia (9)	Experiment (3271)	63	1.1	6.3	−22.2	0.71
Georgia (9)	Gainesville (3621)	59	−0.3	5.9	−22.2	0.86
Georgia (9)	Glennville (3754)	59	4.4	6.1	−17.2	0.41
Georgia (9)	Moultrie (6087)	60	4.7	6.2	−17.8	0.43
Georgia (9)	Rome (7600)	59	−0.3	6.5	−22.8	0.85
Georgia (9)	Warrenton (9141)	58	1.5	6.1	−19.4	0.60
Georgia (9)	West Point (9291)	58	0.8	6.4	−22.2	0.78
Mississippi (22)	Brookhaven City (1094)	59	3.0	6.9	−16.7	0.63
Mississippi (22)	Clarksdale (1707)	59	0.5	6.4	−18.3	0.80
Mississippi (22)	Columbia (1865)	59	3.5	7.0	−15.6	0.59
Mississippi (22)	Poplarville ES (7128)	58	5.0	6.7	−16.1	0.45
Mississippi (22)	St. University (8374)	58	1.0	6.6	−21.1	0.74
Mississippi (22)	University (9079)	59	−0.8	7.1	−25.0	0.88

b. *Direct fit of Type I EVD*

As mentioned in section 2d, one alternative approach is to fit the Type I EVD directly to the observed extremes. Using the maximum likelihood estimates of the location and scale parameters, A and B, of the Type I [see (6)] provides a much better correspondence with the observed relative frequencies of the same two extreme events treated in section 4a (Fig. 4). Now it appears, however, that these distributions have a tendency to underestimate the observed frequencies. This effect is apparently somewhat stronger for the minimum temperature event E_m (Fig. 4b) than for the maximum temperature event E_M (Fig. 4a).

c. *Direct fit of GEV distribution*

To investigate whether the Type I EVD can be improved upon, the GEV distribution (4) was also fitted to the same observed extremes at each station, using maximum likelihood to estimate the parameters A and B, as well as the shape parameter k. These distributions produce improved estimates of the relative frequencies of the minimum temperature extreme event E_m (Fig. 5b); the tendency for underestimation is somewhat less than that displayed in Fig. 4b. On the other hand, the use of the three-parameter distribution does little to change the probability estimates for the maximum temperature extreme event E_M (Fig. 5a) from those (Fig. 4a) that were obtained using the two-parameter Type I (recall from section 2c that the Type I is a special case of the GEV in which $k = 0$).

As an example, Fig. 6 shows the fit of the Type I and III EVDs to the empirical distribution of July maximum temperature for one site in the Midwest (i.e., Berne, Indiana) and to January minimum temperature for one site in the Southeast (i.e., Blakely, Georgia). Figure 6a indicates that the fitted Type III is virtually identical to the Type I for Berne maximum temperature, whereas Fig. 6b suggests that the Type III improves the fit somewhat over that of the Type I for Blakely minimum temperature.

It is important to recognize that allowing the shape parameter of the GEV distribution to vary spatially would be inconsistent with our statistical model for regional climate differences. But the Type I EVD is still just an asymptotic approximation [recall (1)]. In fact, the exact distribution of this maximum (or minimum) is better approximated by the Type III, with a shape parameter k that gradually decreases to zero as the sequence length n increases toward infinity (i.e., still allowing for convergence to the Type I). For the extreme temperature events treated here, with $n = 31$, this so-

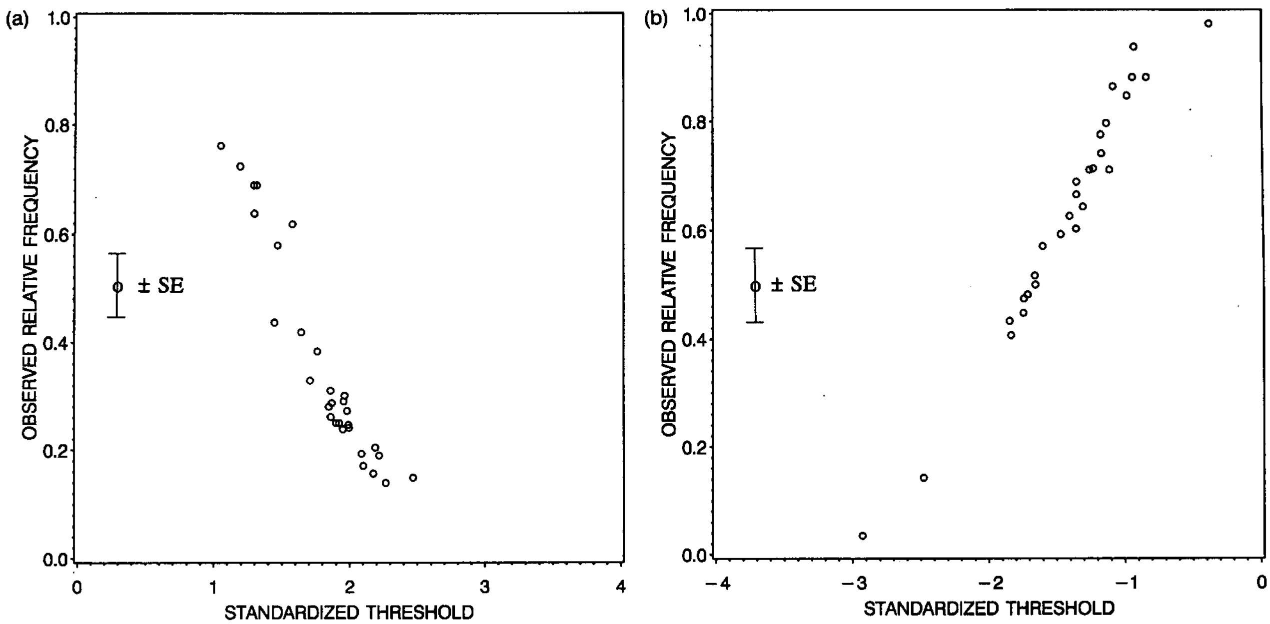

FIG. 2. Observed relative frequency of extreme event vs standardized threshold $(c - \mu)/\sigma$: (a) maximum temperature in July exceeding $c_M = 37.8°C$, and (b) minimum temperature in January falling below $c_m = -6.7°C$. Also shown is approximate size of ± standard error [based on event probability of 0.5 and sample size of ~75 for (a) and ~55 for (b)].

called penultimate approximation leads to $k \approx 0.20$ (Reiss 1989, p. 172; originally proposed by Fisher and Tippett 1928).

For the July maximum temperature series, the estimates of the shape parameter k are close to zero in most cases, with the largest value being 0.27. In particular, these estimates of k are smaller than two standard errors for a majority of the stations (typically, the standard errors range between 0.05 and 0.10). Moreover, the largest estimate of k is still only about one standard error above the value of $k = 0.20$ for the penultimate approximation. Hence, these results are es-

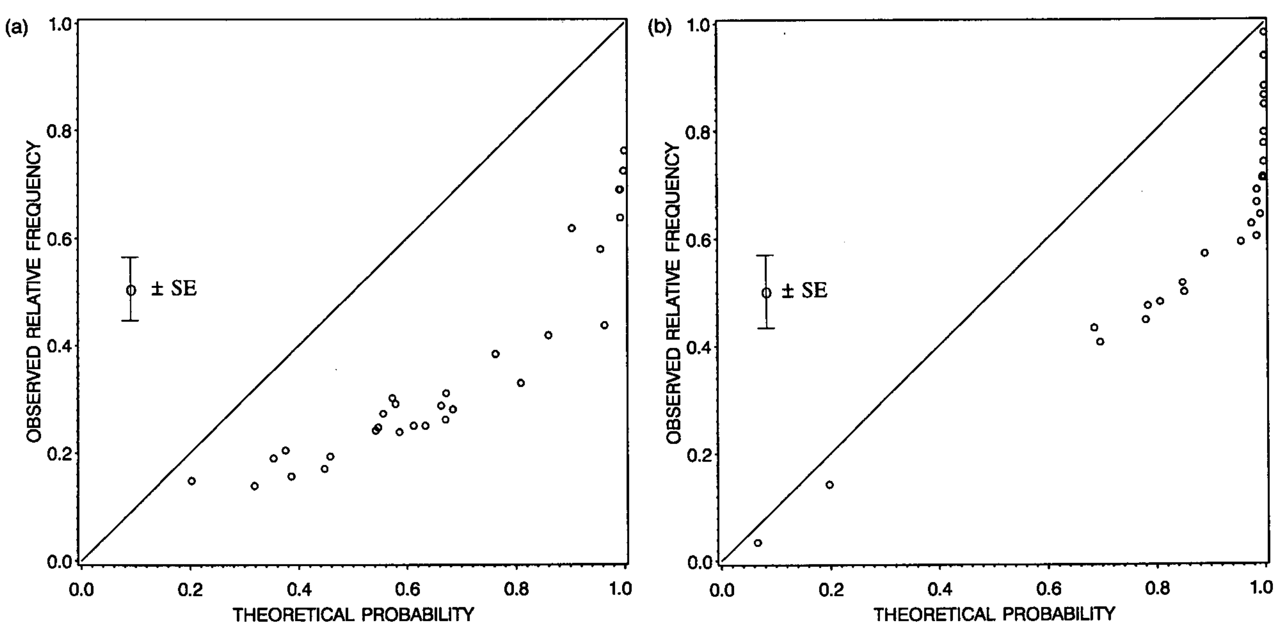

FIG. 3. Observed relative frequency of extreme event vs probability estimated using Type I EVD with theoretical normalizing constants for a normal parent distribution, for the same two extreme temperature events as in Fig. 2.

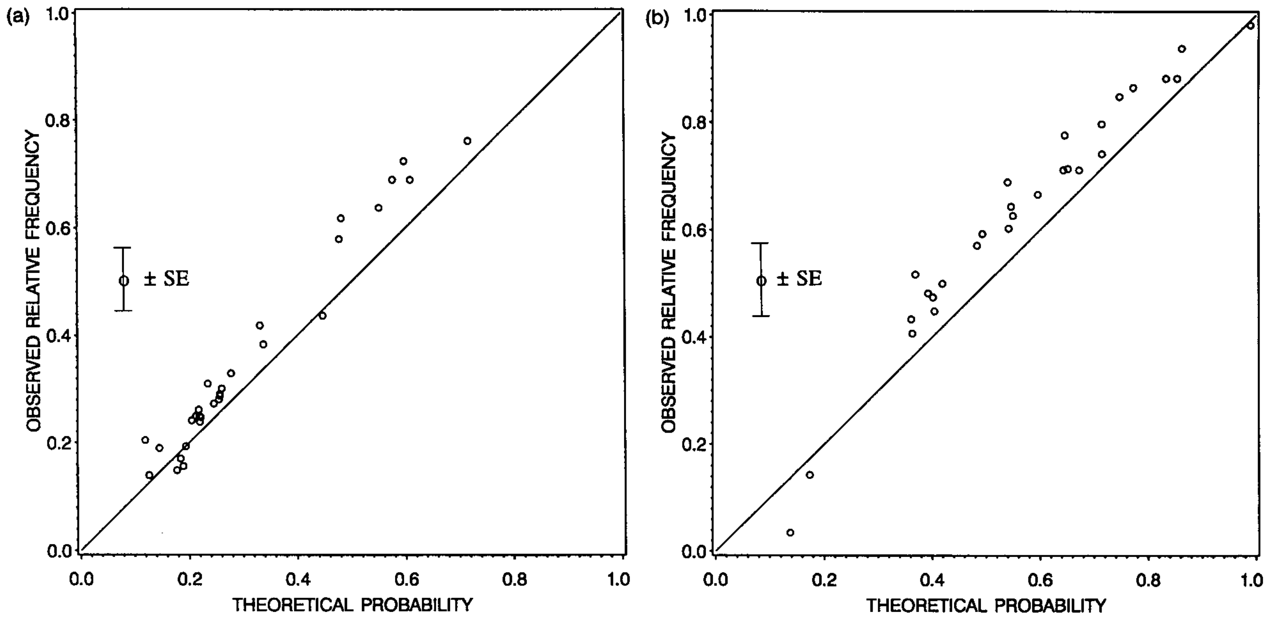

FIG. 4. Observed relative frequency of extreme event vs probability estimated using Type I EVD, with location and scale parameters estimated by method of maximum likelihood, for the same two extreme temperature events as in Fig. 2.

sentially consistent with the apparent lack of improvement in the GEV distribution over the Type I EVD already noted (see Fig. 5a).

In contrast, most of the estimated values of the shape parameter k for the January minimum temperature are greater than 0.20. Virtually all of the estimates are more than two standard errors above zero, with several being more than two standard errors above 0.20 (these standard errors range between 0.06 and 0.13). Again, these results are consistent with the improvement of the GEV distribution over the Type I EVD already noted (see Fig. 5b).

Figure 7 is based on the same comparison of observed and estimated extreme minimum temperature

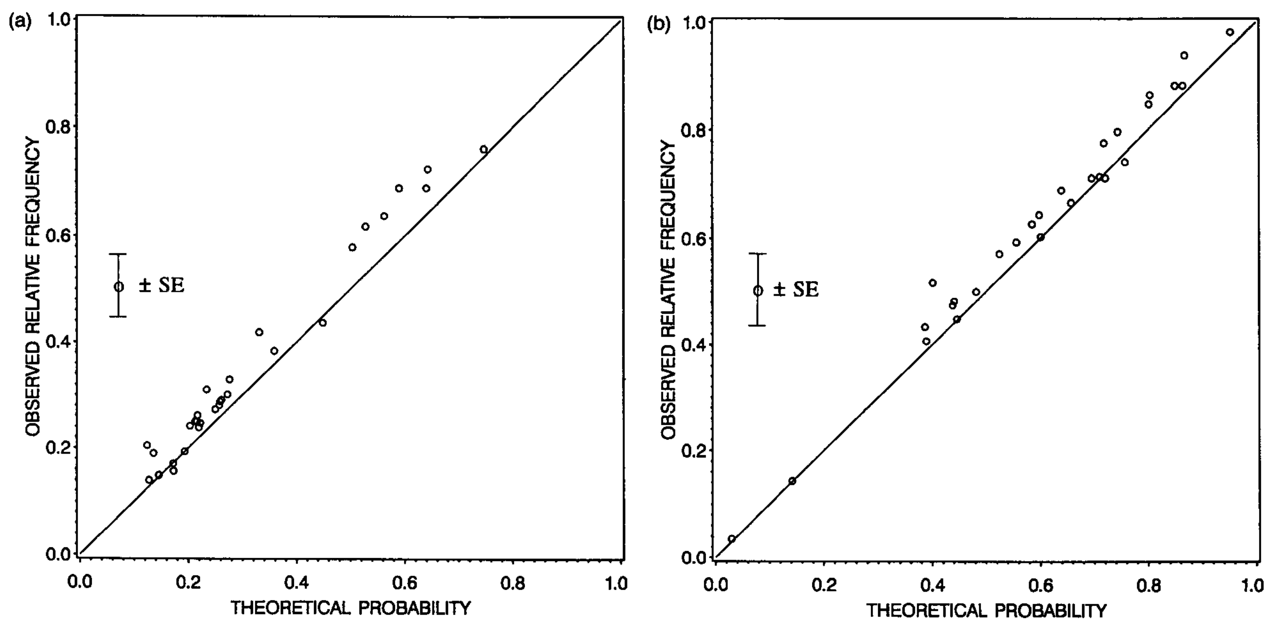

FIG. 5. Observed relative frequency of extreme event vs probability estimated using GEV distribution, with location, scale, and shape parameters estimated by method of maximum likelihood, for the same two extreme temperature events as in Fig. 2.

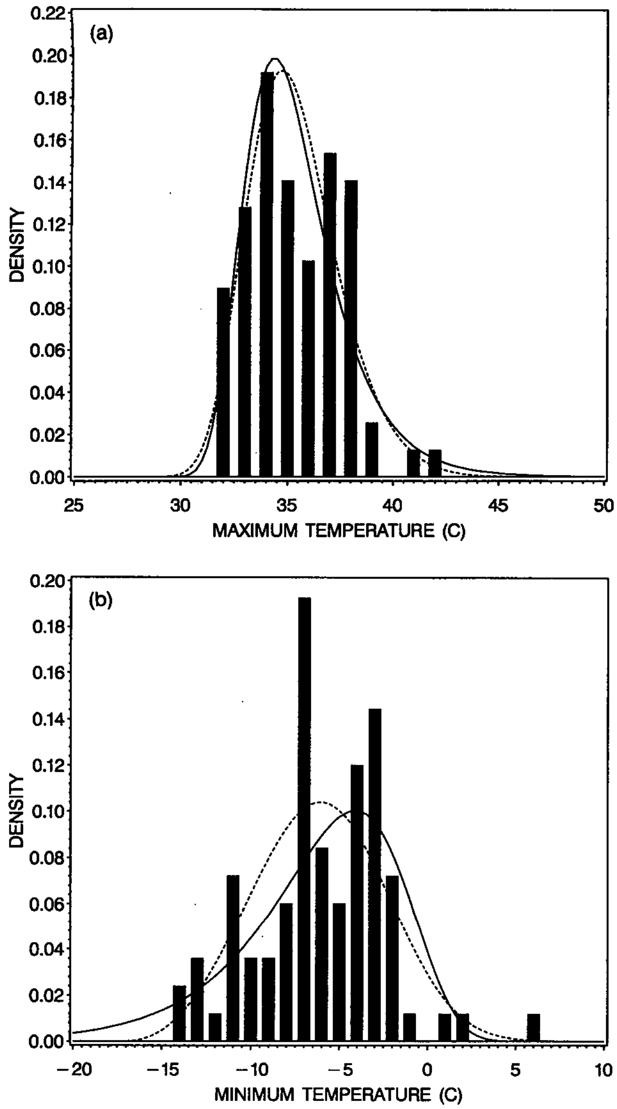

FIG. 6. Empirical distribution of July monthly maximum temperature at (a) Berne, Indiana, and January minimum temperature at (b) Blakely, Georgia. Fitted Type I (solid curve) and Type III (dashed curve) EVDs also included.

event probabilities as in Fig. 5b, but now the shape parameter k has been constrained to always equal the penultimate value of 0.20. It is evident that this penultimate approximation achieves nearly as much improvement over the Type I EVD (Fig. 4b) as the GEV distribution with k free to vary from station to station (Fig. 5b). This simpler form of regional analysis of extremes (i.e., common shape parameter) is necessary in order for our statistical model for spatial differences in climate to be valid. Another possibility, not treated here, would be to constrain the shape parameter to be spatially invariant but not necessarily equal to 0.20.

d. Interpretation

It has been demonstrated that the apparent better fit of the Type III EVD over the Type I can be attributed primarily to the effect of finite sample sizes. The question remains as to why the Type I fit directly to the extreme temperature values is so much better than the same distribution with the normalizing constants theoretically determined for a parent normal df F. To examine this issue further, the fitted location and scale parameters A and B are compared with their theoretical values as specified by the relationship (7). Figure 8 illustrates the discrepancies between these two approaches. The theoretical values of the location parameter A are systematically larger than the corresponding maximum likelihood estimates (Fig. 8a). Moreover, the theoretical values of the scale parameter B are smaller than the corresponding maximum likelihood estimates (Fig. 8b) but with a less systematic discrepancy than for the location parameter.

Interestingly enough, this discrepancy for the location parameter (which also occurs for the minimum temperature extremes) is consistent with the possibility that high-level temperature exceedances are clustered (Leadbetter et al. 1989). We note that such "clustering" cannot be explained simply by the fact that daily temperature time series are autocorrelated. In particular, a normally distributed, first-order autoregressive process (e.g., as employed by Katz and Brown 1994) is only capable of producing "apparent" clustering, which gradually disappears as the exceedance threshold in-

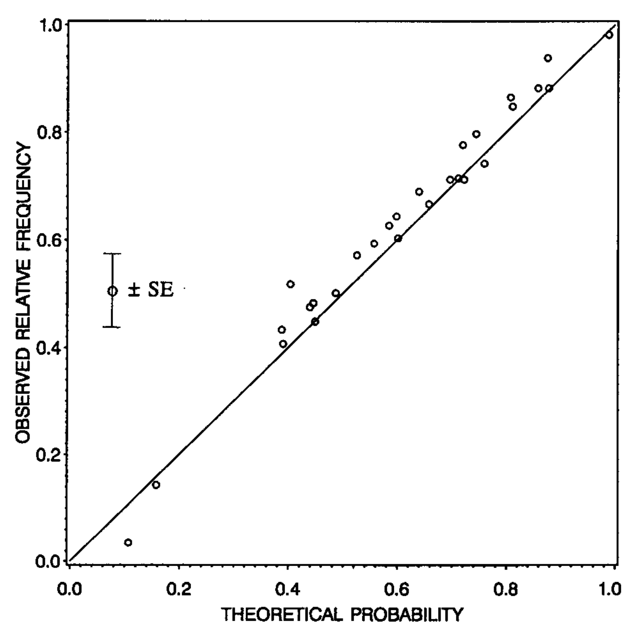

FIG. 7. Same as Fig. 5b except for shape parameter k of GEV distribution constrained to equal the penultimate value of 0.20 for all stations.

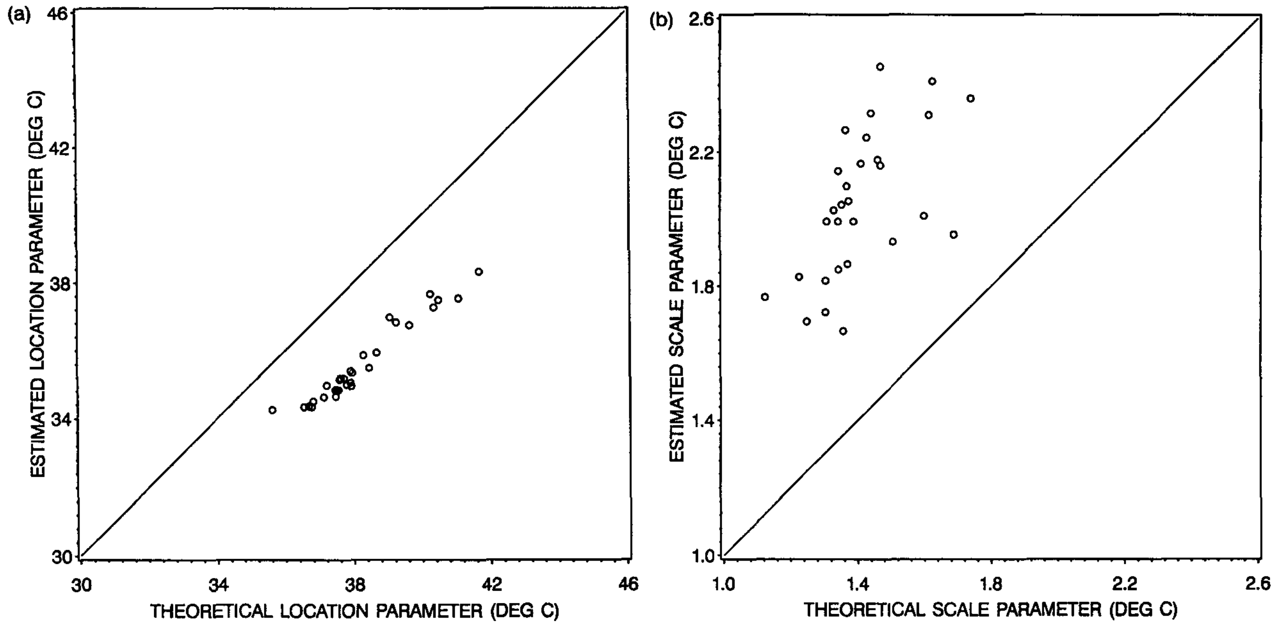

FIG. 8. Comparison of theoretically derived normalizing constants based on normal parent distribution and direct estimation of parameters of Type I EVD by method of maximum likelihood for July maximum temperature: (a) estimated location parameter A vs $\mu + \sigma b_n^*$, and (b) estimated scale parameter B vs σ/a_n^*.

creases. Genuine clustering requires a dependence of a fundamentally different nature than that associated with conventional time series models.

But clustering is not the only possible source of the differences illustrated in Fig. 8. For instance, it may be that the assumption of a parent normal df is inappropriate for the extreme tails. If this were indeed the case, then the expressions (5b) for the normalizing constants, a_n^* and b_n^*, would no longer hold.

5. Implications

The results of this regional analysis of extreme temperature events have potential implications concerning how the frequency of extreme temperature events might change as part of a changing climate. The proposed statistical model for spatial differences in climate is consistent with the satisfactory fit of the Type I EVD to extreme high temperature events in the Midwest and of the Type III, with common shape parameter, to extreme low temperature events in the Southeast. If we were to contemplate applying the same statistical model to temporal changes in climate, then this spatial invariance in distributional shape should give us encouragement about temporal shape invariance as well.

Given the present lack of reliable information about how the frequency of extreme events would change along with other climate statistics, one way of addressing this issue is to conduct a sensitivity analysis. Katz and Brown (1992) performed such a sensitivity analysis, making use of the Type I EVD to quantify how the probability of an extreme temperature event would change as the mean and standard deviation of the corresponding daily temperature distribution are changed. The results of the present paper provide support for reliance on the Type I in determining the sensitivity of extreme high temperature events, but indicate that use of the Type III might provide better sensitivity estimates for extreme low temperature events. It is straightforward to establish that if the Type I were replaced by the Type III, then the sensitivity of extreme low temperature events to changes in either the mean or standard deviation would be even greater than that obtained by Katz and Brown (1992).

This work could be expanded upon in several respects. For example, an analogous treatment of extreme precipitation events could be attempted. A start in this direction is the study by Katz and Garrido (1994). They have performed a regional analysis of extreme precipitation events defined on a monthly or seasonal timescale. In this application, the positive skewness characteristic of the distribution of total precipitation is taken into account by applying a power transformation to the data to obtain an approximately normal distribution.

Of course, the spatial analog for climate change considered here leaves much to be desired from a physical point of view. Nevertheless, social scientists have argued that even quite imperfect analogs are valuable in assessing the potential impacts on society of climate change (Glantz 1988). In this regard, other analogs exist that could be candidates for evaluating the proposed statistical model for climate change. For instance, the so-called heat island effect associated with the growth of cities constitutes a real temporal climate change that has occurred on a local, rather than global, scale. In this regard, Tarleton and Katz (1993) have established that a change in standard deviation, as well as the well-documented change in mean, is necessary to explain the observed trends in the frequency of extreme temperature events in urban environments (Balling et al. 1990).

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APPENDIX

Simulation Study

A simulation study was designed to mimic the scatterplot, shown in Fig. 2a, for the observed relative frequency of extreme high temperatures in July exceeding 37.8°C versus the standardized threshold. For each of these 30 sites in the Midwest, a simulated outcome from a binomial distribution, denoted by $B(N; p)$, was generated by Monte Carlo methods. The number of trials N of the binomial experiment was taken as the number of years of observation for the particular station (listed in Table 1). The binomial probability p was derived from a Type I EVD evaluated at the standardized threshold value $(c - \mu)/\sigma$ for the site. Rather than attempting to provide a satisfactory fit to the scatterplot (Fig. 2a), the parameters of the Type I were chosen just to approximate the range of observed relative frequencies.

Figure 9 shows the outcomes of these 30 binomial simulations, conducted independently for each site, as well as the underlying Type I EVD. The deviation of each point from the smooth curve represents a simulated sampling error. It is evident that this scatter is as much or more than that of Fig. 2a. Next, the entire simulation experiment was replicated several times (i.e., repeating the binomial simulation for each station to obtain new outcomes). These additional plots are not shown here, but Fig. 9 is typical of the extent of scatter.

As a formal test of regional homogeneity, this simulation study would have some drawbacks. Most importantly, the monthly maxima at the individual sites are surely cross correlated (although not to as great an extent as for the maximum temperatures for a single day). Hence, it is unrealistic to conduct the binomial simulations independently for each station. Less importantly, the visual pattern obtained does depend on the assumed form of the underlying distribution, making it difficult to quantify any comparisons between Figs. 2a and 9. Nevertheless, it should be kept in mind that the magnitude of the sampling errors is determined by the number of trials N of the binomial experiment, not by the particular shape of the curve.

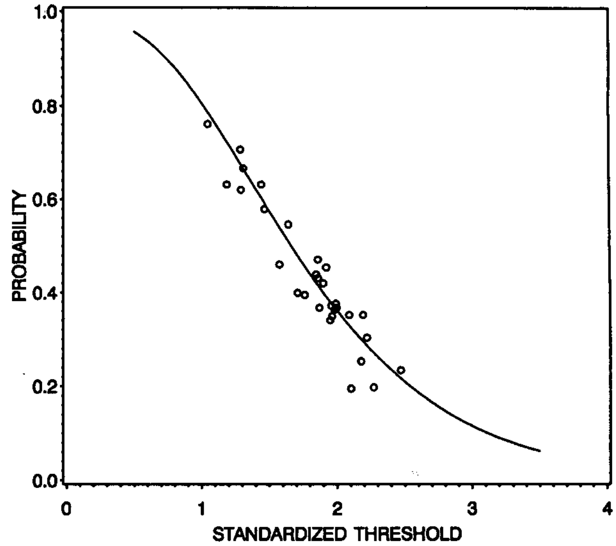

FIG. 9. One trial of simulation study: each circle represents outcome of individual binomial experiment with probability based on Type I EVD (solid curve) and standardized thresholds as in Fig. 2a.

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