NOTES AND CORRESPONDENCE

Global Temperature Deviations as a Random Walk

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Surface air temperature is the main parameter to represent the earth’s contemporary climate. Several historical temperature records on a global/monthly basis are available. Time series analysis shows that they can be modeled via autoregressive moving-average models closely connected to the classical random walk model. Fitted models emphasize a nonstationary character of the global/monthly temperature deviation from a certain level. This means that the short-term (in comparison with infinity) temperature trends are inevitable and may have little in common with a currently increasing carbon dioxide amount.

Gordon (1991) assumes that the earth’s climate system is subjected to periodic shocks consisting of random warming and cooling impulses. The impulses may be random in magnitude and frequency and may occur in very different timescales. Unfortunately, his reasoning is not followed by an adequate statistical analysis of existing temperature records. The following calculations show that a reasonable understanding of the contemporary global mean climate (in terms of the surface air temperature) is attainable, assuming random forcing to the climate system and treating temperature deviation as a response to it. The forcings occur due to volcanic eruptions, redistribution of cloudiness, variations in snow- and ice-covered areas, changes in solar output, etc. Their impact cannot be estimated directly from changes of the earth’s radiation budget at the top of the atmosphere because measurements represent mixture of the forcings and responses.

The method of fitting autoregressive, integrated moving-average models (Box and Jenkins 1976) is used to represent the temporal variability of two global/monthly average records of the surface air temperature anomalies [Hansen and Lebedeff 1988, hereafter referred to as GISS (Goddard Institute for Space Studies); Vinnikov et al. 1990, hereafter referred to as VGL; NASA 1992]. The time series of temperature anomalies \( x_t, t = 1, \ldots, n \) represent periods from 1880 to 1988 and from 1881 to 1990, respectively, and the anomalies are calculated respective to the mean value of the period 1951–1980. The total number of months in series \( n \) equals 1297 for GISS and 1320 for VGL.

Sample autocorrelations for series \( x_t \) and \( w_t = x_t - x_{t-1} \), up to the lag 40 months, are shown in Fig. 1. There is a very slow decay in the autocorrelations for \( x_t \). Sample autocorrelations for \( w_t \) indicate that the first-order moving-average model

\[
 w_t = a_t - \theta a_{t-1},
\]

(1)

can be appropriate to represent the temporal variability of monthly increments \( w_t \). Here \( a_t \) is white noise and \( \theta \) is a coefficient. For \( \theta \), least-squares fitting gives values of 0.566 for GISS and 0.586 for VGL.

It can be presented as

\[
 x_t = a_t + \lambda \sum_{j=1}^{\infty} a_{t-j},
\]

(2)

where \( \lambda = 1 - \theta \) (Box and Jenkins 1976). Equation (2) opens a nonstationary nature of the century’s record of temperature deviations: \( x_t \) can be represented as a weighted sum of independent identically distributed impulses. If \( \lambda = 1 \), then we get the classical random walk case (see, e.g., Feller 1970 for details).

Any process (2) can be interpreted as a random walk overshadowed by white noise, that is, to consider a process

\[
 x_t = y_t + b_t,
\]

(3)

where \( y_t \) is a “clean” random walk \( y_t = c_t + \sum_{j=1}^{\infty} c_{t-j} \) and \( b_t \) is white noise, independent on \( c_t \). Mathematically both processes, \( b_t \) and \( c_t \), are white noises, but physically they have different substance. Process \( b_t \) can be interpreted as a measurement error, while \( c_t \) stands

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for random impulses generated by the earth–climate system (as a response to the sequence of forcings).

Using $\lambda$ and $\sigma^2$ estimates one can calculate (Box and Jenkins 1976) $\sigma^2 = \lambda \sigma^2_0$ and $\sigma^2 = \sigma^2_0 (1 - \lambda) / \lambda^2$. The results are as follows: the estimate of $\sigma^2_0$ equals 0.0448 for GISS and 0.0393 for VGL, and $\sigma^2_0$ equals 0.0149 for GISS and 0.0115 for VGL. It is reasonable to assume that $b$, is a process with decreasing variance (recent estimates are more accurate than previous), but the approach is unable to use the idea. Anyway, one can take $\sigma^2_0$ as a basis in estimating possible monthly increments of the temperature anomaly.

This elementary approach explains an essential part of variance (58% for GISS and 62% for VGL) but does not pass any (linear) independence test for residuals (not shown). Physically, this means that feedback loops longer than one month are important in generating a temperature response to a (radiative) forcing in the earth–climate system. Several new terms are to be involved in order to pass some test.

As a result the model

$$w_t - A_1 w_{t-1} - A_{12} w_{t-12} + A_1 A_{12} w_{t-13} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$  \hspace{1cm} (4)

gives a satisfactory result.

Table 1 shows estimates of coefficients and residual variance for both time series. The last row shows that the inclusion of new terms gains only 3% in terms of residual variance in comparison with the first approach.

One test for residuals is applied following Box and Jenkins (1976). For an adequate model, sample autocorrelations for residuals have approximately normal distribution with zero mean value and variance equal to $1/n$. Sample autocorrelations for actual residuals are shown in Fig. 2. Dotted lines show the 95% confidence interval for zero correlation. Only one estimate does not fit in the interval, showing that in the first approximation the test is passed.

The result means that the global/monthly mean temperature anomaly is most likely a nonstationary process. In principle, the second approach brings along little in comparison with the first one: the model still emphasizes that a global/monthly mean temperature deviation $x_t$ from a long-term average equals a weighted sum of independent, identically distributed random variables (see, e.g., Box and Jenkins 1976 for details). Only the weights have changed. The nonstationarity explains all trends and (apparent) periods found in the last century's variability of global mean temperature (e.g., Schlesinger and Ramankutty 1994). It is impossible empirically to separate the impact of one particular forcing (e.g., that due to an increase of CO₂ amount) from the (possibly random) sequence of all existing forcings in the earth–climate system. More accurate modeling involving main feedback loops is necessary to ease such a separation. The random walk essence also warns that the conclusion

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**Table 1. Estimated parameter values for both temperature series and models.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model (2)</th>
<th>Model (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>GISS</td>
<td>0.718</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>GISS</td>
<td>0.097</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>GISS</td>
<td>0.566</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>GISS</td>
<td>1.253</td>
</tr>
<tr>
<td>Initial variance</td>
<td>GISS</td>
<td>-0.269</td>
</tr>
<tr>
<td>Residual variance percent</td>
<td>0.0792</td>
<td>0.0671</td>
</tr>
<tr>
<td>Residual variance percent</td>
<td>42</td>
<td>38</td>
</tr>
</tbody>
</table>
about net positive feedback (Mitchell 1989; Gordon 1991) after a forced temperature change is probably premature. Global temperature measurements can give us only an integrated feedback to forcings during some time interval, and this parameter changes its sign similar to a trajectory of a one-dimensional random walk.

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REFERENCES