Estimating the Role of Local Evaporation in Precipitation for a Two-Dimensional Region

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ABSTRACT

Budyko's model for estimating the contributions of locally evaporated and advected moisture to regional precipitation is extended to two dimensions. It is shown that a simple extension by analogy of the one-dimensional Budyko's formula to a two-dimensional region is inconsistent unless the flow in the region is parallel and uniform. The correct extension based on the two-dimensional equations of conservation of water vapor in the region leads to a generalization of Budyko's formula that includes a correction factor depending on the atmospheric flow structure. A general procedure for calculating the correction factor for a given atmospheric flow field is presented. Calculations of the correction factor for specific flow structures show that the deviations of the flow from the rectilinear structure can significantly affect the degree to which the local evaporation contributes to precipitation.

1. Introduction

The hydrological cycle is increasingly recognized for its possible importance in climate change and climate variability. One of the classical issues in this respect is the relative contributions of locally evaporated versus externally advected atmospheric water vapor to the precipitation for a given region.

The precipitation over a large region is composed of two components: the advective precipitation resulting from the flux of external (advective) water vapor formed by evaporation outside the given region and the internal component resulting from the flux of local water vapor formed by local evaporation. What portion of the precipitation in the region is formed from advected moisture and what portion from regional evaporation, that is, from recycled moisture, has been extensively discussed in the literature [see Brubaker et al. (1993) for the history of the problem]. The degree of precipitation recycling determines both a role for land-surface hydrology in the regional climate and a role of climate in the formation of surface and subsurface water resources and is a measure of a feedback in hydrologic and climatologic processes.

Early studies of precipitation recycling involved limited data analysis and little theoretical description of the recycling process. The practical need to improve the reliability of hydrological computations resulted in a more detailed and profound study of the hydrological cycle with the use of mathematical models. The modern procedure for studying the hydrological cycle was proposed by Budyko and Drozdov (1953) (see also Budyko 1974). They developed a model (commonly called Budyko's model) to estimate the contributions of locally evaporated and advected moisture to regional precipitation for the region traversed by the parallel air flow. The model is one dimensional and the process is described along a single streamline. Budyko's model is based on the following assumptions: (i) the mean evaporation and mean precipitation rates are characteristic of all points of the streamline, (ii) the atmosphere is assumed to be fully mixed so that the relationship between the sums of precipitation formed from local versus external water vapor is equal to that between the corresponding quantities of vapor molecules in the atmosphere, and (iii) the moisture content of the air (both locally evaporated and advected) varies linearly as the air moves across the region—this is simply a consequence of the one-dimensional equation of conservation of moisture content solved under assumption (i). Budyko and Drozdov (1953) obtained a simple formula expressing the ratio of total precipitation to precipitation due to advected moisture (recycling coefficient) as a function of regional evaporation, inflow of atmospheric moisture, and the linear scale of the region measured along streamlines.

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Drozdov and Grigor'eva (1965) developed a generalization of Budyko's model by waiving assumption (i) that the mean evaporation and mean precipitation rate are constants equal to their average values. In their model (also one dimensional) arbitrary variations of evaporation and precipitation fluxes along streamlines are allowed, and as a result of applying the equation of water vapor conservation, the concentration of water vapor of each origin varies nonlinearly along streamlines. Different formulas for the recycling coefficient are obtained by assigning specific distributions of evaporation and precipitation rates.

Lettau et al. (1979) developed a model that is also based on the one-dimensional equation of conservation of water vapor but they rejected assumption (ii) of Budyko's model that water vapor molecules of external and local origin have an equal probability of being precipitated. In their model, the precipitation is divided into two parts in such a way that the recycled part of precipitation depends only on the evaporative flux and the advective part of precipitation depends on the moisture content. Such a presentation of precipitation involves some additional regional parameters that should be prescribed.

Eltahir and Bras (1994) developed a numerical model to estimate precipitation recycling with allowance for the seasonal and spatial variability of the recycling process and applied this model in studying the hydrological cycle in the Amazon Basin. The model is based on the assumption of a fully mixed atmosphere and the equations of conservation for the moisture of both local and advective origin applied to a box of the rectangular grid covering a given area of the region. The estimation procedure consists of the iterations using a trial and error technique to determine a distribution of the local precipitation recycling ratio from which the recycling ratio for the total area is obtained.

Budyko (1974) applied his formula to the European part of Russia, obtaining a local component of the precipitation equal to 12% of the total on an annual basis. This contradicted early views suggesting that the contribution of evaporation from a land region to precipitation in the same region is the most significant. Similar results have been obtained in other studies using Budyko's model. Sellers (1965) applied Budyko's formula to three regions with very different areas—the state of Arizona (excluding the Colorado River), European Russia, and the United States and Canada combined—and found that the percentage of the total annual precipitation coming from evaporation from the ground is 6% for Arizona, 11% for European Russia, and 27% for Canada and United States. Brubaker et al. (1993) applied a modified Budyko formula, with the area of the region replacing the linear scale of the territory, to estimate recycling in four large intercontinental regions of the world, covering parts of Eurasia, North America, South America, and Africa. They obtained for the fraction of precipitation derived from evaporation from within the region the following annual means: 11% for the Eurasian region, 24% for the North American and South American regions, and 31% for the African region. It is also appropriate to mention here the studies (Rodriguez-Iturbe et al. 1991a,b; Entekhabi et al. 1992), where Budyko's formula was used to represent the coupling between the land surface and the atmosphere in the systems of equations describing nonlinear dynamics of soil moisture.

On the basis of recent studies it is believed that calculations with the use of Budyko's formula apparently underestimate the actual role of recycling. Studies involving control experiments with general circulation models (GCM) (Shukla and Mintz 1982; Koster et al. 1986; Shukla et al. 1990) have documented the important role of land evaporation in rainfall on continental spatial scales. There are observational indications that local evaporated moisture supplies a large fraction of precipitation for some continental regions. [See, for example, a study of the water vapor budget over central North America by Zangvil et al. (1993a,b) or studies of precipitation recycling in the Amazon Basin by Molion (1975) and Marques et al. (1977), which estimate that about 50% of precipitation in the Amazon Basin is contributed by evaporation within the basin.] Note, however, that Eltahir and Bras (1994) argue against these estimates, which are at variance with their estimate of 25%–35%, explaining the contradiction by an inaccurate description of the hydrologic cycle in the Amazon Basin assumed implicitly in those studies.

It could be expected that the use of the generalized model of Drozdov and Grigor'eva (1965), which takes into account the effects of the inhomogeneity of the precipitation and evaporation fluxes, should improve estimates made within the framework of Budyko's approach. However, the computations of moisture cycle components with the use of Drozdov and Grigor'eva formulas for European Russia, presented in Shiklomanov (1989), gave the same estimate of 11% for recycled precipitation as did Budyko (1974) and Sellers (1965) calculations. Drozdov and Grigor'eva (1965) also reported that the discrepancies between yearly estimates were within the accuracy limits for the calculations. However, Drozdov and Grigor'eva observed that for individual months the discrepancies may become appreciable. Detailed comparison of the precipitation recycling estimates, based on Budyko's formula and one of Drozdov and Grigor'eva formulas, is given in Brubaker et al. (1993). It is apparent from these two formulas that the estimation of precipitation recycling according to Budyko's model underestimates the rate when compared to the estimation by the Drozdov and Grigor'eva generalization. However, the calculations of Brubaker et al. (1993) have shown that the underestimate is rather small if one does not consider extreme values of parameters of the model. This may well explain the aforementioned agreement of the results obtained for European Russia with the use of these two models.

Another possibility to improve estimations of precipitation recycling within the framework of Budyko's approach is realized in the present paper. It stems from the observation that an application of Budyko's one-dimensional formula to a two-dimensional domain is inconsistent unless the airflow is parallel and uniform, which is unrealistic for sufficiently large regions. Hence, a generalization of Budyko's formula for two-dimensional land regions is needed for a correct estimation of precipitation recycling. The main purpose of our paper is to develop a method for estimation of the contribution of local evaporation to precipitation for two-dimensional regions within the framework of Budyko's theory.

The outline of the paper is as follows: in the next section we discuss Budyko's model and give arguments showing the inconsistency of drawing a simple analogy of Budyko's formula to a two-dimensional region. The two-dimensional recycling model, developed within the framework of Budyko's approach, is presented in section 3. Analytical and numerical results of determining the recycling coefficient for different flow patterns are considered in sections 3 and 4. Basic assumptions of the theory and possible further extensions are discussed in section 5. The numerical method for determination of the recycling coefficient is described in appendix A. A list of symbols used in the text is given in appendix B.

2. Budyko's approach

Let us consider an atmospheric control volume into which water vapor is brought by air currents with a moisture influx F^+ through the sides of the volume. The water vapor content w in the air, moving across the region with a horizontal velocity $\mathbf{V}$, varies within the region decreasing due to precipitation with a vertical flux P and increasing due to evaporation with a vertical flux E. Here w is the vertically integrated water vapor depth (precipitable water) and the air velocity $\mathbf{V} = (U, V)$ is the vertical average of the wind vector weighted according to the specific humidity profile.

The water vapor content is composed of an advective portion w_a and an evaporative portion w_m:

$$w = w_a + w_m, \tag{2.1}$$

and likewise the precipitation P is composed of the parts P_a and P_m of advective and local (evaporative) origins:

$$P = P_a + P_m. \tag{2.2}$$

The problem consists in estimating the relative contributions of advective moisture and local evaporation to precipitation for a given domain. This relation could be characterized, for instance, by Budyko's recycling coefficient β, which is the ratio of total to advected average precipitation:

$$\beta = \frac{\bar{P}}{\bar{P}_a}. \tag{2.3}$$

Here, and everywhere, bars above letters denote a horizontal average over the region.

Budyko and Drozdov (1953) obtained a simple formula expressing β through the influx F^+, mean evaporation flux E, and a length scale of the territory L [the derivation of the formula has been reproduced in Budyko (1974)]. We will reproduce Budyko's arguments below.

Budyko considered a parallel air flow with a constant velocity U traversing a land region with a length scale L measured along streamlines. Air enters the region normal to the region boundary so that the influx $F^+ = w^+U$, where w^+ is the moisture content in the air entering the region. The following assumptions are made:

(i) The vertical flux quantities P_a, P_m, P, and E are treated as constants equal to their average values.

(ii) The ratio of sums of precipitations formed from the external water vapor to that formed from the local water vapor is equal to the ratio of advected to evaporated moisture present in the air

$$\frac{P_a}{P_m} = \frac{\bar{w}_a}{\bar{w}_m} \text{ or } \frac{P}{P_a} = \frac{\bar{w}}{\bar{w}_a}. \tag{2.4}$$

Here and hereafter bars over P_a, P_m, and P are omitted in view of the assumption (i). The assumption (2.4) means that the moisture of different origins mix thoroughly and the precipitation water is drawn proportionally to the abundance of atmospheric moisture of each origin.

(iii) The concentrations of advected, locally derived, and total moisture change linearly as the air masses traverses the region, or, in other words, the average of a moisture flux is the arithmetic mean of the incoming and outgoing moisture:

$$\bar{w}_a U = \frac{F^+ + F^+ - P_a L}{2} = F^+ - \frac{P_a L}{2} \tag{2.5}$$

$$\bar{w} U = \frac{F^+ + F^+ + (E - P)L}{2} = F^+ + \frac{(E - P)L}{2}. \tag{2.6}$$

(The corresponding relation for the concentration of locally derived moisture is the direct consequence of these two equations.) Budyko's formula is easily obtained from these equations by introducing the relations (2.3) and (2.4) into (2.6) and subsequently eliminating P_a and $\bar{w}_a$ from the relation obtained and (2.5), as follows:

$$\beta = 1 + \frac{EL}{2F^+}. \tag{2.7}$$

Before developing an extension of Budyko's approach to two dimensions, we consider a simple extension by analogy, which may seem plausible but as shown in section 3 is inconsistent. It is based on the assumption that the average horizontal flux of advected (total) moisture over the region is an arithmetic mean of the incoming and outgoing advective (total) moisture, as in the one-dimensional case, even though the flow in the region is not parallel and the velocity is not constant. That leads to formulas that are identical to (2.5) and (2.6) with the area of the region A replacing L (see, for example, Brubaker et al. 1993); the corresponding expression for Budyko's recycling coefficient has the form

$$\beta = 1 + \frac{EA}{2F^+}, \quad (2.8)$$

which is identical to (2.7) with A replacing L.

Such a simple extension by analogy of the formulas (2.5), (2.6), and (2.7) to a two-dimensional region is unreasonable. In the one-dimensional case, the possibility to take the average moisture flux as an arithmetic mean of incoming and outgoing moisture follows from the linear distribution of moisture content with the coordinate that is a direct consequence of the assumption of uniform and parallel flow, the assumption (i), and the one-dimensional equations of conservation of water vapor

$$\frac{d(w_a U)}{dX} = -P_a \quad (2.9a)$$

$$\frac{d(wU)}{dX} = E - P. \quad (2.9b)$$

In the two-dimensional case the relation between averaged, incoming and outgoing moisture depends on the regional moisture distribution, which will depend on the structure of the airflow in the region.

Thus, it is evident that a procedure for estimating the precipitation recycling for a two-dimensional region within the framework of Budyko's approach should proceed from assumptions (i) and (ii) and the two-dimensional equations of conservation of water vapor. Such a procedure is developed in the next section.

3. Extension of Budyko's approach to two dimensions

a. Basic relations

Consider a rectangular region: $0 \leqslant X \leqslant L$, $0 \leqslant Y \leqslant H$, where X and Y are horizontal coordinates. In accordance with the framework of Budyko's approach, the basis of the theory consists of assumptions (i) and (ii) stated in the previous section and of the two-dimensional equations of conservation of water vapor

$$\frac{\partial(wU)}{\partial X} + \frac{\partial(wV)}{\partial Y} = E - P, \quad (3.1a)$$

$$\frac{\partial(w_a U)}{\partial X} + \frac{\partial(w_a V)}{\partial Y} = -P_a, \quad (3.1b)$$

where the U and V fields are given and, in accordance with assumption (i), the mean evaporation E and the mean precipitation rates P and P_a are treated as characteristics of all points within the region. Equations (3.1) have to be complemented by the initial conditions giving the moisture content distribution in the air entering the region

$$w = w^+ f(Y), \quad w_a = w^+ f(Y) \quad \text{at} \quad X = 0, \quad (3.2)$$

where w^+ is a characteristic value for the moisture content.

The assumption (ii) of a fully mixed atmosphere, given before by the relation (2.4), with allowance for (2.3), can be expressed in the form

$$P = \beta P_a, \quad \bar{w} = \beta \bar{w}_a, \quad (3.3)$$

which is more convenient for subsequent calculations.

The system of basic relations needed for calculating the recycling coefficient β is completed by the relations for calculation of the average moisture concentrations $\bar{w}$ and $\bar{w}_a$ and the influx F^+ through moisture and flow velocity fields, as follows:

$$\bar{w} = \frac{1}{LH}\int_0^L dX \int_0^H w(X, Y)dY,$$

$$\bar{w}_a = \frac{1}{LH}\int_0^L dX \int_0^H w_a(X, Y)dY \quad (3.4a)$$

$$F^+ = -\int_{\Gamma_{in}} w_\gamma \mathbf{V}_\gamma \cdot \mathbf{n}_\gamma d\gamma, \quad (3.4b)$$

where in the boundary integral (3.4b) w_γ and $\mathbf{V}_\gamma = (U_\gamma, V_\gamma)$ are respectively the moisture content and the flow velocity vector on the boundary, $\mathbf{n}_\gamma$ is the outward unit normal vector, and Γ_{in} denotes a part or parts of the boundary across which the atmospheric motion is inward.

b. Procedure for determining the recycling coefficient

The framework of the theory is formed by the relations (3.1)–(3.4). The procedure for derivation of the recycling coefficient from the relations (3.1)–(3.4) consists of the following steps.

1) The solutions $w(X, Y)$ and $w_a(X, Y)$ of the equations (3.1) for a given flow fields U and V subject to the boundary conditions (3.2) are found.

2) The horizontal averages $\bar{w}$ and $\bar{w}_a$ and the influx F^+ are calculated from (3.4) on the basis of the solutions obtained.

3) Each of the horizontal averages $\bar{w}$ and $\bar{w}_a$ is expressed through the influx F^+ by eliminating the char-

acteristic value of the moisture content w^+ from the expressions.

4) The expression for the recycling coefficient β is obtained by eliminating the quantities P, P_a, $\bar{w}$, and $\bar{w}_a$ from the expressions for $\bar{w}$ and $\bar{w}_a$ and two relations (3.3).

It should be noted that, in general, four equations for five variables P, P_a, $\bar{w}$, $\bar{w}_a$, and β are not sufficient to make such an elimination. Therefore, it is not possible to know beforehand whether such calculations would result in a formula for the recycling coefficient that, like Budyko's formula, does not include dependence on the precipitation rate. It appears, however, that the relations expressing $\bar{w}$ and $\bar{w}_a$ through the other quantities have such a structure that the elimination of $\bar{w}_a$ implies the elimination of P_a, and the resulting formula for β has almost the same structure as Budyko's formula

$$\beta = 1 + \frac{EA}{2F^+} R, \qquad (3.5)$$

differing from it by a correction factor R.

The procedure is shown in a general form suitable both for analytical and numerical calculations in appendix A. To make the perception easier, in the next subsection all the steps can be seen in the example of a linear diffluent flow for which the result can be obtained by means of quite simple analytical calculations (this example is interesting in itself as the diffluent flow obtained by superimposing the linear deformation field on the rectilinear structure is commonly used in the literature).

c. Calculating the recycling coefficient—Example of the linear diffluent flow field

If the linear deformation field is superimposed on the rectilinear structure, the resulting diffluent flow field is

$$U = U^* - kX, \quad V = kY \qquad (3.6a)$$

or

$$U = U^* - kX, \quad V = k(Y - H/2). \qquad (3.6b)$$

Both the diffluent flow fields (3.6a,b), shown in Figs. 1a,b, produce exactly the same results. In what follows we will deal with the field (3.6a).

It should be noted that although we restrict ourselves to nondivergent flows for which $\partial U/\partial X + \partial V/\partial Y = 0$, there are no principal difficulties in considering divergent flows.

Next, the relations (3.1), (3.2), (3.4), and (3.6) are written in a nondimensional form using the following dimensionless variables

$$x = X/L, \quad y = Y/H,$$
$$u = U/U^*, \quad v = (V/U^*)L/H. \qquad (3.7)$$

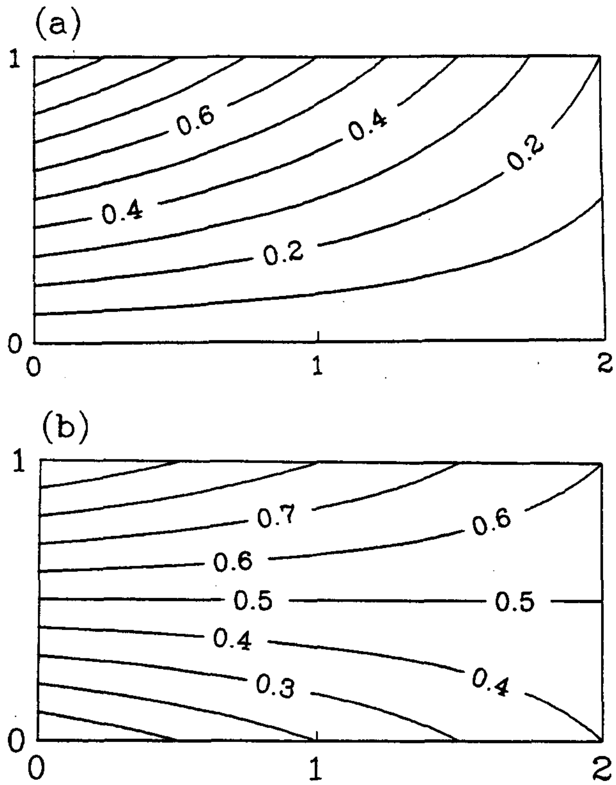

FIG. 1. Dimensionless streamfunction ψ for the linear diffluent flow fields: (a) (3.6a); (b) (3.6b). The numbers on the horizontal and vertical axes are the values of the dimensionless x and y coordinates, respectively.

The equations (3.1) with the flow fields given by (3.6a) take, with the nondimensional variables (3.7), the following forms

$$(1 - \epsilon x)\frac{\partial w}{\partial x} + \epsilon y \frac{\partial w}{\partial y} = q \qquad (3.8a)$$

$$(1 - \epsilon x)\frac{\partial w_a}{\partial x} + \epsilon y \frac{\partial w_a}{\partial y} = q_a, \qquad (3.8b)$$

where

$$\epsilon = kL/U^*, \quad q = (E - P)L/U^*, \quad q_a = -P_a L/U^*. \qquad (3.9)$$

To facilitate the comparison with the one-dimensional case, we assume that $0 \leqslant \epsilon < 1$ — the deviation from the basic flow does not exceed it.

In what follows we use the same enumeration for the steps of the procedure as that in section 3b. The steps 1 to 3 of the procedure are aimed at obtaining the expressions for $\bar{w}$ and $\bar{w}_a$ on the basis of solutions of (3.8) subject to the initial conditions (3.2). Since (3.8b) is almost identical to (3.8a) excluding the constant right-hand side, and the initial conditions (3.2) for w and w_a are also identical, we will only show the calculations resulting in the expression for $\bar{w}$ and obtain $\bar{w}_a$ replacing $(E - P)$ by $-P_a$ in the expression for $\bar{w}$.

1) The solution of the quasi-linear equation (3.8a) subject to the boundary condition (3.2) is

$$w = -(q/\epsilon)\ln(1 - \epsilon x) + w^+ f[y(1 - \epsilon x)]. \quad (3.10)$$

Consider first the simplest case of the uniform moisture distribution in the stream entering the region, corresponding to $f(y) = 1$ in the boundary conditions (3.2). Then we obtain from (3.10) the moisture distribution that does not depend on y, as follows

$$w = -(q/\epsilon)\ln(1 - \epsilon x) + w^+. \quad (3.11)$$

2) The dimensionless form of the formula (3.4a) with L and H replaced by unity is used for the calculation of $\bar{w}$. Then the result of substitution of (3.11) into (3.4a) is

$$\bar{w} = (q/\epsilon^2)[(1 - \epsilon)\ln(1 - \epsilon) + \epsilon] + w^+. \quad (3.12)$$

The expression for influx F^+ is easily obtained from (3.4b), (3.6a), and (3.11) as

$$F^+ = w^+ U^* H. \quad (3.13)$$

3) Eliminating w^+ from (3.12) with the help of (3.13) and introducing the expression (3.9) for q into the relation obtained, one can represent the expression (3.12) for $\bar{w}$ in the form

$$\bar{w} = \frac{F^+}{U^* H}\left[1 + \frac{(E - P)A}{2F^+} R(\epsilon)\right], \quad (3.14)$$

where $A = LH$ is the area of the region and $R(\epsilon)$ is given by

$$R = (2/\epsilon^2)(1 - \epsilon)\ln(1 - \epsilon) + 2/\epsilon. \quad (3.15)$$

The corresponding expression for $\bar{w}_a$ has the form that is identical to (3.14) with $(-P_a)$ replacing $(E - P)$, as follows

$$\bar{w}_a = \frac{F^+}{U^* H}\left[1 + \frac{(-P_a)A}{2F^+} R(\epsilon)\right]. \quad (3.16)$$

4) After eliminating P and $\bar{w}$ from (3.14) with use of (3.3), the relation (3.14) is represented as

$$\beta\left[\bar{w}_a + \frac{P_a A}{2U^* H} R(\epsilon)\right] = \frac{F^+}{U^* H}\left[1 + \frac{EA}{2F^+} R(\epsilon)\right], \quad (3.17)$$

from which $\bar{w}_a$ can be eliminated with the help of (3.16). One can see that the elimination of $\bar{w}_a$ implies also elimination of P_a, which results in the formula (3.5) with the correction factor R given by (3.15).

Analysis of the expression (3.15) for values of $\epsilon \ll 1$ gives

$$R \approx 1 + \epsilon/3 + \epsilon^2/6, \quad (3.18)$$

which shows that for $\epsilon = 0$ when the flow is parallel (one-dimensional case), the formula (3.5) becomes a straightforward modification of Budyko's formula (2.9). Analysis of (3.15) for finite values of ϵ shows that R increases monotonically with the increase of ϵ approaching the limiting value $R = 2$ as $\epsilon \to 1$ (see Fig. 2).

d. Nonuniform moisture distribution at the entrance of the region

Here we will present an example showing how a violation of the uniformity of the entrance moisture distribution influences the recycling coefficient β (or the correction factor R). Taking again the airflow in the form (3.6a), we consider the initial moisture distribution, linear in y, as follows:

$$f(y) = 1 + \lambda y. \quad (3.19)$$

Then the solution of (3.8a) subject to the boundary condition (3.2) has the form (3.10), which being specified for the case (3.19) gives

$$w = -(q/\epsilon)\ln(1 - \epsilon x) + w^+[1 + \lambda y(1 - \epsilon x)]. \quad (3.20)$$

The subsequent transformations made in the same way as for the case of the uniform initial distribution lead to the formula (3.5) with the correction factor of the form

$$R = \frac{(2 + \lambda)[(1 - \epsilon)\ln(1 - \epsilon) + \epsilon]}{\epsilon^2(1 + \lambda/2 - \lambda\epsilon/4)}, \quad (3.21)$$

from which it follows that R increases with increasing λ [the value of λ in (3.19) should be rather small since relative variations of w in the air entering the

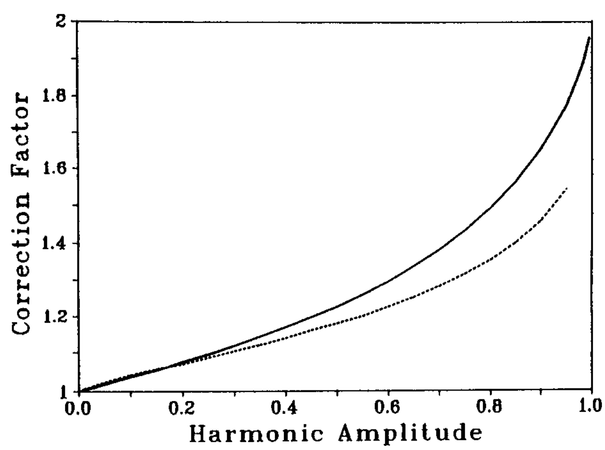

FIG. 2. Correction factor R, as a function of the amplitude ϵ of the deviation from a parallel flow, for the linear diffluent flow fields (3.6a,b) (solid line) compared to that for the diffluent flow field (4.1a) obtained by superimposing an even double Fourier harmonic on a parallel flow (dashed line).

region cannot be large]. Similar calculations carried out for some other types of nonuniformity in $f(y)$ show that, for example, a sine-shape distortion of the uniform distribution, $f(y) = 1 + \lambda \sin(\pi y)$, $(0 \leq y \leq 1)$, does not practically affect the value of R while a cosine-shape distortion causes the same effect as the linear one (3.19).

4. Recycling coefficient for some flow structures

The recycling coefficient (or the correction factor) for any flow structure, given either analytically or as data at grid points, can be obtained with the help of the numerical method described in appendix A. The method has been verified by comparison of the results with the exact expressions for the correction factor (3.15) and (3.21) and other exact solutions.

Below we present the results of applying the method to flow structures that are composed of a basic parallel flow and one or several double Fourier harmonics of the form

$$\psi_1 = y - (\epsilon/\pi)\cos(\pi x)\cos(\pi y) \quad (4.1a)$$

$$\psi_2 = y + (\epsilon/\pi)\sin(\pi x)\cos(\pi y) \quad (4.1b)$$

$$\psi_3 = y - (\epsilon/\pi)\cos(\pi x)\sin(\pi y) \quad (4.1c)$$

$$\psi_4 = y + (\epsilon/\pi)\sin(\pi x)\sin(\pi y) \quad (4.1d)$$

$$\psi_{22} = y + (\epsilon/\pi)\sin(2\pi x)\cos(\pi y), \quad (4.1e)$$

where ψ is the dimensionless streamfunction defined by the relations

$$u = \partial\psi/\partial y, \quad v = \partial\psi/\partial x. \quad (4.2)$$

The corresponding flow structures are shown in Figs. 3a–3g.

Results of numerical calculations of the correction factor for the structures $\psi_1^{(+)}$, $\psi_2^{(+)}$, and ψ_4 (Fig. 4) give a significant increase in the value of R as compared with the one-dimensional case (in all the calculations the moisture distribution in the air entering the region was uniform). The values of R for the structures $\psi_1^{(-)}$, $\psi_2^{(-)}$, and ψ_3 (they are not shown in Fig. 4) do not differ from unity by more than several percent in the entire interval $0 \leq \epsilon \leq 0.9$.

Results of calculations for airflow structures, including more than one double Fourier harmonics, show that a complication of the flow structure causes an increase in the correction factor (see Fig. 5).

Note that the procedure for a numerical calculation of the correction factor presented in appendix A is equally suitable for application of the analytical approximate methods. If it is desirable to have simple analytical expressions for the correction factor R, corresponding to the flows given by Fourier harmonics (4.1), one can use, for example, the eigenfunction method. Application of the eigenfunction method to the case of a meandering current system ψ_4 [(4.1d), Fig. 3f] results in the simple formula

$$R = 1/(1 - \epsilon^2/4). \quad (4.3)$$

This formula describes rather accurately the dependence of R on ϵ found with the help of the numerical method (curve 4 in Fig. 4): for $\epsilon \leq 0.8$ the difference does not exceed 2%. The formulas corresponding to the other flow fields (4.1) are slightly more complicated.

Thus, calculations of the recycling coefficient for different flow patterns show that the correction factor to Budyko's formula can significantly differ from unity, and hence the deviations of the flow from the rectilinear structure can significantly increase the role the recycling process plays in precipitation formation.

Let us now consider how the idealized flow fields used in the previous analysis are connected with the flow fields observed in the real world. One can see that the flows (4.1) shown in Fig. 3 are types of large-scale circulation patterns (typical dimensions of the order of a few thousand kilometers) with the rotational and the deformative components much larger than the divergent component. The patterns $\psi_1^{(+)}$ and $\psi_1^{(-)}$ (Figs. 3a,b) obtained by superimposing a deformation field on a parallel flow are easily recognized as typical diffluent and confluent flows. Note, however, that this divergence of the contour lines do not necessarily mean divergence in the hydrodynamical sense ($\mathrm{div}\,\mathbf{v} \neq 0$). The patterns of the types ψ_2, ψ_3, and ψ_4 may arise as a result of superimposing disturbances in the form of rotational flows or represent the part of a wave structure. For example, the flow shown in Fig. 3f may correspond to smoothed synoptic-scale perturbations common in the westerlies and having a wavelike character with a typical wavelength of 2000 to 3000 km.

Of course, the flow structures occurring in nature are usually more complicated than the idealized patterns discussed above. However, such patterns may represent the result of harmonic analysis of the atmospheric flows being the main terms of the expansion. Therefore, the corresponding results have also some general meaning and may be applied to both tropical and midlatitude flow fields. It should be emphasized that the applications of our method are by no means confined to the examples given in the paper. The method in its general form described in appendix A can be used for any observed flow field given either as data at grid points or analytically.

5. Discussion

All the arguments leading to our recycling model were within the framework of Budyko's approach, and so the model suffers from the same limitations (except the assumption of the rectilinear flow) as the original Budyko model; namely, (i) the model deals with all the fluxes E, P, and P_a as constants equal to their av-

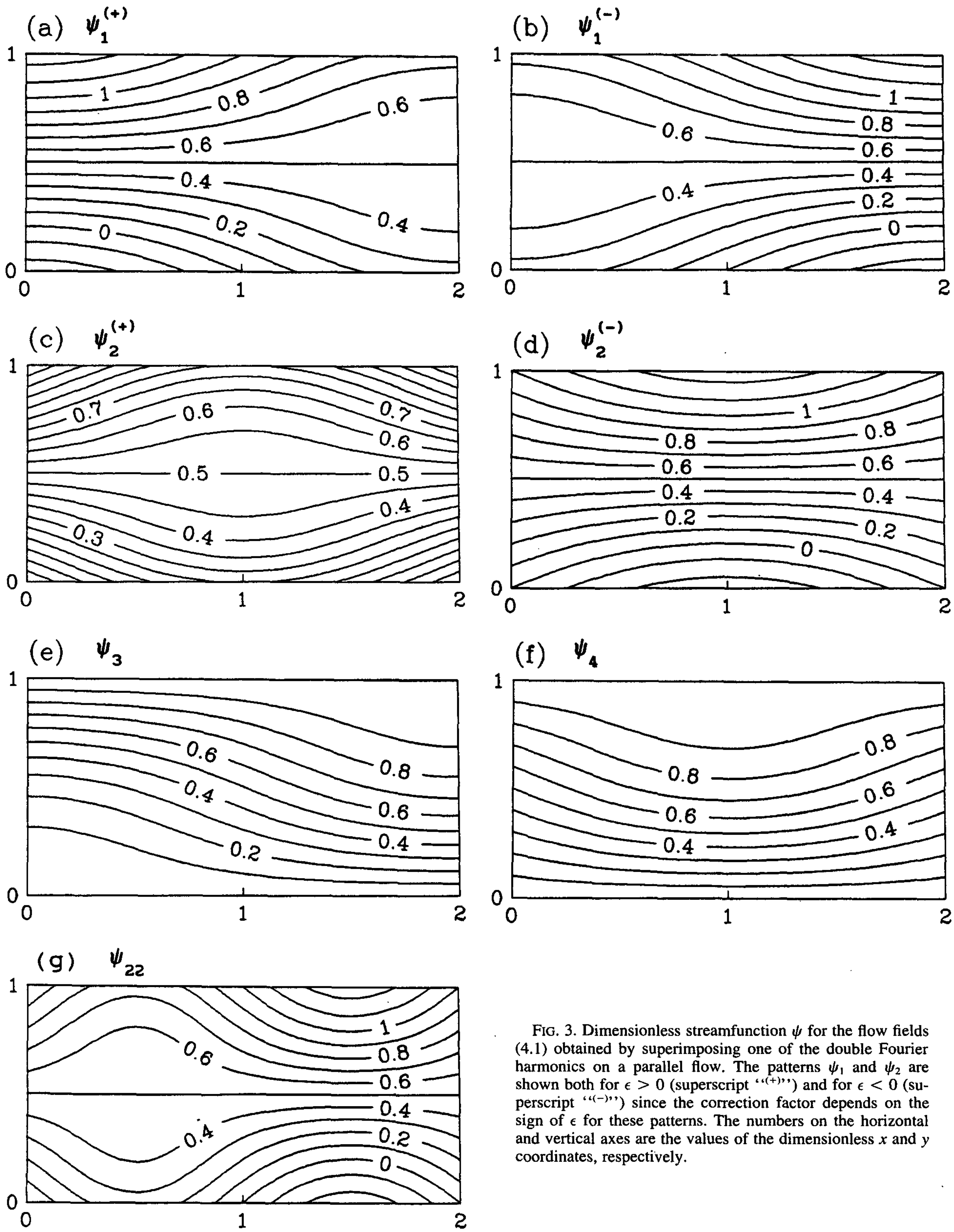

FIG. 3. Dimensionless streamfunction ψ for the flow fields (4.1) obtained by superimposing one of the double Fourier harmonics on a parallel flow. The patterns ψ_1 and ψ_2 are shown both for $\epsilon > 0$ (superscript "(+)") and for $\epsilon < 0$ (superscript "(−)") since the correction factor depends on the sign of ϵ for these patterns. The numbers on the horizontal and vertical axes are the values of the dimensionless x and y coordinates, respectively.

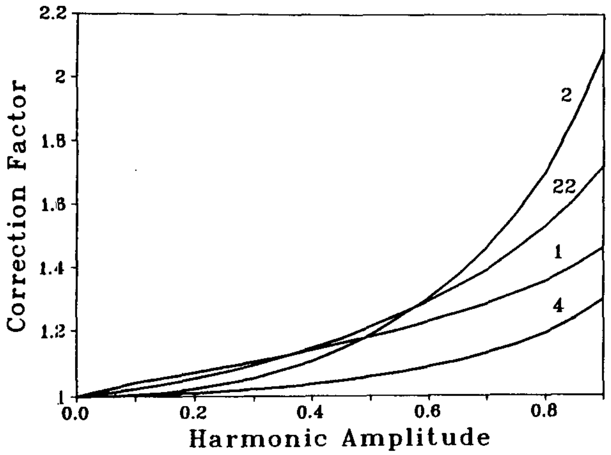

FIG. 4. Correction factor R as a function of the amplitude ϵ for some of the flow fields (4.1) shown in Fig. 3. Numbers on lines correspond to flow fields: 1, flow field $\psi_1^{(+)}$; 2, flow field $\psi_2^{(+)}$; 4, flow field ψ_4; 22, flow field ψ_{22}.

erage values, and (ii) the assumption of a fully mixed atmosphere expressed in the form (2.4) is accepted. Here we will discuss these assumptions and consider possible extensions of our method.

First we shall give some considerations concerning assumption (ii) of a fully mixed atmosphere. Apparently the atmosphere above most land regions is well mixed vertically (see, for example, discussion in Eltahir and Bras 1994), so that the assumption of a fully mixed atmosphere applied to a small area of the region (local form) is quite well justified. However, in Budyko's model, the assumption of well mixing is not used locally but applied to the horizontal averages as (2.4) suggests. This implies that some horizontal mixing occurs, which is sometimes hard to justify for large scales. On the other hand, using the condition of well mixing locally implies that the precipitation fluxes P and P_a are variable within the region, which leads to the model where the assumption (i) of the uniform flux distributions is also rejected.

The effect of an inhomogeneity of the fluxes is expected to be very significant for large scales: the larger the scale, the more the inhomogeneity effects. Note, however, that the results of applying the one-dimensional nonuniform generalization of Budyko's model by Drozdov and Grigor'eva (1965) to some specific regions did not reveal a large difference as compared with the results of calculations using Budyko's formula; see references made in section 1. The comparative study of these two models [but with the use of only one of possible final formulas of Drozdov and Grigor'eva (1965)] made by Brubaker et al. (1993) shows that the discrepancies should not be large if influx, evaporation, and precipitation rates are of the same order.

One could expect that a combined effect due to both nonrectilinear flow structure and the inhomogeneity of the fluxes should be much more significant. Therefore, an extension of Budyko's model both to two dimensions and to nonuniform flux distributions makes sense and should improve estimates of precipitation recycling based on Budyko's approach. In addition, as discussed above, the limitations of Budyko's model imposed by assumptions (ii) can be also overcome by permitting the flux distributions to be nonuniform. Such an extension of Budyko's model could be obtained by means of a generalization of our method to nonuniform flux distributions. Because of inevitable complications, this should be a subject for a separate study.

6. Summary

It was shown above that the search for the correct way to estimate the contribution of the local evaporation to regional precipitation within the framework of Budyko's approach has led to a two-dimensional extension of Budyko's formula that contains a correction factor depending on the atmospheric flow structure over the region. A direct method (suitable both for analytical and numerical calculations) to determine the correction factor from observational data has been developed. It can be used both for data given at grid points and for data given analytically as the result of harmonic analysis of the atmospheric flows. Calculations of the correction factor for specific flow structures have shown that the correction factor can significantly differ from unity, and hence the deviations of the flow from the rectilinear structure can significantly affect the degree to which the local evaporation contributes to precipitation.

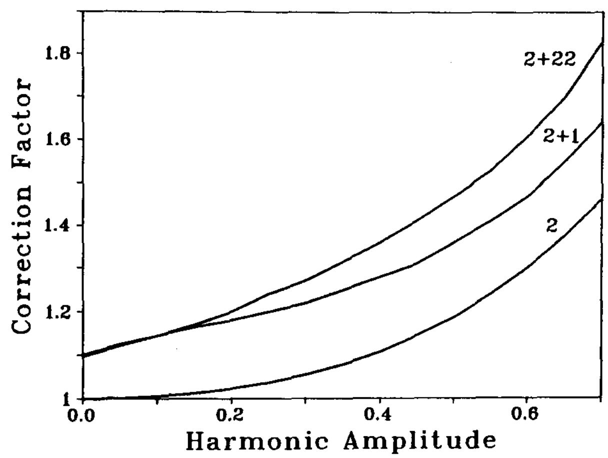

FIG. 5. Correction factor R as a function of the amplitude ϵ for the flows including two Fourier harmonics compared to that for the flow including one harmonic. Numbers on lines correspond to flow fields: 2 + 1, flow field $\psi_2^{(+)}$ combined with the even harmonic from $\psi_1^{(+)}$; 2 + 22, flow field $\psi_2^{(+)}$ combined with the harmonic from $\psi_{22}^{(+)}$; 2, flow field $\psi_2^{(+)}$.

This also shows that the slightly modified Budyko formula with the area of the region replacing the linear scale of the domain cannot be considered as a correct extension of Budyko's formula to two dimensions. It is trivial (the same formula) for the case of a parallel flow if the evaporative flux referring to a unit length is just replaced by that referring to a unit area; in terms of our model the correction factor is equal to unity in this case. It is not correct in other cases since the correction factor differs (in many cases significantly) from unity. Thus, correct estimations of the recycling coefficient for two-dimensional regions should include the correction factor calculated by using the proposed procedure on the basis of the flow fields taken from observations.

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APPENDIX A

Numerical Procedure for Calculation of the Correction Factor

For simplicity we restrict ourselves, as in section 3, to a rectangular region and nondivergent flows, but there are no principal difficulties in extending the procedure to a region of an arbitrary shape and divergent flows. Equations (3.1) written with use of the dimensionless variables (3.7) take the form

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = q \qquad \text{(A.1a)}$$

$$u\frac{\partial w_a}{\partial x} + v\frac{\partial w_a}{\partial y} = q_a, \qquad \text{(A.1b)}$$

where q and q_a are defined in (3.9). The conditions at the entrance side of the region have the form (3.2). Following the procedure, described in section 3b and demonstrated on the simple example in section 3c, we will retain the same enumeration for the steps.

1) The solution of the quasilinear equation (A.1a) subject to the initial condition (3.2) is written in the form

$$w = qW(x, y) + w^+ f[\zeta(x, y)], \qquad \text{(A.2)}$$

where $W(x, y)$ is the solution of the problem

$$u\frac{\partial W}{\partial x} + v\frac{\partial W}{\partial y} = 1, \quad W(0, y) = 0 \qquad \text{(A.3)}$$

and $\zeta(x, y)$ is the solution of the problem

$$u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = 0, \quad \zeta(0, y) = y. \qquad \text{(A.4)}$$

2) The average moisture contents are calculated as

$$\bar{w} = q\bar{W} + w^+\bar{f}, \qquad \text{(A.5)}$$

where

$$\bar{W} = \int_0^1 dx \int_0^1 W(x, y)dy,$$

$$\bar{f} = \int_0^1 dx \int_0^1 f[\zeta(x, y)]dy. \qquad \text{(A.6)}$$

The formula for calculating the influx F^+ is written as

$$F^+ = (HU^*)(qG_2 + w^+G_1), \qquad \text{(A.7)}$$

where

$$G_1 = -\int_{\Gamma_{\text{in}}} f_\gamma \mathbf{v}_\gamma \cdot \mathbf{n}_\gamma d\gamma, \quad G_2 = -\int_{\Gamma_{\text{in}}} W_\gamma \mathbf{v}_\gamma \cdot \mathbf{n}_\gamma d\gamma. \qquad \text{(A.8)}$$

In the boundary integrals (A.8), we use the same notation as in the integral (3.4b).

3) After eliminating w^+ from (A.5) with the help of (A.7) and introducing the expression (3.9) for q in (A.5), the expression for $\bar{w}$ is represented in the form

$$\bar{w} = \frac{F^+\bar{f}}{U^*HG_1}\left[1 + \frac{(E-P)A}{2F^+}R\right], \qquad \text{(A.9)}$$

where

$$R = 2(\bar{W}G_1/\bar{f} - G_2) \qquad \text{(A.10)}$$

and $A = LH$ is the area of the region. The expression for $\bar{w}_a$ obtained as the result of replacing $(E - P)$ by $(-P_a)$ is

$$\bar{w}_a = \frac{F^+\bar{f}}{U^*HG_1}\left[1 + \frac{(-P_a)A}{2F^+}R(\epsilon)\right]. \qquad \text{(A.11)}$$

4) After eliminating P and $\bar{w}$ from (A.9) with the use of (3.3), the relation (A.9) is represented as

$$\beta\left[\bar{w}_a + \frac{P_aA\bar{f}}{2U^*HG_1}R(\epsilon)\right] = \frac{F^+\bar{f}}{U^*HG_1}\left[1 + \frac{EA}{2F^+}R(\epsilon)\right]. \qquad \text{(A.12)}$$

By eliminating $\bar{w}_a$ (and P_a together with it) from (A.11) and (A.12), the formula for the recycling coefficient β is easily obtained in the form (3.5), where the correction factor is given by (A.10).

Thus, the numerical procedure for the calculation of R includes the numerical solution of the problems (A.3) and (A.4) for given u and v fields (the solutions can be obtained by the finite-difference or finite-element methods) and numerical calculation of the integrals (A.6) and (A.8) on the basis of the calculated fields of W and ζ. If the entrance moisture distribution is uniform ($f = 1$), the amount of calculations is reduced since one does not need to calculate the ζ field and the corresponding integrals.

APPENDIX B

List of Symbols

$f(y)$	Moisture distribution at the entrance to the region
k	Deformation rate in the diffluent flow fields (3.6)
$\mathbf{n}_\gamma$	Outward unit normal vector on the boundary
q, q_a	Quantities defined by Eqs. (3.9)
u, v	Dimensionless horizontal velocity components defined by (3.7)
w	Vertically integrated water vapor (precipitable) depth
w^+	Characteristic value of the moisture content at the entrance side of the region
x, y	Dimensionless horizontal coordinates [(3.7)]
A	Area of the region
E	Evaporation flux
F^+	Influx of atmospheric moisture
G_1, G_2	Integrals defined by (A.8)
H	Dimension of the region along the Y axis
L	Dimension of the region along the X axis
P	Precipitation flux
R	Correction factor to Budyko's formula [(3.5)]
U, V	Vertical averages of the wind vector components weighted according to the specific humidity profile
U^*	Horizontal velocity scale
$\mathbf{V}$	$= (U, V)$
$W(x, y)$	Variable defined by (A.2) and (A.3)
X, Y	Horizontal coordinates
β	Budyko's recycling coefficient [(2.3)]
γ	Part of the boundary
ϵ	Relative amplitude of a deviation from the rectilinear flow
ζ	Variable defined by (A.2) and (A.4)
λ	Parameter of the distortion of the initial moisture distribution [(3.19)]
ψ	Dimensionless streamfunction defined by (4.2)
ψ_i	($i = 1, 2, 3, 4$) dimensionless streamfunctions for the flow fields (4.1a–d)
$\psi_i^{(+)}$	$= \psi_i$ for $\epsilon > 0$
$\psi_i^{(-)}$	$= \psi_i$ for $\epsilon < 0$
ψ_{22}	Dimensionless streamfunction for the flow field (4.1e)
Γ_{in}	Part or parts of the boundary across which the atmospheric motion is inward [(3.4b) and (A.8)]

Subscripts

a	Parts of the water vapor content or precipitation of advective origin
m	Parts of the water vapor content or precipitation of local (evaporative) origin
γ	Moisture contents and flow velocities on the boundary

Superscript

"$\overline{\ }$"	Horizontal averages over the region

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