Wind Measurement Capabilities of a Doppler Radiosonde System (The Safesonde)

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(Manuscript received 18 June 1974, in revised form 16 October 1974)

ABSTRACT

The Safesonde is a minimum weight, cost, and hazard sounding system for measurement from surface to 15 km. It consists of a simple transmitter carried by a shaped superpressure balloon and tracked by 5 receivers. The accuracy of wind determination as a function of receiver location is described.

1. Introduction

There is and for many years has been an urgent need for a balloon sounding system which is safe, simple, accurate, and operable by a single unskilled worker. For most applications, soundings can be limited to 15 km. This is not a serious limitation since the satellite infrared sounder can provide adequate data in the stratosphere (except for the equatorial belt). Special observing systems can be used for special needs without burdening the standard system with unneeded capabilities which increase complexity, cost, and hazard. At altitudes above 15 km any object will be hazardous for impact with an SST. At altitudes below 15 km, great care must be exercised in design to minimize hazard for impact with a jet aircraft (see p. 35 of the referenced report by COSPAR Working Group VI, 1970).

The system described here is still in the concept stage although the basic hardware and software have been demonstrated (Gage and Jasperson, 1974a, b). The objective of this paper is to present for critical review the basic system accuracy as a function of geometry.

2. System characteristics

The Safesonde is designed to provide wind, temperature, humidity and other data from the surface to the lower stratosphere without hazard to aircraft. The radiosonde should be an electronic idiot, since it is expendable; the ground system should have no moving parts, since machines are becoming more costly at an exponential rate, exceeded only by the rate at which solid-state devices are decreasing in cost. The first requirement interdicts intelligent radiosondes that receive and retransmit; the second requirement excludes radars and tracking receivers.

The Safesonde system requires the placement of four transponder stations and a master station spaced in an area of 5 to 10 km dimension. The transponders may be placed on a pole or on a roof-top. The central station consists of transmitting and receiving antennae and a desk-top electronics installation. Placement is not critical, but positions must be surveyed.

A sounding requires a few minutes of preparation and preflight data entry. An inflation housing can be used outdoors or wheeled out from an office prior to launch. No special shelter is required.

Soundings take 25 min with ascent rate of 10 m/s achieved by the shaped, superpressure balloon. The balloon has a volume of 1 m^3 and weighs 150 g.

3. Mathematical model

The Safesonde transmits at a VHF or UHF frequency. There is no stability requirement on this transmission. Each of the transponder stations receives a signal from the Safesonde and provides a measurement of this signal to the master station. The measured frequency is the transmitted frequency from the sonde minus the frequency change due to radial motion (Doppler) plus the offset in the measurement due to any offset in the local oscillator. Thus for the station S_j, $j=1, \ldots, n$, we can write

$$f_{jM}=f_T-\Delta f_j+f_0, \tag{1}$$

where f_{jM} is the number of cycles measured in one time unit; f_T is the number of cycles transmitted by the Safesonde in one time unit; Δf_j is the change in the number of received cycles at S_j due to a change in radial distance in one time unit; and f_0 is the frequency of the local oscillator, which is identical for all stations since all local oscillators are slaved to a transmission from the master station,

$$\Delta R_j=-\Delta f_j\cdot k, \tag{2}$$

[1] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

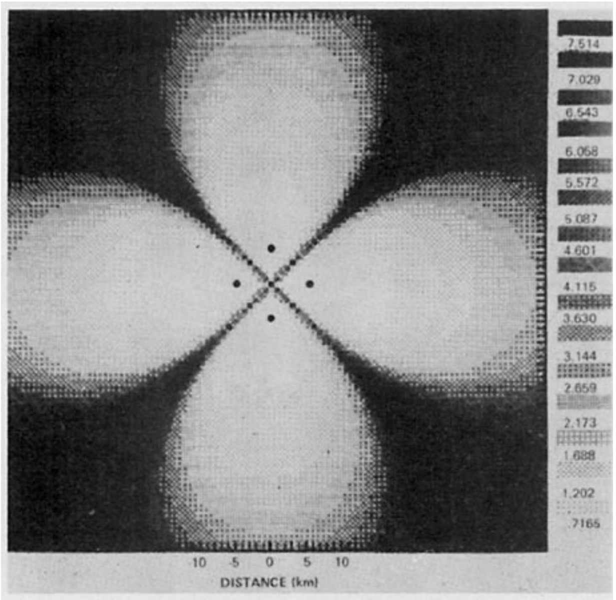

FIG. 1. Horizontal velocity errors (m/sec) at 5 km altitude (4 station–symmetrical).

where ΔR_j is the radial distance change in meters from station S_j to the Safesonde in one time unit

$$k=\frac{300}{\text{Safesonde frequency (MHz)}}.$$

Combining (1) and (2) we obtain

$$\Delta R_j = k(f_{jM}-f_T-f_0), \quad j=1,\ldots,n. \tag{3}$$

However, since the transmission frequency is not assumed to be stable, it is necessary to eliminate f_T from Eq. (3). This can be done by differencing two equations in (3), i.e.,

$$\Delta R_i - \Delta R_j = k(f_{iM}-f_{jM}), \quad i\neq j=1,2,\ldots,n. \tag{4}$$

Let (x,y,z) be the position of the sonde and (X_j,Y_j,Z_j) be the position of the jth station, in a three-dimensional space with given coordinate system. Then the radial distance R_j of the sonde from station S_j can be expressed as

$$R_j^2=(x-X_j)^2+(y-Y_j)^2+(z-Z_j)^2. \tag{5}$$

Also, since ΔR_i is the change in the radial distance, we can write

$$\Delta R_i = \int_{t_0}^{t} \frac{\partial R_i}{\partial t} dt.$$

Using this and the fact that only $(n-1)$ equations of (4) are linearly independent, they may be written as

$$\frac{\partial R_i}{\partial t}-\frac{\partial R_n}{\partial t}=k\left(\frac{\partial f_{iM}}{\partial t}-\frac{\partial f_{nM}}{\partial t}\right), \quad i=1,\ldots,(n-1). \tag{6}$$

If we set

$$a_j=(x-X_j)/R_j, \quad b_j=(y-Y_j)/R_j, \quad c_j=(z-Z_j)/R_j,$$

then differentiation of both sides of (5) with respect to t yields

$$\frac{\partial R_j}{\partial t}=a_jU+b_jV+c_jW, \tag{7}$$

where U, V are the horizontal components and W is the vertical component of the sonde velocity. Using (7) we may now write equation (6) in terms of U, V, and W:

$$(a_i-a_n)U+(b_i-b_n)V+(c_i-c_n)W = k(\dot{f}_{iM}-\dot{f}_{nM}), \quad i=1,\ldots,(n-1). \tag{8}$$

These equations can now be solved for U, V, and W, provided n is at least four and the $\dot{f}_i$ on the right side of (8) are known. In the following we give a statistical procedure for estimating these time derivations and their variances.

4. Statistical analysis

In this section we shall assume that the observations on the cycle counts of the transmission received from the sonde are made every 10 s. To find the time derivatives $\dot{f}_{iM}$ involved in (8), we must find a differentiable function which adequately describes these data. For this purpose we carry out analyses under two different assumptions and present the wind error estimates which are conservative.

a. *Linear fit*

In this case we assume that the observed data f_{iM} from station S_i can be described by a linear function

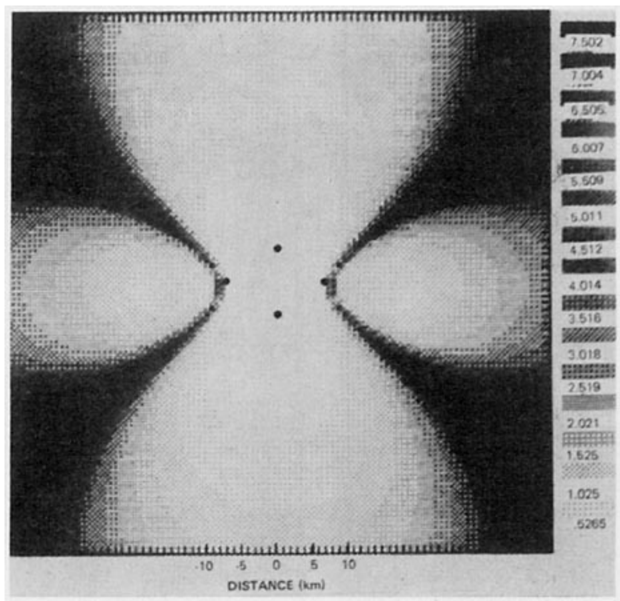

FIG. 2. Horizontal velocity errors (m/s) at 5 km altitude (4 station–unsymmetrical).

of time over a period of one minute. This assumption will be valid if

- the cycle count function $f_i(t)$ is linear in t over one minute, i.e., the sonde is moving with a component of constant velocity in the direction of station S_i over a period of one minute, and
- the transmission frequency f_T varies linearly in t over a period of one minute.

Under this assumption we can write the following statistical model:

$$f_{iM}(t)=\alpha_1+\alpha_2 t+e(t),$$

where $e(t)$ is a random error such that

$$E(e(t))=0,\quad \text{and}\quad \mathrm{Var}(e(t))=\mathrm{Var}(f_{iM}(t))=\sigma_i^2. \tag{9}$$

Since this model is assumed to be valid over a period of one minute, we have seven observations at time points 0, 10, ..., 60. These observations can be assumed to be statistically independent. Then the variance of the regression estimate $\hat{\alpha}_2$ of α_2 is

$$\begin{aligned}\mathrm{Var}(\dot{f}_{iM}(t))&=\mathrm{Var}(\hat{\alpha}_2)\\&=\sigma_i^2/\sum (t_i-\bar{t})^2=0.000357\cdot\sigma_i^2.\end{aligned} \tag{10}$$

b. Quadratic fit

Here we assume that the observed data f_{iM} from station S_i can be described by a quadratic function of time t over a period of two minutes. This assumption will be valid if

- either $f_i(t)$ or f_T is not linear in t and both are at most quadratic in t over a period of two minutes.

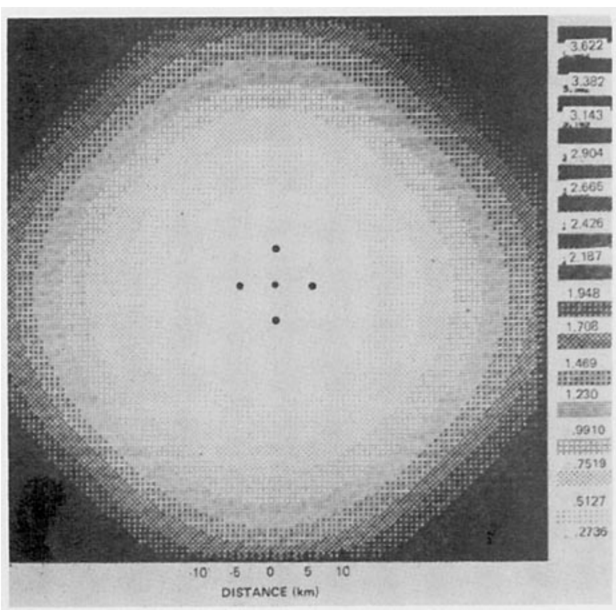

FIG. 3. Horizontal velocity errors (m/s) at 5 km altitude (5 station–symmetrical).

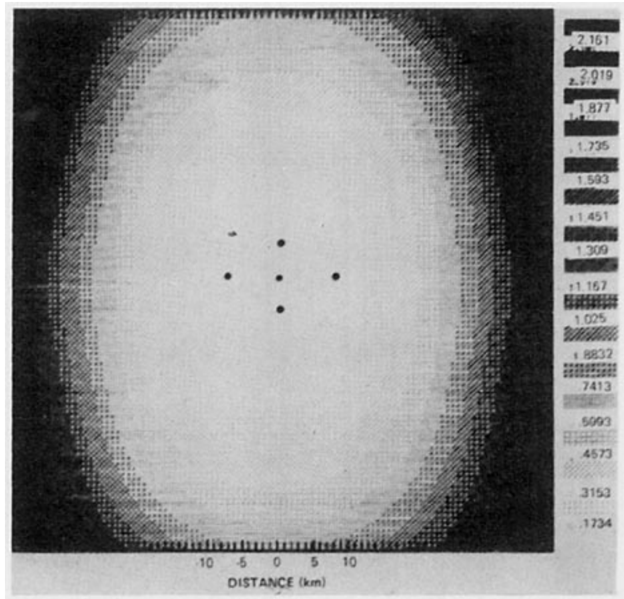

FIG. 4. Horizontal velocity errors (m/s) at 5 km altitude (5 station–unsymmetrical).

The assumption that $f_i(t)$ is quadratic in t implies that the sonde is moving with a component of constant acceleration towards the station S_i. Thus

$$f_{iM}(t)=\beta_1+\beta_2 t+\beta_3 t^2+e(t),$$

with the same conditions on $e(t)$ as in (9).

The time derivative $\dot{f}_i(t)$ is now approximated as

$$\dot{f}_i(t)=\hat{\beta}_2+2\hat{\beta}_3 t,$$

where $\hat{\beta}_i$ are least square estimates of β_i. Therefore,

$$\mathrm{Var}(\dot{f}_i(t))=\mathrm{Var}(\hat{\beta}_2)+4t^2\,\mathrm{Var}(\hat{\beta}_3)+4t\,\mathrm{Cov}(\hat{\beta}_2,\hat{\beta}_3). \tag{11}$$

Assuming the 13 observations to be statistically independent, the covariance matrix of $\hat{\beta}_i$, $i=1, 2, 3$, is seen to be

$$\begin{bmatrix}0.516483516 & -0.016483516 & 0.000109890\\ & 0.000774226 & -0.000005994\\ & & 0.000000050\end{bmatrix}\sigma_i^2.$$

Eq. (11) represents a parabola which is concave downward with a minimum at $t=60$, and increases as the absolute difference $|t-60|$ increases; its maximum value is achieved at the end point $t=0$ and $t=60$. The minimum variance is min $\mathrm{Var}(\dot{f}_i(t))=0.000055665\cdot\sigma_i^2$. The variance in the linear case (10) is greater by a factor of 6.4 than the quadratic case. The latter, however, restricts estimation only at the middle of the 2-min interval.

c. Error in velocity components

Gage and Jasperson (1974a) point out that the instances of cycle slippage will occur only if the signal-to-noise ratio becomes too small. However, with an

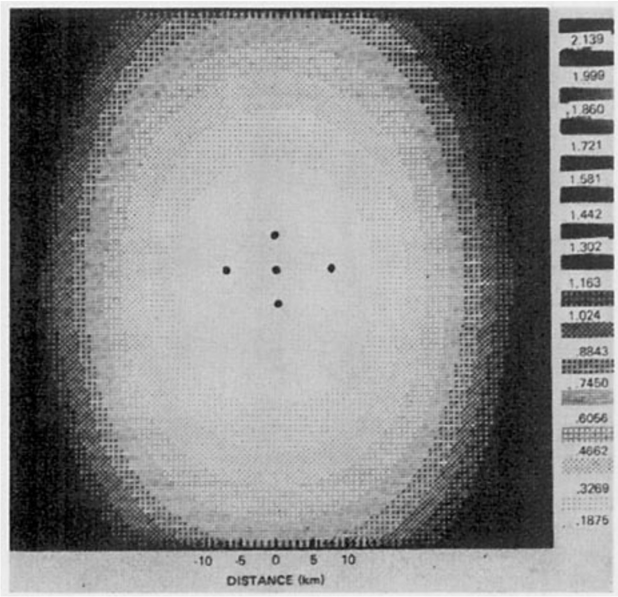

FIG. 5. Horizontal velocity errors (m/s) at 10 km altitude (5 station–unsymmetrical).

adequate signal-to-noise ratio, cycle count should be accurate. Thus the only error in our cycle count will be the round-off error, the distribution of which should be the same for all receivers. It is, therefore, reasonable to assume here that the variance of the number of the cycle counts for each station is the same, i.e., $\sigma_i^2=\sigma^2$, $i=1, \ldots, n$.

We now revert to Eqs. (8) for the estimation of the wind components. Eqs. (8) can be written in the matrix notation as

$$\begin{bmatrix} \dot{f}_{1M} & -\dot{f}_{nM} \\ \dot{f}_{2M} & -\dot{f}_{nM} \\ \vdots & \\ \dot{f}_{n-1,M} & -\dot{f}_{nM} \end{bmatrix} = (1/k) \begin{bmatrix} a_1 & -a_n & b_1 & -b_n & c_1 & -c_n \\ a_2 & -a_n & b_2 & -b_n & c_2 & -c_n \\ \vdots & & \vdots & & \vdots & \\ a_{n-1} & -a_n & b_{n-1} & -b_n & c_{n-1} & -c_n \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

or

$$\dot{\mathbf{f}}=(1/k)\mathbf{C}\mathbf{V},$$

where $\mathbf{C}$ is the coefficient matrix and $\mathbf{V}$ is the velocity vector. The least square solution for $\mathbf{V}$ is given by

$$\mathbf{V}=k(\mathbf{C}'\mathbf{\Sigma}_{\dot{\mathrm{f}}}^{-1}\mathbf{C})^{-1}\mathbf{C}'\mathbf{\Sigma}_{\dot{\mathrm{f}}}^{-1}\dot{\mathbf{f}}, \tag{12}$$

where $\mathbf{\Sigma}_{\dot{\mathrm{f}}}$ is the covariance matrix of the vector $\dot{\mathbf{f}}$. Let $\mathbf{\Sigma}$ be the covariance matrix for $\mathbf{V}$; then

$$\mathbf{\Sigma}=k^2(\mathbf{C}'\mathbf{\Sigma}_{\dot{\mathrm{f}}}^{-1}\mathbf{C})^{-1}. \tag{13}$$

To evaluate $\mathbf{\Sigma}_{\dot{\mathrm{f}}}$ we note that the (i,j) element of $\mathbf{\Sigma}_{\dot{\mathrm{f}}}$ is given by

$$\mathrm{Cov}(\dot{f}_i-\dot{f}_n, \dot{f}_j-\dot{f}_n)=\begin{cases}\mathrm{Var}(\dot{f}_i)+\mathrm{Var}(\dot{f}_n) & \text{if } i=j\\ \mathrm{Var}(\dot{f}_n) & \text{if } i\neq j\end{cases}$$
$$=\begin{cases}2\delta\sigma^2 & \text{if } i=j\\ \delta\sigma^2 & \text{if } i\neq j,\end{cases} \tag{14}$$

since σ_i^2 are assumed to be equal; here δ equals 0.0003571 or 0.0000557 according as the number of cycle-counts is assumed to be linear or quadratic in time. The matrix $\mathbf{\Sigma}_{\dot{\mathrm{f}}}$ is a patterned matrix and its inverse is available in a closed form.

The standard error of the estimates of U, V, and W are respectively given by σ_{ii}, $i=1, 3$, the diagonal elements of $\mathbf{\Sigma}$. We define the horizontal wind error e_h and the vertical wind error e_w as

$$e_h=(\sigma_{11}+\sigma_{22})^{\frac{1}{2}} \quad \text{and} \quad e_w=\sigma_{33}^{\frac{1}{2}}. \tag{15}$$

These errors have been computed for various geometrical configurations of the transponder stations. The results are discussed in the following section.

5. System geometry and error analysis

In order to determine the geometric dilution of accuracy for a network of stations the following assumptions are used:

a) Safesonde frequency is 300 MHz, i.e., $k=1$.
b) The standard error of the number of cycles of the signal transmission is the same for each station and is equal to one cycle, i.e., $\sigma=1$.
c) The time unit is 10 s. This will yield seven observations in one minute and 13 in two minutes.

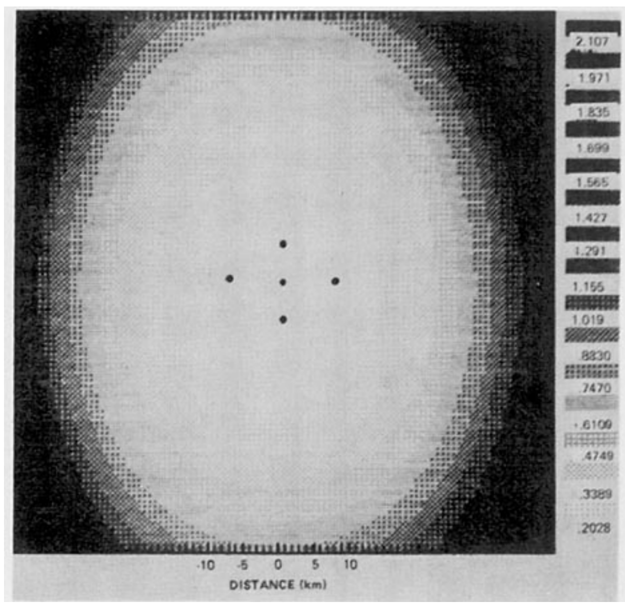

FIG. 6. Horizontal velocity errors (m/s) at 15 km altitude (5 station–unsymmetrical).

d) The number of cycles is linear over a one-minute period, i.e., $\delta=0.0003571$. Even though a model which is quadratic in time over a period of two minutes seems more reasonable, the one that is linear over a period of one minute was used in computation so as to retain a conservative result. To convert the results to the quadratic case, the given results should be divided by 2.55.

Some general remarks are necessary to point out the generality of the procedure used to derive the error curves.

The factor δ involved in deriving the covariances in (14) depends on the model that should be used to describe the data. The example of linear and quadratic fits is used to illustrate as the order of magnitude of δ one should expect. It is clear from (13) that the error e_h and e_w are directly proportional to δ. Hence the shape of the error curves is independent of the value of δ. However, the scale of the errors is directly proportional to δ. In actual practice we will use the process of "continuous smoothing" (Passi, 1974; Passi and Olson, 1974) instead of a linear or quadratic fit. Errors in wind measurements using Omega signals were derived using a quadratic fit (Passi, 1973) and the results were found to be conservative when compared against experimental results obtained (Passi and Olson, 1974).

These error illustrations were made using the assumption of one count standard error in the cycle-count per 10 sec sample, i.e., $\sigma=1$. However, if $\sigma=r$ counts, the shape of these error curves will essentially remain the same but the errors will be multiplied by a factor r.

It should be noted that the covariance matrix $\mathbf{\Sigma}$ of the velocity vector $\mathbf{V}$ in (13) depends on the coefficient matrix $\mathbf{C}$; this in turn depends $a_i, b_i, c_i, i=1, \ldots, n$. These coefficients, by definition, do not depend on the absolute distance of the sonde from the ith station but on the relative distances in the X, Y, and Z direction, e.g., $a_i=(X-X_i)/R_i$, etc. Therefore, it follows that the scale in the illustrations may be proportionately changed in all the directions without affecting the error structure.

Figures 1, 2, and 3 represent the error in the horizontal wind field at an altitude of 5 km for different station configurations. Figure 1 with four stations located symmetrically 5 km from the central point illustrates a serious defect in a four-station network. It would appear that four stations would suffice for a unique solution since we solve for four unknowns, the velocity components in three dimensions and the unknown Safesonde transmitter frequency. However, there are areas within the network where errors become very large. In Fig. 2 with two stations moved to 7.5 km the area of poor resolution moves away from the center and provides improved resolution in a plane orthogonal to the extended network dimension. Figure 3 illustrates the advantage of a fifth station. This configuration is identical to Fig. 1 except that a fifth station is located

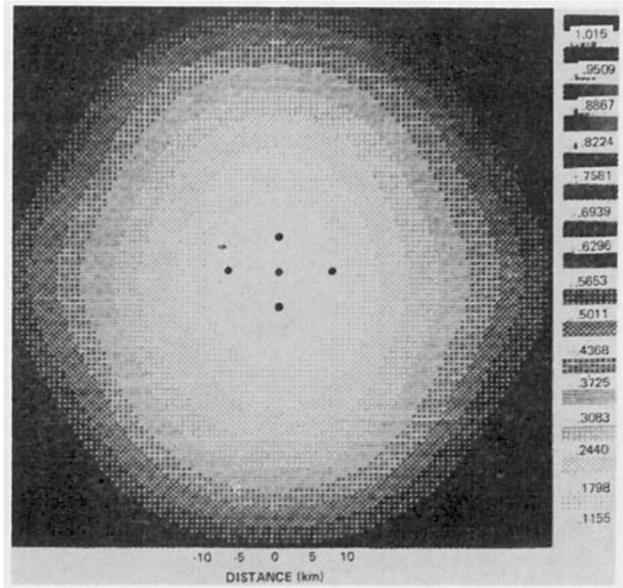

FIG. 7. Vertical velocity errors (m/s) at 10 km altitude (5 station–unsymmetrical).

at the center point. The regions of geometric dilution within the network disappear and we see only a well-behaved deterioration of accuracy as we move away from the network.

Figures 4–7 illustrate the system errors for placement of five stations in an area with dimension of 10 by 15 km. An arbitrary location of 5 stations within an equivalent area will not make significant changes. Figure 4 pictures the horizontal wind error at an altitude of 5 km for the unsymmetric five station system. The rising balloon will still be within the network at this altitude and wind errors will not exceed 0.1 m/s for one-minute averages. The 10-km wind error shown in Fig. 5 is similar to the 5-km error with a slight increase. Since the balloon may now be outside the network, errors can increase to 0.2 m/s. At 15 km altitude (Fig. 6), the horizontal wind error at a distance of 32 km reaches 1 m/s. This distance will be exceeded only with strongest wind conditions. Figure 7 illustrates the vertical velocity error at 10 km altitude. This is similar to the vertical velocity errors at all altitudes from 1 km to 15 km. Below 1 km vertical errors are large at all locations removed from one of the stations. Launch is normally near a station, so the error is never serious.

Altitude may be derived by integration of the measured vertical velocities. If the one-minute average is in error by 0.2 m/s, the maximum error in absolute altitude during a 25-min flight should not exceed 60 m. A continuous integration without Doppler count loss has been used with the METRAC system to reduce the altitude error. There is no requirement with the Safesonde system for pressure measurement since temperature and humidity data together with altitude measurements can be used to compute pressure more accurately than it can be measured with a low-cost sensor. (A typical radiosonde pressure element has an error in pressure-altitude of 30 m near the ground, increasing to 100 m at 15 km height.)

These error illustrations were made using assumptions of standard error, averaging time, and network dimensions, which may be changed for a particular application. In areas of light winds, both ascent rate and network dimensions may be reduced. Averaging times may be reduced at lower altitudes where errors are least and better definition may be desired.

Acknowledgments. The basic concept of a Doppler radiosonde was first described by A. D. Belmont and M. S. Ulstad of Control Data Corporation in a proposal in 1968. The technique, now called the METRAC Positioning System, was successfully demonstrated in March 1974 as a high-precision location system for use in urban pollution studies (Gage and Jasperson, 1974a, b).

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