Distinguishing Between CAT and Non-CAT Areas by Use of Discriminant Function Analysis

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ABSTRACT

The formulation of algebraic functions, involving synoptic-scale atmospheric parameters as variables, capable of predicting clear-air turbulence within 7000 ft sub-layers of the stratosphere was attempted. The data sample used was composed of 153 turbulent and non-turbulent regions identified from 46 stratospheric flights of the XB-70 aircraft over the western United States during the period March 1965 to November 1967, and the values of 69 synoptic-scale parameters determined from rawinsonde data associated with each of the regions. After the XB-70 regions and the values of the synoptic-scale parameters were grouped into one or more of five overlapping categories, or sub-layers, determined by the altitude of the aircraft at the time the turbulence or non-turbulence was reported, discriminant function analysis was employed in each sub-layer to construct functions which could discriminate the turbulent from the non-turbulent regions. Those discriminant functions yielding the best results in each sub-layer were tested from 23 stratospheric flights of the YF-12A aircraft over the same area during the period March 1970 to January 1972. For each sub-layer, five discriminant functions yielding the best results were used to derive a forecasting procedure. This procedure correctly identified approximately 85% of the turbulent and non-turbulent regions in each of the five sub-layers.

1. Introduction

Since the advent of military and commercial stratospheric aircraft flights, an adequate procedure to forecast the spatial and temporal occurrences of stratospheric clear-air turbulence has needed to be developed. Aviators need to be warned of the locations and altitudes of CAT, since the vertical accelerations accompanying this mesoscale phenomenon can lead to discomfort for pilots and passengers, and possibly damage to the structure of the aircraft as well. However, a successful forecasting procedure is difficult to develop without a firm foundation for the theory of CAT, which has not been established.

The objective of this research is to determine, from a statistical approach, algebraic functions involving selected synoptic-scale parameters that would indicate areas and altitudes of CAT in the stratosphere over the western United States. Although CAT is created by mesoscale atmospheric processes, synoptic-scale analysis procedures are used since measurements on this scale are much easier to obtain on a regular basis than on the mesoscale. Moreover, it has been shown by Scoggins et al. (1972) from statistical and synoptic approaches that there is an interrelationship between meso- and synoptic-scale phenomena.

Discriminant function analysis was chosen as the statistical approach, since it proved to be reasonably successful in previous studies (Fisher, 1936; Miller, 1962; Cox, 1973). The functions were determined from combinations of synoptic-scale parameters and stratospheric turbulence data. Two samples were obtained so that one sample could be employed to construct the discriminant functions, while the other could be employed to test the results of the discriminant functions. This was done to assure the representativeness of the dependent sample on a comparison basis.

2. Description of data

a. Aircraft

The turbulence data employed in this research were provided by the NASA Flight Research Center, Edwards, Calif., and were obtained by two instrumented supersonic aircraft while in flight in the stratosphere over the western United States. The XB-70 aircraft, which has been flown at speeds up to Mach 3.0 and altitudes over 70,000 ft, obtained turbulence data from 46 flights during the period March 1965 to November 1967 (Fulton, 1968). These data were used in this research as the dependent
sample. Data from 23 flights of the YF-12A aircraft obtained during the period March 1970 to January 1972 were used in this research as the independent sample.

Each aircraft was instrumented with a NASA VGH recorder which continually provided such information as aircraft altitude, airspeed, and normal accelerations at the center of gravity of the aircraft. The peak-to-peak normal accelerations, measured in g-units, were taken to be direct indications of the intensities of the encountered turbulence. The response of the two aircraft to turbulence may be slightly different, thus indicating different turbulence intensities. However, this would not influence the results of this study since it is based upon the present and the non- intensity of turbulence. These normal accelerations were encountered by the two aircraft along planned routes, which differed with each mission. At no time were the pilots interested in searching for areas of turbulence.

The XB-70 and YF-12A aircraft flight routes and encounters of turbulence were superimposed on geographical maps of the western United States (Ehrenberger, 1968). Since the turbulence was encountered in scattered patches, circular areas approximately 200 n mi in diameter were drawn to encompass clusters of reported turbulence. Circular areas of the same size were superimposed over extensive portions of the flight routes where no turbulence was encountered to represent non-turbulent geographical regions. It was assumed that the mean values of the atmospheric parameters within each circle were representative of the entire encircled region. As a result of the procedure, the 46 XB-70 flights yielded 94 turbulent and 78 non-turbulent regions, while the 23 YF-12A flights yielded 18 turbulent and 22 non-turbulent regions.

b. Meteorological

Since most of the data obtained from the aircraft missions were collected during mid-day, the preceding 1200 GMT and succeeding 0000 GMT rawinsonde data for the corresponding XB-70 and YF-12A flight days were utilized to analyze 300, 200, and 100 mb constant-pressure maps. Although large portions of the flight routes were well above the 100 mb surface, rawinsonde data above that level were not utilized since the accuracy of the wind data there deteriorates.

On the basis of data from the constant-pressure charts, values of 69 atmospheric parameters, characterizing each turbulent and non-turbulent region, were evaluated. Those parameters involving partial derivatives were computed from a square grid of 162.7 km spacing; those involving temporal changes were computed from the 1200 GMT to 0000 GMT time period; and the remainder were determined from the 1200 GMT information.

3. Method of data analysis

a. Theory

The method of discriminant function analysis, a form of linear regression analysis developed by Fisher (1936), has been employed in several studies to predict such non-numerical predictands as ceiling heights (Miller, 1962), occurrences of precipitation (Sassman and Allen, 1957), and occurrences of stratospheric CAT (Cox, 1973). From a given sample, this analysis formulates functions, composed of any number of terms involving parameters hypothesized to be related to an event, capable of producing information which, to a certain degree, would discriminate between the occurrences (Event 1) and the non-occurrences (Event 2) of that parameter. Those functions that best discriminated the cases of Event 1 and Event 2 can be used in a forecasting procedure.

The general form of the linear discriminant function is

\[ L = c_0 + c_1 X_1 + c_2 X_2 + \cdots + c_k X_k, \]  

(1)

where \( X_1, X_2, \ldots, X_k \) are numerical predictors and represent synoptic-scale parameters in this research. The greater the number of terms in this expression, the greater the probability of obtaining a function that perfectly discriminates Event 1 from Event 2 of a data sample. However, it should be emphasized that the greater the number of terms in the discriminant function, the greater the probability of limiting the success of the function to the dependent sample only. In this research, the optimal number of variables was considered to be three.

As an example, a perfect linear discriminant function containing two variables,

\[ L = c_0 + c_1 X_1 + c_2 X_2, \]

represents a line in the \( X_1, X_2 \) plane which perfectly separates Event 1 from Event 2 or, in this research, the turbulent from the non-turbulent regions. When the sign of the functional value, \( L \), is positive, the region where \( X_1 \) and \( X_2 \) were measured is classified as a case of Event 1. It is a case of Event 2 when the sign of \( L \) is negative. An example of discrimination by a two-variable, linear discriminant function is illustrated in Fig. 1.

The construction of a discriminant function for a data sample begins by computing the coefficients \( c_1, c_2, \cdots, c_k \) of Eq. (1). These coefficients are chosen in a manner to maximize the quantity

\[ T^* = \left( \frac{L_1 - L_2}{S_L} \right)^2, \]

(2)

where \( L_1 \) and \( L_2 \) are the average functional values for Event 1 and Event 2, respectively, and \( S_L \) is the standard deviation of \( L \) computed from the sums of
The values of these coefficients are determined from the following set of simultaneous equations:

\[
\frac{c_1x_1^2 + c_2x_1x_2 + \cdots + c_kx_kx_k}{N_1N_2} = \frac{N_1N_2d_1}{(N_1+N_2)^2}, \tag{3}
\]

\[
\frac{c_1x_2x_1 + c_2x_2^2 + \cdots + c_kx_kx_k}{N_1N_2} = \frac{N_1N_2d_2}{(N_1+N_2)^2}, \tag{4}
\]

\[\vdots\]

\[
\frac{c_1x_kx_1 + c_2x_kx_2 + \cdots + c_kx_k^2}{N_1N_2} = \frac{N_1N_2d_k}{(N_1+N_2)^2}, \tag{5}
\]

where \(x_i = (X_i - \bar{X}_i^*)\) is the deviation of the parameter \(X_i\) from the mean of the combination of Groups 1 and 2, \(\bar{X}_i^*\); \(d_i = (\bar{X}_i - \bar{X}_i^*)\) is the difference between the mean of the parameter \(X_i\) of Group 1 and the mean of the parameter of Group 2; and \(N_i\) is the total number of cases in Group \(i\).

After the \(k\) equations and \(k\) unknowns are solved by determinantal methods, the coefficients are substituted into Eq. (6) so that \(c_0\), the corrective coefficient, can be determined where

\[
c_0 = -c_1x_1^* - c_2x_2^* - \cdots - c_kx_k^*, \tag{6}
\]

and \(x_i^* = (\bar{X}_i - X_i)/2\) is the mean of the sum of means of the parameter \(x_i\) of Groups 1 and 2. After the corrective term has been substituted into Eq. (1), the linear discriminant function is in applicable form (Panofsky and Brier, 1958).

Since the square of some of the atmospheric parameters (e.g., the square of the vertical shear term in the denominator of the Richardson number) is related to turbulence, discriminant functions with squared terms also were constructed. One disadvantage of the linear discriminant function is that it does not discriminate between Events 1 and 2 when nonlinearities are important. Figure 2 schematically illustrates the improvement of the discrimination when the nonlinear function is used. The broken line, AB, represents the best linear discriminant function involving variables \(X_1\) and \(X_2\). This line failed to discriminate perfectly the occurrences of Events 1 and 2, since it incorrectly identified two occurrences of Event 1 and one occurrence of Event 2.

b. Stratification of data

Since it was an objective to determine altitudes where CAT would occur, the categorization of the data into groups representing stratospheric sub-layers was necessary. The synoptic data pertaining to the regions in a group could, therefore, be utilized to determine discriminant functions that would discriminate between the occurrences and non-occurrences of CAT within that particular sub-layer. Sub-samples were created by establishing five overlapping 7000 ft thick sub-layers from the entire 40,000–67,000 ft layer sampled by the two aircraft. The turbulent and non-turbulent regions of the XB-70 and YF-12A samples and the values of the 69 synoptic parameters corresponding to each were separated into one or more of the following sub-layers: 40,000–47,000; 45,000–52,000; 50,000–57,000; 55,000–62,000; and 60,000–67,000 ft. By overlapping each sub-layer with adjacent sub-layers by 2000 ft, some of the flights and synoptic data were used for more than one sub-layer. This enabled the summation of the number of turbulent and non-turbulent reports in each sub-layer to be greater than the total number of reports in the overall data sample.

c. Selection of parameters

The determination of synoptic parameters and variable combinations used in the discriminant analysis of turbulence is difficult, since our understanding of CAT mechanisms is limited. For greatest efficiency, the variables used in one discriminant function should be independent and each variable would, ideally, demonstrate some bimodality with respect to the occurrence and non-occurrence of turbulence. However, in reality, most parameters are at least slightly correlated physically, temporally, or spatially and few
parameters demonstrate a satisfactory degree of bimodality.

The selection procedure suggested by Miller (1962) was considered initially, but there was no assurance that the best predictors were obtained. This led the authors to consider a simpler approach. The first variable combinations used in this research were selected as the result of a statistical study and adopted from Cox (1973). Correlation matrices, comprised of correlation coefficients between pairs of the 69 variables for the turbulent, non-turbulent, and combined turbulent and non-turbulent regions were utilized. Most of the two-variable combinations were selected on the basis of two requirements. The first was that the absolute value of the correlation coefficients between variables for the combined turbulent and non-turbulent regions be less than 0.30. The second was that the absolute values of the correlation coefficients between variables for the turbulent and non-turbulent regions be greater than or equal to 0.30. Other two-variable and all three-variable combinations were chosen on the basis of their physical relationships to CAT. Using this selective procedure, 112 variable combinations were selected.

A computer program was designed to construct 112 discriminant functions from the variable combinations of each sub-layer and calculate the functional values from synoptic parameters from the XB-70 data sample. The functions resulting from the 112 variable combinations were screened by a process based upon the ability of the functions to identify both the turbulent and non-turbulent regions of this data sample in the appropriate sub-layer. Any function was considered to have potential as a turbulent predictive equation if at least 60% of both the turbulent and non-turbulent regions were correctly identified. As a result, for each sub-layer, approximately 15 of the 112 functions were considered to have potential. An attempt was made to improve the results by replacing one of the variables in the three-variable combination by other variables and adding a third variable to the two-variable combination. Generally, results were improved by the addition of a third variable to the two-variable combination. As a result, for each sub-layer, approximately a dozen linear discriminant functions, correctly identifying approximately 70% of the turbulent and non-turbulent regions, were constructed.

To attempt to further improve the results, another computer program was designed to construct nonlinear discriminant functions from the three-variable combinations whose linear functions yielded the best results in the appropriate sub-layers. Each of these three-variable combinations was used in the program three times, but each time a different variable was squared. If the results of a non-linear function were better than those of its linear counterpart, the non-linear function was retained and the linear function discarded. In each sub-layer, the results of only one or two of the nonlinear functions were better than those of the linear discriminant functions involving like variables.

Upon completion of the discriminant function analysis, those functions retained for each sub-layer were applied to the independent YF-12A sample. The percentages of the turbulent and non-turbulent regions correctly identified were calculated. For each sub-layer, the best five functions were selected for use in a forecasting procedure and are listed in Table 1. (The functions follow the form of Eq. (1) and the symbols are explained at the end of this table.) The CAT Index, I, derived by Colson and Panofsky (1965) is given by

$$I = (\Delta V)^2 \left( 1 - \frac{R_i}{R_{crit}} \right)$$

where $\Delta V$ is the magnitude of the vector change in wind through the layer over which the Richardson number, Ri, is calculated, and $R_{crit}$ is the critical value taken as 0.25 in this paper.

It was reasoned that the forecasting capability of the group of five discriminant functions for a sub-layer would be better than any single function. Therefore, the occurrence or non-occurrence of CAT could be predicted in a region of a stratospheric sub-layer from the simultaneous values of the five best discriminant functions computed from atmospheric parameters measured in that region of the sub-layer. CAT was expected in regions of sub-layers when the simultaneous values of four or five of the selected discriminant functions, designed for those sub-layers, were greater than zero. On the other hand, non-turbulence would be expected if the simultaneous values of four or five of the selected discriminant functions were less than zero. However, no predictive conclusions would be obtained if the simultaneous values of three of the discriminant functions were either all greater or less than zero.

4. Results

The percentages of the turbulent and non-turbulent regions in each sub-layer correctly discriminated by the appropriate discriminant functions chosen are shown in Table 2. In addition, this table gives the number of turbulent and non-turbulent regions in the sub-layers of the XB-70 (dependent) and YF-12A (independent) samples. Some of the functions could not be evaluated for a particular non-turbulent region in the YF-12A sample due to missing data. Therefore, some of the verifications computed for the non-turbulent regions of the YF-12A sample are based upon a number one less than the total number.

The five discriminant functions selected for each sub-layer yielded satisfactory results for the data of the XB-70 sample, especially for the lowest three sub-layers. Only one two-variable function discrimi-
Table 1. The five discriminant functions selected for each sub-layer.*

<table>
<thead>
<tr>
<th>Sub-layer</th>
<th>Function 1</th>
<th>Function 2</th>
<th>Function 3</th>
<th>Function 4</th>
<th>Function 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>40,000-47,000 ft</td>
<td>$v_1$</td>
<td>$T_1$</td>
<td>$\partial (-\mathbf{V} \cdot \mathbf{V})/\partial t$</td>
<td>$c_0$</td>
<td>$c_1$</td>
</tr>
<tr>
<td></td>
<td>$V_2$</td>
<td>$\gamma_1$</td>
<td>$c_0$</td>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>45,000-52,000 ft</td>
<td>$H_3$</td>
<td>$V_2$</td>
<td>$\partial (-\mathbf{V} \cdot \mathbf{T})/\partial t$</td>
<td>$8.656$</td>
<td>$9.394 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$H_1$</td>
<td>$\theta_1$</td>
<td>$c_0$</td>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>50,000-57,000 ft</td>
<td>$V_3$</td>
<td>$T_2$</td>
<td>$\partial (-\mathbf{V} \cdot \mathbf{V})/\partial t$</td>
<td>$1.840$</td>
<td>$1.230 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$V_2$</td>
<td>$\gamma_2$</td>
<td>$c_0$</td>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>55,000-62,000 ft</td>
<td>$T_3$</td>
<td>$\partial (-\mathbf{V} \cdot \mathbf{V})/\partial t$</td>
<td>$1.110$</td>
<td>$2.685 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$T_1$</td>
<td>$\theta_2$</td>
<td>$c_0$</td>
<td>$c_1$</td>
<td>$c_2$</td>
</tr>
<tr>
<td>60,000-67,000 ft</td>
<td>$\partial T_1/\partial t$</td>
<td>$\partial (-\mathbf{V} \cdot \mathbf{V})/\partial t$</td>
<td>$1.395$</td>
<td>$2.498 \times 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

* List of symbols

- $V$: scalar wind speed (m/s)
- $v$: meridional wind speed (m/s)
- $\xi$: relative vorticity (s$^{-1}$)
- $\Gamma$: temperature lapse rate (°C m$^{-1}$)
- $H$: Richardson number

Subscripts 1, 2, and 3 refer to the 100, 200, and 300 mb levels, respectively.

nated well enough to be included in the forecasting procedure. For the combined XB-70 and YF-12A data sample, most of the 25 functions discriminated a larger percentage of the non-turbulent than the turbulent regions. These same functions were applied to the independent YF-12A sample and the results also are listed in Table 2. The few poor verification percentages of the functions, when applied to the YF-12A sample, resulted partly because of the small number of non-turbulent regions in the 40,000-47,000 and 45,000-52,000 ft sub-layers and turbulent regions in the 60,000-67,000 ft sub-layer of the YF-12A sample.

The percentages of turbulent and non-turbulent regions for the XB-70 (dependent) and YF-12A (independent) data samples in each sub-layer, correctly identified by the forecasting procedure, are shown in Table 3. Also shown for each sub-layer is the number of regions where no predictive conclusions (NPC) were obtained by the forecasting procedure.

The results for both data samples were generally in good agreement with the verification percentage exceeding eighty in most instances. Statistical variations in the relatively small sample sizes (10 to 60) in each sub-layer could easily account for the differences noted.

5. Conclusions

The degree of success of the discriminant functions derived in this research will depend partly upon the representativeness of the data in the two samples. It will depend also upon the effect of the unequal number of turbulent and non-turbulent regions in some of the sub-layers of the samples. Moreover, for greatest efficiency the variables in a combination should be independent and should demonstrate some bimodality with respect to the occurrences and non-occurrences of turbulence. However, in reality, few atmospheric parameters satisfy these criteria.
Table 2. Verification percentages of the turbulent (T) and non-turbulent (NT) regions for the discriminant functions presented in Table 1. The number of turbulent and non-turbulent regions in each sub-layer for the two samples is indicated inside the parentheses. XB-70 represents dependent sample; YF-12A independent or test sample.

<table>
<thead>
<tr>
<th></th>
<th>F1 T</th>
<th>F1 NT</th>
<th>F2 T</th>
<th>F2 NT</th>
<th>F3 T</th>
<th>F3 NT</th>
<th>F4 T</th>
<th>F4 NT</th>
<th>F5 T</th>
<th>F5 NT</th>
</tr>
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<tr>
<td>40-47×10^3 ft</td>
<td>(XB-70: 33T, 10NT; YF-12A: 14T, 1NT)</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>XB-70</td>
<td>83.3</td>
<td>90.0</td>
<td>81.8</td>
<td>90.0</td>
<td>83.8</td>
<td>80.0</td>
<td>81.8</td>
<td>70.0</td>
<td>78.8</td>
<td>70.0</td>
</tr>
<tr>
<td>YF-12A</td>
<td>66.7</td>
<td>00.0</td>
<td>75.0</td>
<td>00.0</td>
<td>85.7</td>
<td>00.0</td>
<td>75.0</td>
<td>00.0</td>
<td>66.7</td>
<td>00.0</td>
</tr>
<tr>
<td>45-52</td>
<td>(XB-70: 33T, 13NT; YF-12A: 15T, 1NT)</td>
<td></td>
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<tr>
<td>XB-70</td>
<td>84.8</td>
<td>92.3</td>
<td>72.7</td>
<td>92.3</td>
<td>78.8</td>
<td>84.6</td>
<td>72.7</td>
<td>76.9</td>
<td>75.8</td>
<td>69.2</td>
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<td>93.3</td>
<td>100.0</td>
<td>92.3</td>
<td>100.0</td>
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<td>100.0</td>
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<tr>
<td>XB-70</td>
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<td>88.2</td>
<td>80.4</td>
<td>70.6</td>
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</tr>
<tr>
<td>YF-12A</td>
<td>92.9</td>
<td>60.0</td>
<td>78.6</td>
<td>40.0</td>
<td>86.7</td>
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<td>71.4</td>
<td>40.0</td>
<td>71.4</td>
<td>60.0</td>
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<tr>
<td>55-62</td>
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<td>XB-70</td>
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<td>83.3</td>
<td>65.7</td>
<td>80.0</td>
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<td>86.7</td>
<td>68.6</td>
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<td>80.0</td>
<td>41.7</td>
<td>70.0</td>
<td>50.0</td>
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<td>60.0</td>
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<tr>
<td>60-67</td>
<td>(XB-70: 33T, 40NT; YF-12A: 3T, 9NT)</td>
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<tr>
<td>XB-70</td>
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<td>70.0</td>
<td>69.7</td>
<td>70.0</td>
<td>69.7</td>
<td>67.5</td>
<td>60.6</td>
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<tr>
<td>YF-12A</td>
<td>66.0</td>
<td>55.6</td>
<td>66.0</td>
<td>55.6</td>
<td>66.0</td>
<td>77.8</td>
<td>33.3</td>
<td>66.7</td>
<td>66.7</td>
<td>77.8</td>
</tr>
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</table>

The results of this research indicate that there is a relationship between selected combinations of synoptic-scale parameters of the upper troposphere and lower stratosphere and stratospheric clear-air turbulence. The relationship can be identified by examination of the signs of the coefficients in the discriminant functions. The functions suggest that CAT most likely would exist in regions of:

1) relatively strong winds at the 300, 200, and 100 mb levels,
2) northerly (meridional) and/or westerly (zonal) wind components,
3) large wind shear in the 200–100 mb layer,
4) large positive horizontal wind shear at the 100 mb level,
5) large positive values of the relative and absolute vorticity,
6) decreasing values of the 100 mb relative vorticity,
7) increasing values of the 300 and 100 mb advection of relative vorticity,
8) low values of the geopotential heights of the 300 and 200 mb surfaces,
9) relatively high values of the temperature at the 100 mb level,
10) decreasing values of the advection of temperature at the 200 mb level,
11) relatively small values of the Richardson number, and
12) decreasing values of the CAT index.

The discriminant functions constructed for the sub-layers of the stratosphere from the selected variable combinations discriminated to a reasonable degree the turbulent and the non-turbulent regions in the appropriate sub-layers of the XB-70 and YF-12A data samples. However, the discrimination was improved when a group of five discriminant functions designed for a sub-layer of the stratosphere was employed to discriminate the regions. These groups of discriminant functions could be used in a CAT forecasting procedure. CAT would be predicted in a stratospheric sub-layer when four or five of the values of the five discriminant

Table 3. Verification percentages (V) of the turbulent and non-turbulent regions in the sub-layers of the XB-70 (dependent) and YF-12A (independent) samples for the turbulence forecasting procedure. Also listed is the number of regions in each layer (N) and the number of regions where no predictive conclusions were obtained (NPC).

<table>
<thead>
<tr>
<th>Sub-layer (ft)</th>
<th>Turbulence</th>
<th>Non-turbulence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XB-70 V(%)</td>
<td>YF-12A V(%)</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>NPC</td>
</tr>
<tr>
<td>40,000-47,000</td>
<td>93.3</td>
<td>33</td>
</tr>
<tr>
<td>45,000-52,000</td>
<td>87.5</td>
<td>33</td>
</tr>
<tr>
<td>50,000-57,000</td>
<td>86.4</td>
<td>46</td>
</tr>
<tr>
<td>55,000-62,000</td>
<td>74.1</td>
<td>35</td>
</tr>
<tr>
<td>60,000-67,000</td>
<td>86.2</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>87.5</td>
<td>9</td>
</tr>
</tbody>
</table>

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functions for that sublayer exceed zero. Results indicate that this method could discriminate approximately 85% of CAT and non-CAT areas.

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REFERENCES


