# Addendum to "Analysis Error as a Function of Observation Density for Satellite Temperature Soundings with Spatially Correlated Errors"

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We wish to acknowledge that a paper by Gandin *et al.* (1972) includes a discussion of the effect of spatially correlated observation errors on analysis error of geopotential height as obtained by optimum interpolation. While this paper predates our work (Bergman and Bonner, 1976), an English translation has only recently been made available to us.

Gandin *et al.* (1972) study the effects of spatially correlated errors for a triangular observing network with the observations at the vertices and the interpolated value obtained at the center of an equilateral triangle. In this case the normalized mean square error of the interpolated value may be expressed directly as

$$\sigma_a^2 = \left\{ 1 - \frac{3\mu^2(h/\sqrt{3})}{1+2\mu(h)+\sigma_\epsilon^2[1+2\rho(h)]} \right\}, \qquad (1)$$

where $\mu(s)$ is the isotropic spatial correlation of observed field, $\rho(s)$ the isotropic spatial correlation of observational errors, $\sigma_\epsilon$ the normalized standard deviation of observational error, and $h$ the length of a side of the triangle.

Assuming that

$$\mu(s) = \exp[-0.195 s^{1.5}] J_0(1.065 s), \qquad (2)$$

where $J_0$ is the zeroth-order Bessel function, and that

$$\rho(s) = \mu(\beta s), \qquad (3)$$

Gandin *et al.* (1972) obtained errors of interpolation as a function of observational spacing $h$ of the three observations for $\beta = 0$, 0.5, 1, 2, 5, 10 and $\infty$ and for $\sigma_\epsilon^2 = 0.005$, 0.02 and 0.2. ($\beta = 0$ corresponds to a uniformly correlated observational error or "bias," whereas $\beta = \infty$ corresponds to no spatial correlation of observational errors.) The results of their calculations are presented in the form

$$(\sigma_a^2 - \sigma_a^2|_{\beta=\infty}) / (\sigma_a^2|_{\beta=\infty})$$

and indicate a dependence of normalized interpolation error $\sigma_a^2$ on observational error correlation which is similar to our results. There is further comparison and discussion of the two extreme cases $\beta = 0$ and $\beta = \infty$. They conclude as we do that a satellite observing system must have *smaller* observational errors than a comparable rawinsonde system if it is to provide the same information content.

They further conclude that it is not possible to reduce the errors of interpolation much below those of the satellite observations, no matter how great the density of observations. This conclusion may be compared with our more precise demonstration that

$$\lim_{h\to 0} (\sigma_a) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + 1}$$

for any number of satellite observations whose errors approach perfect correlation as the separation between them becomes small.

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## REFERENCES

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